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Amnon Levy  
*University of Wollongong*

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## **Economics Working Paper Series 2012**

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### **A Stock Targeting International Carbon-Tax Rule with Uncertainty and Diminishing Compliance**

Amnon Levy  
School of Economics  
University of Wollongong  
Email : [levy@uow.edu.au](mailto:levy@uow.edu.au)

*This paper is a revised version of WP 11-09*

WP 12 - 09

*October 2012*

# A Stock Targeting International Carbon-Tax Rule with Uncertainty and Diminishing Compliance

Amnon Levy  
School of Economics  
University of Wollongong  
Wollongong, NSW 2522  
Australia  
Amnon\_levy@uow.edu.au

A rule for setting a tax on carbon emissions to limit their atmospheric stock to a predetermined level is developed for a world inhabited by uncoordinated, myopic, expected utility maximizing agents. In all locations, the mean of the marginal product of the carbon emitting input diminishes and the variance increases as climate deteriorates. The rule is illustrated for a world divided into poor countries and rich countries. The poor countries' costs of non-compliance with the tax, in terms of per capita utility loss from diminished reputation, are negligible. The rich countries' costs of non-compliance and, consequently, inclination to pay the globally set tax can be substantial but not identical. The number of complying rich countries decreases with the tax level, but at a rate that is moderated by the range of the rich countries' loss of per capita utility from abstinence.

*JEL Classification:* Q52, Q54

*Keywords:* Carbon Emissions; Climate Change; Uncertainty; Tax; Compliance

## **1. Introduction**

The atmosphere is an indivisible, open access, essential natural resource. In the absence of property rights, private formation of markets for the externalities created by polluting emissions into this natural resource, climate change in particular, is impossible. The policy measures for controlling polluting emissions are classified as quantity-based instruments and price-based instruments. Theoretical comparisons of these instruments have followed Weitzman's (1974) analysis of stock-based externalities. Within a general stochastic benefit-cost framework, his analysis has led to the proposition that price control is superior (inferior) to quantity control if, and only if, the sum of the curvature of the cost curve and the curvature of the benefit curve is positive (negative) around the optimal output, and that the efficiency gap between the said instruments is intensified by the variance of the slope of the benefit curve. In the context of greenhouse-gas emissions, Pizer (2002), Hoel and Karp (2002), Newell and Pizer, (2003) and Fischer and Newell (2008) have provided arguments in favour of price-based instruments.

A tax on greenhouse-gas emissions has been implemented by a small number of rich nations including Scandinavian and North European countries, Canada, New Zealand and, most recently, Australia. Levy's (2011) conceptual analysis of the effects of abstaining rich and poor countries' free-riding, overstated expectations and guilt on the efficiency of carbon tax implemented by a group of willing rich countries has led to the proposition that a one-sided implementation of a tax on greenhouse-gas emissions does not necessarily reduce global emissions. Nor does it necessarily reduce the tax-paying group's emissions when the members' utilities are directly affected by the engendered environmental damage. In view of this proposition and the indivisibility of the atmosphere, and consistent with the catastrophe-avoidance argument for capping the concentration of greenhouse gases at 450 particles per million (in order to limit global warming to 2° Celsius), this paper outlines a tax-based procedure for constraining the stock of greenhouse gases in a world inhabited by stakeholders facing different economic conditions with uncertainty intensified by climate change. To simplify the analysis the paper ignores the differences between the various greenhouse gases and refers to them as carbons. The analysis incorporates a global atmospheric stock target into the determination of a flat carbon tax for effectively moderating climate change. The incorporation takes into account that

countries cannot be forced to comply. The stakeholders are classified as poor, hence unable and/or unwilling to pay, countries and as rich countries with different costs of non-compliance.

