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Leasing or repositioning empty containers? Determining the time-varying guide leasing prices for decision making

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Abstract
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Keywords
prices, decision, making, determining, guide, leasing, time-varying, containers?, empty, repositioning

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Leasing or Repositioning Empty Containers? Determining the Time-varying Guide Leasing Prices for Decision Making

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Abstract Repositioning empty container and leasing container from leasing company are mainly the outcomes of imbalance in trade flows. How to make decision on repositioning or leasing remains a challenge in running shipping service efficiently and economically. To address such challenge, this study aims to derive guide leasing prices. If the realistic leasing price is lower than the guide price, shipping companies should consider leasing containers from leasing company; otherwise repositioning empty container is a more economical option. Both time-varying short-term and long-term guide leasing prices are determined in a shipping network with consideration of empty container devanning and laden container transportation. A two-stage approach is applied to model development. At the first stage, an empty container repositioning model is proposed with consideration of holding, transportation and lifting-on/off costs. Then, inverse optimization technique is applied to deriving the second model in order to determine the guide leasing prices. A realistic shipping network is adopted to compare long-term guide leasing prices among different ports and short-term guide leasing prices on different paths between a pair of ports. The results show that short-term guide leasing prices at a deficit port vary on different paths and they are correlated to the schedule of vessels.

Keywords Empty container repositioning; Container leasing; Guide leasing price; Cargo routing; Inverse optimization

1 Introduction
With the rapid growth of world economy and global trade, a fast growing number of containers are utilized among seaports and inland depots worldwide. In 2017, container shipping took approximately sixty percent of the value in seaborne trade worldwide, and the quantity of goods carried by containers has risen from around 102 million metric tons in 1980 to about 1.83 billion metric tons in 2017. A forecasting indicated that the global shipping container market was expected to grow at a compound growth rate of 8.3 percent between 2017 to 2025, and the market size of shipping containers would reach 11 billion by 2025 (Staista 2020).

Due to the imbalance of world trade in different regions, inequality between import and export containers exists, which results in that some ports have surplus empty containers, while some are in shortage of them (Song and Dong, 2015). Empty container repositioning (ECR), as a feasible solution for container management, optimizes the storage and movement of empty containers in order to maintain a balance between supply and demand of empty containers. Shipping companies may consider to reposition their containers from surplus depots to deficit ones if necessary, but it usually causes expensive cost and consumes a lot of resources, like vessels and empty slots occupied at ports. Besides ECR, leasing containers from a container leasing company is an alternative option commonly adopted by shipping companies. According to Dong and Song (2012a), the number of containers controlled by container leasing companies raised from approximately 6.7 million TEUs in 2001 to 10.7 million in 2009, and the share of container lessors was estimated in a range of 40-50% among the world container fleet over the period between 2001 and 2007. Although ECR has been widely studied in the literature, very limited studies have investigated the ECR problem with consideration of leasing activities simultaneously. More importantly, there exists a trade-off between ECR and container leasing, which lacks proper investigation.

The major contributions of this paper are threefold: First, we propose a two-stage approach to derive both long-term and short-term guide leasing prices of empty containers. Those guide leasing prices are helpful for shipping companies to make decision on repositioning and/or leasing empty containers. If the realistic leasing price is below the guide price, shipping companies should
consider leasing containers from leasing company; otherwise repositioning empty container will be a more economic option. Secondly, different from fixed leasing price considered in most literature studies, it is assumed that the guide leasing price is time-varying since the leasing cost varies at different time periods. Thus, both long-term and short-term guide leasing prices on various paths between any pair of ports change over time in this study. Finally, besides leasing activities, multiple practical operations, such as cargo routing, container devanning and the processes of empty containers becoming laden, are taken into account in the model, which were ignored in the literature more or less.

The remainder of this paper is organized as follows: Section 2 reviews the relevant literature; A two-stage approach consisting of two models is developed to determine the guide leasing prices in Section 3, and the solution approach is introduced as well; In Section 4, numerical experiments based on a realistic shipping network are conducted to validate the proposed approach; Section 5 concludes this study with a discussion of future research direction.

2 Literature review

Over the past decades, there has been an extensive investigation of ECR. Based on the way of transporting empty containers, relevant studies can be grouped into three categories: ECR in inland transportation with focus on empty container movement among inland depots and customers (Song and Earl, 2008; Brackers et al., 2013; Xie et al. 2017), ECR in intermodal transportation with at least two transportation modes involved (Choong et al., 2002; Olivo et al., 2005; Zhang et al., 2018), and ECR in maritime transportation only considering empty container flow among seaports, which falls under the radar of this paper.

Prior research on ECR in maritime transportation mainly combined ECR problem with shipping network design, ship deployment, cargo routing, and policy strategies. Cruijssen et al. (2007) suggested container sharing to reduce cost and to increase productivity, however, Song and Carter (2009) showed that the optimization of cargo routing problem was more effective than container sharing. Song and Dong (2011) discussed an ECR policy with flexible destination ports under different demand patterns. Meng and Wang (2011) investigated the joint problem of shipping network design and ship deployment with consideration of ECR. Song and Dong (2012) considered multiple service routes in the ECR problem with cargo routing, which was formulated as a two-stage shortest-path problem, and a two-stage heuristic-rule was proposed to solve multiple cases at different scales. Dong and Song (2012b) considered the container fleet sizing problem with uncertain customer demand, and developed a simulation-based tool to optimize container fleet size and the ECR policy. Akyüz and Lee (2016) integrated the ship deployment and cargo routing problems into a path-flow based mixed integer linear programming model considering constraints like transit time and vessel speed. Column generation algorithm was combined with the branch and bound algorithm to solve the problem. Wu et al. (2019) investigated a joint problem of shipping network design and ship deployment with consideration of container life span, where the life stage of containers used for a certain voyage was determined. Moreover, some studies focused on the substitution of standard containers with foldable containers to reduce the cost of container repositioning and to save storage space both on vessels and at ports. However, the price of purchasing foldable container is much more expensive (Shintani et al., 2010; Moon et al., 2013; Kuzmicz and Pesch, 2019).

