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Abstract

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COMPLEX ORTHOGONAL SPREADING SEQUENCES USING MUTUALLY ORTHOGONAL COMPLEMENTARY SETS

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Abstract: *This paper presents a new family of complex spreading sequences designed using mutually orthogonal(MO) complementary sets. Based on the technique described in this paper, the correlation properties of sets of sequences are compared to well-known Walsh-Hadamard sequence sets. Further improvement of correlation qualities can be achieved by employing a diagonal modification method. We also present simulation results of an asynchronous multiuser CDMA system using the modified sequences.*

1 Introduction

Orthogonal sequences are of a great practical interest for the current and future direct sequence(DS) code-division multiple-access(CDMA) systems where the orthogonality principle can be used for channels separation, e.g. [1]. However, most of the known sets of orthogonal spreading sequences possess very poor aperiodic cross-correlation characteristics, i.e. when there is any misalignment between the sequences corresponding to different users, the cross-correlation between such misaligned sequences is usually quite high, resulting in significant multi-access interference(MAI) problems where it is impossible to guarantee sequence alignment due, for example to different propagation delays. The issue of misalignment due to different propagation delays can be mitigated by the use of the so called sequences with zero (or low) correlation zones [2], but this approach usually requires a significant increase in the spreading ratio and the sets of such sequences are rather small for the given sequence length. Another possible solution to this problem can be use of orthogonal polyphase spreading sequences, like those proposed in [4], which for some values of their parameters can exhibit a reasonable compromise between autocorrelation and cross-correlation functions. Polyphase spreading sequences are rather difficult to implement as this require an analog phase modulator. In the paper we introduce quadri-phase orthogonal spreading sequences derived from mutually orthogonal (MO) complementary sets [3] and modified using a diagonal modification method proposed in [5] to achieve good aperiodic cross-correlation characteristics.

2 Construction technique using MO complementary sets of sequences

In this section, we briefly introduce complex MO complementary sequences. For a general background reading, we refer the reader to [3].

Let A_i be a complex sequence. Let $\psi_{A_iA_i}$ denote the aperiodic autocorrelation function of the sequence A_i and $\psi_{A_iA_i}(k)$ be the kth element in the sequence $\psi_{A_iA_i}$. Sequences $(A_i, 1 \le i \le M)$ are complementary if and only if they have zero aperiodic autocorrelation, except for the zero shift:

$$\sum_{i=1}^{M} \psi_{A_iA_i}(k) = 0, \quad k \neq 0 \tag{1}$$

Let $\psi_{A_iB_i}$ denote the cross correlation function of the complex sequences A_i, B_i, $1 \le i \le M$. Let $\psi_{A_iB_i}(k)$ denote the kth element in the sequence $\psi_{A_iB_i}$ and let $(A_i, 1 \le i \le M)$ be a complementary set of sequences. The set of sequences (B_i) is said to be a mate of the set (A_i) if

1. the length of A_i is equal to the length of B_i, for $1 \le i \le M$,
2. the set (B_i) is also complementary, and
3. $\sum_{i=1}^{M} \psi_{A_iB_i}(k) = 0, \quad \forall k$

A collection of complementary sets of sequences is said to be *mutually orthogonal* if any two in this collection are mates to each other. The construction of bipolar MO complementary set of more than two sequences are described in [3]. Here, we focus on synthesizing larger complex MO complementary sets.

Let $\widetilde{A}$ denote the reverse and conjugate of the sequence A, and $-A$ denote the negation of the sequence A. For brevity, we denote sequence element 1 by +, -1 by -, i by i and -i by j for complex sequences. A larger number of MO sets of sequences with bigger size can be generated by the following theorems [3].

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Theorem 2.1 *Let $A_i, 1 \leq i \leq M$ be a complementary set consisting of an even number of sequences of the same length. Then $\{\widetilde{A}_2, -\widetilde{A}_1, \widetilde{A}_4, -\widetilde{A}_3, \ldots, \widetilde{A}_M, -\widetilde{A}_{M-1},\}$ is one of its mates.*

Theorem 2.2 *Let A be a matrix whose columns are MO complementary sets. Let $B = \left[\begin{array}{c|c} A \otimes A & -A \otimes A \\ \hline -A \otimes A & A \otimes A \end{array}\right]$, where $\otimes$ denotes interleaving. Then, the columns of B are also MO complementary sets. The interleaving of two sequences $X = \{x_1, x_2, \ldots\}$ and $Y = \{y_1, y_2, \ldots\}$ is defined as $X \otimes Y = \{x_1, y_1, x_2, y_2, \ldots\}$.*

