1990

Optimal taxation policies for a nonrenewable but quasi-infinite energy resource: a two-period framework

Claus-Hennig Hanf
Christian Albrechts University of Kiel

Dodo J. Thampapillai
University of Wollongong

UNIVERSITY OF WOLLONGONG

DEPARTMENT OF ECONOMICS

OPTIMAL TAXATION POLICIES FOR A NONRENEWABLE BUT QUASI-INFINITE ENERGY RESOURCE: A TWO-PERIOD FRAMEWORK

Claus-Hennig Hanf
Department of Agricultural Economics
Christian Albrechts University of Kiel
Kiel, Germany (F.R) D2300

and

Dodo J. Thampapillai
Department of Economics
University of Wollongong
Wollongong NSW 2500 Australia

Working Paper 90-1,

Co-ordinated by Dr Charles Harvie & Di Kelly
PO Box 1144 [Northfields Avenue], Wollongong NSW 2500 Australia
Phone: [042] 270 725 or 270 555. Telex 29022. Cable: UNIOFWOL. Fax [042] 270 477

ISSN 1035-4581
ISBN 08-6418-105-1

ABSTRACT

This paper deals with the formulation of a resource extraction tax for a resource that is finite yet relatively abundant in its availability. The basis for the formulation is the higher extraction costs that are imposed on future generations as a result of present extraction. The optimal size of tax is determined by the trade-off between present losses and future gains.

1. INTRODUCTION

Any nonrenewable resource is finite in its availability. Theoretical frameworks concerning extraction strategies for such resources have relied on the premise that: extraction during a given time period would be at the expense of resource requirements during subsequent time periods, resulting in inter-temporal or inter-generational conflicts, (Hartwick and Olewiler 1986, Thampapillai and Hanf 1988). Hence resource conservation is recognised as a pertinent objective in these frameworks, and the concept of user coosts has been introduced to resolve intergenerational conflicts. User costs are defined as the net benefit that is denied to a future generation due to extraction for the present generation (McInerney 1976). These user costs, when determined, can be translated into a tax to be imposed on present users of the resource, to prompt resource conservation. However, with some nonrenewable resources, the size of the resource stock is large enough to warrant the relegation of the user cost concept to only long term analyses. For example, Brain and Schuyers (1981) illustrate that the size of recoverable deposits of coal in Australia would be sufficient for Australia's energy requirements for at least 200 years. Resources of this type could be referred to as nonrenewable but quasi-infinite. With such energy resources, (for example, coal, uranium and oil-shale), although the relevance of the user cost concept is theoretically valid, its impact is likely to be negligible in the context of positive discount rates.

Although the extraction of the nonrenewable resources that are quasi-infinite does not readily present conflicts pertaining to exhaustion, another type of conflict arises from their extraction. Given that the resource is embedded in layers beneath the ground, the extraction of the initial layers is relatively inexpensive, compared to the extraction of the subsequent layers. The increasing cost is a function of not merely the depth of the resource layer, but also the quality of the resource ore as extraction progresses. For example, the grades of all metalic ores currently in use are far below the grades of those exploited in the past. That is, the amount of ore which has to be moved and processed, in order to obtain a unit of the resource increases with extraction. Hence, even if the extraction of these so called quasi-infinite resources may not deprive future generations of resource availability, their present extraction imposes a higher extraction cost on future generations. The intergenerational conflict therefore, concerns the possible reduction in future welfare due to increases in extraction costs. So, an optimal extraction policy for the present time period will be based on an analysis of future costs.

The object of this paper is to illustrate a conceptual framework for the optimal extraction of a quasi-infinite resource. The policy instrument considered is a tax on present extraction as compensation for the higher extraction costs that are imposed on the future generation. The analysis is set in the context of two time periods namely a present and a future time period. Given that the tax is based on the costs of extraction, these costs are briefly discussed, prior to the illustration of the taxation framework.