The outlining of the tax-based procedure for constraining the atmospheric carbon stock is based on five premises. The first one is that the controllable carbon emissions are a by-product of the production process of goods and services. The second premise is the existence of a climate-wise ideal concentration of carbons in the atmosphere. The larger the deviation from that ideal concentration is, the less favourable the climate and, in turn, the smaller the output mean and the larger the output variance. The third premise is that stakeholders are aware of these adverse effects of the deviations from the climate-wise ideal stock on their output and are risk averse and rational, but uncoordinated and myopic. Their carbon emissions are determined by a maximization of their individual expected periodical utility from their random net periodical revenues. The fourth premise is that the international planner takes into account the stakeholders' expected periodical utility maximizing carbon emissions and the periodical changes in the natural environment's absorptive capacity of carbons. In analogy to the common practice of central banks with regard to interest rate, the international planner sets the carbon tax at the beginning of each period at a level required for achieving an atmospheric carbon stock target. The fifth premise is that the stakeholders are sovereign countries in various stages of economic development. They have different ability and inclination to pay for carbon emissions. They cannot be forced to pay the carbon tax set by the international planner. Hence, the higher the tax level is the smaller the number of complying countries. Yet, potential loss of international reputation encourages compliance.

Taking into account the first, second and third premises, section 2 derives the effect of a carbon tax on the emissions of the countries' representative agents. With the fourth and fifth premises in mind, section 3 and 4 incorporate the agents' expected utility maximizing carbon emissions into the formulation of the relationship between compliance and the carbon tax level and, subsequently, into the motion equation of the atmospheric carbon stock and the formulation of the periodical carbon tax rate required for limiting the atmospheric stock to a predetermined level with an allowance for a negative relationship between the number of complying countries and the tax level. Section 4 provides derives the stock-targeting carbon tax for a special case where countries are classified into two groups: poor countries with negligible loss of

per capita utility from non-compliance and rich countries with different levels of loss of per capita utility from non compliance. Section 5 concludes.

## 2. Climate driven uncertainty and expected utility maximizing emissions

Consider a world where the carbon emission of the representative agent (per capita carbon emission) of each country  $i = 1, 2, 3, \dots, N$  at period  $t$ ,  $e_{it}$ , is proportional to his use of energy,  $c_{it}$ . That is,

$$e_{it} = \alpha_i c_{it} \quad (1)$$

where the positive scalar  $\alpha_i$  reflects the emission-intensity of country  $i$ 's representative agent's energy consumption. The larger the proportion of "dirty" sources of energy (e.g., brown coal) and the less advanced the production technology are, the greater  $\alpha_i$ . The goods and services produced by the representative agent's energy consumption at period  $t$  is represented by a composite output,  $y_{it}$ . The composite output is, for tractability, proportional to the energy consumption, but uncertain:

$$y_{it} = (a_{it} + \varepsilon_{it})c_{it}. \quad (2)$$

The scalar  $a_{it}$  is positive and represents the expected marginal product of energy in country  $i$ . It is assumed to diminish with the deterioration of the climate engendered by the divergence of the current stock of carbons in the atmosphere from the climate-wise ideal level. The additional factor,  $\varepsilon_{it}$ , is a zero-mean normally distributed random deviation from this expected marginal product caused by random disturbances in the representative agent's production environment. That is,  $\varepsilon_{it} \sim \mathcal{N}(0, \sigma_{it}^2)$ . It is assumed that  $\sigma_{it}^2$  is finite, but increases with the deterioration of the climate caused by an increasing deviation from the ideal atmospheric carbon stock.

As countries do not have identical location and topography (and, consequently, exposure to the sun and oceans), the ideal atmospheric carbon stock is country-specific. Consistent with the said assumptions, country  $i$ 's ideal atmospheric carbon stock is defined as the stock level associated with both the smallest marginal product's variance,  $\sigma_i^2$ , and the largest expected marginal product,  $a_i$ , of energy for its representative agent. With  $S_{t-1}$  denoting the atmospheric carbon stock at the

beginning of period  $t$ ,  $S_i^o$  the ideal stock, and  $\beta_i > 0$  the sensitivity of country  $i$ 's marginal product's variance to climate change, the variance of country  $i$ 's representative agent's marginal product at  $t$  is expressed as:

$$\sigma_{it}^2 = \sigma_i^2 [1 + \beta_i (S_{t-1} - S_i^o)^2]. \quad (3)$$

The representative agent's expected marginal product of energy decreases from the maximum level,  $a_i$ , with the deviation of the actual atmospheric carbon stock from the ideal stock. This assumption is formally represented by

$$a_{it} = a_i / [1 + \gamma_i (S_{t-1} - S_i^o)^2] \quad (4)$$

where  $\gamma_i > 0$  reflects the sensitivity of country  $i$ 's representative agent's expected marginal product to climate change.

