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ON TESTING THE JOINT HYPOTHESIS OF SHORT TERM INTEREST RATE:
A SIGNAL EXTRACTION APPROACH

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ABSTRACT

In this paper, I shall implement a methodology to identify and measure the *ex ante* real interest rate from the *ex post* real interest rate that involves application of signal-extraction techniques from the engineering literature. Diagnostic tests indicate that these methods are quite successful in capturing the essence of the time series properties of the *ex ante* real rate. The estimated model indicate that the variance of the *ex ante* real rate is not zero and the *ex ante* real rate may even follow a random walk stochastic process.
1. INTRODUCTION

In a renowned article, Fama (1976) tests the joint hypothesis that the *ex ante* real interest rate is constant over time and the Treasury bill market is efficient in the rational expectations sense. By combining the assumption that the *ex ante* real rate is constant under market equilibrium with the observation that sample autocorrelation of the *ex post* real rate is small (not significantly different from zero), Fama concludes that his joint hypothesis cannot be rejected.

Nelson and Schwert (1977) criticise Fama's methodology. Since the *ex post* real rate consists by definition of the *ex ante* real rate plus a pure forecasting error which will be serially uncorrelated if the market is efficient, in such a market any autocorrelation in the *ex post* real rate can be attributed to autocorrelation in the *ex ante* real rate. The lack of serial correlation in the *ex post* real rate is also consistent with both purely random variation in the *ex ante* real rate and market inefficiency in the form of forecast errors that are larger than necessary given available information. They further demonstrate that the low autocorrelation observed in the *ex post* real rate is indicative of strong autocorrelation and sizeable variation in the *ex ante* real rate since the *ex ante* real rate is being overlaid with forecast errors when we observe the *ex post* real rate.

Nelson and Schwert try many combinations of autocorrelation and variance for the *ex ante* real rate that are consistent with low autocorrelation in *ex post* real rate. They find that even if the *ex ante* real rate were a random walk process, the sample autocorrelation of *ex post* real rate would tend to be small relative to the variance of the forecasting errors.

The objective of this study is to reexamine the short-term interest rate model under the signal extraction framework of the state space model. The observable *ex post* real rate is decomposed into the sum of two unobservable processes, the signal and the noise. The signal represents the *ex ante* real rate which cannot be observed directly because it is contaminated by the noise (i.e., the forecasting error). The Kalman filter will be used to estimate the signal components of the underlying series. Unlike most Kalman filter applications, however, where parameters are assumed to be known so that recursive estimates of the "state" of the system are obtained, in this paper Kalman's recursive equations are used to compute the likelihood function for any given values of parameters occurring anywhere in the state space.
representation. Nonlinear optimisation technique can then be used to find the maximum likelihood estimates. The recursive residuals, or innovations, will be examined to judge the adequacy of fit of the model.

The paper will continue as follows: The State Space Model and the methodology of the Kalman filtering will be described in section 2. Section 3 will be the implementation of the State Space Model. Data source will be described in Section 4. In section 5, the empirical results will be presented and section 6 will offer some concluding comments.

2 STATE SPACE MODEL AND THE KALMAN FILTER

2.1 State Space Model

State space models of random processes are based on the Markov property, which implies the independence of the future of a process from its past, given the present state. In such a system, the state of the process summarises all the information from the past that is necessary to predict the future. Let \( X(t) \) referred to as the state vector be an \( m \times 1 \) vector of the system at time \( t \). This set of \( m \) state variables, which change overtime, may be "signals". In most cases, the signal will not be directly observable, being subject to systematic distortion as well as contamination by "noise". The \( n \) variables that are actually observed are defined by an \( n \times 1 \) vector, \( Y \), and they are related to the state variables, \( X(t) \), by a measurement equation,

\[
Y(t) = M(t) X(t) + V(t) \quad t = 1, 2, ..., T \tag{2.1}
\]

where \( M(t) \) is the \( n \times m \) matrix of known coefficients. The \( n \times 1 \) vector of measurement errors, \( V(t) \), has a zero mean and the covariance matrix \( R(t) \).

Although the state vector, \( X(t) \), is not directly observable, its movements are assumed to be governed by a well-defined process. This process is defined by the transition equation,

\[
X(t) = \phi(t) X(t-1) + G(t) U(t) \quad t = 1, 2, ..., T \tag{2.2}
\]
where $\phi(t)$ and $G(t)$ are fixed matrices of order $m \times m$ and $m \times q$ respectively, and $U(t)$ is a $q \times 1$ vector of random disturbances, with mean zero and covariance matrix $Q(t)$.