In contrast to a large body of investigations on ECR problem and its variants, there are a limited number of ECR studies with consideration of container leasing activity. Cheung and Chen (1998) developed a two stage network model for the stochastic empty container allocation problem with consideration of short-term leasing of empty containers. However, no returning containers to lessors was taken into account in their model. Moon et al. (2010) considered both container leasing and purchasing activities, and proposed a hybrid genetic algorithm to solve large scale problems, which was extended to both standard and foldable containers by Moon and Hwajin (2016). Dong and Song (2012a) developed a model to optimize the length of container leasing term
with consideration of multi-voyages and turn-around time. Wong et al. (2015) investigated the ECR problem with uncertain upsurge demand, where shipping companies could balance empty container flow by leasing in and out empty containers. Zheng et al. (2016) developed a multi-stage optimization approach to derive the perceived leasing price at each port under the assumption that shipping companies could make long-term leasing to maintain the balance of container flow at any port. Wang et al. (2017) addressed the assignment of ship capacity in a given shipping network while taking ECR into account, and developed a network simplex algorithm based on the flow network model. They assumed that long-term leased-in container could be treated as owned containers and short-term leased containers would be returned to lessors after emptied. Jeong et al. (2018) investigated the empty container management strategy in a two-way four-echelon container supply chain, and proposed different leasing prices at domestic and overseas ports.

To our best knowledge, the vast majority of existing studies on ECR problem with leasing activities treated leasing price as fixed and considered long-term leasing only, however it was important to consider time-varying leasing price and short-term leasing activity to reflect the dynamic shipping market. Moreover, some practical operations like container devanning and the processes of empty containers becoming laden were ignored in the model formulation, which actually affected the ECR and leasing activities more or less. This study is motivated by the issues raised heretofore that reflect a need to develop a model to capture the dynamic features of various leasing activities with more practical operation constraints.

3 Model formulation

In this section, an ECR model is proposed first, followed by the derived dual of ECR model. With applying inverse optimization technique, the corresponding inverse optimization model is formulated. Based on the ECR and cargo routing plan generated from the ECR model, the guide leasing prices can be obtained by solving the inverse optimization model. The complete modelling framework is illustrated in Figure 1.

3.1 Problem description and settings

In this study, we consider a shipping company that provides weekly shipping service among a set of ports with a fleet of vessels. Each vessel operates according to a predetermined timetable to pick up and drop off containers at origin, transshipment and destination ports. Due to trade imbalance, import ports may accumulate unnecessary empty containers and become surplus ports; while export ones often face shortage of empty containers and result in deficit ports. If there are not enough empty containers at a deficit port, shipping companies have to reposition empty containers from other ports or lease in empty containers from container lessors to meet customer demand. With different costs, shipping company faces a trade-off between repositioning...
and leasing empty containers.

As defined in most literature studies, all containers are measured by TEU in this paper. The shipping network with scheduled shipping service routes is assumed unchanged in the planning horizon, and empty containers are ready for reuse once they are unloaded from ships (Akyüz and Lee, 2016; Wu et al., 2019). In this study, the demand for empty containers refers to customers’ request for satisfying transportation consignment need. Shipping companies could lease in long-term containers whenever owned empty containers are not enough to meet customers’ demand, and the leased containers are treated as owned containers after leasing in. For short-term leasing, empty containers should be returned to agreed destination immediately after usage. It takes time to load cargoes and to make laden containers ready for delivering, and it costs several days for customers to return empty containers. Additionally, three assumptions are given below prior to model development.

**Assumption 1.** All the demand allocated to a service route is satisfied at a certain time before vessel departure event occurs, and all the empty containers are returned at a certain time after unloading.

**Assumption 2.** Turn-around time is the period from the time that a laden container is unloaded from a vessel until the moment that the laden container becomes empty for reuse or repositioning.

**Assumption 3.** Lead time is the period from the time that an empty container is picked up by a customer from a shipping company until the moment that the empty container turns into a laden one and is ready for delivering.

**Assumption 4.** The demand for empty containers at each port is given in advance, and the transportation demand for laden containers precedes over the empty ones.

The following notations are introduced for the model development in sub-sections.

### Sets
- **$P$** set of ports, $i \in P$
- **$V$** set of vessels, $v \in V$
- **$T$** planning horizon, $t \in T$
- **$R$** set of service routes, $r \in R$
- **$P_r$** set of ports in service route $r$
- **$R_i$** set of service routes covering port $i$
- **$V_r$** set of vessels deployed in service route $r$
- **$\text{Path}_{ij}$** set of paths from port $i$ to port $j$, $p_{ij} \in \text{Path}_{ij}$, $i \neq j$

### Parameters
- **$G_v$** capacity of vessel $v$
- **$d_{ij,n}$** demand of empty container from port $i$ to port $j$ at $n$th week, $i \neq j$
- **$C_{i}^{\text{empty,hold}}$** unit empty container holding cost per period at port $i$
- **$C_{i}^{\text{on}}$** unit lifting on cost per container at port $i$
- **$C_{i}^{\text{off}}$** unit lifting off cost per container at port $i$
- **$C_{ij}^{\text{empty},r}$** unit transport cost per empty container from port $i$ to port $j$, $i,j \in P_r$, $i \neq j$
- **$C_{ij}^{\text{laden},r}$** unit transport cost per laden container from port $i$ to port $j$, $i,j \in P_r$, $i \neq j$
- **$C_{p_{ij},t}^{\text{short}}$** unit short-term container leasing cost from port $i$ to port $j$ along path $p_{ij}$ at time period $t$, $i \neq j$
- **$C_{i,t}^{\text{long}}$** unit cost for long-term container leasing from port $i$ at time period $t$
- **$\xi_1$** lead time of empty container becoming laden container
- **$\xi_2$** turn-around time of returning empty container from customers to liner
- **$\tau_{ij}^{\text{travel},r}$** travel time from port $i$ to port $j$ along route $r$, $i \neq j$
waiting time at transit port $k$ between port $i$ to port $j$ with two different
routes $r$ and $r'$, $i \neq j$, $r \neq r'$
route where vessel $v$ is deployed
1 if vessel $v$ arrives at port $i$ at time period $t$ and has not called at port $j$ in
this trip, but will call at port $j$ along route $r$ later; 0 otherwise, $i \neq j$
1 if there is a vessel that starts path $p_{ij}$ at time period $t$; 0 otherwise, $i \neq j$
1 if vessel $v$ starts the $n$th path segment $p_{ij}^n$ of $p_{ij}$ at time period $t$; 0
otherwise, $i \neq j$, $1 \leq n \leq 3$