As  $e_{it} = \alpha_i c_{it}$  implies that  $c_{it} = e_{it} / \alpha_i$ , country  $i$ 's representative agent's production function can be portrayed as:

$$y_{it} = \left[ \frac{a_i}{\alpha_i [1 + \gamma_i (S_{t-1} - S_i^o)^2]} + (\varepsilon_{it} / \alpha_i) \right] e_{it}. \quad (5)$$

With the price of energy for country  $i$ 's representative agent at  $t$  being equal to  $q_{it}$ , the imputed price of his carbon emissions before tax is  $q_{it} / \alpha_i$ . With the price of his composite output at  $t$  being equal to  $p_{it}$  and with his carbon tax rate being equal to  $\tau_{it} \geq 0$ , the net revenue of country  $i$ 's representative agent at  $t$  is:

$$\pi_{it} = p_{it} \left[ \frac{a_i}{\alpha_i [1 + \gamma_i (S_{t-1} - S_i^o)^2]} + (\varepsilon_{it} / \alpha_i) \right] e_{it} - [(q_{it} / \alpha_i) + \tau_{it}] e_{it}. \quad (6)$$

This net revenue is normally distributed with

$$E(\pi_{it}) = \left[ \frac{p_{it} a_i}{\alpha_i [1 + \gamma_i (S_{t-1} - S_i^o)^2]} - (q_{it} / \alpha_i) - \tau_{it} \right] e_{it} \quad (7)$$

and

$$\text{VAR}(\pi_{it}) = \{ p_{it}^2 \sigma_i^2 [1 + \beta_i (S_{t-1} - S_i^o)^2] / \alpha_i^2 \} e_{it}^2. \quad (8)$$

Country  $i$ 's representative agent derives utility from his net revenue. For tractability, his utility function is taken to be negative exponential,  $u_{it} = 1 - \exp(-R_i \pi_{it})$ . In which case his expected utility can be expressed as

$$Eu_{it} = 1 - m(\pi_{it} | -R_i) \quad (9)$$

where  $m$  is the moment generating function of  $\pi_{it}$  and  $R_i > 0$  the representative agent's degree of absolute risk aversion.<sup>1</sup> It is postulated that each country's representative agent is myopically rational – at the beginning of every period  $t$  he chooses a carbon emission level that maximizes his current expected utility *per se*. With a normally distributed net revenue, his current expected utility function can be further expressed as

$$E(u_{it}) = 1 - e^{-0.5R_i V_{it}} \quad (10)$$

where  $V_{it} = E(\pi_{it}) - 0.5R_i \text{VAR}(\pi_{it})$ .

As  $-R_i \{p_{it}^2 \sigma_i^2 [1 + \beta_i (S_{t-1} - S_i^o)^2] / \alpha_i^2\} < 0$ , the second-order condition for maximum expected utility is satisfied. In consideration of (7) and (8), the expected utility maximizing emission level,  $e_{it}^*$ , for a given carbon tax rate,  $\tau_{it}$ , should satisfy the following condition:

$$\frac{p_{it} a_i}{\alpha_i [1 + \gamma_i (S_{t-1} - S_i^o)^2]} - (q_{it} / \alpha_i) - \tau_{it} - R_i \{p_{it}^2 \sigma_i^2 [1 + \beta_i (S_{t-1} - S_i^o)^2] / \alpha_i^2\} e_{it}^* = 0. \quad (11)$$