2. THE MARGINAL COSTS OF RESOURCE EXTRACTION

The usual assumption in economic analyses is that the marginal cost (MC) curve is positively sloped in the short run and U-shaped or L-shaped in the long run. Hence in long term analyses, production is assumed to occur in the flat portion of the L-shaped function or at the minimum of the U-shaped function. Further, in the context of oligopolistic and monopolistic situations, production is assumed to occur in the decreasing portion of the long run marginal cost (LRMC) function. Such assumptions are reasonable in defining the costs of extracting renewable resources. This is because, the cost curves of each period may be regarded as replicas of one another, at least in a stable economy. However, these assumptions are not relevant in the analyses of nonrenewable resource extraction. This is due to the fact that with nonrenewable resources, the marginal costs in a given period, MC_t, are influenced by not only the volume of extraction in that period (X_t), but also the total volume of extraction in all preceding periods (X). That is:

$$MC_t = t(X_t, X). \tag{1}$$

Further, given that the quality of the resource declines with the progression of extraction.

$$\partial MC_t/\partial X > 0. \tag{2}$$

Hence, the assumption herein is that the LRMC function for non-renewable resources is one that is positively sloped. That is, the costs of extraction increase as the volume of extraction increases. In fact, this assumption, which is central to the discussion that follows, does not deviate too far from the reality that prevails with respect to the extraction of nonrenewable resources.

For purposes of convenience and simplicity, the following assumptions are >>> made.

(i) The long run marginal cost of extraction in period t, namely ($LRMC_t$), is a linear function and is defined as:

$$LRMC_t = a + b(X_t) + b \sum_{i=1}^{t-1} X_{t-i}, \tag{3}$$

where X_t is the extraction quantity in period t, and a and b are the intercept and the gradient respectively. Hence, the LRMC function for the initial time period is:

$$LRMC_0 = a + bX_0. \tag{4}$$

(ii) The demand for the resource is also described by a linear function and is assumed to remain unchanged during the two time periods. That is:

$$P_t = c - dX_t, \tag{5}$$

where c and d are respectively the intercept and the gradient

(iii) The resource is neither imported nor exported.

(iv) The extraction of the resource does not result in any externalities or external effects, with the exception of the higher extraction cost that is imposed on the subsequent period.

3. THE DERIVATION OF THE OPTIMAL TAX

Suppose now that a uniform tax, T, is imposed on extraction during the intitial period. The supply function for this period, namely S_T, will hence be a function to the left of the LRMC as shown in Figure 1. Sinced the tax is uniform, S_T can be defined as:

$$S_T = \{(a + T) + bX_0\} \tag{6}$$

That is, the tax raises the price and reduces the extraction quantity during the initial time period. The optimal extraction quantities for the initial time period, with and without the tax, namely $\tilde{X}_{0T}$ and $\tilde{X}_0$, respectively can be defined by the equilibrium between the demand and the relevant supply functions. That is:

$$\tilde{X}_{0T} = (c - a)(b + d)^{-1} - T(b + d)^{-1}, \tag{7}$$

$$\tilde{X}_0 = (c - a)(b + d)^{-1}. \tag{8}$$

If $T > 0$, then $\tilde{X}_{0T} < \tilde{X}_0$. That is introducing a tax T in the initial period, results in a lowered level of production. The welfare loss during the initial period due to taxation, WLT, is represented by the area of triangle ABC in Figure 1, and can be defined as:

$$WLT = (\tilde{X}_0 - \tilde{X}_{0T})\, T\frac{1}{2}, \text{ or} \tag{9}$$

using (7) and (8) as:

$$WLT = \frac{1}{2}T\{(c - a)(b + d)^{-1} - [(c - a)(b + d)^{-1} - T(b + d)^{-1}]\}$$
$$= \frac{1}{2}T^2(b + d)^{-1} \tag{10}$$

The marginal welfare loss with respect to the tax is:

$$\frac{dWLT}{dT} = T(b + d)^{-1} \tag{11}$$

This definition of the marginal welfare loss is considered subseqently in formulating the optimal tax.