Variables

$x_{ij,t}^r$ number of empty containers moved from port $i$ to $j$ along route $r$ at time
period $t$, $i \neq j$
y_{p_{ij},t}$ number of laden containers transported from origin port $i$ to destination
port $j$ along path $p_{ij}$ at time period $t$
$y_{p_{ij},t}^{\text{own}}$ number of owned empty containers to satisfy the demand from origin port
$i$ to destination port $j$ through path $p_{ij}$ at time period $t$, $i \neq j$
l_{short}^{p_{ij}}$ number of empty short-term leased containers to satisfy the demand from port
$i$ to port $j$ through path $p_{ij}$ at time period $t$, $i \neq j$
s_{i,t}$ inventory of empty containers at port $i$ at time period $t$
$Cap_{t}^{v}$ remaining capacity of vessel $v$ at time period $t$
l_{long}^{t}$ number of empty long-term leased containers at port $i$ at time period $t$

3.2 ECR model

Laden containers are allowed to be transported to the destination port using multiple service
routes but under the limitation of three service routes with transshipment twice at most (Song
and Dong, 2012). This reflects the fact that transshipment not only increases lifting on/off cost
but also causes extra delay and waiting cost at transshipment ports. In this study, path is defined
as a sequence of service routes and transshipment points connecting two ports. Path$_{ij}$ represents
a set of path $p_{ij}$ connecting port $i$ and $j$, which can be divided into three types depending on
the number of transshipment ports involved: direct service $p_{ij} = (i \xrightarrow{r} j)$ denotes a direct path
from port $i$ to port $j$ on route $r$ without transshipment; transshipment once $p_{ij} = (i \xrightarrow{r} k \xrightarrow{r'} j)$
means two routes $r$ and $r'$ are adopted to connect port $i$ with transshipment point $k$, and point
$k$ with port $j$, respectively; and transshipment twice $p_{ij} = (i \xrightarrow{r} k \xrightarrow{r'} k' \xrightarrow{r''} j)$ includes two
transshipment points $k$ and $k'$ and three corresponding routes $r$, $r'$ and $r''$ in sequence to connect
port $i$ and $j$. Particularly, path with transshipment can be split into a sequence of segments
according to the number of transshipment points, and each segment represents a direct route.
Let $p_{ij}^n$ denote the $n$th segment of path $p_{ij}$, thus we have $p_{ij} = p_{ij}^1 + \ldots + p_{ij}^n$, $1 \leq n \leq 3$. Since the schedules and routes of vessels are fixed and regular services are provided by shipping companies
in the planning horizon, the corresponding travel time on each path can be calculated. Taking a
path $p_{ij}$ with two transshipment points $k$ and $k'$ and three corresponding routes $r$, $r'$ and $r''$ as
instance, the corresponding cumulative travel time covering multiple segments can be calculated.
Let $\tau_{p_{ij}}^{c_1}$ be the cumulative travel time covering the first segment $p_{ij}^1$ thus

$$\tau_{p_{ij}}^{c_1} = \tau_{ik}^{\text{travel},r}$$

(1)

Similarly, let $\tau_{p_{ij}}^{c_2}$ be the cumulative travel time covering the first two segments, $p_{ij}^1$ and $p_{ij}^2$, then

$$\tau_{p_{ij}}^{c_2} = \tau_{ik}^{\text{travel},r} + \tau_{ikk'}^{\text{wait},rr'} + \tau_{kk'}^{\text{travel},r'}$$

(2)

and $\tau_{p_{ij}}^{c_3}$ be the cumulative travel time covering all the segments, $p_{ij}^1$, $p_{ij}^2$ and $p_{ij}^3$, such that

$$\tau_{p_{ij}}^{c_3} = \tau_{ik}^{\text{travel},r} + \tau_{ikk'}^{\text{wait},rr'} + \tau_{kk'}^{\text{travel},r'} + \tau_{kkj}^{\text{wait},rr''} + \tau_{kj}^{\text{travel},r''}$$

(3)

Thus, $\tau_{p_{ij}}^{c_3}$ can be treated as the total travel time on path $p_{ij}$, $\tau_{p_{ij}}$. If the path is a direct service,
then $\tau_{p_{ij}}^{c_2} = \tau_{p_{ij}}^{c_3}$. If one transshipment operation is involved, then $\tau_{p_{ij}}^{c_2} = \tau_{p_{ij}}^{c_3}$. In the
meanwhile, the start time of each segment, \( \tau^{s,1}_{p_{ij}}, \tau^{s,2}_{p_{ij}} \) and \( \tau^{s,3}_{p_{ij}} \) can be calculated. Moreover, let \( C^{laden}_{p_{ij}} \) be the total cost of transporting goods from port \( i \) to port \( j \) along path \( p_{ij} \), which includes the transportation cost on routes, waiting cost for transshipment, and lifting-on/off cost at origin, destination and transshipment ports.

Let \( In^r_{i,t} \) and \( Out^r_{i,t} \) be the numbers of containers repositioned in and out at port \( i \) at time period \( t \), respectively, then we have:

\[
In^r_{i,t} = \sum_{r \in R_i} \sum_{j \in P_{i,r}} \sum_{v \in V^v_r} \beta^r_{ij,t}(t-\tau^r_{ij}) x^r_{ji}(t-\tau^r_{ij})
\]

\[
Out^r_{i,t} = \sum_{r \in R_i} \sum_{j \in P_{i,r}} \sum_{v \in V^v_r} \beta^{v,r}_{ij,t} x^r_{ij,t}
\]

Let \( In_{i,t} \) be the number of containers that flow in port \( i \) at time period \( t \) in Eq.(6). The first term on the right-hand-side of Eq.(6) represents the long-term leased-in empty containers at time \( t \); the second term represents the returned empty containers from customers at port \( i \) at time \( t \), which is equal to the arriving laden containers at time \( t - \xi_2 \); and the third and fourth terms represent the short-term leased-in empty containers and the arriving ones at time \( t \), respectively.

