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A market risk model for asymmetric distributed series of return

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Abstract

In this paper we propose to model short-term interest rates by taking into consideration both the asymmetric properties of returns, using Pearson's type IV distribution, and the time-varying volatility, using GARCH models. We show that conditional skewness is negatively related to spot price interest rates and that negative conditional skewness can lead the process to generate steady returns.

Key Words: Short-term interest rates; Pearson IV; GARCH; Conditional skewness

I. Introduction

During the last two decades, a vast literature on dynamic volatility models as well its applications have shed light on forecasting speculative price changes and predicting their magnitude. There is a consensus that dynamic volatility models fit well with the properties of daily price changes of the speculative assets, namely volatility clustering. Starting with the seminal work of Engle(1), a family of dynamic models have been developed, known as (G)ARCH, that capture the clustering effect present in financial time series. Later the GARCH family volatility models have been enhanced in order to accommodate some additional properties like asymmetric volatility, i.e. negative returns and price crashes have a larger impact on the volatility increase than positive price changes (for reviews see Refs. 2, 3). Early GARCH type of models assumed that the conditional density of the error term was normal. This assumption was probably due to the computation simplicity offered by a conditional likelihood function offered by a normally distributed error term. However, the changing values of the second moment imply that the unconditional distribution of returns is leptorkutotic. This property, although conformed to the fat tails that characterize most of financial property changes, does not capture the entire tail characteristics. Further investigation, however, revealed that the empirical conditional density of financial time series is heavy-tailed and thus deviate from the normality assumption (for a short review, see Ref. 4). A second generation of ARCH models emerged where the normality assumption in the error term was either relaxed or replaced by a different density which allowed for heavier tails. One study suggested a GARCH model with t-distribution in the error term with the degrees of freedom being as a parameter itself and had to be estimated jointly with the others, in the variance and mean equation, parameters.(5) Another study recommended a semi-parametric ARCH model with non parametric density function for the error term.(6)

More recent developments in modeling the volatility of financial assets has introduced a further flexibility where the density of the error term is allowed to exhibit, in addition to fat-tail-ness (kurtosis), asymmetry (skewness). Jondeau and Rockinger(7) suggested that the error term follows a Gram-Charlier expansion where the skewness and kurtosis are directly appearing as a parameter. These densities present a major challenge in practice. In fact the set of parameters for which the densities is well defined (positive) is limited and far from obvious.(8) An improved model using entropy densities has been proposed(9) and where skewness and kurtosis appear as parameters of the model. Unfortunately no closed form of these densities is possible; they are only defined as solutions of some integral equations. Brannas and Nordman(10) considered GARCH models where the error follows either a Pearson IV or a log-generalized gamma distribution. In both models, the error density allows for changes in the skewness and kurtosis via the density parameters.

In this paper, we will investigate the applicability of the GARCH with a log-generalized gamma error density and Pearson IV for the short-term Yen interest rate. In fact, the behavior of short term interest rates has been of major interest among finance scholars and understanding the dynamics of short-term interest rates is of fundamental importance for many financial applications. Among others, in the pricing of interest rate derivatives and designing hedging strategies, a number of researchers have investigated the link between macroeconomic variables and the short end of the yield curve (see for example Ref. 11). In finance, pricing fixed income securities and interest rate derivatives, designing hedging strategies, all depend on the dynamic behavior of the term structure of interest rate. In general, the existing models for pricing interest rate and bonds assume either that interest rates follow a random walk with constant volatility, or that interest rates are mean-reverting. However, what makes peculiar a series of short term interest rates are some distributional properties not common among other financial time series. Nevertheless, the most atypical property is a lower bound found when the rates are at such low levels like the current US dollar and Japanese Yen. Under these circumstances, the conditional density function is asymmetric truncated to the left. Therefore, there is a need to allow for fat tails, skewness and kurtosis in order to understand investors' behavior and their preferences for moments and its implications in risk management. Given this need, there is an increasing interest to study distributions that allow enough flexibility in accommodating broad range of distributional properties of the data observed in practice, with a particular eye on the Pearson's type IV distribution that covers a wide region of skewness-kurtosis plane (for a short review, see Ref. 12).