Consider now the next time period, which is illustrated in Figure 2. The LRMC of extraction during this period will depend on whether or not a tax had been imposed during the initial period. For example, if a tax were not imposed, then a larger quantity of the resource would have been extracted during the intial period,

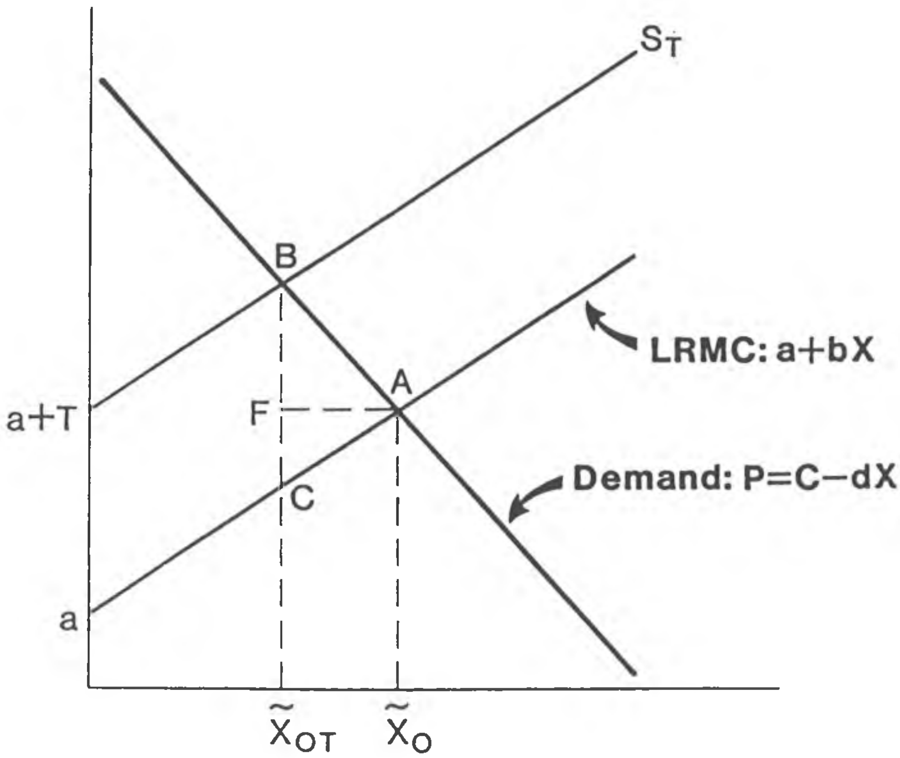

Figure 1: The effect of a tax on the initial period.