\[
In_{i,t} = \sum_j \sum_{r \in Path_{ji}} \left( \varphi_{p_{ji},(t-\tau_{p_{ji}}-\xi_2)} y_{p_{ji},(t-\tau_{p_{ji}}-\xi_2)} + p_{ji}(t-\tau_{p_{ji}}-\xi_2) \right) + In^r_{i,t}
\]

Let \( Out_{i,t} \) be the number of containers that flow out of port \( i \) at time period \( t \) in Eq.(7). The first term on the right-hand-side of Eq.(7) represents the empty containers returned to lessors at port \( i \) at time \( t \); the second term represents the empty containers used to satisfy customer demand; and the third term represents the empty containers repositioned out at time \( t \);

\[
Out_{i,t} = \sum_{P \ni p_{ji}} \sum_{Path_{ji}} \left( \varphi_{p_{ji},(t-\tau_{p_{ji}}-\xi_2)} y_{p_{ji},(t-\tau_{p_{ji}}-\xi_2)} + p_{ji}(t-\tau_{p_{ji}}-\xi_2) \right) + Out^r_{i,t}
\]

In addition, \( On^v_{i,t} \) in Eq.(8) is defined as the number of containers loaded on vessel \( v \) at time period \( t \), wherein the first and second terms on the right-hand-side denote the loaded laden containers and loaded empty containers on vessel \( v \) at time \( t \), respectively.

\[
On^v_{i,t} = \sum_{i \in P} \sum_{j \in P_{i,v}} \sum_{Path_{ij}} \sum_{n \leq 3} \left( \gamma^v_{p_{ji},n} y_{p_{ji},(t-x^v_{p_{ji},n})} + \sum_{i \in P} \sum_{j \in P_{i,v}} \beta^{v,r}_{ij,t} x^r_{ij,t} \right)
\]

Similarly, \( Off^v_{i,t} \) in Eq.(9) denotes the number of containers unloaded from vessel \( v \) at time period \( t \), wherein the first and second terms on the right-hand-side represent the unloaded empty and laden containers from vessel \( v \) at time period \( t \), respectively.

\[
Off^v_{i,t} = \sum_{i \in P_{i,v}} \sum_{j \in P_{i,v}} \left( \beta^{v,r}_{ij,t}(t-\tau^r_{ij}) x^r_{ij,t}(t-\tau^r_{ij}) \right) + \sum_{i \in P} \sum_{j \in P_{i,v}} \sum_{Path_{ij}} \sum_{n \leq 3} \left( \gamma^v_{p_{ji},n} y_{p_{ji},(t-x^v_{p_{ji},n})} \right)
\]

In this subsection, an ECR model is proposed with multiple practical considerations, such as cargo routing, container devanning, and the processes of empty containers becoming laden. The ECR problem aims at minimizing the total cost \( J \) of a specific liner carrier in the whole time horizon \( T \) with optimal solution \( a = \{ x^r_{ij,t}, y^v_{p_{ji},n}, P_{i,v}, l^r_{ij,t}, P_{i,v} \} \). The total cost consists of laden container transportation cost, container holding cost, empty container repositioning cost, and empty container leasing-in cost. This problem can be formulated as an integer programming (IP) model, denoted as model [ECR] as follows:
[ECR] \[
\min_a J(a) := \min \sum_{r \in R} \sum_{t \in T} \sum_{j \in \mathcal{P}} \sum_{v \in \mathcal{V}} \left( x_{ij,t}^r C_{ij}^{\text{empty},r} + \sum_{i \in \mathcal{P}} \left( s_{i,t} C_t^{\text{empty,hold}} + l_{i,t} C_t^{\text{long}} \right) \right) \\
+ \sum_{i \in \mathcal{P}} \sum_{j \in \mathcal{P}} \sum_{v \in \mathcal{V}} \sum_{\text{Path}_{ij}} \left( y_{ij,t}^{\text{laden}} C_{ij}^{\text{laden}} + y_{ij,t}^{\text{short}} C_{ij}^{\text{short}} \right)
\]
\[
\text{s.t. } y_{ij,t}^{\text{laden}} = p_{ij,t}^{\text{own}} + l_{ij,t}^{\text{short}}
\]
\[
\sum_{t = \lceil t \rceil - 6}^{t} \sum_{\text{Path}_{ij}} \left( \varphi_{ij,t}^{\text{laden}} y_{ij,t}^{\text{laden}} \right) = d_{ij,t}
\]
\[
s_{i,(t-1)} - \sum_{r \in R_i} \sum_{j \in \mathcal{P}_i} \sum_{v \in \mathcal{V}} \left( \beta_{ij,t}^v x_{ij,t}^r \right) - \sum_{j \in \mathcal{P}_i} \sum_{\text{Path}_{ij}} \left( \varphi_{ij,(t+1)}^{\text{own}} y_{ij,t}^{\text{laden}} \right) \geq 0
\]
\[
\text{Cap}_{i}^v = \text{Cap}_{i}^v(t-1) - \text{On}_{i}^v + \text{Off}_{i}^v
\]
\[
s_{i,t} = s_{i,(t-1)} + \text{In}_{i,t} - \text{Out}_{i,t}
\]
\[
x_{ij,t}^r, y_{ij,t}^{\text{laden}}, y_{ij,t}^{\text{short}}, x_{ij,t}^r, s_{i,t} \geq 0
\]

Constraint (11) guarantees that the number of laden containers at time \( t \) should be equal to the number of owned and leased-in empty containers utilized to meet customer demand. Constraint (12) ensures that the number of laden containers transported within a week is equal to the customer demand, where \( \tilde{t} = \lceil t/7 \rceil \). Constraint (13) is used to regulate that the total number of owned empty containers repositioned out and used to meet customer demand does not exceed the inventory in the previous time period. The initial inventory of empty containers at each port is given as \( s_{i,0} \) at port \( i \). Constraint (14) describes the flow balancing of empty and laden containers on vessel \( v \) at time period \( t \), and ensures that the number of loaded containers is no more than the capacity of vessel. \( \text{Cap}_{i}^v \) denotes the initial capacity of vessel \( v \), which is equal to its maximal capacity \( G^v \). Constraint (15) reflects the inventory change of empty containers at port \( i \) at time period \( t \). Finally, constraint (16) shows non-negative variables.