The main objective of this paper is to model short-term interest rates by taking into consideration non-normality of returns using Pearson's type IV distribution coupled with the GARCH model to account for dynamic volatility. Also, the model aims to build on previous work on interest rates modeling to show how negative conditional skewness could serve better evaluation short-term interest rates. The paper is organized as follows: Section 2 presents the methodology; Section 3 describes the data and presents the results; Section 4 concludes.

II. Methodology

Let Yt be the series of interest rates and Rt the series of log-returns given by Rt = 100 □ log(Yt/Yt−1). It is desired to investigate how well the returns fit the following model will allows for asymmetric error distribution and also allows for time varying skewness. The basic model can be written as follows:

Rt = μRt−1 + ut, (1)

ut = t ht,

where t follows a standard Pearson IV density with parameters (r,δ,a). Precisely, the density of t is given by

$$f_t(x) = c^{-1}\sigma\left(1+\frac{(\sigma x+\mu)^2}{a^2}\right)^{-(1+r/2)}\exp\left[-\delta_t \arctan\left(\frac{(\sigma x+\mu)}{a}\right)\right], \tag{2}$$

where, $\mu = -\delta_t a/r,\ \sigma^2 = a^2(r^2+\delta_t^2)/(r^2(r-1))$ and $c = a\int_{-\pi/2}^{\pi/2}\cos(w)^r \exp(-\delta_t w)dw$.

Note that all the time variation in ft is introduced through the parameter δt. As argued in Brannas and Nordman (2003), the parameter δ is closely related to the skewness, therefore it chosen to introduce time variation in the skewness through this parameter in the following way:

δt = ω1 + α1Yt−1 + β1δt−1 (3)

The skewness of this distribution is given by $s_t = -\frac{-4\delta_t}{r-2}\sqrt{\frac{r-1}{r^2+\delta_t^2}}$.

One of the purposes of the GARCH models is to capture leptokurtosis in the data series that arise due to dynamic volatility. The volatility satisfies the classical GARCH equation outlined next.

ht = ω2 + β2ht−1 + α2ut2 −1 (4)

We chose the Pearson IV density because it offers large degree of asymmetry and extensive tail behavior. In fact, for δ = 0, the Pearson IV becomes the Student distribution. The parameter δ controls the degree of asymmetry and for positive delta the skewness is negative while it is positive for negative delta. The parameter r controls the tail behavior and small values of r indicate heavier tail.

Pearsons Type IV distribution appears to be a suitable candidate to estimate the unknown true distributions of the GARCH innovations fairly accurately. This distribution was first introduced in the GARCH context by Premaratne and Bera(13) for modeling asymmetry and fat-tail. Yan(14) also uses Pearsons Type IV distribution and adopts autoregressive conditional density (ARCD) models to accommodate time-varying parameters.

All parameters of the AR(1)-GARCH model with the Pearsons Type IV distribution are estimated by the method of maximum likelihood (ML). The ML estimation is carried out in MATLAB.

III. Data and Results

We will illustrate our methodology with the use of a numerical example. We collected daily rates for the three months Japanese Yen yield rates. Our data source is the International bond market and the data set in this study covers the period January 2nd 1995 until February 24th 2009. The three months yield series is among the most liquid among the range of maturities, it is characterized by volatility clusters and as we will show its conditional density is asymmetric. The level of short term yield reached historical lows during the above period separated by only few basis points from the, theoretical, zero bound. The time trend of the series is shown on figure 1.

In Table I we report the descriptive statistics of the return series. The kurtosis has a high value but for such a low level rates even small movements of the rates will look very excessive. As we can see, the skewness is positive and moderate which mistakenly can be interpreted as a tendency for the rates to go down. We believe that this could be due to the large negative movements during the first half of the period examined.