Consequently, the expected utility maximizing carbon emission level in period  $t$  for the representative agent of country  $i$  is equal to the ratio of his climate-change eroded expected marginal net revenue to his climate-change intensified marginal cost of risk-bearing:

$$e_{it}^* = \frac{\frac{p_{it} a_i}{\alpha_i [1 + \gamma_i (S_{t-1} - S_i^o)^2]} - (q_{it} / \alpha_i) - \tau_{it}}{R_i \{p_{it}^2 \sigma_i^2 [1 + \beta_i (S_{t-1} - S_i^o)^2] / \alpha_i^2\}}. \quad (12)$$

### 3. Compliance, abstinence and stock-targeting carbon tax

In analogy to the practice of central banks with regard to controlling inflation by periodical interest-rate adjustments, the proposed control scheme is based on a flat uniform tax on carbon emissions,  $\hat{\tau}_t$ , set at the beginning of each period by an international planner at a rate required for achieving an atmospheric carbon stock target,  $\hat{S}_t$ . The setting of the tax takes into account that compliance is not universal. Non-compliance generates different levels of loss of international reputation for

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<sup>1</sup> See Freund (1956) for original suggestion and Hammond (1974) for the generality of this simplification.



countries. The consideration of potential loss of international reputation provides a rationale for the practicality of an internationally set tax on carbon emissions.

It is beneficial for country  $i$  to comply as long as the carbon tax is lower than a threshold level,  $\tilde{\tau}_{it}$ . At this threshold level, country  $i$ 's representative agent's maximum expected utility under compliance is equal to his maximum expected utility under abstinence ( $\tau_{it} = 0$ ). That is,

$$Eu_i(\pi_i(e_{it}^*(\tilde{\tau}_{it}), \tilde{\tau}_{it})) = Eu_i(\pi_i(e_{it}^*(\tau_{it} = 0), \tau_{it} = 0)) - \psi_{it} \quad (13)$$

where  $\psi_{it} \geq 0$  is the current loss of per capita utility for country  $i$  from diminished international reputation. Recalling (10) and using the envelope theorem,<sup>2</sup> the representative agent's maximum expected utility decreases with the carbon tax rate:

$$\frac{\partial Eu_i(\pi_i(e_{it}^*(\tau_{it}), \tau_{it}))}{\partial \tau_{it}} = -0.5R_i e^{-0.5R_i V_i(e_{it}^*(\tau_{it}), \tau_{it})} e_{it}^* < 0. \quad (14)$$

In conjunction with equality (13),  $\partial Eu_i(\pi_i(e_{it}^*(\tau_{it}), \tau_{it})) / \partial \tau_{it} < 0$  implies that  $d\tilde{\tau}_{it} / d\psi_{it} > 0$  and  $\lim_{\psi_{it} \rightarrow 0} \tilde{\tau}_{it} = 0$ .

The global planner is assumed to have perfect information about each country's technology, vulnerability to climate change, degree of risk aversion, and potential loss from non compliance and about the natural depletion rate of atmospheric carbon stock. In setting the periodical carbon tax he takes into account the countries' decisions on compliance and, subsequently, emissions. In conjunction with (12), his chosen  $\hat{\tau}_t$  satisfies:

$$\hat{S}_t = \sum_{i_c=1}^N L_{i_c} e_{i_c,t}^*(\tau_{i_c,t} = \hat{\tau}_t) + \sum_{i_{nc} \neq i_c}^N L_{i_{nc}} e_{i_{nc},t}^*(\tau_{i_{nc},t} = 0) + (1 - \delta_t) S_{t-1}. \quad (15)$$

In this equality, the countries indicated by  $i_c$  pay the globally set carbon tax as their threshold levels are larger than the carbon tax set by the global planner (i.e.,  $\tilde{\tau}_{i_c,t} > \hat{\tau}_t$ ). The countries indicated by  $i_{nc}$  do not comply as the carbon tax set by the global planner exceeds their threshold levels (i.e.,  $\tilde{\tau}_{i_{nc},t} < \hat{\tau}_t$ ).  $L_{i_c}$  and  $L_{i_{nc}}$  denote the populations of the complying countries and non-complying countries, respectively.

