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Abstract
Optimal control theory is employed to characterize the socially optimal trajectory of the royalty per channel and the number of royalty-paying users of state-owned spectrum for broadcasting. The spectrum royalty is set by an omniscient public planner to maximize the sum of the discounted consumers' utilities over an infinite planning horizon. The number of broadcasters adjusts over time to profits, while the quality of the industry's service is determined by variety and reception. The trade-off between the benefits of greater variety and the costs of intensified interferences associated with the number of broadcasters is central to the analysis. The convergence of the socially optimal trajectory of the royalty per channel and the number of broadcasters to a steady state and the comparative statics of the steady state are analyzed.

Keywords: Broadcasting; Royalties; Spectrum; Optimal Control

JEL Classification : C61, C62, D61, K23, L52

This paper is based on UOW Economics Working Paper 12-10. It provides an improved and more general analysis.

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1. Introduction

Consumers' satisfaction from over-the-air (OTA) broadcasts typically increases with the broadcasts' variety and the clarity of their reception. While an expansion of the number of OTA broadcasters increases the variety of programs, the increased spectral congestion intensifies interferences thereby decreasing reception clarity. This variety-reception trade-off is the focal theme of our theory of optimal pricing of the broadcasting spectrum, and is motivated by the ensuing background description of OTA broadcasting industries in technologically advanced countries.

Due to the public good nature of OTA broadcasts and their educational, cultural and political impacts, and because of scarce bandwidth and high sunk costs, entry and content rules have long been tightly regulated in all major OECD countries (Steiner 1952; Webbink 1973; Spence and Owen 1977). Until the late 1970s the television broadcasting industries in OECD countries comprised only a handful of licensed and highly protected public and commercial firms. Since 1980 alternative transmission techniques, such as satellite and cable, have created a more favorable environment for entry into the television broadcasting industry. Yet opportunities for entry have been mainly taken up at local levels and the market-shares of nation-wide OTA television broadcasters have remained high (Motta and Polo, 1997; Caves, 2006).

Radio broadcasting industries, on the other hand, have been less concentrated and more localized, but entry into these industries has also remained highly regulated due to sunk costs and a tight spectrum constraint. In many European metropolitan areas, for example, OTA radio broadcasts are provided by 20 to 80 FM stations and by similar numbers of AM stations. Metropolitan areas in Italy offer the largest FM variety. With only fifty kHz separation between stations, their FM broadcasting spectrum is the most congested in Europe. The European Conference of Postal and Telecommunications Administrations, a coordination agency, recently re-emphasized the implications of this trade-off between content variety and reception clarity (CEPT, 2010).[2]

The adoption of digital transmission technologies, which are spectrally efficient, has expanded the scope for program variety in the television and radio broadcasting industries (Adda and Ottaviani, 2005). For example, the adoption of digital technology by American OTA broadcasters in 2007 and the subsequent turning off of analogue signals in 2008 have freed significant UHF spectrum. Even so, these spectral gains have not relaxed the spectrum constraint in the US television broadcasting industry, as the spectral dividend from the digital switchover was mainly auctioned off to large telecommunications carriers to accommodate the deployment of 4G mobile-phone networks[3]. Similar diversion of the digital switchover's spectral dividend is expected in other OECD countries. The situation is more complex for radio broadcasting, as only a few countries have successfully adopted and rolled out digital platforms for radio transmissions, and even fewer have clear digital switchover plans for analogue radio broadcasting.

In sum, buffer zones between broadcasters' bands still have to be reduced if new entrants are to be accommodated. As a result, expansion of the OTA broadcasting industry can be expected to intensify the variety-reception trade-off in the pursuit of overall service quality. The variety-reception trade-off is thus likely to be most prominent under a deregulatory scheme that allows free entry and exit, both features we incorporate in our theoretical model. Spence (1976), Spence and Owen (1977), and Mankiw and Whinston (1986) have analyzed market failure arguments against free entry, based on the role of commercial broadcasters' business models. The market price of quality programs (variety) under free entry may fail to induce supply if the value of programs to consumers exceed costs but advertising revenues do not. The public good argument may favor a regulatory approach by which public providers

[2] "The available spectrum…constitutes a limited resource that is used intensively in Europe. In many countries the introduction of new FM services is difficult and may lead to an unacceptable degradation of existing services." (CEPT, 2010, Section 1, P. 5). "The FM spectrum is in many areas overcrowded and may be reaching saturation if the high quality of reception and existing coverages must be retained. This results in FM services increasingly being interference-limited by design or otherwise and these higher interference levels may have to be accepted to allow the introduction of many more additional services" (ibid. P. 5).

[3] Hazlett & Muñoz (2009) advocate increased competition in wireless markets and stress a tension between welfare-maximizing frequency allocations and government rent-extraction policies.

are funded to supply program variety, although the relative value of regulation in this context is disputed by Borenstein (1988).

More recent theoretical contributions take a more positive view of free entry in broadcasting. For instance, Berry and Waldfogel (1999) argue that free entry in the radio broadcasting market is justified as long as consumption benefits compensate welfare (revenue) losses to incumbents stations. Cunningham and Alexander (2004) show that lower industry concentration produces direct (improved programming choice) and indirect (lower advertising prices) welfare gains. If advertising is also modeled as a bad (a nuisance inducing consumers to switch channels), then free entry may have positive welfare effects depending on the substitutability of programs and the relative benefits of programs and advertising (Anderson and Coate 2005).

In view of recent trends in broadcasting spectrum deregulation (De Vany, 1998; Hazlett, 2008) and the variety-reception trade-off, our theoretical analysis examines the possibility of convergence of the number of broadcasters and the royalty per channel to a steady state and the comparative statics of the steady state when entry and exit are motivated by economic profit. We treat the broadcasting spectrum as a state-owned, time-invariant, scarce natural resource. As in the case of any state-owned natural resource, governments are entitled to royalties on the use of spectrum. Therefore, in addition to the direct benefits from the quality of the service provided by the broadcasting industry, there are indirect benefits, namely, the public goods and services financed by the states' royalties on this natural resource. These indirect benefits and the aforesaid variety-reception trade-off form the centerpieces of our theoretical analysis.

We construct an optimal control model—a novel departure from the literature extant—in which the state's time-varying royalties are chosen by an omniscient central planner so as to maximize the sum of the discounted direct and indirect benefits stemming from the use of the broadcasting spectrum. The model accounts for the fact that the number of broadcasters adjusts to the profit from broadcasting at a rate moderated by sunk costs, where profit is assumed to rise with the quality of the industry's service. Importantly, it also accounts for the variety-reception tradeoff, to wit, that on the one hand, entry increases variety, heightens competition and, in turn, raises service quality, while on the other, entry increases spectral congestion and the intensified interferences lower service quality.

We show that a unique steady-state number of broadcasters and royalty per channel exist, that the steady state is a local saddle point, and that the optimal solution converges to the steady state along the stable manifold. In contrast to the observed consolidation and return to concentration in the aftermath of deregulatory reforms in the United States, Italy, Germany and Japan (Noam, 1992; Motta and Polo, 1997; Hazlet, 2005), our analysis reveals that an optimally controlled royalty per channel will lead the OTA broadcasting industry to converge to a steady state with a larger number of broadcasters, higher quality, and a higher royalty. What's more, our analysis of the basic control model shows that the steady state number of broadcasters is less than the number that maximizes the broadcasts' quality at each point in time. Interestingly, this implies that quality rises with the number of broadcasters in a neighborhood of the steady state. That is, in a neighborhood of the steady state, the program variety effect dominates the reception clarity effect when the number of broadcasters increases.