We estimated the conditional volatility model in Eq. 3 by maximizing the likelihood function. The parameter estimates with standard error and the t statistic are shown in Table II. The coefficient of the mean and the volatility equations are found to be significant and correlations at all lags are found to be in significant implying that the log return series are stationary. In fact, the results show that there is a highly significant autoregressive term in the mean equation, which suggests that there is an own-spillover of the mean of the short-term interest rate. In addition, all the coefficient s in the variance equation are significant. In addition to own past innovations (arch-effects), represented by the coefficient $\alpha 2$,there is a significant degree of volatility persistence, displayed by the coefficient β, an indication that large volatility increases do last at least the following day. The parameters $w1$, $\alpha 1$, $\beta 1$, in the conditional skewness, Eq. 4, determine the shape of the conditional skewness which is shown in figure 2. These coefficient are significant, with the exception of the coefficient $\alpha 1$, indicating that the skewness changes over time. Moreover, there is significance of the presence of heavy tails indicated by the parameter r.

The results clearly indicate that the change in the direction of trends in the yield rates has affected the dynamics of the return-volatility and displayed time-varying conditional skewness. In fact, the conditional skewness is negative within the time horizon studied. Moreover, looking at both Fig. 1 and Fig. 2, one could suggest that the 3-month interest rates are negatively correlated with the conditional skewness. The fact is that this correlation is indeed negative, −0.978, which suggests that the heavier the tail of the left side of the return distribution is the more the return process can generate steady returns. Hence, it is proposed to employ negative conditional skewness as part of the criteria to evaluate daily short-term interest rates, bearing that investors are aware about the conditional skewness. Therefore, the model proposed for this study could add value in helping understanding why investors could prefer negative skewness in contrast with the consensus in financial theory that investors prefer positive skewness. Under the rubric of behavioral finance, the model could be used to show how the utility of skewness, or the preference for negative skewness, could be relevant in financial markets trading fixed-income instruments that offer asymmetric properties.

IV. Conclusion

This paper proposes a model for the short-term interest rates that takes into consideration non-normality of returns using Pearson's type IV distribution coupled with the GARCH model to account for dynamic volatility. The model builds on previous work on interest rates modeling and shows how negative conditional skewness could drive the interest rate return process to generate steady returns. This in turn could serve better the evaluation of short-term interest rates and could contribute in understanding the utility of skewness in investors' preferences.

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Fig. 1: Daily rates of Japanese 3-month interest rates from January 2nd, 1995 to February 24th, 2009.