<sup>2</sup> By differentiating (10) at the agent's optimal emission level and considering the necessary condition, 
$$\frac{\partial Eu_i(\pi_i(e_{it}^*(\tau_{it}), \tau_{it}))}{\partial \tau_{it}} = 0.5R_i e^{-0.5R_i V_i(e_{it}^*(\tau_{it}), \tau_{it})} \frac{\partial V_i(e_{it}^*(\tau_{it}), \tau_{it})}{\partial \tau_{it}} = 0.5R_i e^{-0.5R_i V_i(e_{it}^*(\tau_{it}), \tau_{it})} \left[ -e_{it}^* + \underbrace{\frac{\partial V_i(e_{it}^*(\tau_{it}), \tau_{it})}{\partial e_{it}}}_{0} \frac{\partial e_{it}^*}{\partial \tau_{it}} \right].$$

The parameter  $0 < \delta_t < 1$  denotes the natural depletion rate of atmospheric carbons through sinking into the Earth's surface, dissemination into space and sequestration by plants. It varies with changes in the absorptive capacity of the oceans and plants, in particular. The following section provides an illustration and a close-form solution for the stock-targeting carbon tax in a world divided into a group of identical poor countries with negligible potential loss of expected utility from non-compliance and a group of technology, location and population wise identical rich countries but with varying potential loss of expected utility from non-compliance due to cultural and/or ideological differences.

#### 4. Illustration: Non-complying poor countries and some complying rich countries

Consider a world with  $N_r$  rich (r) countries and  $N_p = N - N_r$  poor (p) countries. The poor countries have the same technology, climate, prices of energy and composite output, and population size. The poor countries' loss of per capita utility from abstinence from the internationally coordinated emission-control scheme is negligible ( $\psi_p \rightarrow 0$ ) due to their low level of per capita emissions and strong expectations of being excused or forgiven. The rich countries are also identical with regard to technology, climate, prices of energy and composite output, and population size. However, the rich countries do not have identical loss of per capita utility from reputation tarnished by abstinence. Due to cultural and/or ideological differences,<sup>3</sup> their levels of potential per capita utility loss form a continuum along a range  $(0, \psi_{\max})$ .

In view of the properties that  $d\tilde{\tau}_{it} / d\psi_{it} > 0$  and  $\lim_{\psi_{it} \rightarrow 0} \tilde{\tau}_{it} = 0$ , the larger the loss of utility range  $(0, \psi_{\max})$  is, the smaller the adverse effect of a carbon-tax increment on the number of complying rich countries in period  $t$  ( $N_{\text{rct}}$ ). Furthermore,

$\lim_{\psi_{\max} \rightarrow \infty} N_{\text{rc}} = N_r$  and  $\lim_{\psi_{\max} \rightarrow 0} N_{\text{rc}} = 0$ . These properties are reflected by the following

explicit form:

$$N_{\text{rct}} = \frac{N_r}{1 + (\theta / \psi_{\max}) \tau_t} \quad (16)$$

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<sup>3</sup> For example, see Berck and Helfand (2011) for a discussion on the effect of culture and ideology on the willingness of North Europeans, Scandinavians and Americans to pay emission tax.

where  $\theta$  is a positive scalar – the rich countries’ non-compliance coefficient. It is further assumed that the non-complying rich countries do not implement domestic emission-control schemes. With this relationship between compliance and carbon tax in mind, the international planner sets the periodical carbon-tax level so as to limit the stock of carbon in the atmosphere at the end of period  $t$  to a targeted level.