Our formal analysis of the OTA broadcasting industry begins by developing the dynamics of the number of broadcasters in section 2, and also includes the development of the variety-reception trade-off. Section 3 argues that the broadcasting industry has a multifaceted effect on consumers' utility: a quality enhanced positive direct effect of the industry service, a negative indirect effect of the industry service by diverting income away from other goods, and a positive effect of the spectrum royalties paid by the industry on the provision of public goods and services. These considerations and the OTA broadcasting industry's dynamics are then incorporated into the construction of the public planner's optimal control problem. In section 4 the necessary and sufficient conditions are derived to set the stage for the qualitative analysis in the ensuing sections.

The unique steady-state solution of the control problem is derived in section 5 and shown to be a local saddle point. Section 6 presents the comparative statics of the steady-state number of OTA broadcasters, quality, and the royalty payment, and provides an economic interpretation of them. Finally, section 7 contemplates an extension of the basic optimal control model in which consumers' aggregate income is dependent on the quality of the service provided by the broadcasting industry.

2. Industry dynamics

Let $n(t)$ denote the number of broadcasters of OTA transmitted programs (or broadcasts) at time t. At every instant t each broadcaster uses a single channel and delivers a single program. Let the broadcasters be technologically and location-wise identical, paying royalties $r(t)$ to the government for using a channel (or spectrum band) at time t. Also let the width of each band be technologically determined and fixed, say $\omega \in \mathbb{R}_{++}$, and let the bands be evenly spread along a fixed homogeneous spectrum space set aside for the broadcasting industry, namely $\hat{S} \in \mathbb{R}_{++}$. As a result, if the number of broadcasters increases, the bands shift and the buffer zones between them are evenly reduced. For tractability, let us further assume that the consumers of the OTA broadcasting industry's service are located at an identical, physically unobstructed distance from the broadcasters, i.e., along a flat circle with the broadcasters at its centre. Accordingly, all broadcasts are equally receivable by all consumers. We assume that the consumers of the broadcasting industry's service are also users of broadcast time. That is, in addition to watching and/or listening to programs, they advertise their services during the programs. The consideration of consuming-using agents simplifies the analysis which, more generally, could have considered the demands of three types of agents: consumers only, users only and consumer-users.

Broadcasters enter the industry as long as profit per broadcaster $\pi(t)$ is positive, i.e.,

$$\dot{n}(t) = \phi \pi(t), \tag{1}$$

where $\phi \in \mathbb{R}_{++}$ is the time-invariant and parametric speed of adjustment, or ease of entry and exit, into the industry. Sunk costs, that is, the fixed costs associated with facilities, equipment and knowledge that are not transferable to other industries, deter entry and exit. Hence the larger the broadcasters' sunk costs are, the lower the speed of adjustment of the number of broadcasters to profit per broadcaster.

From the perspective of the consumers, the quality of the aerially transmitted programs, say $Q(t)$, rises with content variety and reception clarity. While the variety of programs is increasing in the number of channels $n(t)$, interferences intensify as the number of channels increases because the buffer zones between the channels diminish. Given that the buffer zones are evenly reduced, reception clarity is positively related to the size of the unused spectrum $S(t)$, which by definition is given by

$$S(t) \stackrel{\text{def}}{=} \hat{S} - \omega n(t). \tag{2}$$

Consequently, the quality of the aerially transmitted broadcasts has two opposing effects resulting from an increase in the number of broadcasters. There is a direct, or variety effect, of the number of channels on quality, which is positive but not increasing. On the other hand, there is an indirect, or interference effect, of the number of channels on quality by way of the reduced separation between channels, which is negative. Furthermore, up to a critical number of channels, say $\tilde{n} \in (0, \hat{S}/\omega)$, the positive variety effect is assumed to dominate the negative interference effect.

Taking the above trade-off and assumptions into consideration, a functional form that captures the content variety and reception clarity effects on the quality of the service of the OTA broadcasting industry is $Q(t) = n(t)S(t)$, or using Eq. (2),

$$Q(t) = n(t)[\hat{S} - \omega n(t)]. \tag{3}$$

The logistic specification captures the opposing effects of the number of broadcasters on quality, because $\partial Q/\partial n = \hat{S} - 2\omega n \gtrless 0$ as $n \lessgtr \tilde{n} \stackrel{\text{def}}{=} \hat{S}/2\omega$ and $\partial^2 Q/\partial n^2 = -2\omega < 0$. Thus the marginal effect of the number of broadcasters on the quality of the service of the broadcasting industry is initially positive, but when more than half of the spectrum's carrying capacity is used the negative interference effect dominates the positive variety effect on the quality of the industry's service. Note that $\tilde{n} \stackrel{\text{def}}{=} \hat{S}/2\omega$ is nothing more than the number of broadcasters that maximizes quality at each t.

Naturally, the demand for broadcasts increases with quality. Consequently, the broadcasting industry's total revenue (TR) from advertisements and subscription fees (in the case of pay TV) is a function of quality as well. For tractability, we take this function to be linear. Letting the price of quality be unity (numéraire), it follows that $TR = Q(t)$, and upon using

Eq. (3), the industry's total revenue may be expressed as a function of the number of broadcasters, namely

$$TR = \hat{S}n(t) - \omega[n(t)]^2. \tag{4}$$

Assuming that consumers do not have favorite channels, the industry's total revenue is equally distributed. For simplicity, the instantaneous operational cost of each channel is taken to be time-invariant and positive, say $c \in \mathbb{R}_{++}$, in which case the industry's profit per broadcaster is given by $\pi(t) = [TR/n(t)] - c - r(t)$. Substituting this expression for $\pi(t)$ in Eq. (1) and making use of Eq. (4), it follows that the differential equation governing the change in the number of broadcasters over time is

$$\dot{n}(t) = \phi[\hat{S} - \omega n(t) - c - r(t)]. \tag{5}$$

Equation (5) shows that a once and for all increase in the royalty per channel by an industry initially in a steady state, i.e., when $\dot{n}(t) = 0$, reduces profit below zero and, subsequently, broadcasters leave the industry. In turn, the variety of programs is reduced but the reception of each broadcast is improved, all the while revenue per broadcaster decreases, causing more broadcasters to leave the industry until profit is driven back to zero.

3. Consumers' utility and public planner's optimal control problem

The consumers are taken to have an aggregate income of $Y(t)$, of which $Q(t)$ is spent on access to, and advertisements in, OTA broadcasts. Consequently, the remainder, $Y(t) - Q(t)$, is spent on private goods and services. The royalties paid by the broadcasters at time t are given by $r(t)n(t)$, and are immediately directed to finance public goods and services.

Consumers derive instantaneous utility from the quality of the service provided by the broadcasting industry $Q(t)$, expenditures on private goods and services $Y(t) - Q(t)$, and the investment of the spectrum's royalties in the provision of public goods $r(t)n(t)$. Thus, the instantaneous utility function $U(\cdot)$ can be written as $U(Q(t), Y(t) - Q(t), r(t)n(t))$. In order to simplify the qualitative characterization of the ensuing optimal control problem, and to put the public goods nature of the royalty payments front and centre, it is henceforth assumed that the instantaneous utility function is of the quasi-linear form. That is, it is assumed that $U(\cdot)$ is linear in the consumers' total spending on goods and services, $Q(t)+[Y(t)-Q(t)]$, and strictly increasing and strongly concave in the government's investment of the spectrum's royalties in the provision of public goods and services, $r(t)n(t)$. Hence,

$$U\big(Q(t),Y(t)-Q(t),r(t)n(t)\big)=\big\{Q(t)+\big[Y(t)-Q(t)\big]\big\}+u\big(r(t)n(t)\big)=Y(t)+u\big(r(t)n(t)\big), \quad (6)$$

where $u(\cdot)\in C^{(2)}$, $u'(\cdot)>0$, and $u''(\cdot)<0$ on $\mathbb{R}_{++}$. Although the quality of the service provided by the broadcasting industry no longer appears in the instantaneous utility function, it is taken into account in the optimal control problem via the adjustment of the number of broadcasters to profit per broadcaster.

