System estimation of generalized working models: a semiparametric approach

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Abstract

System estimation of a complete model of \( m \) generalized working equations of the Phillips/Tukey form with unknown exponent function is often unsatisfactory due to computational instability encountered in practical studies. A semiparametric economic-theoretical procedure is proposed. Some illustrations of the new procedure are also given.
1. INTRODUCTION

A complete model of $m$ generalized working (GW) equations was proposed by Laitnen, Theil and Raparla in 1983 to improve the functional flexibility in modelling consumer behaviour. The integrability properties of the GW model, in relation to an indirect to an indirect utility function with mean of order $\theta$, were demonstrated by Tran Van Hoa (1983). In its explicit form, each GW equation is intrinsically nonlinear as Philips (1958) and Tukey (1957) proposed, and in theory, each GW equation can be estimated to give meaningful results after Jacobian transformation by means of the statistical procedure of the Box-Cox (1964) data analysis. In an earlier application of the GW model (see Tran Van Hoa, 1985), the nonlinearity of the GW equation has been shown to represent the effects of income distribution on consumer’s behaviour and can be estimated by a system method. In empirical studies, this estimation method is often unsatisfactory due to frequent computational instability. However, if the nonlinearity represents the effects of income distribution, it is related conceptually and functionally to an index of economic inequality. The true index of economic inequality and the exact form of this function are however unknown.

Existing estimation methods can be used to deal with these kinds of equations in a system framework include: the Theil-Goldberger mixed theory, using approximated stochastic constraints; Bayesian analysis with diffuse or proper prior; and, the semiparametric (SP) approach. The first two are parametric estimations and require, as usual, additional parametric and distributional information. In the usual treatment of the SP approach, serious theoretical and practical problems are as yet unresolved. These include the problems of zero asymptotic efficiency, unconventional function form, shrinking band width assumption, greater computational expenses, undesirability of a particular shape estimate, large sampling size, and stronger regularity conditions than in a pure parametric estimation (see Robinson, 1988).

This paper provides a new and practical finite-sample semiparametric estimation procedure for complete models of GW equations, having as the nonlinear exponent an unknown function of an index of economic inequality. The procedure is simple, statistically stable and economically and theoretically justifiable. Some illustrations of the procedure with Australian household data are also reported.
2. COMPLETE GW MODELS & SEMIPARAMETRIC ESTIMATION

Consider a complete model of \( m \) GW equations relating expenditure \( (X_i) \) to total expenditure in income \( M \) for the \( i \)th good

\[
X_i = (\alpha_i - \beta_i/\theta) M^{1-\theta} + (\beta_i/\theta) M, \tag{1}
\]

with \( \sum (\alpha_i - \beta_i/\theta) = 0 \) and \( \sum \beta_i/\theta = 1, (i = 1, ..., m) \). It is well known (Tran Van Hoa, 1983) that (1) can be derived from maximizing, subject to the budget constraint, an indirect utility function \( U \) of the form:

\[
U = \left[ \frac{M^\theta - a(P)^\theta}{c(P)^\theta - a(P)^\theta} \right], \tag{2}
\]

in which \( \theta \) lies in the domain \( (1, -\infty) \), and \( a(P) \) and \( c(P) \) are two positive linear homogeneous functions of commodity prices \( P \).

The integrable GW equation in (1) can be written in budget shares \( W_i = X_i/M \) as:

\[
W_i = B_i M^{-\theta} + A_i, \tag{3}
\]

with \( \sum B_i = 0 \) and \( \sum A_i = 1 \). In (3) when the cross equation parametric (adding-up) restrictions are not imposed, the unrestricted equations each has the functional form proposed originally by Phillips (1958) for his well known studies on unemployment and money wage rates, and also by Tukey (1957) for his work on data transformation in statistics. These equations are subsets of a more general class of Box-Cox (1964) analysis.

In Tran Van Hoa (1985), it has been demonstrated that \( \theta \) in (1) to (3) above is in fact an exponent describing the shape of the Lorenz curve, which exerts distributive influence on consumer's behaviour and on the measurement of household income. In this context, \( \theta \) can be approximated by a real-value function, in the range \( (1, -\infty) \), of an index of economic inequality [denoted by say \( g \), and therefore, \( \theta = f(g) \)]. Since an exact functional form of \( f(g) \) is not known, we follow conventional procedures in the SP estimation (see Robinson, 1988, p. 46) and approximate \( f(g) \) in our case by a first-order or planar approximation function. Thus, \( \theta = f(g) \) can be written simply as...
\theta = 1 - g \quad \quad (4)

When \( g \) is assumed to be a Gini index or any index that has similar positive, real and finite values [see Jenkins (1988) for a recent survey of inequality indexes], it is clear that \( \theta \) in (4) lies in the domain \((1,0)\) and, as \((1,0) \in (1, -\infty)\), the linear function (4) \( a \textit{fortiori} \) satisfies the integrability conditions imposed on (3) or (1).

In the case where \( g \) is a Gini index, the necessary coefficient can be obtained by any of the methods of covariance from household income distribution data and integrated in (3) for system estimation. Different values for \( g \) for different cohorts of households can also be obtained from disaggregated household income distribution data available from household expenditure and income surveys.

Below, we report our empirical findings (Table 1) on an application of this new estimation procedure to 10 complete models of GW equations for 13 commodities for Australia for 1984. The 10 models consist of 5 models for household services and 5 for household goods. For both 'services' and 'goods' separately, the models are also disaggregated according to the sources of income of the households (see detail in Table 1). All data are from the household expenditure survey 1984, made available by the Australian Bureau of Statistics (ABS).