It is assumed that the disturbances in both the measurement and transition equations have zero mean, are serially uncorrelated and are uncorrelated with the initial state vector, $X(0)$. These assumptions can be summarised:

$$
\begin{bmatrix}
V(t) \\
U(t)
\end{bmatrix} \sim \begin{bmatrix} 0 \\ R(t) \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ Q(t) \\ 0 \end{bmatrix}
$$

and

$$
E[X(0)V'(t)] = 0, \quad E[X(0)U'(t)] = 0 \quad t = 1, 2, \ldots, T
$$

where' denotes the transpose of the matrix in question.

The above representation of a linear dynamic model is known as the state space form. Although at first sight it may appear to have no particular advantages, in fact, the state space model includes the traditional regression, ARIMA time-series models, Bayesian forecasting (see Harrison and Stevens, 1976), and models with time-varying coefficients as special cases.

### 2.2 The Discrete Kalman Filter

The Kalman filter is a method for estimating the state vector of a linear dynamic system from noisy observations. It consists of a set of equations which allows an estimation to be updated once a new observation becomes available. This process is carried out in two steps. The first step consists of forming the optimal predictor of the next observation, given all the information currently available. This is carried out by means of the prediction equations. Once the new observation is available, it is then incorporated into the estimation of the state vector by means of the updating equations.

The Kalman filter provides an optimal estimator for the problem of prediction and updating. By optimal, we mean it is a minimum mean square (linear) estimator. The one-period prediction problem for this system is to predict the state variables $X(t)$ on the basis of the information that is available at time $(t-1)$. This information set consists of the current and
previous observations and is denoted by \( Y(t-1) = [y(t-1), y(t-2), ..., y(1)] \). Under certain conditions, \([1]\) the optimal predictor is the conditional expectation of \( X(t) \) given \( Y(t-1) \), i.e.,
\[
X(t/t-1) = \mathbb{E}[X(t)/Y(t-1)]
\]  

Given the state space model in (2.1) - (2.4), let us assume that the minimum mean square estimator of the unknown state vector \( X(t) \) based on all information up to and including time \( t-1 \) be \( X(t/t-1) \). For convenience, we will write \( X(t/t) = X(t) \). The estimation \( X(t/s) \) is referred to as prediction if \( t > s \) and as filtering if \( t = s \) and as smoothing if \( t < s \). Denote the error \( [X(t) - X(t/t-1)] \) by \( \epsilon(t/t-1) \) and let \( P(t/t-1) \) be the associated error covariance matrix. Similarly, \( Y(t/t-1) \) will be the predicted observation at time \( t-1 \) based on all the observation up to time \( t-1 \).

The recursion proceeds through the following steps (Kalman, 1960). Given the state vector and its covariance matrix at any sample time, say \( t-1 \), we can predict the state vector one observation into the future by using the transition equation (2.2). The prediction is
\[
X(t/t-1) = \phi(t) X(t-1)
\]  

Because the expected value of the stochastic component of the transition is zero, the error covariance matrix of the prediction is
\[
P(t/t-1) = \phi(t) P(t-1) \phi'(t) + G(t) Q(t-1) G'(t)
\]  

From the observation equation and the predicted state, we can predict the next observation
\[
Y(t/t-1) = M(t) X(t/t-1)
\]  

When the next observation, \( Y(t) \), becomes available, we can compare the prediction with the actual data. The difference is the recursive residual, or "innovation",
\[
I(t) = Y(t) - Y(t/t-1) = M(t) [X(t) - X(t/t-1)] + V(t)
\]
The residuals are important. Later they will be used to check the fit of the model and estimate the parameters through the non-linear optimisation program. For now, we will use the information in the residual to complete the recursion.

The innovation covariance matrix is computed as

$$C(t) = M(t) P(t/t-1) M'(t) + R(t)$$ \hspace{1cm} 2.10

Since the observation in time period $t$ is known, we can update the predicted state, $X(t/t-1)$, and its associated covariance by the following equations:

$$X(t/t) = X(t) = X(t/t-1) + K(t) I(t)$$ \hspace{1cm} 2.11
$$P(t/t) = P(t) = P(t/t-1) - K(t) M(t) P(t/t-1)$$ \hspace{1cm} 2.12

$$= [I_n - K(t) M(t)] P(t/t-1)$$

where $K(t)$ is an unknown "weighting" matrix, called Kalman gain matrix, satisfying the following equation:

$$K(t) = P(t/t-1) M'(t) C(t)^{-1}$$ \hspace{1cm} 2.13
$$= P(t/t-1) M'(t) [M(t) P(t/t-1) M(t)' + R(t)]^{-1}$$

This completes the recursion.