In the proposed framework, the number of broadcasters is controlled through the state equation (5) by the royalty per channel, the latter of which is set by an omniscient central planner to maximize the consumers' utility over an infinite horizon. The public planner's decision problem is

$$V(\theta)\stackrel{\text{def}}{=}\max_{r(\cdot)}\int_0^{+\infty}\big[Y(t)+u\big(r(t)n(t)\big)\big]e^{-\rho t}\,dt \quad (7)$$

$$\dot{n}(t)=\phi[\hat{S}-\omega n(t)-c-r(t)],\ n(0)=n_0>0,$$

where $\rho>0$ is the planner's time-preference rate, $\theta\stackrel{\text{def}}{=}(\gamma,n_0)\stackrel{\text{def}}{=}(c,\phi,\rho,\hat{S},\omega,n_0)\in\mathbb{R}^6_{++}$, and $V(\cdot)$ is the intertemporal indirect utility function. As $Y(t)$ is exogenous, it does not influence the solution to problem (7), thus the objective functional in Eq. (7) may be replaced by $\int_0^{+\infty}u\big(r(t)n(t)\big)e^{-\rho t}\,dt$ for the purpose of deriving an optimal control.

4. Necessary and sufficient conditions for the optimal control of royalties

Recalling that $\hat{S}$ denotes the fixed amount of spectrum available to the broadcasting industry, it is assumed that $n(t)\in(0,\hat{S}/\omega)$ for all $t\in[0,+\infty)$. That is, we assume that there exist broadcasters of OTA transmitted programs and that they number less than the number that would use the entire spectrum available to the industry. A necessary and sufficient condition for the steady-state number of broadcasters and royalty per channel to be positive is $\hat{S}>c$, as may be verified by inspection of Eqs. (16) and (17) below. Accordingly, $\hat{S}>c$ will be subsequently maintained. It will be shown in section 5 that the steady-state number of broadcasters is less than $\hat{S}/\omega$ without imposing any further assumptions on problem (7). As a result, the state variable constraints $n(t) \ge 0$ and $n(t) \le \hat{S}/\omega$ do not bind along an optimal solution in a neighborhood of the steady state and thus may be safely ignored for our purposes.

The current-value Hamiltonian associated with problem (7) is defined as

$$H(n,r,\lambda;\gamma) \overset{\text{def}}{=} u(rn) + \lambda\phi[\hat{S} - \omega n - c - r] \tag{8}$$

where the costate variable λ is the planner's current value shadow price of the number of broadcasters. By Theorems 14.3 and 14.9 of Caputo (2005), an interior solution to problem (7) must satisfy the following necessary conditions:

$$H_r(n,r,\lambda;\gamma) = u'(rn)n - \phi\lambda = 0, \tag{9}$$

$$\dot{\lambda} = \rho\lambda - H_n(n,r,\lambda;\gamma) = [\rho + \phi\omega]\lambda - u'(rn)r, \tag{10}$$

$$\dot{n} = H_\lambda(n,r,\lambda;\gamma) = \phi[\hat{S} - \omega n - c - r],\ n(0) = n_0 > 0, \tag{11}$$

$$\lim_{t\to+\infty} H(n,r,\lambda;\gamma)e^{-\rho t} = 0. \tag{12}$$

In order to derive a qualitative characterization of a solution to the decision problem postulated in Eq. (7), the necessary conditions in Eqs. (9)–(11) are reduced to a pair of ordinary differential equations in (n,r).

To accomplish the reduction, first differentiate Eq. (9) with respect to t. Then substitute Eqs. (10) and (11) in the resulting differential equation, along with $\lambda = \phi^{-1}u'(rn)n > 0$ from Eq. (9), to arrive at

$$\dot{n} = \phi[\hat{S} - \omega n - c - r],\ n(0) = n_0, \tag{13}$$

$$\dot{r} = \frac{u'(rn)[\rho n + \phi\omega n - \phi r] - \phi[u'(rn) + u''(rn)rn][\hat{S} - \omega n - c - r]}{n^2 u''(rn)}. \tag{14}$$

Equations (13) and (14) form the backbone of the analysis in the ensuing two sections. A solution of problem (7) must satisfy them and the transversality condition given in Eq. (12).

Let $\left(n^*(t;\theta), r^*(t;\theta)\right)$ be a solution of the necessary conditions given in Eqs. (9)–(12), with corresponding current value costate variable $\lambda^*(t;\theta)$. By Theorem 14.4 of Caputo (2005), $\left(n^*(t;\theta), r^*(t;\theta)\right)$ is a solution of problem (7) if $\lim_{t\to+\infty} e^{-\rho t}\lambda^*(t;\theta)[n^*(t;\theta) - n(t)] \le 0$ for every admissible time path $n(t)$ and if $H\left(\cdot,\cdot,\lambda^*(t;\theta);\gamma\right)$ is concave in (n,r) for all $t \in [0,+\infty)$ over an open convex set containing all admissible values of (n,r).

Denote a steady state solution of the necessary conditions by $\left(n^{ss}(\gamma), r^{ss}(\gamma)\right)$, with corresponding current value costate variable $\lambda^{ss}(\gamma)$. Then the transversality condition $\lim_{t\to+\infty} e^{-\rho t}\lambda^*(t;\theta)[n^*(t;\theta)-n(t)]\leq 0$ holds with an equality if $\lim_{t\to+\infty} n(t)$ exists and if $\left(n^*(t;\theta), r^*(t;\theta)\right) \to \left(n^{ss}(\gamma), r^{ss}(\gamma)\right)$ as $t \to +\infty$. In order to see the veracity of this claim, note that (i) $\left(n^*(t;\theta), r^*(t;\theta)\right) \to \left(n^{ss}(\gamma), r^{ss}(\gamma)\right)$ as $t \to +\infty$ implies that $\lambda^*(t;\theta) \to \lambda^{ss}(\gamma)$ as $t \to +\infty$ by way of the fact that $\lambda = \phi^{-1}u'(rn)n > 0$ from Eq. (9), (ii) $\lim_{t\to+\infty} e^{-\rho t} = 0$, and (iii) the fact that the limit of a product of functions equals the product of their individual limits when each function possesses a limit. Moreover, if $\left(n^*(t;\theta), r^*(t;\theta)\right) \to \left(n^{ss}(\gamma), r^{ss}(\gamma)\right)$ as $t \to +\infty$, then the necessary transversality condition given in Eq. (12) also holds, and for basically the same reasons as just indicated.