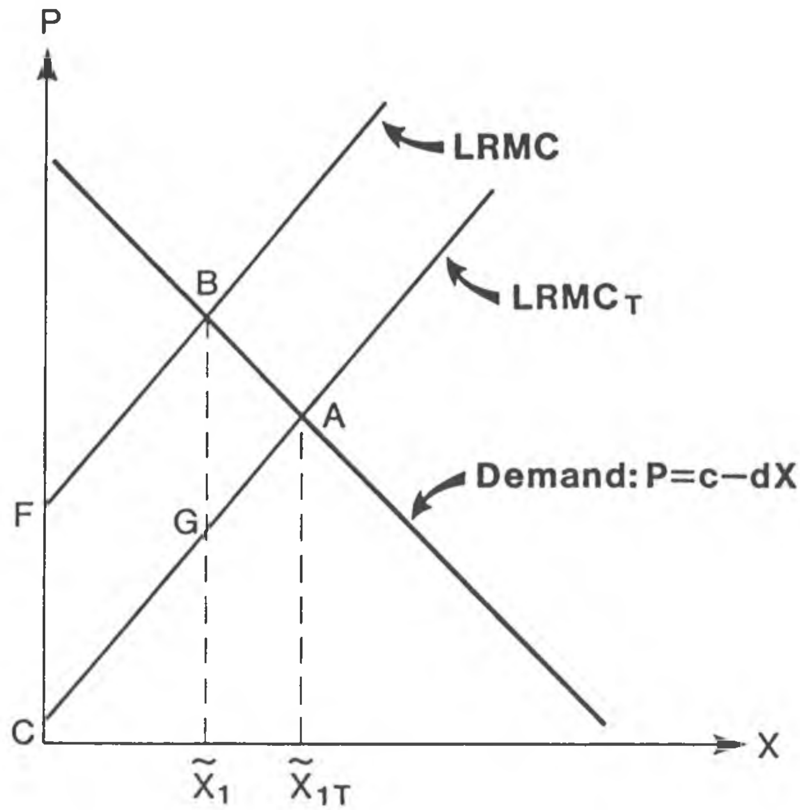

Figure 2: The effect of a tax on the second period.

relative to the situation of having a tax during that period. So, the intercept, of the LRMC for the subsequent time period would be higher without tax than with a tax. Hence in Figure 2, two possible marginal cost functions are shown for the subsequent time period, namely:

(i) LRMC - representing extraction costs without tax during the initial time period, and

(ii) $LRMC_T$ - representing extraction costs with a tax during the intial time period.

However, note that both these functions LRMC and $LRMC_T$ are derived from the same function, namely that described in (3) above. The difference in their intercepts is due to the imposition of a tax during the intital time period. So, the distances labelled FC in Figures 1 and 2 are identical, and the two marginal cost functions for the second time period can be defined as follows, using (3), (7) and (8) as follows.

$$LRMC = (a + \tilde{X}_0 b) + bX_1 \qquad (12)$$

$$= \{a + [(c - a)(b + d)^{-1}] b\} + bX_1$$

$$LRMC_T = (a + \tilde{X}_{0T} b) + bX_1 \qquad (13)$$

$$= \{a + [(c - a)(b + d)^{-1} - T(b + d)^{-1}]b\} + bX_1$$

The welfare gain during the subseqent time period, WGT, due to taxation in the initial period, is represented by the area ABFC in Figure 2. This area consists of a parallelogram GBFC and a triangle ABG. Let the area of the parallelogram be A_1, which is:

$$A_1 = (FC)\,\tilde{X}_1, \text{ where} \qquad (14)$$

FC is the difference between the intercepts of LRMC and $LRMC_T$; and

$\tilde{X}_1$ is the extraction quantity in the absence of a tax during the intital period.

From (12) and (13) above,

$$FC = bT\,(b + d)^{-1}, \text{ and} \tag{15}$$

by equating (5) and (12):

$$\tilde{X}_1 = (c - a)\,(b + d)^{-1} - b(c - a)\,(b + d)^{-2}. \tag{16}$$

Hence,

$$A_1 = bT\,(c - a)\,(b + d)^{-2} - b^2T\,(b + d)^{-3}\,(c - a). \tag{17}$$

Let the area of triangle ABG be A_2 , and can be defined using (15) above as:

$$A_2 = \left(\tilde{X}_{1T} - \tilde{X}_1\right) bT\,(b + d)^{-1}\,\frac{1}{2}, \text{ where} \tag{18}$$

$\tilde{X}_{1T}$ is the extraction quantity in the presence of a tax on extraction during hte initial period.

Since, by equating (5) and (13),

$$\tilde{X}_{1T} = (c - a)\,(b + d)^{-1} - b(c - a)\,(b + d)^{-2} + bT\,(b + d)^{-2} \tag{19}$$

and $\left(\tilde{X}_{1T} - \tilde{X}_1\right)$ becomes $\{bT\,(b + d)^{-2}\}$, (18) can be rewritten as:

$$A_2 = \frac{1}{2}\,b^2T^2\,(b + d)^{-3}. \tag{20}$$

Hence the welfare gain during the second time period, WGT, which is $(A_1 + A_2)$ can be defined by adding (17) and (20) as:

$$WGT = bT\,(c - a)\,(b + d)^{-2} + \frac{1}{2}\,b^2\,T^2\,(b + d)^{-3} - b^2\,T(b + d)^{-3} \tag{21}$$

From equation (21) it is evident that a welfare gain may occur only if $b > 0$. Hence, we will confine the analysis below to only cases where $b > 0$.