Based on the estimate $X(t)$, a prediction of $X(t+1)$ can be made by (2.6) again. The calculation goes on recursively as each new observation $y(t)$ becomes available. Equations (2.6) and (2.7) make up the prediction equations, while those in (2.11) and (2.12) are the updating equations. These equations are called the Kalman filter. When all the observations have been processed, recursive technique may be applied in reverse to solve the problem of smoothing.

3. STATE SPACE IMPLEMENTATION OF THE MODELS

The difference between the market interest rate and the subsequently observed rate of inflation is the \textit{ex post} real interest rate. The \textit{ex post} real interest rate consists, by definition, of
the *ex ante* real interest rate and a pure forecasting error. The hypothesis of market efficiency implies that these forecasting errors must be serially random. Therefore, observing the *ex post* real rate is equivalent to observing the *ex ante* real rate with random measurement error.

Fama's model can be outlined with the help of the following equations:

\[ R(t) = i(t) + P^e(t) \]  
\[ P(t) = P^e(t) + \varepsilon(t) \]

where \( R(t) \) is the nominal interest rate, \( i(t) \) is the *ex ante* real rate, \( P^e(t) \) is the expected rate of inflation, \( P(t) \) is the actual rate of inflation, and \( \varepsilon(t) \) is the market's forecast error which has the following stochastic properties under the efficient market hypothesis:

\[ E(\varepsilon(t)) = 0, \quad E(\varepsilon^2(t)) = \sigma^2_e, \quad E(\varepsilon(t)P^e(t)) = 0, \quad \text{and} \quad E(\varepsilon(t)R(t-i)) = 0. \]

Subtracting Equations (3.1) from (3.2) yields

\[ r(t) = R(t) - P(t) = i(t) - \varepsilon(t) \]

where \( r(t) \) is the *ex post* real rate.

Since the *ex post* real rate is the difference between the *ex ante* real rate and the market forecasting error of the rate of inflation, we can put it into the signal extraction framework of the state space model by dividing the *ex post* real rate into two components, the signal and the noise. In the model, the signal represents the *ex ante* real rate which cannot be observed directly because it is contaminated by the noise (i.e., the forecasting error). Then the procedure described in section two can be applied to estimate the variance of the signal and that of the noise.

As mentioned in Nelson and Schwert, the low autocorrelation observed in the *ex post* real rate could indicate strong autocorrelation and sizeable variation in the *ex ante* real rate, because the latter is overlaid with forecast errors when we observe the former. Therefore, we first suppose that the *ex ante* real rate rather than being constant is a stochastic process with first order serial correlation coefficient \( \phi \). We then have \( i(t) = \phi i(t-1) + v(t) \) in our transition equation, where \( \phi \) is a scalar. It has been shown (Nelson and Schwert, 1977) that in an
efficient market, the first order serial correlation coefficient for the \textit{ex post} real rate will be related to $\phi$ by

$$\text{corr} \ [r(t), r(t\pm1)] = \frac{\phi \sigma_i^2}{\sigma_i^2 + \sigma_e^2} = \phi \frac{1}{(1+1/s)}$$

where $\sigma_i^2$ and $\sigma_e^2$ are the variances of the \textit{ex ante} real rate and the forecasting error respectively, and $s = \sigma_i^2 / \sigma_e^2$ will be the ratio of the signal to the noise.

If the variance of the forecast error is large relative to the variance of the \textit{ex ante} real rate (i.e., $1/s$ is large), the first order autocorrelation in the \textit{ex post} real rate may be considerably less than $\phi$. The first order autocorrelation in the \textit{ex post} real rate will approach zero as a limiting case (i.e., $1/s$ approaches infinity), while approaching $\phi$ at the other extreme. Thus, it is of our interest to estimate the first order serial correlation coefficient, $\phi$, and the variance of \textit{ex ante} real rate and that of the forecast errors.

An alternative hypothesis is that the \textit{ex ante} real rate follows a random walk with no long run mean and unbounded variance. This implies that the \textit{ex post} real rate is generated by the process.

$$r(t) = r(t-1) + \nu(t) - \epsilon(t) + \epsilon(t-1)$$

which could be thought of as a random walk with an autocorrelated disturbance. The theoretical autocorrelations for $r$ are undefined (as was shown in Nelson and Schwert) but sample autocorrelations are of course readily computed for any finite data series. If the variance of $\nu(t)$ is small relative to that of the forecast errors, $\epsilon(t)$, we would expect to have a small sample autocorrelation of $r(t)$ as was implied by the Box-Jenkins analysis.[2]