Let us now turn to the concavity requirement on the current value Hamiltonian $H\left(\cdot,\cdot,\lambda^*(t;\theta);\gamma\right)$. Because an interior solution and $u''(rn) < 0$ have been assumed, it follows that $H_{nn}\left(n,r,\lambda^*(t;\theta);\gamma\right) = u''(rn)r^2 < 0$ and $H_{rr}\left(n,r,\lambda^*(t;\theta);\gamma\right) = u''(rn)n^2 < 0$. Hence, if the determinant of the Hessian matrix of $H\left(\cdot,\cdot,\lambda^*(t;\theta);\gamma\right)$ with respect to (n,r) is positive, then a solution $\left(n^*(t;\theta), r^*(t;\theta)\right)$ of the necessary conditions is the solution to the decision problem (7). As is readily verified, the aforesaid determinant is positive if and only if $-\left[u''(rn)rn/u'(rn)\right] > 1/2$. Accordingly, the so-called Arrow-Pratt degree of relative risk aversion in a stochastic framework being greater than one-half is equivalent to the determinant of the Hessian matrix of $H\left(\cdot,\cdot,\lambda^*(t;\theta);\gamma\right)$ with respect to (n,r) being positive.

Summing up, we have shown that if (i) a solution to the necessary conditions given in Eqs. (9)–(12) exists and converges to a steady state solution of the said necessary conditions, (ii) the Arrow-Pratt coefficient of relative risk aversion is greater than one-half, and (iii) the limit as the time horizon goes to infinity of the number of broadcasters exists, then there exists a unique optimal solution of the posed optimal control problem that converges to the steady state solution of the necessary conditions. Moreover, the unique optimal solution is fully characterized by Eqs. (13) and (14).

In the next section we show that (i) a unique steady state solution of the necessary conditions (13) and (14) exists, (ii) a closed-form expression for $\left(n^{ss}(\gamma), r^{ss}(\gamma)\right)$ can be derived, and (iii) that $\left(n^{ss}(\gamma), r^{ss}(\gamma)\right)$ is a local saddle point.

5. The steady state and local stability

The steady state values $\left(n^{ss}(\gamma), r^{ss}(\gamma)\right)$ are by definition the solution to Eqs. (13) and (14) when $\dot{n}=0$ and $\dot{r}=0$, and are therefore the solution to the system of linear equations

$$\begin{bmatrix} -\omega & -1 \\ \rho+\phi\omega & -\phi \end{bmatrix}\begin{bmatrix} n \\ r \end{bmatrix}=\begin{bmatrix} c-\hat{S} \\ 0 \end{bmatrix}, \tag{15}$$

seeing as $\phi>0$ and $u'(rn)>0$. Recalling that $\hat{S}>c$, the unique solution to this system is

$$n=n^{ss}(\gamma)\stackrel{\text{def}}{=}\frac{\phi[\hat{S}-c]}{\rho+2\phi\omega}>0, \tag{16}$$

$$r=r^{ss}(\gamma)\stackrel{\text{def}}{=}\frac{[\rho+\phi\omega][\hat{S}-c]}{\rho+2\phi\omega}>0. \tag{17}$$

Note that even though we have not specified the functional form of $u(\cdot)$, which is defined over the public goods and services, we have nonetheless been able to derive a unique closed-form solution for the steady state values of the number of broadcasters and the royalty per channel. Furthermore, the implied unique steady state value of quality is by definition given by $Q^{ss}(\gamma)\stackrel{\text{def}}{=}\hat{S}n^{ss}(\gamma)-\omega[n^{ss}(\gamma)]^2$, or

$$Q=Q^{ss}(\gamma)\stackrel{\text{def}}{=}\frac{\phi[\hat{S}-c][\hat{S}\rho+\phi\hat{S}\omega+\phi c\omega]}{[\rho+2\phi\omega]^2}>0, \tag{18}$$

upon using Eqs. (16) and (17).

At this juncture it is worthwhile to pause and examine two features of the steady state solution. First, observe that $n^{ss}(\gamma)<\bar{n}\stackrel{\text{def}}{=}\hat{S}/2\omega$, a fact that may be established using a proof by contradiction. In other words, the consumers' lifetime utility maximizing number of broadcasters in the steady state is less than the number that maximizes the quality of the broadcasting industry at each point in time. Second, using Eq. (3), it then follows that $\partial Q/\partial n=\hat{S}-2\omega n>0$ in a neighborhood of the steady state. That is, the quality of the services provided by the broadcasting industry is an increasing function of the number of broadcasters in a neighborhood of the steady state. This means that in a neighborhood of the steady state, the program variety effect of the number of broadcasters dominates the reception clarity effect.

As shown heretofore, the optimal solution of the control problem converges to the steady state in the limit of the planning horizon. It is thus logical to determine if the steady state can indeed be approached in the said limit, particularly from an industry with an initially small number of broadcasters. Once convergence to the steady state is proven to be possible, the effects of the model parameters on the steady-state solution can be investigated.

The local stability of the steady state solution $(n^{ss}(\boldsymbol{\gamma}), r^{ss}(\boldsymbol{\gamma}))$ is determined by examining the Jacobian matrix of Eqs. (13) and (14) evaluated at $(n^{ss}(\boldsymbol{\gamma}), r^{ss}(\boldsymbol{\gamma}))$, namely,

$$J \stackrel{\text{def}}{=} \begin{bmatrix} \frac{\partial \dot{n}}{\partial n} & \frac{\partial \dot{n}}{\partial r} \\ \frac{\partial \dot{r}}{\partial n} & \frac{\partial \dot{r}}{\partial r} \end{bmatrix}_{\substack{n=n^{ss}(\boldsymbol{\gamma}) \\ r=r^{ss}(\boldsymbol{\gamma})}} = \begin{bmatrix} \underset{(-)}{-\phi\omega} & \underset{(-)}{-\phi} \\ \underset{(?)}{\frac{[\rho+2\phi\omega][u'(rn)+rnu''(rn)]-\phi r^2 u''(rn)}{n^2 u''(rn)}} & \underset{(+)}{\rho+\phi\omega} \end{bmatrix}_{\substack{n=n^{ss}(\boldsymbol{\gamma}) \\ r=r^{ss}(\boldsymbol{\gamma})}} . \quad (19)$$

Using the fact that $\phi r^{ss}(\boldsymbol{\gamma}) \equiv [\rho+\phi\omega]n^{ss}(\boldsymbol{\gamma})$ from Eqs. (16) and (17), it is somewhat tedious but straightforward to show that $|J| = \phi[\rho+2\phi\omega]u'\left(r^*(\boldsymbol{\gamma})n^*(\boldsymbol{\gamma})\right) / [n^*(\boldsymbol{\gamma})]^2 u''\left(r^*(\boldsymbol{\gamma})n^*(\boldsymbol{\gamma})\right) < 0$. Hence, by Theorem 13.6 of Caputo (2005), the steady state is a local saddle point. The local phase-diagram in the (n,r)-plane surrounding the steady state can now be constructed.

By definition, and using Eq. (13), the $\dot{n}=0$ isocline is given by those values of n and r that satisfy the linear equation $r=[\hat{S}-c]-\omega n$. As a result, the $\dot{n}=0$ isocline is a downward sloping straight line in the (n,r)-phase plane with vertical intercept $[\hat{S}-c]>0$ and horizontal intercept $[\hat{S}-c]/\omega>0$.