### 3.3 Inverse optimization model

As a similar work, Zheng et al. (2016) derived the long-term and fixed perceived leasing price. In this study, we aim to derive both long-term and short-term guide leasing prices, which are time-varying to reflect the dynamic market. In order to determine such dynamic prices, an Inverse Optimization (IO) technique is adopted (Ahuja and Orlin, 2001). Let \( \min P(x,c) : x \in X \) be the objective function of an optimization problem, where \( X \) represents the feasible region and \( c \) is a parameter vector representing cost. Let \( \tilde{x} \) denote a feasible solution, the IO approach aims to conduct minimal adjustment of the parameter values of vector \( c \) to make \( \tilde{x} \) optimal to the problem \( \min P(x,c) : x \in X \). In the model [ECR], both long-term and short-term guide leasing prices are input as adjustable parameters. In this subsection, an IO model is developed based on the model [ECR] to minimize parameter adjustment of the guide prices in the model [ECR].

Let \( a^* \) be the optimal solution to model [ECR] where \( l_{i,t}^{\text{long}}, l_{ij,t}^{\text{short}} \neq 0 \), and \( \tilde{a} \) be a feasible solution when \( l_{i,t}^{\text{long}}, l_{ij,t}^{\text{short}} = 0 \), thus demands are met only by repositioning empty containers. The goal of the IO model is to adjust the parameter values to make \( \tilde{a} \) optimal and the adjustment minimal. Let \( a_{\text{dual}} = \{ \lambda_{ij,t}, \theta_{ij,t}, \eta_{t}, \psi_{i,t}, \omega_{i,t}, \psi_{i,t}^*, \omega_{i,t}^* \} \) denote the dual variables associated with constraints (10)-(14), and \( a^*_{\text{dual}} = \{ \lambda_{ij,t}^*, \theta_{ij,t}^*, \eta_{t}^*, \psi_{i,t}^*, \omega_{i,t}^* \} \) be the optimal solution to the dual problem of model [ECR]. Then, according to the primal-dual complementary slackness conditions, additional sets are incorporated as follows.

\[
Z_{ijtr} = \{(r, i, j, t) | x_{ij,t}^r > 0, \forall r \in R, i \in P, j \in P, t \in T \}
\]
\[ Z_{p_{ij},t}^{2} = \{(p_{ij}, t) | y_{p_{ij},t}^{own} > 0, \forall p_{ij} \in Path_{ij}, i \in P, j \in P, t \in T \} \tag{18} \]
\[ Z_{p_{ij},t}^{3} = \{(p_{ij}, t) | y_{p_{ij},t}^{laden} > 0, \forall p_{ij} \in Path_{ij}, i \in P, j \in P, t \in T \} \tag{19} \]
\[ Z_{i,t}^{4} = \{(i, t) | s_{i,t} > 0, \forall i \in P, t \in T \} \tag{20} \]
\[ Z_{v,t}^{5} = \{(v, t) | Cap_{v}^{i} > 0, \forall v \in V, t \in T \} \tag{21} \]
\[ Z_{i,t}^{6} = \left\{ (i, t) | s_{i,(t-1)} - \sum_{r \in R_{i}, j \in P} \sum_{v \in V^{r}} \beta_{v}^{i,j}(\omega_{j,(t+\tau_{j}^{i})}) - \sum_{j \in P_{p_{ij}} \in Path_{ij}} \varphi_{p_{ij},(t+\xi_{i})} y_{p_{ij},t}^{own} > 0, \forall i \in P, t \in T \right\} \tag{22} \]

Let \( \varepsilon_{p_{ij},t}^{short} \) and \( \varepsilon_{p_{ij},t}^{long} \) be the adjustment of \( C_{p_{ij},t}^{short} \) and \( C_{p_{ij},t}^{long} \) respectively; \( \Phi_{i,t} \) be the adjustment of \( C_{i,t}^{long} \); and \( \sigma = \{\tilde{C}_{p_{ij},t}, \tilde{C}_{i,t}^{long}\} \) are adjusted parameters of \( \sigma = \{C_{p_{ij},t}^{short}, C_{i,t}^{long}\} \), thus
\[ \tilde{C}_{p_{ij},t}^{short} = C_{p_{ij},t}^{short} + \varepsilon_{p_{ij},t}^{short} - \varepsilon_{p_{ij},t}^{short} \tag{23} \]
\[ \tilde{C}_{i,t}^{long} = C_{i,t}^{long} + \varepsilon_{i,t}^{long} - \varepsilon_{i,t}^{long} \tag{24} \]

Particularly, \( \sigma = \tilde{\sigma} \) when \( a = a^{*} \) and \( \tilde{a} = a^{*} \). In addition, we have \( \sigma \geq \tilde{\sigma} \), therefore \( \varepsilon_{p_{ij},t}^{short}, \varepsilon_{i,t}^{long} = 0 \). Using IO technique, a model \([IO]\) based on the model \([ECR]\) can be formulated below.