Theorem 2.3 *Let $(A_{11}, A_{21}, \ldots, A_{M1},), (A_{12}, A_{22}, \ldots, A_{M2}), \ldots, (A_{1k}, A_{2k}, \ldots, A_{Mk})$ be k mutually orthogonal complementary sets of sequences. Let $\triangle = \begin{pmatrix} A_{11} & A_{12} & & A_{1k} \\ A_{21} & A_{22} & & A_{2k} \\ \vdots & \vdots & \ldots & \vdots \\ A_{M1} & A_{M2} & & A_{Mk} \end{pmatrix}$, and $\triangle\pi_r$ denote a matrix obtained from $\triangle$ by a permutation of the columns of $\triangle$. Let H be a $t \times s$ orthogonal complex orthogonal matrix. Define $\triangle' = \begin{pmatrix} h_{11}(\triangle\pi_1) & h_{12}(\triangle\pi_1) & & h_{1s}(\triangle\pi_1) \\ h_{21}(\triangle\pi_2) & h_{22}(\triangle\pi_2) & & h_{2s}(\triangle\pi_2) \\ \vdots & \vdots & \ldots & \vdots \\ h_{t1}(\triangle\pi_t) & h_{t2}(\triangle\pi_t) & & h_{ts}(\triangle\pi_t) \end{pmatrix}$, where $h_{ij}(\triangle\pi_k)$ is a matrix with entries obtained by multiplying all sequences in $\triangle\pi_k$ by h_{ij}. Then the columns of $\triangle'$ are MO complementary sets of sequences.*

Example 2.1 *In this example, we show the construction of bigger MO complementary sets using Theorem 2.3.*

Let $\triangle = \begin{bmatrix} + & + & j & i & & i & j & + & + \\ + & + & i & j & & i & j & - & - \\ j & i & + & + & & + & + & i & j \\ i & j & + & + & & - & - & i & j \end{bmatrix} \stackrel{def}{=} \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \\ A_{31} & A_{32} \\ A_{41} & A_{42} \end{bmatrix}$. Note that $\triangle$ is a 4×2 matrix of sequences. The first column of $\triangle$, $\begin{bmatrix} + & + & j & i \\ + & + & i & j \\ j & i & + & + \\ i & j & + & + \end{bmatrix}$, and the second column of $\triangle$, $\begin{bmatrix} i & j & + & + \\ i & j & - & - \\ + & + & i & j \\ - & - & i & j \end{bmatrix}$, are both complementary because $\psi_{A_{11}A_{11}} + \psi_{A_{21}A_{21}} + \psi_{A_{31}A_{31}} + \psi_{A_{41}A_{41}} = \{0, 0, 0, 16, 0, 0, 0\}$ and $\psi_{A_{12}A_{12}} + \psi_{A_{22}A_{22}} + \psi_{A_{32}A_{32}} + \psi_{A_{42}A_{42}} = \{0, 0, 0, 16, 0, 0, 0\}$. $\triangle$ is MO complementary sequences sets since the sum of cross correlation is zero for the two columns in $\triangle$, which means $C^{(2)} \equiv \psi_{A_{11}A_{12}} + \psi_{A_{21}A_{22}} + \psi_{A_{31}A_{32}} + \psi_{A_{41}A_{42}} = \{0, 0, 0, 0, 0, 0, 0\}$.

Let $H = \begin{pmatrix} + & i \\ + & j \end{pmatrix}$, $\triangle\pi_1 = \triangle$, $\triangle\pi_2 = \begin{bmatrix} i & j & + & + & & + & + & j & i \\ i & j & - & - & & + & + & i & j \\ + & + & i & j & & j & i & + & + \\ - & - & i & j & & i & j & + & + \end{bmatrix}$. Theorem 2.3 gives

$$\triangle' = \begin{pmatrix} + & + & j & i & & i & j & + & + & & i & i & + & - & & - & + & i & i \\ + & + & i & j & & i & j & - & - & & i & i & - & + & & - & + & j & j \\ j & i & + & + & & + & + & i & j & & + & - & i & i & & i & i & - & + \\ i & j & + & + & & - & - & i & j & & - & + & i & i & & j & j & - & + \\ i & j & + & + & & + & + & j & i & & + & - & j & j & & j & j & - & + \\ i & j & - & - & & + & + & i & j & & + & - & i & i & & j & j & + & - \\ + & + & i & j & & j & i & + & + & & j & j & + & - & & - & + & j & j \\ - & - & i & j & & i & j & + & + & & i & i & + & - & & + & - & j & j \end{pmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{13} & B_{14} \\ B_{21} & B_{22} & B_{23} & B_{24} \\ B_{31} & B_{32} & B_{33} & B_{34} \\ B_{41} & B_{42} & B_{43} & B_{44} \\ B_{51} & B_{52} & B_{53} & B_{54} \\ B_{61} & B_{62} & B_{63} & B_{64} \\ B_{71} & B_{72} & B_{73} & B_{74} \\ B_{81} & B_{82} & B_{83} & B_{84} \end{bmatrix},$$

where the sum of cross correlation functions of the first and kth columns of $\triangle'$ is zero, i.e. $C^{(k)} \equiv \psi_{B_{11}B_{1k}} + \psi_{B_{21}B_{2k}} + \ldots + \psi_{B_{81}B_{8k}} = \{0, 0, 0, 0, 0, 0, 0\}, \quad k = 2, 3, 4.$

The MO complementary sets of sequences have applications in multicarrier CDMA communication systems for minimizing MAI on a nonfading channel [6]. The following example shows that how we construct a quadri-phase orthogonal spreading sequences with length 32 by using MO complementary sets.