To achieve a targeted atmospheric carbon stock,  $\hat{S}_t$ , the periodical carbon-tax level should satisfy the following equality:

$$\hat{S}_t = N_{\text{rc}} L_r e_{\text{rc}}^* + [N_r - N_{\text{rc}}] L_r e_{\text{rat}}^* - (N - N_r) L_p e_{\text{pt}}^* + (1 - \delta_t) S_{t-1}. \quad (17)$$

Recalling (12), the carbon emissions of the representative agent of a rich cooperative (rc) country are

$$e_{\text{rc}}^* = \frac{\frac{p_{\text{r}} a_{\text{r}}}{\alpha_{\text{r}} [1 + \gamma_{\text{r}} (S_{t-1} - S_{\text{r}}^{\circ})^2]} - (q_{\text{r}} / \alpha_{\text{r}}) - \hat{\tau}_t}{R_{\text{r}} \{p_{\text{r}}^2 \sigma_{\text{r}}^2 [1 + \beta_{\text{r}} (S_{t-1} - S_{\text{r}}^{\circ})^2] / \alpha_{\text{r}}^2\}} \quad (18)$$

the carbon emissions of the representative agent of a rich abstaining (ra) country are

$$e_{\text{rat}}^* = \frac{\frac{p_{\text{r}} a_{\text{r}}}{\alpha_{\text{r}} [1 + \gamma_{\text{r}} (S_{t-1} - S_{\text{r}}^{\circ})^2]} - (q_{\text{r}} / \alpha_{\text{r}})}{R_{\text{r}} \{p_{\text{r}}^2 \sigma_{\text{r}}^2 [1 + \beta_{\text{r}} (S_{t-1} - S_{\text{r}}^{\circ})^2] / \alpha_{\text{r}}^2\}} \quad (19)$$

and the carbon emissions of the representative agent of a poor country are

$$e_{\text{pt}}^* = \frac{\frac{p_{\text{p}} a_{\text{p}}}{\alpha_{\text{p}} [1 + \gamma_{\text{p}} (S_{t-1} - S_{\text{p}}^{\circ})^2]} - (q_{\text{p}} / \alpha_{\text{p}})}{R_{\text{p}} \{p_{\text{p}}^2 \sigma_{\text{p}}^2 [1 + \beta_{\text{p}} (S_{t-1} - S_{\text{p}}^{\circ})^2] / \alpha_{\text{p}}^2\}}. \quad (20)$$

By substituting (16), (18), (19) and (20) into (17), the stock-targeting carbon-tax level should satisfy:

$$\begin{aligned} \hat{S}_t = & \frac{N_r}{1 + (\theta / \psi_{\text{max}}) \hat{\tau}_t} L_r \frac{\frac{p_{\text{r}} a_{\text{r}}}{\alpha_{\text{r}} [1 + \gamma_{\text{r}} (S_{t-1} - S_{\text{r}}^{\circ})^2]} - (q_{\text{r}} / \alpha_{\text{r}}) - \hat{\tau}_t}{R_{\text{r}} \{p_{\text{r}}^2 \sigma_{\text{r}}^2 [1 + \beta_{\text{r}} (S_{t-1} - S_{\text{r}}^{\circ})^2] / \alpha_{\text{r}}^2\}} \\ & + \left[ N_r - \frac{N_r}{1 + (\theta / \psi_{\text{max}}) \hat{\tau}_t} \right] L_r \frac{\frac{p_{\text{r}} a_{\text{r}}}{\alpha_{\text{r}} [1 + \gamma_{\text{r}} (S_{t-1} - S_{\text{r}}^{\circ})^2]} - (q_{\text{r}} / \alpha_{\text{r}})}{R_{\text{r}} \{p_{\text{r}}^2 \sigma_{\text{r}}^2 [1 + \beta_{\text{r}} (S_{t-1} - S_{\text{r}}^{\circ})^2] / \alpha_{\text{r}}^2\}} \\ & + (N - N_r) L_p \frac{\frac{p_{\text{p}} a_{\text{p}}}{\alpha_{\text{p}} [1 + \gamma_{\text{p}} (S_{t-1} - S_{\text{p}}^{\circ})^2]} - (q_{\text{p}} / \alpha_{\text{p}})}{R_{\text{p}} \{p_{\text{p}}^2 \sigma_{\text{p}}^2 [1 + \beta_{\text{p}} (S_{t-1} - S_{\text{p}}^{\circ})^2] / \alpha_{\text{p}}^2\}} + (1 - \delta_t) S_{t-1}. \end{aligned} \quad (21)$$

