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DEM analysis of angular ballast breakage under cyclic loading

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Railway ballast particles undergo significant amount of breakage under repeated train load. Breakage of ballast particles, especially highly angular fresh ones, causes an increase in settlement, contributing to track degradation. The quantitative analysis of the influence of breakage on the stress-strain properties of ballast can be performed either experimentally or numerically. Numerical modeling has the advantage of simulating ballast breakage subject to various types of loading and different boundary conditions for a range of material properties. In this paper, ballast breakage under cyclic loading is simulated using a 2D discrete element method (DEM) utilizing the software *PFC^{2D}*. A new subroutine is developed and incorporated in the *PFC^{2D}* analysis to study ballast breakage and to quantify breakage in relation to particle size distribution. The influence of confining pressure on both breakage and permanent deformation is also studied and compared with laboratory observations. The findings of this paper provide an insight into the true ballast behavior under cyclic loading and are expected to assist railway practitioners in developing suitable design criteria for track stability.

Keywords: Ballast; Breakage; Angularity; Cyclic loading; Discrete element method

1. Introduction

To date, very limited fundamental research has been carried out to explain particle breakage in granular media under cyclic (repeated) loads. In cases where particle breakage is approximated, most of the studies found in the literature often relate to spherical or circular particles (Cheng *et al.* 2004; Lim and McDowell 2005; Lobo-Guerrero and Vallejo 2005, 2006). The breakage model used by McDowell *et al.* (1996) is based on Weibull statistics of fracture (Weibull 1951), and the probability of splitting of particles increases with the applied (macroscopic) stress, but decreases with any increase in the co-ordination number and with any reduction in grain size. This model is limited to static loading and for uniform grains only. While it may not be obvious from past studies how the coordination number is varied with time to incorporate degradation, there is no doubt that the coordination number is important in estimating the extent of breakage. Again complex situations may arise due to the removal of particles randomly from locations other than where the stress concentrations occur. The research by Cheng *et al.* (2004) followed McDowell *et al.* (1996) using the same statistical breakage approach, where their model was validated by comparing the sand behaviour based on a critical state theory. The scope of their analysis was limited by the absence of particle angularity and a realistic well-graded grain size distribution. Lobo-Guerrero and Vallejo

(2005, 2006) analysed breakage under cyclic loading for circular particles. Their analysis also lacks the incorporation of angular particles, and is further limited by the very small number of cycles considered.

Confining pressure and angularity are two important factors that affect ballast breakage under cyclic deviatoric stress. Confining pressure applied to a real track is derived from the gravitational weight of particles and the depth of the ballast layer, its internal formation and the resulting lateral stress. The usual ballast depth of about 300 mm and the presence of crib ballast provides a confining pressure which is usually less than 20 kPa. This implies that under certain circumstances of low confining pressure, typical of railway track environments, dilatation takes place freely, leading to significant breakage of sharp (angular) corners of ballast particles, as demonstrated by experimental studies (Lackenby 2006). It is mainly due to considerable stress concentrations at sharp corners during dilatation. In this situation, the rail track may be affected by excessive permanent deformation due to particle rearrangement. The high angularity of ballast particles provides better interlocking, which helps to prevent excessive dilatation. At the same time, the stress concentrations occur at the corners of angular particles, leading to corner breakage (Indraratna *et al.* 2005).

This study employs a DEM approach to study the effect of angular ballast breakage on the stress-strain behaviour of a ballast foundation under different confining pressures. The property of angularity that enables particle interlocking contributes to enhanced confining pressure, which is absent in the case of rounded (spherical) particles that offer much less

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internal friction. Progressive images of breakage and the grain assembly deformations at various cycles are presented to describe the breakage mechanisms in the ballast foundation. Comparative results of the grain assembly deformation capturing breakage at different loading conditions provide an indication of the acceptable levels of breakage in rail track environments.

2. Numerical analysis

The numerical simulation presented here is based on discrete element modelling (DEM), where the coarse angular grains are best represented as distinct particles. The dynamic analysis of Cundall and Strack (1979) in the modelling of circular or spherical particles is considered as the pioneering work in DEM. The particle flow code (*PFC*) which is a recent development from their initial efforts is now widely used in particulate mechanics. However, the limitation of *PFC* modelling is related to its ability to correctly represent highly angular particles and the exclusion of breakage characteristics at a high number of load cycles. The approach followed by the authors in modelling the angularity of the ballast particles and defining a breakage criterion are described below.