By the implicit function theorem, the slope of the $\dot{r}=0$ isocline in a neighborhood of the steady state is

$$-\left.\frac{\partial \dot{r}/\partial n}{\partial \dot{r}/\partial r}\right|_{\substack{n=n^{ss}(\boldsymbol{\gamma}) \\ r=r^{ss}(\boldsymbol{\gamma})}} = -\left.\frac{[\rho+2\phi\omega][u'(rn)+rnu''(rn)]-\phi r^2 u''(rn)}{n^2 u''(rn)[\rho+\phi\omega]}\right|_{\substack{n=n^{ss}(\boldsymbol{\gamma}) \\ r=r^{ss}(\boldsymbol{\gamma})}}, \quad (20)$$

which may be positive or negative in general. However, it is readily shown from Eq. (19) and the fact that $|J|<0$, that

$$-\left.\frac{\partial \dot{r}/\partial n}{\partial \dot{r}/\partial r}\right|_{\substack{n=n^{ss}(\boldsymbol{\gamma}) \\ r=r^{ss}(\boldsymbol{\gamma})}} > -\left.\frac{\partial \dot{n}/\partial n}{\partial \dot{n}/\partial r}\right|_{\substack{n=n^{ss}(\boldsymbol{\gamma}) \\ r=r^{ss}(\boldsymbol{\gamma})}} . \quad (21)$$

That is to say, the slope of the $\dot{r}=0$ isocline in a neighborhood of the steady state is greater than the slope of the $\dot{n}=0$ isocline in a neighborhood of the steady state.

The upshot of the two preceding paragraphs is that the local phase diagram can take either of two configurations, as depicted in Figures 1 and 2 below. Note that the vector field in each figure follows from inspection of the signs of the elements of the Jacobian matrix in Eq. (19).

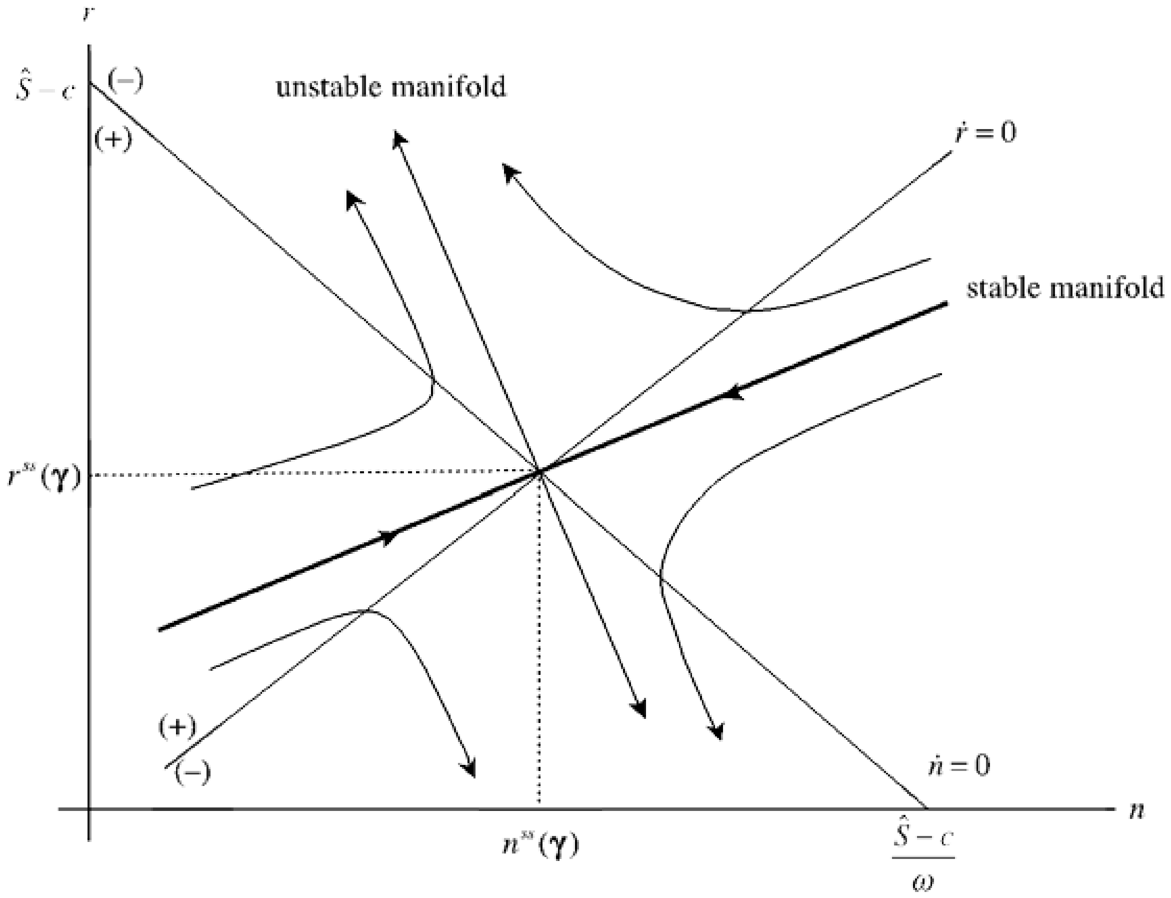

Figure 1

As seen in Figure 1, the only trajectories that converge to the steady state in the limit of the planning horizon are the pair corresponding to the stable manifold, indicated by the thicker pair of lines. Given that the aforementioned sufficient conditions hold, the stable manifold represents the unique solution to problem (7) once an initial condition is specified, as the values of the number of broadcasters and royalty per channel along it satisfy all the necessary conditions. It therefore follows that the optimal time-path of the number of broadcasters and the royalty per channel are either both monotonically increasing, or monotonically decreasing, functions of time. In view of the fact that the optimal time-path of quality is given by $Q^*(t;\theta) = \hat{S}n^*(t;\theta) - \omega[n^*(t;\theta)]^2$, it follows that $\dot{Q}^*(t;\theta) = [\hat{S} - 2\omega n^*(t;\theta)]\dot{n}^*(t;\theta)$. Hence, because $n^{ss}(\gamma) < \bar{n} \stackrel{\text{def}}{=} \hat{S}/2\omega$, quality is either a monotonically increasing or monotonically decreasing function of time and mimics the qualitative behavior over time of the number of broadcasters in a neighborhood of the steady state.

For example, starting from a highly regulated industry with a relatively small number of broadcasters, say $n_0 < n^{ss}(\gamma)$, the left upward sloped arm of the stable manifold is the one that is relevant for the public planner. Along this trajectory the number of broadcasters and quality increase over time despite the fact that the royalty per channel is increasing over time as well. The economic explanation for this seemingly anomalous result is that given the relatively small number of initial broadcasters and quality, the initial and subsequent royalties are relatively low, thereby allowing broadcasters to earn a positive profit. The resulting profit induces entry by other broadcasters and thus raises quality. The optimal royalty per channel set by the public planner in this case permits the broadcasting industry to approach the steady state along this trajectory by setting a relatively low initial royalty, followed by an ever increasing royalty.

The situation is rather different in Figure 2, which also depicts a local saddle point for the steady state. The optimal time-paths of the number of broadcasters, quality, and the royalty per channel are monotonic functions of time, as in Figure 1, but in this case the first two variables move in the opposite direction of the third over time, as the stable manifold is now downward sloping. As before, consider a highly regulated industry with a relatively small number of broadcasters so that $n_0 < n^{ss}(\gamma)$, so that the left downward sloped arm of the stable manifold is the one that is relevant for the public planner. Here, the more natural case results: namely, the optimal policy has the royalty falling over time and the number of broadcasters and quality increasing over time. The economic explanation for this result is that in contrast to the prior case, the small number of initial broadcasters and quality is accompanied by a relatively high initial royalty. In which case, the planner must facilitate the broadcasters' extraction of profits by lowering the royalty over time in order to attract additional broadcasters into the industry and increase quality.