\[
[IO] \quad \min_{b} Q(b) := \min \sum_{t \in T} \sum_{i \in P} \sum_{j \in P_{p_{ij}} \in Path_{ij}} \varepsilon_{p_{ij},t}^{short} + \sum_{t \in T} \sum_{i \in P} \varepsilon_{i,t}^{long} \tag{25}
\]
\[
\text{s.t.} \quad \sum_{v \in V^{r}} \left( \beta_{v}^{i,j}(\omega_{j,(t+\tau_{j}^{i})}) - \omega_{i,t} - \omega_{i,t} - \psi_{t} - \psi_{t}^{(t+\tau_{j}^{i})} \right) \leq C_{ij}^{empty,r}, \forall (i, j, t, r) \notin Z_{ijtr}^{1} \tag{26}
\]
\[-\lambda_{p_{ij},(t+\xi_{i})} + \varphi_{p_{ij},(t+\xi_{i})}(\omega_{j,(t+\tau_{p_{ij}}+\xi_{j}+\xi_{i})} - \omega_{i,t} - \omega_{i,t}) \leq 0, \forall (p_{ij}, t) \notin Z_{p_{ij},t}^{2} \tag{27}
\]
\[ \varphi_{p_{ij},t} \theta_{p_{ij},t} + \sum_{v \in V^{n}} \sum_{t \leq 3} \left( \beta_{v}^{i,j}(\omega_{j,(t+\tau_{j}^{i})}) - \omega_{i,t} - \omega_{i,t} - \psi_{t} - \psi_{t}^{(t+\tau_{j}^{i})} \right) \leq C_{ij}^{laden}, \forall (p_{ij}, t) \notin Z_{p_{ij},t}^{3} \tag{28}
\]
\[ \eta_{i,(t+1)} + \omega_{i,(t+1)} - \omega_{i,t} \leq C_{i}^{empty,hold}, \forall (i, t) \notin Z_{it}^{1} \tag{29}
\]
\[ \psi_{t}^{(t+1)} - \psi_{t} \leq 0, \forall (v, t) \notin Z_{vt}^{5} \tag{30}
\]
\[ \eta_{i,t} = 0, \forall (i, t) \notin Z_{it}^{5} \tag{31}
\]
\[ \sum_{v \in V^{r}} \left( \beta_{v}^{i,j}(\omega_{j,(t+\tau_{j}^{i})}) - \omega_{i,t} - \omega_{i,t} - \psi_{t} - \psi_{t}^{(t+\tau_{j}^{i})} \right) \leq C_{ij}^{empty,r}, \forall (i, j, t, r) \in Z_{ijtr}^{1} \tag{32}
\]
\[-\lambda_{p_{ij},(t+\xi_{i})} + \varphi_{p_{ij},(t+\xi_{i})}(\omega_{j,(t+\tau_{p_{ij}}+\xi_{j}+\xi_{i})} - \omega_{i,t} - \omega_{i,t}) = 0, \forall (p_{ij}, t) \in Z_{p_{ij},t}^{2} \tag{33}
\]
\[ \varphi_{p_{ij},t} \theta_{p_{ij},t} + \sum_{v \in V^{n}} \sum_{t \leq 3} \left( \beta_{v}^{i,j}(\omega_{j,(t+\tau_{j}^{i})}) - \omega_{i,t} - \omega_{i,t} - \psi_{t} - \psi_{t}^{(t+\tau_{j}^{i})} \right) \leq C_{ij}^{laden}, \forall (p_{ij}, t) \in Z_{p_{ij},t}^{3} \tag{34}
\]
\[ \eta_{i,(t+1)} + \omega_{i,(t+1)} - \omega_{i,t} = C_{i}^{empty,hold}, \forall (i, t) \in Z_{it}^{1} \tag{35}
\]
\[ \psi_{t}^{(t+1)} - \psi_{t} = 0, \forall (v, t) \in Z_{vt}^{5} \tag{36}
\]
\[ \eta_{i,t} \geq 0, \forall (i, t) \in Z_{it}^{5} \tag{37}
\]
\[ \omega_{i,t} \leq \tilde{C}_{i,t}^{long} \tag{38} \]
\[ -\lambda_{p_{ij},(t+\xi_{i})} \leq \tilde{C}_{p_{ij},t}^{short} \tag{39} \]
By solving the model \([IO]\), we can obtain both \(\tilde{C}_{\text{long}}^{i,t}\) and \(\tilde{C}_{\text{short}}^{p_{ij},t}\), which are the long-term guide leasing price at port \(i\) and the short-term guide leasing price between port \(i\) and port \(j\) along path \(p_{ij}\), respectively.

### 3.4 Solution method

In this study, model \([ECR]\) is formulated as an IP model and model \([IO]\) is a linear programming (LP) model, which can be solved efficiently by various existing solvers, such as Gurobi. The solution procedure is illustrated as follows:

1. Initialization: Shipping network, vessel’s capacity and schedule, weekly demand between any pair of ports, and the number of owned containers at each port are provided as input;
2. Calculating service paths: Based on the given parameter values in Step 1, service paths can be calculated with corresponding transportation cost \(C_{\text{laden}}^{p_{ij}}\) and transporting time \(\tau_{p_{ij}}\) of any specific path \(p_{ij}\);
3. Solving the model \([ECR]\): Assume \(l_{\text{long}}^{i,t}, l_{\text{short}}^{p_{ij},t} = 0\), an optimal solution, the vector \(a\) and the total cost \(J(a)\), can be obtained by solving the model \([ECR]\). The initial solution represents transportation planning without leasing activities;
4. Solving the model \([IO]\): With the assumption \(C_{\text{long}}^{i,t}, C_{\text{short}}^{p_{ij},t} = 0\) and the solution obtained in Step 3 as input parameters, the model \([IO]\) can be solved to derive the optimal guide leasing prices.

### 4 Numerical experiments

In this section, a real-world shipping network connecting Asia and West Coast of North America is employed for multiple case studies. All the experiments are conducted on a desktop with Intel i7, 2.6GHz CPU, 16G memory and 64-bit windows 10, and are solved in Matlab with Gurobi.

The geographical distribution of a set of ports with multiple shipping service routes is illustrated in Figure 2. The network contains six service routes and twenty ports. The transportation consignments from Asian ports to ports in North America are assumed to follow the uniform distribution \(U(160, 180)\) and that the ones from North America to Asian follow \(U(80, 90)\). Additionally, we assume that the transportation consignments between two countries within the same region follow \(U(50, 70)\). For all the routes, no transportation consignments happen within the same country. The weekly laden container transportation consignments are randomly generated for each selected shipping line. We assume that \(|V| = 37\) container vessels are deployed and scheduled in the shipping network to provide weekly transportation service. \(G^v\) is randomly drawn in \([3000, 5500]\), and all the holding costs and handling costs for both empty and laden containers are the same at each port: \(C_{i}^{\text{empty,hold}} = $6/\text{day}, C_{i}^{\text{on}} = C_{i}^{\text{off}} = $50/\text{TEU}, \forall i \in P\). The costs for transporting empty and laden containers are the same, \(C_{ij}^{\text{laden,r}} = C_{ij}^{\text{empty,r}} = $140/\text{week}\). Shipping companies are able to either reposition or to lease empty containers if necessary, and allow exporters and importers to use containers for a week. The length of planning horizon is set at \(T = 525\) days. No demand is considered out of the planning horizon.
Figure 2. A shipping network connecting Asia and West Coast of North America