2.1 Particle generation with clumping

An assembly of ballast particles of irregular shapes was considered and the angularity of the particles was modelled by clumping two to nine circular particles together to form single particles of twelve different sizes. The clumps are created randomly so that they can be generated at different locations. The contact forces developed within the assembly were computed using *PFC^{2D} Ver. 3.1* (Itasca 2004). Through the doctoral research of the first author, a new subroutine was developed to consider breakage and was incorporated in the *PFC^{2D}*. A brief description of the subroutine, written using the *FISH* programming code available in *PFC^{2D}*, is presented in this section.

Employing the *CLUMP* option of the *PFC^{2D}* program, a preliminary analysis was conducted using four different particle shapes, as shown in figure 1. Considering different radii of the constituent circular disks, the sizes of the particles were also changed to generate the assembly of a desired particle size distribution. The particle generation in the *PFC^{2D}* program can be of random or fixed sizes. In this study, the particle sizes are limited to between 19 mm and 53 mm.

2.2 Simulation of granular assembly

A 300 mm by 600 mm box was considered within which the ballast particles of unit thickness of sizes varying between 19 mm and 53 mm were generated. The ballast sizes were set in accordance with random quantification and described by the particle size distribution (PSD) plot as shown in figure 2, which is representative of the current industry practice in Australia

Figure 1. Particle sizes and shapes considered in the numerical modelling.

(Standards Australia 1996). The subroutine was developed to get the PSD value of the particles for the sieve sizes recommended by Standards Australia (1996). The sieving can be conducted for any desired number of cycles to obtain the updated PSD. The difference of this PSD from the original PSD plot represents the extent of breakage (figure 2). Initial void ratio of the sample was set to 0.67, which produced a moderately dense assembly (2500 kg/m³) that is typical of the type of ballast used commonly in New South Wales, Australia (i.e. latite basalt). Contact normal stiffness (k_n) of the disks was set to 5×10^9 N/m, and the contact shear stiffness (k_s) was set to half of the contact normal stiffness because results are not affected significantly for different ratios of k_n/k_s . A recent paper by Yang et al. (2006) also found that k_n/k_s ratios have little effect on the compressive strength of the material created by bonded particles. Therefore, $k_n/k_s = 2$ was assumed in this analysis following a linear contact law. Under these conditions, the angular aggregates are expected to behave in a manner whereby the rotations of the particles are insignificant, allowing mainly sliding along the particle surfaces. As particles were not restricted to a circular shape, a realistic friction coefficient could be assumed to define the particle roughness. In this test, a moderate friction coefficient of 0.5 was used.

Figure 2. Typical variations in particle size distribution at different confining pressure.

2.3 Loading conditions

The assembly was then subjected to biaxial loading by confining the sample within four platens generated using the code *PFC^{2D}*. The top platen was moved up and down to exert the cyclic loading while keeping the bottom platen fixed. The test was carried out in a strain-controlled fashion by specifying the velocity of the top platen. During all stages of the test, the velocities of the left and right walls were controlled automatically by a numerical servo-mechanism implanted through the *FISH* function that maintained a constant confining pressure within the sample. The left and right platens provided the confining pressure in the range 10–50 kPa. The axial strain was measured from the differences of position between the top and bottom platens. The volumetric strain was computed from the addition of strains in both the horizontal and the vertical directions. The experimental results show that the maximum deviatoric stress that can be applied varies depending on the confining pressure (Salim and Indraratna 2004). Given the typical 25–30 tonne axle load of trains, a maximum cyclic deviatoric stress of 500 kPa was found to be appropriate for the analysis. In this strain-controlled test, sub-stepping (i.e. accelerating the platen to achieve the desired deviatoric stress in n steps) has been introduced to keep the applied cyclic deviatoric stress in the range between q_{min} 50 kPa and q_{max} 550 kPa.

The critical time step depends upon the ballast stiffness (k) and size (mass, m) and is given by (Cundall and Strack 1979):

$$t_{crit} = \sqrt{m/k} \quad (1)$$

The above equation shows that the smaller are the ballast sizes, the lower is the allowable time step, and therefore the higher the computational time. The test was continued for cyclic loading under a stable cyclic deviatoric stress up to 6000 cycles.