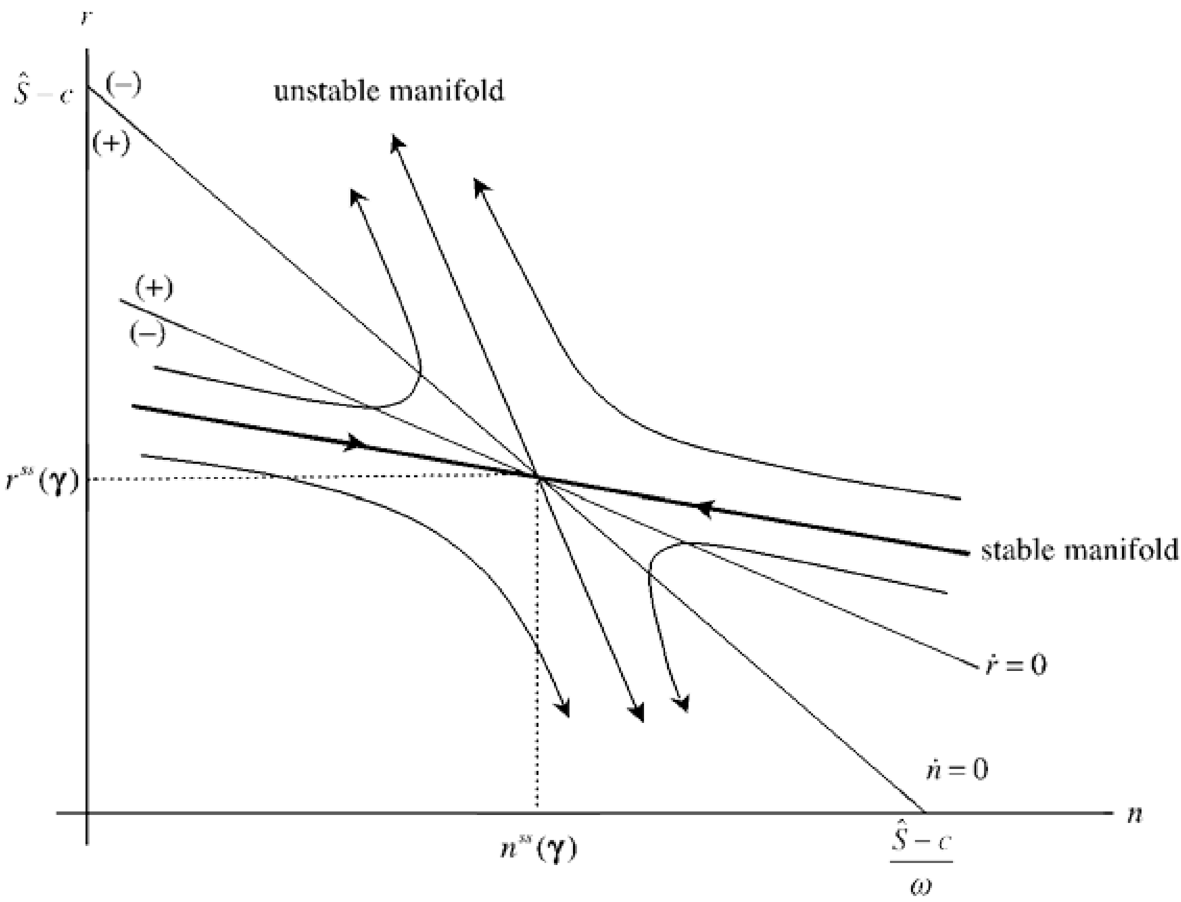

Figure 2

6. Comparative statics of the steady state

The effects of the model parameters on the steady-state number of broadcasters and royalty per channel are obtained by differentiation of Eqs. (16) and (17). The steady state comparative statics for quality follow from differentiation of Eq. (18). Note, however, that because $n^{ss}(\gamma) < \bar{n} \overset{\text{def}}{=} \hat{S}/2\omega$, the steady state comparative statics for quality have the same sign as those for the number of broadcasters and thus need not be computed.

Consider first the effect of an increase in the cost of operating a channel on the steady-state number of broadcasters and royalty per channel, namely,

$$\frac{\partial n^{ss}(\gamma)}{\partial c} = \frac{-\phi}{\rho + 2\phi\omega} < 0, \quad \frac{\partial r^{ss}(\gamma)}{\partial c} = \frac{-[\rho + \phi\omega]}{\rho + 2\phi\omega} < 0. \tag{22}$$

Thus as costs increase, the planner attempts to help the broadcasters by lowering the royalty. Even so, profit declines and some broadcasters leave the industry, resulting in fewer broadcasters in the new steady state along with lower quality. Moreover, royalty revenue $r^{ss}(\boldsymbol{\gamma})n^{ss}(\boldsymbol{\gamma})$ declines and the consumers are worse off, for by Theorem 14.10 of Caputo (2005),

$$\frac{\partial V(\boldsymbol{\theta})}{\partial c} = \int_0^{+\infty} e^{-\rho t} \frac{\partial H}{\partial c}\big(n(t), r(t), \lambda(t); \boldsymbol{\gamma}\big)\Big|_{\substack{\text{optimal} \\ \text{solution}}} dt = -\phi \int_0^{+\infty} e^{-\rho t} \lambda^*(t;\boldsymbol{\theta})\, dt < 0, \tag{23}$$

and $\lambda^*(t;\boldsymbol{\theta}) \equiv \phi^{-1} u'\big(r^*(t;\boldsymbol{\theta}) n^*(t;\boldsymbol{\theta})\big) n^*(t;\boldsymbol{\theta}) > 0$ from Eq. (9).

The effect of an increase in the planner's rate of time preference on the steady-state number of broadcasters and royalty per broadcaster is given by

$$\frac{\partial n^{ss}(\boldsymbol{\gamma})}{\partial \rho} = \frac{-\phi[\hat{S} - c]}{[\rho + 2\phi\omega]^2} < 0,\ \frac{\partial r^{ss}(\boldsymbol{\gamma})}{\partial \rho} = \frac{\phi\omega[\hat{S} - c]}{[\rho + 2\phi\omega]^2} > 0, \tag{24}$$

seeing as $\hat{S} > c$. Therefore as the planner becomes more impatient, the steady state royalty payment increases while the steady state number of broadcasters and quality decrease.

The effect of the speed of entry and exit to profit per broadcaster on the steady-state number of broadcasters and royalty per broadcaster is

$$\frac{\partial n^{ss}(\boldsymbol{\gamma})}{\partial \phi} = \frac{\rho[\hat{S} - c]}{[\rho + 2\phi\omega]^2} > 0,\ \frac{\partial r^{ss}(\boldsymbol{\gamma})}{\partial \phi} = \frac{-\rho\omega[\hat{S} - c]}{[\rho + 2\phi\omega]^2} < 0. \tag{25}$$

Hence, as broadcasters become more responsive to changes in profit per broadcaster, the planner lowers the steady state royalty per channel. This has the effect of increasing profit per broadcaster, which in turn draws more broadcasters into the industry and raises quality. Whether consumers are better or worse off is not clear, however, as

$$\frac{\partial V(\boldsymbol{\theta})}{\partial \phi} = \int_0^{+\infty} e^{-\rho t} \lambda^*(t;\boldsymbol{\theta}) \dot{n}^*(t;\boldsymbol{\theta})\, dt \gtrless 0. \tag{26}$$

If $n_0 < n^{ss}(\boldsymbol{\gamma})$, then it follows from Figures 1 and 2 that $\dot{n}^*(t;\boldsymbol{\theta}) > 0$ and $\dot{Q}^*(t;\boldsymbol{\theta}) > 0$ for all $t \in [0, +\infty)$. Therefore, in this case, consumers are indeed better off as broadcasters become more responsive to changes in profit per broadcaster.