By multiplying both sides of this equality by  $[1 + (\theta / \psi_{\max}) \hat{\tau}_t]$  and rearranging terms, the international planner's rule for setting the atmospheric stock targeting carbon tax is:

$$\hat{\tau}_t = \frac{\left[ \frac{N_r L_r \alpha_r \left( \frac{p_{rt} a_r}{[1 + \gamma_r (S_{t-1} - S_r^0)^2]} - q_{rt} \right)}{R_r p_{rt}^2 \sigma_r^2 [1 + \beta_r (S_{t-1} - S_r^0)^2]} \right] + \left[ \frac{(N - N_r) L_p \alpha_p \left( \frac{p_{pt} a_p}{[1 + \gamma_p (S_{t-1} - S_p^0)^2]} - q_{pt} \right)}{R_p p_{pt}^2 \sigma_p^2 [1 + \beta_p (S_{t-1} - S_p^0)^2]} \right] - \hat{S}_t + (1 - \delta_t) S_{t-1}}{\frac{\theta}{\psi_{\max}} [\hat{S}_t - (1 - \delta_t) S_{t-1}] + \left[ \frac{N_r L_r \alpha_r \left[ \alpha_r - \left( \frac{\theta}{\psi_{\max}} \right) \left( \frac{p_{rt} a_r}{[1 + \gamma_r (S_{t-1} - S_r^0)^2]} - q_{rt} \right) \right]}{R_r p_{rt}^2 \sigma_r^2 [1 + \beta_r (S_{t-1} - S_r^0)^2]} \right] - \left[ \frac{(N - N_r) L_p \frac{\theta}{\psi_{\max}} \alpha_p \left( \frac{p_{pt} a_p}{[1 + \gamma_p (S_{t-1} - S_p^0)^2]} - q_{pt} \right)}{R_p p_{pt}^2 \sigma_p^2 [1 + \beta_p (S_{t-1} - S_p^0)^2]} \right]} \quad (22)$$

An inspection of equation (22) reveals that the derived carbon tax decreases with the stock target,  $\hat{S}_t$ , with the stock's natural depletion rate,  $\delta_t$ , and with the prices of energy in the rich countries and in the poor countries,  $q_{rt}$  and  $q_{pt}$ . The derived carbon tax rises with the expected marginal product of energy in the rich and the poor countries attainable under ideal climate,  $a_r$  and  $a_p$ , with the emission-intensities of the poor and rich countries' energy consumption,  $\alpha_p$  and  $\alpha_r$ , and with the abstaining poor countries' population,  $(N - N_r)L_p$ . Despite the tax-increasing effect of the energy consumption's emission-intensities, the carbon tax does not necessarily rise with the rich countries' population,  $N_r L_r$ . It can decrease with the population of the rich countries when the value of the rich countries' non-compliance coefficient ( $\theta$ ) is sufficiently low and as their range of per capita utility loss from abstinence ( $\psi_{\max}$ ) is sufficiently large, as the burden is shared by a sufficiently large number of complying rich countries.

The inspection of equation (22) also reveals that the derived carbon tax decreases with the poor countries' representative agents' risk-bearing-cost coefficient,  $R_p p_{pt}^2 \sigma_p^2 [1 + \beta_p (S_{t-1} - S_p^0)^2]$ . More specifically, the carbon tax decreases with the poor countries' degree of absolute risk aversion,  $R_p$ , with the variance of their marginal output under ideal climate,  $\sigma_p^2$ , and with the sensitivity of their marginal product's variance to climate change,  $\beta_p$ . As only some of the rich countries comply, the rich countries' representative agents' risk-bearing-cost coefficient,