Example 2.2 Starting from a set of complex complementary sequence $\{A_1, A_2, A_3, A_4\}$ of length 4, let $A_1 = [+ + j\ i]$, $A_2 = [+ + i\ j]$, $A_3 = \widetilde{A}_1$, $A_4 = \widetilde{A}_2$ and $B_1 = \widetilde{A}_2$, $B_2 = -\widetilde{A}_1$, $B_3 = A_2$, $B_4 = -A_1$. The following steps

1): constructing $AB_{4\times8} = \begin{pmatrix} A_1 & B_1 \\ A_2 & B_2 \\ A_3 & B_3 \\ A_4 & B_4 \end{pmatrix}$, columns of $AB_{4\times8}$ are MO complementary sets according to theorem 2.1,

2): let $AB_{4\times8}\pi_1$ denote a matrix obtained from $AB_{4\times8}$ by permutation of the two columns of $AB_{4\times8}$, since $H = \begin{pmatrix} + & i \\ + & j \\ - & i \\ - & j \end{pmatrix}$ is an orthogonal matrix, then $AB_{16\times16} = \begin{pmatrix} AB_{4\times8} & i * AB_{4\times8} \\ AB_{4\times8} & j * AB_{4\times8} \\ -AB_{4\times8}\pi_1 & i * AB_{4\times8}\pi_1 \\ -AB_{4\times8}\pi_1 & j * AB_{4\times8}\pi_1 \end{pmatrix}$ is mutually orthogonal from Theorem 2.3. Also, $AB_{16\times16}$ is an orthogonal matrix which satisfies $AB_{16\times16} * AB^H_{16\times16} = 16I_{16}$, here

$(.)^H$ denotes conjugate transpose.

3): finally, $AB_{32\times32} = \begin{pmatrix} AB_{16\times16} & i * AB_{16\times16} \\ AB_{16\times16} & j * AB_{16\times16} \end{pmatrix}$

generate quadri-phase MO complementary sets $AB_{32\times32}$. Longer orthogonal sequences can be obtained by repeating step 3.

The matrix $AB_{32\times32}$ defines the set of 32-chip spreading sequences which are characterized by the following correlation parameters: $C_{max} = 0.9325$, $A_{max} = 0.5590$, $R_{CC} = 0.7269$, $R_{AC} = 2.0938$. The above parameters, used for measuring "good" signatures for CDMA systems, were introduced by Oppermann and Vucetic in [4]. R_{CC} denotes the average mean-square value of cross-correlation for all sequences in the set. Similarly, R_{AC} can be used for comparison of the autocorrelation properties of the set on the same basis as their cross-correlation properties. It is desirable to have both R_{CC} and R_{AC} as low as possible, as the higher R_{CC} value results in stronger MAI, and an increase of R_{AC} impedes synchronization. However, decreasing the R_{AC} value causes an increase of R_{CC}, and vice versa. We also need to consider the maximum value of peaks in both aperiodic cross-correlation functions over the whole set and in the aperiodic autocorrelation functions, denoted as C_{max} and A_{max}, respectively. For the comparison, the corresponding values for the Walsh-Hadamard sequences of length 32 are: $C_{max} = 0.9688$, $A_{max} = 0.9688$, $R_{CC} = 0.7873$, $R_{AC} = 6.5938$. The new 32-chip sequences from Example 2.2 have lower values of R_{AC} and R_{CC} resulting in less MAI and better synchronization properties.

3 Sequence modification method

Further improvement to the values of correlation parameters of the sequence sets based on MO complementary sets, can be obtained using the method proposed in [5]. The method is based on the fact that an orthogonal matrix H must satisfy the condition $HH^H = NI$, where I is the $N \times N$ identity matrix and $(.)^H$ denotes conjugate transpose. The modification is achieved by taking another orthogonal $N \times N$ matrix D_N, and the new set of sequences is constructed as $W_N = HD_N$ which is also orthogonal. It has been shown that the correlation properties of the modified sequences defined by W_N can be significantly different to those of the original sequences [5].
In [7], a simple class of orthogonal matrices of any order is introduced as diagonal matrices. To preserve the normalization of the sequences, the elements of the diagonal matrix D_N, being in general complex numbers, must of the form:

$$d_{m,n} = \begin{cases} 0 & for\ m \neq n \\ exp(i\phi_m) & for\ m = n \quad m, n = 1, ..., N \end{cases} \tag{2}$$