Widening the channels' spectral bands (i.e., increasing ω) decreases the steady-state number of broadcasters (and thus channels) as well as the steady-state royalty per channel, as

$$\frac{\partial n^{ss}(\boldsymbol{\gamma})}{\partial \omega} = \frac{-2\phi^2[\hat{S} - c]}{[\rho + 2\phi\omega]^2} < 0,\ \frac{\partial r^{ss}(\boldsymbol{\gamma})}{\partial \omega} = \frac{-\rho\phi[\hat{S} - c]}{[\rho + 2\phi\omega]^2} < 0. \tag{27}$$

Moreover, royalty revenue $r^{ss}(\boldsymbol{\gamma})n^{ss}(\boldsymbol{\gamma})$ falls and consumers are worse off because

$$\frac{\partial V(\boldsymbol{\theta})}{\partial \omega} = -\phi \int_0^{+\infty} e^{-\rho t} \lambda^*(t;\boldsymbol{\theta}) n^*(t;\boldsymbol{\theta}) dt < 0. \tag{28}$$

Therefore, widening the channels' spectral bands is qualitatively equivalent to an increase in the operating costs per channel.

For the final steady state comparative statics calculation, we examine the effect of an increase in the amount of spectrum available to the broadcasting industry, that is,

$$\frac{\partial n^{ss}(\boldsymbol{\gamma})}{\partial \hat{S}} = \frac{\phi}{\rho + 2\phi\omega} > 0, \quad \frac{\partial r^{ss}(\boldsymbol{\gamma})}{\partial \hat{S}} = \frac{\rho + \phi\omega}{\rho + 2\phi\omega} > 0. \tag{29}$$

In comparing Eq. (22) to Eq. (29), it is seen that the effect of an increase in operating costs per channel on the steady state number of broadcasters, quality, and the royalty per channel, is identical to that of a decrease in the amount of spectrum available to the broadcasting industry. Moreover,

$$\frac{\partial V(\boldsymbol{\theta})}{\partial \hat{S}} = \phi \int_0^{+\infty} e^{-\rho t} \lambda^*(t;\boldsymbol{\theta}) dt > 0. \tag{30}$$

so that consumers are better off if more spectrum is allocated to the broadcasting industry.

7. Extension: consumers' income sensitivity to broadcasts' quality

As the total effect of the quality of the broadcasting industry service on the consumers' aggregate income is not clear, the previous sections ignored the issue. Doing so helped sharpen the focus on the implications of the tradeoff between variety and reception for the allocation of spectrum for broadcasting. In extending the analysis to this issue, we consider the possibility that the quality of the broadcasting industry service has two opposing effects on aggregate income, implying that the sign of $dY\big(Q(t)\big)/dQ$ is not clear *a-priori*. On the one hand, broadcasts disseminate information that enhances knowledge, forms standards of performance and generates transactions. On the other, broadcasts divert consumers' time from work and active investment in human and social capitals. These opposing effects are intensified by the quality of the broadcasts. With $\varphi_1 \in \mathbb{R}_{++}$ indicating the information dissemination effect of broadcasts, $\varphi_2 \in \mathbb{R}_{++}$ the production-effort diversion effect, and $\hat{Y} \in \mathbb{R}_{++}$ the aggregate income attainable when the said effects offset one another, the consumers' aggregate income may be expressed as $Y\big(Q(t)\big) = \hat{Y} + \varphi Q(t)$, where $\varphi \stackrel{\text{def}}{=} \varphi_1 - \varphi_2$.

Because we incorporate the sensitivity of the consumers' aggregate income to broadcasts' quality into the analysis, consumers' aggregate income appears as a determinant of the consumers' income directed to the consumption of private goods. We therefore relax the assumption that the marginal utilities from spending on the quality of the broadcasting industry's service and on private goods are identical, and instead denote them by $\alpha \in \mathbb{R}_{++}$ and $\beta \in \mathbb{R}_{++}$, respectively. Consequently, and in consideration of Eq. (3), the consumers' instantaneous utility function $U(\cdot)$ is now of the form

$$\begin{aligned} U\big(Q(t), Y(t)-Q(t), r(t)n(t)\big) &= \alpha Q(t) + \beta\big[Y\big(Q(t)\big) - Q(t)\big] + u\big(n(t)r(t)\big) \\ &= \beta\hat{Y} + \big[\alpha + \beta[\varphi - 1]\big]\big[\hat{S}n(t) - \omega n(t)^2\big] + u\big(n(t)r(t)\big). \end{aligned} \tag{31}$$

Equation (31) replaces Eq. (6) when the effects of the quality of the broadcasting industry's service on the consumers' aggregate income are contemplated, as in the present case.

Our interest in what follows is in deducing the effects of the preceding extension on the local dynamics and steady state comparative statics of optimal control problem (7). The exposition will be greatly simplified by relying on some basic results from Caputo (1997, 2005 Chapter 18) pertaining to the class of exponentially discounted, autonomous, infinite horizon optimal control problems with one state variable and one control variable, of which problem (7) is a special case. Note that assumptions (A.1)–(A.7) from Caputo (1997, 2005 Chapter 18) hold in the present case.

As was the case when the effects of quality on aggregate income were ignored, the steady state is a local saddle point by Theorem 18.1 of Caputo (2005). Moreover, in a neighbourhood of the steady state, the phase diagram is qualitatively identical to Figure 1. As a result, the economic interpretation of the local dynamics that applied in the basic model applies to the extended model and thus need not be repeated. What differs from the basic model is that the steady state number of broadcasters may be greater than or less than the number that maximizes quality at each point in time, i.e., $n^{ss}(\gamma) \gtrless \tilde{n} \stackrel{\text{def}}{=} \hat{S}/2\omega$. That is, it is not necessarily the case that in the extended model, steady state quality rises with the number of broadcasters, in contrast to the basic model. In what follows, we focus on the steady state comparative statics of the parameters specific to the extended version of the model, to wit (α, β, φ).

By Corollary 18.2.1 of Caputo (2005), the effect of the consumers' aggregate income sensitivity to the quality of broadcasters on the steady-state number of broadcasters and royalty per channel is given by

$$\operatorname{sign}\left[\frac{\partial n^{ss}(\boldsymbol{\gamma})}{\partial \varphi}\right]=\operatorname{sign}\left[\hat{S}-2\omega n^{ss}(\boldsymbol{\gamma})\right],\ \operatorname{sign}\left[\frac{\partial r^{ss}(\boldsymbol{\gamma})}{\partial \varphi}\right]=-\operatorname{sign}\left[\hat{S}-2\omega n^{ss}(\boldsymbol{\gamma})\right]. \tag{32}$$

Thus an increase in the effect of the broadcasting industry's service quality on aggregate income increases the number of broadcasters and decreases the royalty per channel in the new steady state if the old steady state number of broadcasters is less than the number that maximizes quality at each point in time. In this case the positive variety effect on the quality of the industry's service dominates the negative congestion effect. Regardless of which effect dominates, however, consumers are better off, as Theorem 14.10 of Caputo (2005) implies that

$$\frac{\partial V(\boldsymbol{\theta})}{\partial \varphi}=\beta\int_0^{+\infty} e^{-\rho t}\left[\hat{S}n^*(t;\boldsymbol{\theta})-\omega[n^*(t;\boldsymbol{\theta})]^2\right]dt>0, \tag{33}$$

seeing as $Q^*(t;\boldsymbol{\theta})=\hat{S}n^*(t;\boldsymbol{\theta})-\omega[n^*(t;\boldsymbol{\theta})]^2>0$.