$R_r p_r^2 \sigma_r^2 [1 + \beta_r (S_{t-1} - S_r^o)^2]$ , does not necessarily reduce the carbon tax set by the international planner. It is only clear that the stock-targeting tax decreases with  $R_r p_r^2 \sigma_r^2 [1 + \beta_r (S_{t-1} - S_r^o)^2]$  when  $\alpha_r - \frac{\theta}{\Psi_{\max}} \left( \frac{p_r a_r}{[1 + \gamma_r (S_{t-1} - S_r^o)^2]} - q_r \right) < 0$ . This property is supported by a large ratio of the non-compliance coefficient ( $\theta$ ) to the range of the potential per capita utility loss from non-compliance ( $\Psi_{\max}$ ) in the group of rich countries, by a low emission-intensity of energy consumption in rich countries ( $\alpha_r$ ), by a large maximum marginal expected marginal product of energy ( $a_r$ ) for rich countries, by a low sensitivity of rich countries' expected marginal product to climate change ( $\gamma_r$ ), by a small deviation from the climate-wise ideal atmospheric stock of carbons for rich countries ( $(S_{t-1} - S_r^o)^2$ ), by a high price for rich countries' composite output ( $p_r$ ), and by a low energy price in rich countries ( $q_r$ ).

The inspection further suggests that the international planner's carbon tax rises with both the rich and poor countries' representative agents' expected marginal profit coefficients  $\{p_r a_r / [(1 + \gamma_r (S_{t-1} - S_r^o)^2)] - q_r\}$  and  $\{p_p a_p / [1 + \gamma_p (S_{t-1} - S_p^o)^2] - q_p\}$ , respectively, and in a rate intensified by the rich countries' non-compliance coefficient ( $\theta$ ) and moderated by the rich countries' range of per capita utility loss from abstinence ( $\Psi_{\max}$ ). This property indicates that the international planner's periodical carbon tax decreases with the rich and poor countries' expected marginal product' sensitivity to climate change,  $\gamma_r$  and  $\gamma_p$ . In conjunction with the previously identified moderating effect of the poor countries' risk-bearing-cost coefficient, this property also indicates that the carbon tax decreases with the deviation of the atmospheric carbon stock from the climate-wise ideal level for the poor countries,  $(S_{t-1} - S_p^o)^2$ . The direction of the overall effect of the deviation of the atmospheric carbon stock from the climate-wise ideal level for the rich countries,  $(S_{t-1} - S_r^o)^2$ , on the international planner's periodical carbon tax is not clear.

Finally, as long as the carbon stock target is set below the actual initial stock minus its natural depletion during the period (i.e.,  $\hat{S}_t < (1 - \delta_t) S_{t-1}$ ), the international planner's periodical carbon tax rises with the rich countries' non-compliance coefficient ( $\theta$ ); whereas for  $\hat{S}_t \gg (1 - \delta_t) S_{t-1}$ , the carbon tax can decrease with  $\theta$ .

These effects are moderated by the rich countries' range of utility loss from abstinence.

## **5. Conclusion**

This paper develops a rule for limiting the atmospheric carbon stock to a predetermined level in a world where climate change is driven by variation in this stock and where the deviations from the ideal stock diminish the mean and increase the variance of the marginal product of the carbon emissions' source input. The derivation took into account that agents are aware of these adverse effects of the deviations from the climate-wise ideal stock on their output and maximize expected utility from the uncertain net revenues, that countries pay an internationally set carbon tax as long as their representative agents' expected utility under compliance is at least as large as that under abstinence. Some countries are poor, hence unable and/or unwilling to pay carbon tax. Due to cultural and ideological differences the loss of utility from abstinence varies across rich countries and the number of the complying rich countries diminishes with the tax rate. For application, the stock-capping carbon tax rate can be numerically solved for the real case, where all the countries are different, with non-identical parameters. For illustration and qualitative analysis, the carbon tax rate was derived for the tractable case where the poor countries are identical, and so are also the rich countries, with the aforesaid exception of non-identical culture and/or ideology.

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