3. SOME EMPIRICAL FINDINGS

The ABS data consists of average weekly household expenditure on each of the 13 commodity groups separately and are grouped into 10 income levels from a sample of 4,494 households. This sample represents an estimated population of 4,967,700 households for Australia as a whole in 1984. From these data, we compute 5 Gini coefficients (see Table 1) for 5 cohorts of households defined by their sources of income, and use these estimates in (4) and (3) to finally estimate, in a system framework, income elasticities and their approximate standard errors.

From the results for 10 complete models of GW equations, as specified above and given in Table 1, we can see: (a) that effects of income distribution as measured by the Gini coefficients are widely divergent; and, (b) that the discrepancy in the magnitude of estimated Engel elasticities for the 5 different cohorts of households is fairly small for both 'goods' and 'services'. However, the similarity is more significant with respect to commodity groups that
are income inelastic than with respect to commodity groups where estimated income elasticities are greater than unity (i.e. alcoholic beverages, clothing and footwear, housing furnishings and equipment, transport and recreation).

It thus appears that luxuries more likely than necessities are affected by the distribution of income. This to us is a fairly plausible empirical finding.

It should also be noted that, although the variations among the estimated income elasticities are not great for the 5 cohorts of households as indicated above, the estimated standard errors of these elasticities show more discrepancy between the cohorts. The effects of income distribution are therefore transmitted more to the dispersion of the elasticities than to their means. In spite of this, we observe no definite association between the magnitude of the Gini coefficients and that of the estimated standard errors.
REFERENCES


Table 1: GW Personal Consumption Patterns: Goods and Services, All Households: Australia 1984

<table>
<thead>
<tr>
<th>Gini Coefficient</th>
<th>(a) (g=0.1078)</th>
<th>(b) (g=0.2492)</th>
<th>(c) (g=0.9990)</th>
<th>(d) (g=0.4744)</th>
<th>(e) (g=0.3393)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Goods</strong></td>
<td>(\eta)</td>
<td>(t)</td>
<td>(\eta)</td>
<td>(t)</td>
<td>(\eta)</td>
</tr>
<tr>
<td>1. Current housing costs</td>
<td>0.89 22.72</td>
<td>0.89 23.12</td>
<td>0.88 18.33</td>
<td>0.88 22.70</td>
<td>0.89 23.12</td>
</tr>
<tr>
<td>2. Fuel and power</td>
<td>0.55 12.52</td>
<td>0.54 13.57</td>
<td>0.52 20.61</td>
<td>0.54 15.70</td>
<td>0.54 14.35</td>
</tr>
<tr>
<td>3. Food</td>
<td>0.89 30.86</td>
<td>0.89 31.79</td>
<td>0.88 37.84</td>
<td>0.86 33.42</td>
<td>0.89 32.42</td>
</tr>
<tr>
<td>4. Alcoholic beverages</td>
<td>1.36 14.51</td>
<td>1.37 15.20</td>
<td>1.51 20.42</td>
<td>1.38 16.46</td>
<td>1.37 15.68</td>
</tr>
<tr>
<td>5. Tobacco</td>
<td>0.82 6.96</td>
<td>0.81 6.85</td>
<td>0.76 6.29</td>
<td>0.80 6.66</td>
<td>0.80 6.77</td>
</tr>
<tr>
<td>6. Clothing &amp; footwear</td>
<td>1.30 9.72</td>
<td>1.31 10.16</td>
<td>1.36 13.06</td>
<td>1.33 10.93</td>
<td>1.32 10.46</td>
</tr>
<tr>
<td>7. Housing furnishing &amp; equipment</td>
<td>1.34 na</td>
<td>1.34 na</td>
<td>1.34 na</td>
<td>1.34 na</td>
<td>1.34 na</td>
</tr>
<tr>
<td><strong>Engel aggregation</strong></td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>No. of households ('000)</td>
<td>2537</td>
<td>315</td>
<td>1203</td>
<td>439</td>
<td>4494</td>
</tr>
<tr>
<td><strong>Services</strong></td>
<td>(\eta)</td>
<td>(t)</td>
<td>(\eta)</td>
<td>(t)</td>
<td>(\eta)</td>
</tr>
<tr>
<td>1. Household services &amp; operation</td>
<td>0.62 25.70</td>
<td>0.61 27.81</td>
<td>0.60 15.96</td>
<td>0.60 27.85</td>
<td>0.61 28.47</td>
</tr>
<tr>
<td>2. Medical care &amp; health</td>
<td>0.90 18.19</td>
<td>0.90 18.17</td>
<td>0.89 14.35</td>
<td>0.90 17.47</td>
<td>0.90 17.98</td>
</tr>
<tr>
<td>4. Recreation</td>
<td>1.10 40.63</td>
<td>1.10 45.58</td>
<td>1.12 79.30</td>
<td>1.11 55.47</td>
<td>1.11 49.25</td>
</tr>
<tr>
<td>5. Personal care</td>
<td>0.87 17.58</td>
<td>0.86 17.26</td>
<td>0.85 16.14</td>
<td>0.86 16.81</td>
<td>0.86 17.07</td>
</tr>
<tr>
<td>6. Miscellaneous</td>
<td>0.86 na</td>
<td>0.87 na</td>
<td>0.92 na</td>
<td>0.89 na</td>
<td>0.88 na</td>
</tr>
<tr>
<td><strong>Engel aggregation</strong></td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Notes: \(\eta = \beta_i W_i + g\): estimated income elasticity; \(t\): estimated \(t\)-value; \(g\): the estimated Gini coefficient with \(g = 0.1078\) for (a) wages and salaries; \(g = 0.2492\) for (b) own business; \(g = 0.9990\) for (c) government cash benefits; \(g = 0.4744\) for (d) 'other sources', and \(g = 0.3393\) for (e) all sources of income combined; na: not applicable due to degeneracy of the omitted equation.