The most commonly used types of spreading sequences are bipolar sequences and quadri-phase sequences. In the first case, the values of $\phi_m, m = 1, 2, ...N$, are limited to $\{0, \pi\}$, and the phase coefficients ϕ_m can take values from the set $\{0, 0.5\pi, \pi, 1.5\pi\}$ in the second case.
To find the best possible modifying matrix D_N, we have to do an exhaustive search of all possible quadri-phase diagonal matrices of length N satisfying (2), and choose the one with which the modified spreading sequences give the best correlation performance.

Example 3.1 In this example we show the results of minimizing C_{max} for the 32-chip sequences set constructed in Example 2.2. After 60000 trials, the lowest value of C_{max} has been achieved by using modifying matrix D_{32} with the following quadri-phase symbols on the diagonal: $[+ \ j \ i \ i + j \ i \ j \ j - i + - i - j \ j \ j \ j + - - i \ i \ i \ i + j \ j \ i - +]$.

The modified sequences have the following correlation parameters: $C_{max} = 0.3494$, $A_{max} = 0.2965$, $R_{CC} = 0.9718$, $R_{AC} = 0.8750$. The results show that a significant improvement in autocorrelation characteristics has been achieved at the expense of slightly degrading in the cross-correlation properties. In Figure 1, we present the plot of the peak magnitudes for the cross-correlation functions between any pair of the modified sequence set, and the peaks in the autocorrelation functions for all the modified sequences, represented as solid line and dotted line, respectively.

4 System simulation results

To assess performance of the derived sequence set when used in an asynchronous CDMA system, we simulated a multiuser asynchronous system with 32 users distinguished by sequences constructed in Example 3.1. For simplicity, we used a BPSK modulation and no additional spreading or error-control coding. The users were fully asynchronous with misalignments between them taking random values from $[0, 31]$ chips. The phases of active users' modulators were also assumed random from $[0, 2\pi)$. Users were assumed to transmit packets of 1024 bytes.

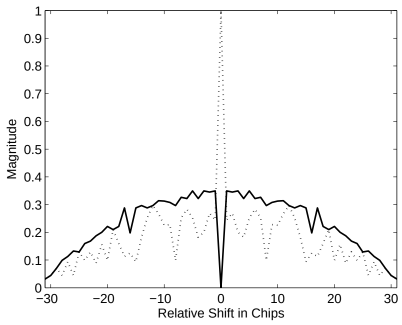

Figure 1: Peak magnitude of aperiodic correlation functions for the sequences constructed in Example 3.1; dotted line - autocorrelation functions, solid line - cross correlation functions

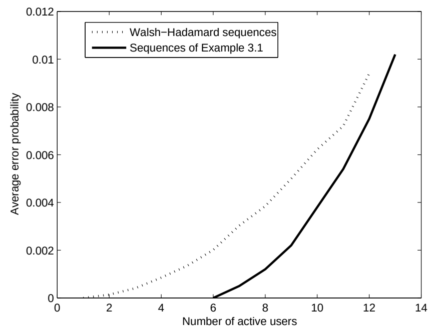

Figure 2: Average BER due to the MAI for the asynchronous CDMA system utilizing sequences from Walsh-Hadamard sequences, and Example 3.1

The misalignments of users' sequences and phases of their modulators were kept constant for the duration of each packet. At the end of each transmitted packet, new constellations of interferers were randomly chosen as well as sequence misalignments and modulator phases. The process was repeated for each of the 32 channels being considered as a channel of interest for 2000 of packets, and an average bit error rate (BER) was then calculated over all 32×2000 results. The number of simultaneously active users was increased from 2 to 13. The channel was assumed to be an additive white Gaussian (AWGN) channel with the SNR set to 20 dB to ensure that MAI was the main source of errors. The implemented receiver was a simple correlative receiver. The resulting average BER vs. the number of active users is plotted in Figure 2. For a comparison, we present there the corresponding results for the Walsh-Hadamard sequences. It is clear that the system utilizing sequences developed in Example 3.1 performs significantly better, in particular for low number of active users.

5 Conclusion

In the paper, we proposed use of the quadri-phase orthogonal spreading sequences derived from mutually orthogonal (MO) complementary sets and modified using a diagonal modification method to achieve good aperiodic cross-correlation characteristics. The sequences designed that way can exhibit a reasonable compromise between autocorrelation and cross-correlation characteristics, and as shown through the simulations lead to good performance when used in an asynchronous multiuser CDMA system. Further research should examine the system performance in a more realistic multipath environment, with or without RAKE receivers.

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