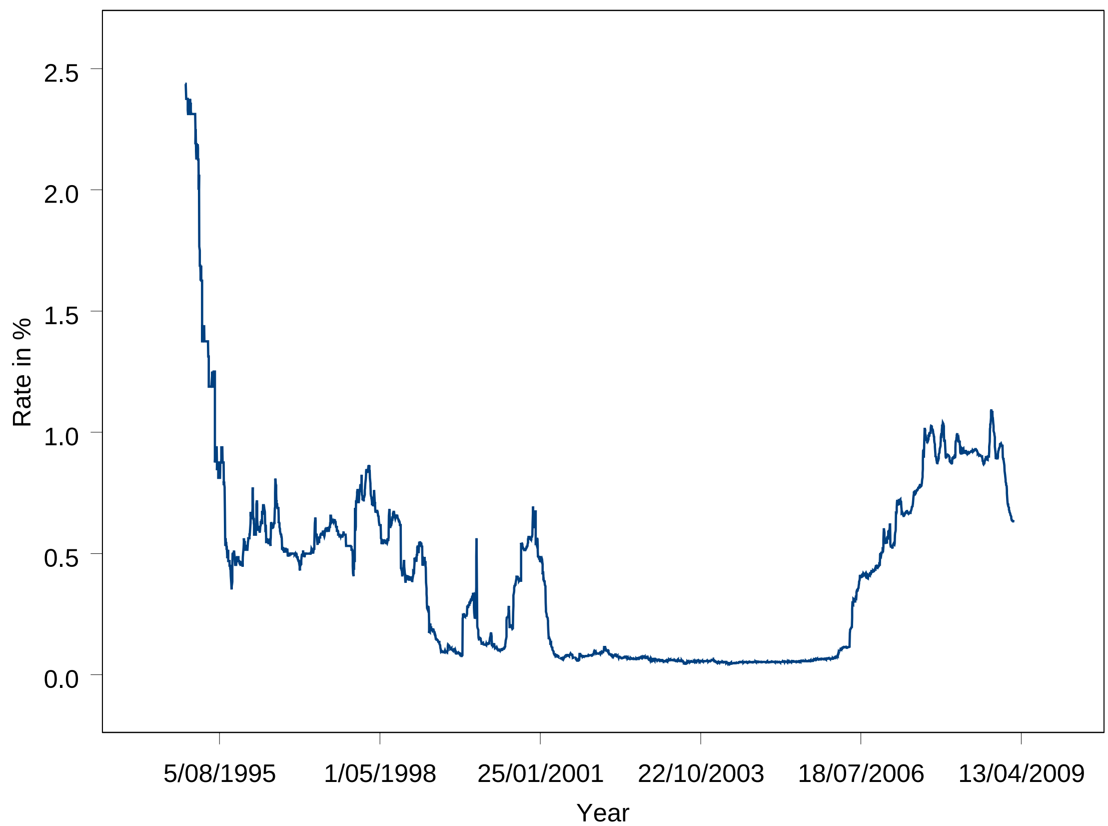

Fig. 2: Conditional Skewness.

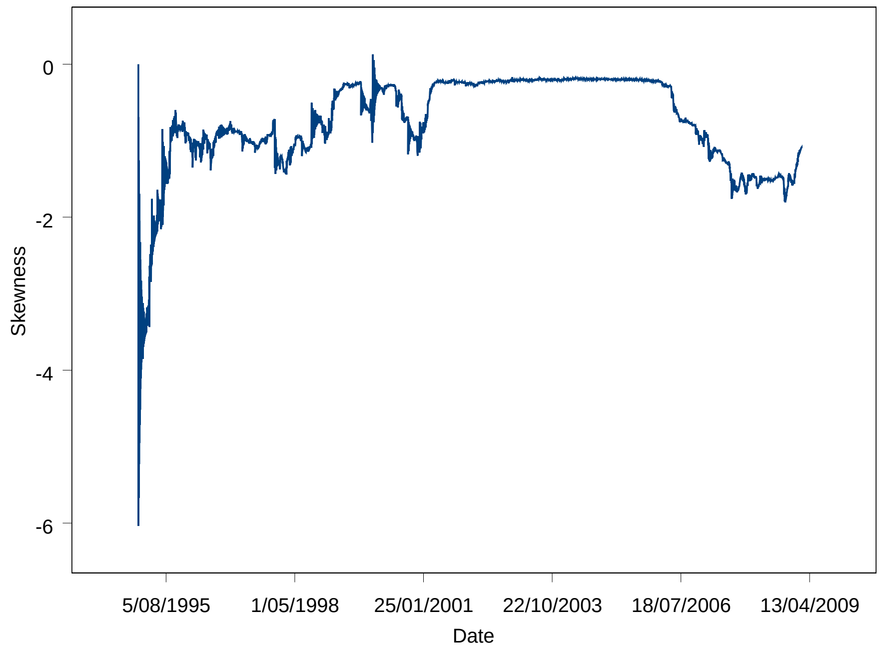

Table I: Descriptive Statistics of the return series (*5% significance level)

Mean	0.0004
SD	0.0456
Skewness	16.9547
Kurtosis	6.769
Maximum	1.7910
Minimum	-0.6444
Jarque-Bera	69135903*

Table II: Parameter estimate for Pearson IV GARCH model (*:1% significance level, **:5% significance level, ***:10% significance level)

Coe□cient	Value	T-Stat
μ	0.128	4.071*
w1	-0.241	−1.307***
α1	0.325	−0.854
β1	0.674	−1.988**
w2	0.205	8.052**
α2	0.325	10.325**
β2	0.674	49.141**
r	1.807	18.005**