Now the marginal gain in welfare during the subsequent period with respect to the tax during the initial period can be defined as:

$$\frac{dWGT}{dT} = b\,(c - a)\,(b + d)^{-2} + b^2T\,(b + d)^{-3} - b^{-2}\,(b + d)^{-3}\,(c - a) \tag{22}$$

As can be seen from (22), an increase in tax prompts an increase in the marginal welfare gain during the subseqent period. At the same time, however, such an increase in tax also causes a welfare loss in the initial period.

A comparison of the marginal welfare loss and gain functions, that is (11) and (22) reveals that the slope of (11) is greater than that of (22) as long as:

$$(b + d)^{-1} > (b + d)^{-3} b^2 \qquad (23a)$$

This is true for $d > 0$. Furthermore, the intercept of (11) is zero, whereas the intercept of (22) is always larger than zero. That is:

$$[b(c - a)(b + d)^{-2}][1 - b(b + d)^{-1}] > 0 \qquad (23b)$$

Hence, the intersection of the marginal loss and marginal gain functions for positive values of T will occur, given that $b > 0$ and $d > 0$. This intersection, that is, the equality between marginal gain and loss, defines the maximization of net welfare gain over two time periods. So, given that $b > 0$ and $d > 0$, the optimal value of the tax for the initial period can be determined by the equality between the marginal welfare loss and gain functions. The optimal tax (T^*) is:

$$T^* = b(c - a)(d + 2b)^{-1} \qquad (24)$$

4. CONCLUDING REMARKS

The following inferences regarding T^*, (with respect to the elasticities of the demand and supply functions of the resource), can be derived from (24):

(i) if $b \to 0$, then $T^* \to 0$, and
if $b \to \infty$, then $T^* \to (c - a)\frac{1}{2}$;

(ii) if $d \to 0$, then $T^* \to (c - a)\frac{1}{2}$, and
if $d \to \infty$, then $T^* \to 0$; and

(iii) if $(c - a) \to 0$, then $T^* \to 0$.

The first and second conditions derived above indicate that the size of the optimal tax tends to be smaller as the demand function becomes more inelastic and/or the supply function becomes more elastic. The third condition reveals that the optimal value of the tax is directly proportional to the size of $(c - a)$. This has implications for the formulation of taxes in subsequent time periods, and can be explained as follows. Note that (c) is a parameter in the demand function representing the willingness to pay for the initial unit of the resource; and (a) is a parameter in the LRMC function, representing the per unit cost of extraction of the initial uinit. The value of (c) will remain constant over time as the demand function is assumed to remain the same. The value of (a) however, increases over time. That is, the value of (a) for the subsequent time period because of extraction during the initial time period. Hence the value of (c - a) becomes smaller over time. So, if a tax policy is formulated for each of the subsequent time periods, in the context of the two-period framework outlined above, the optimal value of the tax will become progressively smaller.

REFERENCES

1. Brian, P. and Schuyers, G. (1981), *Energy and the Australian Economy*, Melbourne, Longman Cheshire.

2. Hartwick, H.M. and Olewiler, N.D. (1986), *The Economics of Natural Resource Use*, New York, Harper and Row.

3. McInerney, J. (1976), 'The simple analytics of natural resource economics', *Journal of Agricultural Economics*, 27, 31-52.

4. Thampapillai, D.J. and Hanf, C-H (1988), 'Transport energy from agricutlure in the European community: some conceptual considerations for economic analysis', *International Journal of Energy Research*, 12, 31-43.