According to the imbalance ratio of laden container flow, around 1.99 in Asia-North America trade (Wang et al., 2017), we can identify some ports in Asia, such as Ningbo and Shanghai ports, as deficit ports, and some in North America, such as Oakland and Seattle ports, as surplus ports. Two case studies with heterogeneous distributions of initial inventory of empty containers are investigated in the subsequent content. In Case 1, it is assumed that initially most empty containers are stored at deficit ports, i.e., Asian ports. In Case 2, most empty containers are accumulated at surplus ports, i.e., North America ports, at initial time period. In both cases, the length of long-term leasing is assumed to be the whole planning horizon, and the length of short-term leasing covers a single trip only. For long-term leasing, shipping company can lease in empty containers at any time period in the planning horizon. Short-term leasing for a specific path only happens once a week since it is assumed that shipping company only releases empty containers to customers at a certain time before the departure of vessels. In the numerical experiments, all the surplus and deficit ports show similar patterns of both long-term and short-term time-varying guide leasing prices, respectively. Due to the space limitation, we only use Oakland port (OAK) and Yantian port (YTN) as representatives of surplus port and deficit port, respectively, to illustrate the results in the following content.

4.1 Case 1: Most empty containers stored at deficit ports initially

The time-varying long-term guide leasing price for a pair of surplus and deficit ports, i.e., OAK and YTN, are shown in Figure 3. It can be found that, at YTN, the guide leasing price is higher than that at OAK at each time period, which indicates that leasing activity is more likely to happen at YTN than OAK. Although the inventory of empty containers at YTN drops quickly, the price at YTN stays below zero within a long time period. During this period, shipping companies at YTN reposition empty containers to satisfy the demand rather than choosing long-term leasing since the latter generates expensive holding cost. The guide leasing price at OAK is always below zero given that the number of laden containers flowing in is larger than that of flowing out. Extra empty containers are accumulated at OAK and then are repositioned to satisfy the demand at deficit ports. Given the initial inventory and the turn-around time of returning empty containers, the inventory at OAK does not raise, but decreases in the initial time period of the planning horizon.
Furthermore, we assume the long-term guide leasing price to be fixed in the planning horizon $T$, and the corresponding results of long-term guide leasing price at each port are shown in Table 1. It can be noticed that, at the surplus ports in North America, the fixed long-term guide leasing prices are always below zero since empty containers are surplus at these ports due to the imbalance of container flow. At the deficit ports in Asia, the fixed guide leasing prices are always above $100. Shipping companies at these ports can lease in empty containers to satisfy customer demand and to reduce the cost of repositioning empty containers. Kaohsiung port presents the highest fixed long-term guide leasing price. Part of the empty container demand at Kaohsiung port is satisfied by containers repositioned from Los Angeles port and OAK with long distance transportation (see route 3 in Figure 2), thus the corresponding repositioning cost is expensive, which leads to a higher guide leasing price than at other ports. It is worthy to mention that the fixed long-term guide leasing price is the maximum value of the time-varying guide leasing price at all ports. For example, the highest price at YTN is $342 (see Figure 3(a)), which is equal to the corresponding price in Table 1.

Table 1. Fixed long-term guide leasing price($)

<table>
<thead>
<tr>
<th>Port</th>
<th>Xiamen</th>
<th>Hong Kong</th>
<th>Yantian</th>
<th>Long beach</th>
<th>Oakland</th>
</tr>
</thead>
<tbody>
<tr>
<td>Guide price</td>
<td>418</td>
<td>322</td>
<td>342</td>
<td>-6</td>
<td>-6</td>
</tr>
<tr>
<td>Port</td>
<td>Ningbo</td>
<td>Shanghai</td>
<td>Pusan</td>
<td>Los Angeles</td>
<td>Tokyo</td>
</tr>
<tr>
<td>Guide price</td>
<td>366</td>
<td>380</td>
<td>110</td>
<td>-6</td>
<td>226</td>
</tr>
<tr>
<td>Port</td>
<td>Qingdao</td>
<td>Kaohsiung</td>
<td>Keelang</td>
<td>Prince Rupert</td>
<td>Vancouver</td>
</tr>
<tr>
<td>Guide price</td>
<td>446</td>
<td>450</td>
<td>410</td>
<td>-6</td>
<td>-6</td>
</tr>
<tr>
<td>Port</td>
<td>Seattle</td>
<td>Kwangyang</td>
<td>Nagoya</td>
<td>Tacoma</td>
<td>Kobe</td>
</tr>
<tr>
<td>Guide price</td>
<td>-6</td>
<td>100</td>
<td>270</td>
<td>-6</td>
<td>298</td>
</tr>
</tbody>
</table>

Following the same shipping network settings, we investigate the time-varying short-term guide leasing prices from YTN to OAK along two paths: path 1 (route 6, YTN-OAK, 20 days) and path 2 (route 2 + route 4, YTN-Ningbo-OAK, 22 days). As shown in Figure 4(a), although the transporting time along path 1 is shorter than that of path 2, which means the leasing term on path 1 is shorter than that along path 2, the guide leasing price along path 1 is higher than...
the price along path 2. This occurs because delivering containers along path 1 costs less than
along path 2 due to shorter transportation time and less transshipment. In the meanwhile, using
owned containers on path 2 can reduce empty container storage time in OAK since it takes two
more days along path 2 than using path 1. Therefore, shipping companies are more willing to
lease containers along path 1 and to use own containers in path 2. Such results indicate that the
short-term guide leasing price is related to the path in use. Similar results for the paths from
OAK to YTN are identified in Figure 4(b).

![Graph showing time-varying short-term guide leasing price]

Figure 4. Time-varying short-term guide leasing price

As shown in Figure 4, the guide leasing prices along both paths for YTN are much higher
than the prices for OAK in almost all the planning horizon. This happens because the short-
term leasing activity at YTN could avoid empty containers accumulated at OAK and reduce
repositioning cost. Since utilizing leased empty containers for shipping from OAK to YTN results
in reduced availability of empty containers at YTN, shipping companies need to lease or reposition
more empty containers to satisfy the demand at YTN, which will generate high cost. Hence, it
is economic for shipping companies to lease in short-term empty containers at YTN rather than
OAK. As time elapses, the short-term guide leasing price at YTN drops because leasing container
generates growing storage cost due to the increasing number of empty containers at OAK. It is
noticeable that the short-term guide leasing prices along both paths raise at the end. It happens
because containers from YTN to OAK would not be repositioned from OAK to other deficit
ports since no demand is considered beyond the planning horizon.