An increase in the consumers' direct instantaneous marginal utility from the quality of broadcasts increases the number of broadcasters and decreases the royalty per channel in the new steady state if in the old steady state the variety effect on the quality of the industry's service dominated the congestion effect, seeing as

$$\operatorname{sign}\left[\frac{\partial n^{ss}(\boldsymbol{\gamma})}{\partial \alpha}\right]=\operatorname{sign}\left[\hat{S}-2\omega n^{ss}(\boldsymbol{\gamma})\right],\ \operatorname{sign}\left[\frac{\partial r^{ss}(\boldsymbol{\gamma})}{\partial \alpha}\right]=-\operatorname{sign}\left[\hat{S}-2\omega n^{ss}(\boldsymbol{\gamma})\right] \tag{34}$$

by Corollary 18.2.1 of Caputo (2005). Consumers are unambiguously better off no matter what signs prevail for the steady state comparative statics, because

$$\frac{\partial V(\boldsymbol{\theta})}{\partial \alpha}=\int_0^{+\infty} e^{-\rho t}\left[\hat{S}n^*(t;\boldsymbol{\theta})-\omega[n^*(t;\boldsymbol{\theta})]^2\right]dt>0. \tag{35}$$

Finally, the effect of an increase in the consumers' instantaneous marginal utility from the private goods is given by

$$\operatorname{sign}\left[\frac{\partial n^{ss}(\boldsymbol{\gamma})}{\partial \beta}\right]=\operatorname{sign}\left[[\varphi-1][\hat{S}-2\omega n^{ss}(\boldsymbol{\gamma})]\right],\ \operatorname{sign}\left[\frac{\partial r^{ss}(\boldsymbol{\gamma})}{\partial \beta}\right]=-\operatorname{sign}\left[[\varphi-1][\hat{S}-2\omega n^{ss}(\boldsymbol{\gamma})]\right]. \tag{36}$$

Hence, if in the old steady state the variety effect dominated the congestion effect and the difference between the information-dissemination effect and effort-diversion effect of the broadcasts’ quality on the consumers’ income is larger than the quality price of broadcasts, then an increase in the instantaneous marginal utility of consumption of private goods increases the number of broadcasters and decreases the royalty per channel in the new steady state. Because

$$\frac{\partial V(\boldsymbol{\theta})}{\partial \beta} = \int_0^{+\infty} e^{-\rho t} \left[Y\left(Q^*(t;\boldsymbol{\theta})\right) - Q^*(t;\boldsymbol{\theta}) \right] dt > 0, \tag{37}$$

consumers are unambiguously better off, as in the two previous cases.

8. Conclusion

Spectrum is a state-owned, time-invariant, scarce natural resource. The advent of digital transmission technologies has done little to relieve broadcasting spectrum constraints. The perennial trade-off between content variety and reception clarity continues to prevail and is likely to be most prominent under a deregulatory scheme. As in the case of any state-owned natural resource, governments are entitled to charge royalties on its use and can direct these revenues to finance public goods and services. Therefore, in addition to the direct benefits from the service provided by the broadcasting industry, additional benefits accrue to consumers from the public goods and services financed by the royalties on this natural resource.

In order to set spectrum royalties efficiently, we developed two optimal control models that take into account the variety-reception trade-off and the entry and exit of broadcasters in accordance with the profit per broadcaster. With the aid of phase-plane analysis, we were able to achieve a qualitative characterization of the consumers’ lifetime utility maximizing time-path of the royalty per channel and the number of broadcasters, including their steady state comparative statics. In the basic model we showed that the steady state number of broadcasters is less than the number that maximizes quality at each point in time, implying that in a neighborhood of the steady state, quality rises with the number of broadcasters. Interestingly, and in contrast to the observed consolidation and return to concentration in the aftermath of past deregulatory reforms in OECD countries, our analysis reveals that the optimal control of the broadcasting industry may gradually lead the industry to a steady state with a larger number of broadcasters, higher quality, and higher royalties.

References

Anderson, S. P., and Coate, S. (2005), "Market Provision of Broadcasting: A Welfare Analysis." *Review of Economic Studies*, 72(4), 947–72.

Adda, J., and Ottaviani, M. (2005), "The Transition to Digital Television", *Economic Policy*, 20(41), 160–209.

Berry, S. T., and Waldfogel, J. (1999), "Free Entry and Social Inefficiency in Radio Broadcasting." *RAND Journal of Economics*, 30(3), 397–420.

Borenstein, S. (1988), "On the Efficiency of Competitive Markets for Operating Licenses." *Quarterly Journal of Economics*, 103(2), 357–85.

Caputo, M. R. (1997), "The Qualitative Structure of a Class of Infinite Horizon Optimal Control Problems," *Optimal Control Applications and Methods*, 18(3), 73–86.

Caputo, M. R. (2005), *Foundations of Dynamic Economic Analysis: Optimal Control Theory and Applications*, Cambridge University Press, Cambridge.

Caves, R. E. (2006), *Switching Channels: Organization and Change in TV Broadcasting*, Harvard University Press Cambridge and London.

CEPT (2009), "Light Licensing, Licence-Exempt and Commons." *ECC Report Nr 132, June 2009*, Electronics Communications Committee (ECC) of the European Conference of Postal and Telecommunications Administrations, Moscow.

Cunningham, B. M., and Alexander, P. J. (2004), "A Theory of Broadcast Media Concentration an Commercial Advertising." *Journal of Public Economic Theory*, 6(4), 557-75.

De Vany, A. (1998), "Implementing a Market-based Spectrum policy." *Journal of Law and Economics*, 41(2), 627–46.

Hazlett, T. W. (2005), "Cable Television." Handbook of Telecommunications Economics, S. Majumdar, I. Vogelsanf, and M. Cave, eds., Elsevier, North Holland., Amsterdam.

Hazlett, T. W. (2008), "Optimal Abolition of FCC Spectrum Allocation." *Journal of Economic Perspectives*, 22(1), 103–128.

Hazlett, T. W. and Muñoz, R.T. (2009), “A Welfare Analysis of Spectrum Allocation Policies”, *RAND Journal of Economics*, 40(3), 424–54.

Mankiw, N. G., and Whinston, M. D. (1986), "Free Entry and Social Inefficiency." *RAND Journal of Economics*, 17(1), 48–58.

Motta, M., and Polo, M. (1997), "Concentration and Public Policies in the Broadcasting Industry: The Future of Television." *Economic Policy*, 12(25), 295–334.

Noam, E. M. (1992), *Television in Europe*, Oxford University Press, New York.

Spence, A. M. (1976), "Product Selection, Fixed Costs, and Monopolistic Competition." *Review of Economic Studies*, 43(2), 217–35.

Spence, A. M., and Owen, B. (1977), "Television Programming, Monopolistic Competition, and Welfare." *Quarterly Journal of Economics*, 91(1), 103–26.

Steiner, P. O. (1952), "Program Patterns and Preferences, and the Workability of Competition in Radio Broadcasting " *Quarterly Journal of Economics*, 66(2), 194–223.

Webbink, D. W. (1973), "Regulation, Profits and Entry in the Television Broadcasting Industry." *Journal of Industrial Economics*, 21(2), 167–76.