2.4 Breakage criterion

Although observations may generally support the fact that larger particles break more easily than smaller particles, some studies clearly demonstrate that breakage is often governed by the angularity (shape) rather than the size (Lackenby 2006). In this study, breakage captures the inevitable occurrence of stress concentrations due to grain angularity. As described earlier, a *FISH* subroutine developed by the authors is included in the *PFC^{2D}* to accommodate breakage. The criterion that defines breakage is based on the splitting tensile strength of cylindrical concrete specimens (ASTM C496/C496M). Previous studies (e.g. Indraratna and Salim 2002) have shown that the tensile strength of latite basalt commonly used for NSW tracks is usually about 10% of the unconfined compressive strength of 130 MPa. Considering fracture and flaws in the real ballast particles, the tensile strength has been reduced to 10 MPa. This tensile strength of ballast has been incorporated into the subroutine as the limiting tensile strength of simulated ballast particles. A particle from the clump is released when the tensile

stress induced by the contact forces acting on the particle is greater than or equal to 10 MPa. This assumed breakage criterion applies only to a particle having a coordination number smaller than or equal to 3. In this case, the loading configuration is assumed to be equivalent to the splitting tensile strength of cylindrical specimens, similar to the approach also described by Lobo-Guerrero and Vallejo (2005). The release of particle from the clump has been assumed to mimic abrasion and the corner breakage phenomenon for real ballast particles. The released particles satisfying the breakage criteria suffer breakage again. At this time, this particle is fractured into eight smaller particles, two particles having half the diameter of the parent particle, two particles with a diameter equal to 0.333 times the diameter of the parent particle, and four particles with a diameter equal to 0.167 times the diameter of the parent particle. This approach is also similar to that of adopted by Lobo-Guerrero and Vallejo (2005).

2.5 Breakage quantification

The breakage index could be quantified from the plot of particle size distributions (Indraratna *et al.* 2005). With the increase in breakage, the particle size distribution plot shifts to the left, and the area generated due to this shift in relation to the original plot is considered as the breakage zone. The breakage potential is considered to be the area between the original particle size distribution plot and an arbitrary reference line connecting the point between the intersection of d_{95} at 100% finer by weight and the minimum particle size of 2.36 mm as explained by Indraratna *et al.* (2005), as shown in figure 3 for clarity.

3. Results and discussions

A cyclic loading analysis was conducted for angular particles. Relatively low confining pressures were used (10, 30 and 50 kPa) in view of real rail track conditions. The generated assembly of angular particles undergoing breakage is shown in figure 4, with

Figure 3. Definition of ballast breakage index (after Indraratna *et al.* 2005).

Figure 4. A few snapshots of assembly deformation including breakage.

a few snapshots at various load cycles. The gradual deformation of the assembly indicates the level of damage caused by cyclic loading. Large deformations are due to damage caused by ballast breakage and rearrangement. A salient feature of this DEM analysis is that it is possible to quantify the influence of breakage on settlement separately for both uncrushable and crushable particles.

The volumetric strain against axial strain of the ballast is shown in figure 5 and the change in volumetric and lateral strains with the number of cycles is shown in figure 6. In figure 5, compression of simulated particles is observed up to 4% because of the collapse of voids initially. Beyond 4% of axial strain, dilatation of the particles is observed significantly, because, at a low confining pressure (e.g. 10 kPa), high dilatation and degradation are attributed mainly to the shearing and attrition of angular projections due to excessive axial and radial (expansive) strains (Indraratna *et al.* 2005). For confining pressure of 30 kPa, slight dilatation is observed around 4 % of axial strain, while at confining pressure of 50 kPa the dilatation observed is insignificant (figure 5(b,c)).

The trend of these results is similar to the experimental observations made by Indraratna *et al.* (2005) and Lackenby (2006), where they recommended a confining pressure exceeding 30 kPa to totally suppress dilatation. Further increase in confining pressure up to 50 kPa does not show a marked increase in compression, which is quite clear from figure 5(b,c), and this is attributed to the maximum cyclic densification that the ballast has already achieved under $\sigma_3 = 30$ kPa. Figure 6(a) also indicates the marginal difference in volumetric compression between $\sigma_3 = 30$ kPa and 50 kPa beyond 2000 cycles. It has also been observed in figure 6(a) that there is a sudden compression followed by immediate dilatation for the $\sigma_3 = 30$ kPa curve close to 1000 cycles. This behaviour can be attributed to the breakage of particles as shown in figure 7, where the BBI is plotted against the number of cycles. It is seen

from figure 7 that the breakage of particles for $\sigma_3 = 30$ kPa at around 1000 cycles is smaller than that for $\sigma_3 = 50$ kPa. Significantly higher breakage is observed at a lower confining pressure of 10 kPa (figure 7). This trend is also similar to that of Indraratna *et al.* (2005), where the maximum breakage of particle corners was observed to occur at a confining pressure less than 25 kPa (figure 8). They have reported that the ballast behaviour is in dilatant unstable degradation zone (DUDZ) at this stage. It is also relevant to note from figure 7 that the breakage index does not increase significantly beyond 3000 cycles for the levels of confining pressure analysed. The plot of lateral strain vs. number of cycles (figure 6(b)) indicates that the dilatation of ballast is due to high lateral strains in the absence of sufficient confinement.