The results of cargo routing from YTN to OAK through different paths are shown in Figure 5,
where most of laden containers are transported through path 1 (route 3, 19 days, departure every
Tuesday), which is the path with minimal travel time. Path 2 (route 6, 20 days, departure every
Saturday) is only used at the beginning and end periods of the planning horizon. It happens
because the majority of empty containers are stored at deficit ports (like YTN) initially, the
adoption of path 2 could avoid empty containers accumulated at OAK at the beginning of
planning horizon. In this case, although the corresponding transportation cost on path 2 is
higher, the storage cost will be reduced. At the end periods of planning horizon, the empty
containers arriving at YTN on Thursday (from Pusan port and Kwangyang port on route 2)
could be used for delivery to OAK (departure every Tuesday) along path 1, however, those empty containers would be stored another five days, which is just two days if using path 2 (departure every Saturday). Therefore, it is more economical to use path 2 at the beginning and end periods of the planning horizon.

Figure 5. Number of laden containers transported through different paths

4.2 Case 2: Most empty containers stored at surplus ports initially

To prove that the guide leasing price is not determined by the initial distribution of empty containers, we assume that most containers are stored at surplus ports initially in this case study. By adopting the same shipping network in Case 1, we derive the corresponding time-varying long-term and short-term guide leasing prices, as shown in Figure 6 and Figure 7, respectively.

Figure 6. Time-varying long-term guide leasing price
Similar to Case 1, the time-varying long-term guide leasing prices at both YTN and OAK are below zero at most time periods. Customer demand at deficit ports could be satisfied by repositioning containers because the amount of containers is enough in the shipping network without consideration of special situations, like container maintenance. Thus, even though empty containers are deficit at YTN, it is not economical for shipping companies to lease long-term containers at time periods with negative guide leasing price. Compared to Case 1, the short-term guide leasing prices on both paths from YTN to OAK in Case 2 is higher at the beginning of $T$ because empty containers are initially stored at OAK in Case 2, and leasing empty containers could reduce repositioning and storage cost. As time elapses, the short-term guide leasing price at YTN on path 2 drops below zero and then maintains a fixed value for a long period since the balance of flow-in and flow-out containers is achieved. Although leasing activity helps to reduce repositioning cost, it increases holding cost simultaneously. To a certain level, leasing is no longer an economical option (with negative guide price), which is then replaced by container repositioning. Hence, repositioning empty container from surplus ports in North America to deficit ports in Asia happens after a certain period, and the corresponding short-term guide leasing price falls below zero and remains stable for a long period before growing at the end of the time horizon. In addition, the fixed long-term guide leasing prices in Case 2 show identical results as in Table 1 in Case 1, which provides evidence that the guide leasing price is not determined by the initial distribution of empty containers.

It is worthy to mention that a large number of numerical experiments (57 totally) have been conducted in this study considering that some parameter values such as the transportation consignments are randomly generated from given distributions rather than fixed values. Due to the space limitation, not all the experiment results are illustrated in this section. In general, almost all the numerical results showed similar trends of both long-term and short-term guide leasing prices as the results in this section. Only a few (3 only) showed slight difference at the beginning of the planning horizon and remained similar trend in most periods of the planning horizon. Further analysis found that the initial inventory of empty containers and the flow-out and flow-in containers had influence on the guide leasing price at the beginning of the planning horizon to some extent. Only under special/extreme scenarios, for example when the flow-out
demand is much larger than the flow-in one or the initial inventory is at an appropriate or optimal level to satisfy the port’s demand, the trends of guide leasing prices at the beginning of the planning horizon would be slightly different from the results shown in this section. Therefore, it is evident from the extensive experiment results that the numerical results illustrated in this section are universal rather than a special case.

5 Conclusion

In this paper, both long-term and short-term time-varying guide leasing prices of empty containers were derived with consideration of cargo routing, container devanning and the processes of empty containers becoming laden. An ECR model was proposed first, based on which an IO model was formulated to derive the guide leasing prices. To validate the model, two case studies in a realistic shipping network were conducted based on different initial inventory distributions of empty containers. The numerical results indicated that the guide leasing price was not determined by the initial distribution of empty containers. The time-varying long-term guide leasing price reflected the dynamic shipping market in a more accurate and practical way than fixed price, which was usually the highest value of the time-varying prices. By comparing the short-term leasing prices along different paths, we found that the short-term leasing price varied along with different paths, and it was related to the schedule of vessels deployed on the routes. Paths with shorter transporting time usually came with higher guide leasing prices, while longer paths showed lower price.

This paper provides a method to derive the guide leasing price to support decision making on repositioning or leasing empty containers along a specific path, by which shipping companies can save their operational costs. If the realistic leasing price is lower than the guide price, shipping companies should consider leasing containers from leasing company; otherwise repositioning empty container is a more economical option. In view of the numerical results, the proposed approach is not only able to trade-off between leasing and repositioning activities, but also able to help shipping company to make decision on which path to transport the laden containers. For example, given a certain pair of origin and destination ports, the path with the most significant difference between the guide and realistic leasing prices (guide leasing price is higher than the corresponding realistic leasing price on this path) should be used by the shipping company to transport laden containers. With the long-term guide leasing price, shipping company is able to optimize the container fleet size at the beginning of planning horizon to reduce repositioning and leasing cost. Furthermore, this study implies that leasing long-term containers is not an economic option when a shipping company has plenty of empty containers.

To simplify the problem, this study has made several assumptions, for example, the devanning process was fixed for customers, and stochastic demand was ignored. Future research will focus on efficient strategy design to allocate slots and to reposition empty containers, and corresponding inventory control. Beyond the fact that individual shipping company could minimize the operational cost by following the guide leasing prices, the proposed approach is extendable to motivate cooperation to allow leasing empty containers among different shipping companies by following the rule that the guide leasing-in and leasing-out prices are identical along the same path. Under such cooperation, the total operation cost of the whole shipping network could be reduced by maximizing the utilization of empty containers.

Conflict of Interest

The authors declare that there is no conflict of interest regarding the publication of this paper.
References