Figures 7 and 8 show the variation of BBI with the number of cycles and the confining pressure, respectively. Figure 7 indicates that most particle breakage occurs at a relatively small number of cycles (less than 2000) for all confining pressure. The initial breakage at $N < 500$ is attributed to particle dilatation, and the subsequent breakage is associated with increased compression at elevated confining pressure. It is noted that at relatively low confining pressures, where dilatation is considerably higher, particle breakage occurs mainly within this dilatation zone, and it is only at much higher confining pressure ($\sigma_3 > 50$ kPa) that breakage occurs under compression (Indraratna *et al.* 2005). The results shown in figure 7 reinforce this argument. Figure 8 plots the change in BBI vs. confining pressure based on this DEM analysis for σ_3 varying from 10 to 50 kPa. This comparison verifies that the minimum breakage of ballast occurs at confining pressures in the vicinity of 30 kPa. The trend of these results is similar to the experimental results observed by Indraratna *et al.* (2005) as shown in figure 8. At increased values of σ_3 (i.e. beyond 30 kPa) the granular assembly undergoes greater compression and less dilatation. At such confining pressure, particles suffer both corner degradation and

Figure 5. Volumetric strain vs. axial strain at different confining pressure: (a) 10 kPa, (b) 30 kPa and (c) 50 kPa.

Figure 6. Volumetric and lateral strains with number of cycles.

Figure 7. Ballast breakage index variations with number of cycles.

Figure 8. Comparison of breakage index variations with different confining pressure.

splitting, whereas in the other tests ($\sigma_3 < 30$ kPa) only corner splitting is found pronounced under dilating conditions (Indraratna *et al.* 2005).

The axial strains are always compressive at all confining pressure leading to settlements. For all three levels of confining pressure, the DEM analysis shows that the maximum settlement of ballast occurs around 2000 cycles and beyond that the increase in settlement is marginal (figure 9(a)). Given that the increase in BBI of ballast is significant within 0–2000 cycles (figure 7), part of the settlement shown in figure 9(a) is attributed to the breakage of ballast. Figure 9(a) also shows the variation of axial strain with the number of cycles, where the maximum axial strain approaching 12% is obtained at 10 kPa confining pressure. Figure 9b compares the settlement with and without breakage at $\sigma_3 = 10$ kPa, where the shaded zone represents the effect of particle breakage on the settlement. This plot shows an additional 20 mm settlement due to breakage, which is equivalent to an extra 3.5% of axial strain. In this case, although the magnitude of BBI appears to be small, the actual impact of breakage on the ballast settlement is substantial.

The permanent strain related to particle rearrangement is significant in the case of a low confining pressure of 10 kPa (figure 9(a)). As explained earlier, a small confining pressure permits greater dilatation of particles. Nevertheless, in all cases of σ_3 , it

was found that breakage zones were located closer to the top loading platen, as shown in figure 10 for a typical analysis at $\sigma_3 = 50$ kPa, because a randomly generated sample has a smaller coordination number at the top, and it is well known that the lower the coordination number, the greater is the chance of breakage.

4. Conclusions

The breakage of ballast under cyclic loading at different confining pressures has been modelled using the discrete element method to observe the effect of breakage on permanent deformation. The model simulates clumps of 2 to 9 particles in 12 different sizes. From the analysis, it has been observed that the breakage of particles has significant influence on the settlement and volumetric strains. The analysis also demonstrated that breakage is a function of confining pressure, where minimum breakage for coarse particles such as ballast occurs around a confining pressure of 30 kPa. For lower confining pressure, breakage occurs under dilating conditions. At much higher confining pressures (50 kPa or more), breakage occurs under contracting conditions. Under dilating conditions, corner breakage is the most pronounced, whereas under compression splitting across particles is expected (Lackenby *et al.* 2007). The numerical results of this study are also in line with the experimental observations reported by Indraratna *et al.* (2005). However, the simulation has been carried out up to 6000 loading cycles only due to limitations on computing time. More realistic harsh railway environments require much greater loading cycles, exceeding 100 000.

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Figure 9. (a) Settlement and axial strain vs. number of cycles at different confining pressure. (b) Settlement vs. number of cycles with and without breakage.

Figure 10. Breakage of particles at locations near the top platen at 50 kPa confining pressure.

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