Fama also tests market efficiency by regressing realised inflation rate, $p(t)$, on the market interest rate, $R(t)$, and past rates of inflation, $p(t-i)$. If $p^+(t)$ represents a piece of information about $p(t)$ which is available to the market at the beginning of period $t$, then a regression of $p(t)$ on $R(t)$ and $p^+(t)$ yield a nonzero coefficient for $p^+(t)$ only if the market is inefficient in its use of available information or if the predictive ability of $R(t)$ is distorted by the
underlying variation of the *ex ante* real rate. Fama chose \( p(t-1) \) as a particular \( p^*(t) \) and found that the coefficient of \( p(t-1) \) was small and not significant. The power of such a test will be low if the \( p^*(t) \) chosen contains little information about \( p(t) \).

In order to provide a more powerful test of the joint hypothesis, we decompose the realised inflation rate into unobservable components: the expected inflation rate, \( p^e(t) \), and the forecasting error, \( w(t) \). By assuming that the expected rate of inflation follows a stochastic process, ie., \( p^e(t) = p^e(t-1) + w(t) \), we obtain the optimal estimate (forecast) of the expected inflation, \( p^e(t) \), in the rational expectations sense. This is the more efficient predictor between the Box-Jenkins extrapolative predictor \( p^*(t) \) and \( p(t-1) \). [3]

4 DATA SOURCE

The one month nominal rate of interest \( R(t) \) used in the tests is the return from the end of month \( (t-1) \) to the end of month \( t \) on the US Treasury bill that matures closest to the end of month \( t \). The US Bureau of Labor Statistics Consumer Price Index (CPI) is used to estimate \( p(t) \), the rate of change in the purchasing power from the end of month \( (t-1) \) to the end of month \( t \). [4] Sources of data are given in Nelson and Schwert (1977). To be consistent with Fama's testing, we would first test with the period from January 1953 through July 1971.

In order to test Fama's joint hypothesis for a longer period, the data set is then extended to December 1982. The one month Treasury bill rates no longer existed after 1974, so the 91-day Treasury bill rates are used instead. Both the 91-day Treasury bill rates and the CPI series are obtained from Data Bank of Department of Economics, University of Washington, Seattle, USA.

5 EMPIRICAL RESULTS AND TESTING

In order to demonstrate that the low autocorrelation observed in the *ex post* real rate could indicate both strong autocorrelation and sizeable variation in the *ex ante* real rate, since the latter is being overlaid with forecast errors when we observe the former, the first estimate assumed the *ex ante* real rate rather than being constant was a stochastic process with first-order serial correlation coefficient \( \phi \). The estimation results are shown in Table 1.
The maximum likelihood estimates of $\sigma_1$ and $\sigma_e$ are 0.164 and 2.310, respectively, and both are significantly different from zero. The maximum likelihood estimate of $\phi$ is 0.985, which is significantly different from zero but not significantly different from one, implying that the $ex \ ante$ real rate follows a random walk rather than a first order autoregressive process.

We then estimated the model assuming the $ex \ ante$ real rate follows a random walk. The estimation results are also reported in Table 1 under model (b). The maximum likelihood estimates of $\sigma_1$ and $\sigma_e$ are 0.121 and 2.325 respectively. All the parameters are significantly different from zero. The $-2 \ln$ maximised likelihood has a value of 615.34. Based on the Akaike's Information Criterion for model selection, this would be the better model of the two. The result confirms that $\sigma_1$ is not equal to zero, and rejects the hypothesis that $ex \ ante$ real interest rate is a constant.

The same pattern of results are observed with the data extended to cover up to December, 1982 (with $T = 360$) and from the quarterly data (1953-1 to 1982-4). The results are shown in Table 1 also. These results further support the hypothesis that $\sigma_1$ is not equal to zero and reject the hypothesis that $ex \ ante$ real interest rate is a constant.

The maximum likelihood estimates of the inflation rate series are presented in Table 3. For the period from January 1953 to July 1971, the standard deviation of the expected inflation rate $\sigma_\mu$ and the forecasting error $\sigma_e$ are 0.309 and 2.284, which are significantly different from zero. When the additional information component is added to the model, the estimated parameters $\phi_1$ and $\sigma_\eta$ are 0.321 and 0.808. The former is significantly different from zero, but the latter is not, providing evidence to support the argument that the market did use additional information which is contained in the past inflation rates. The extension of the model to a longer period is done with both monthly and quarterly observations. Similar results are obtained for both the periods from January 1953 to December 1982 ($T = 360$) and the quarterly data of 1953-1 to 1982-4 ($T = 120$) (see Table 3.)

Regressions of the inflation rates on the nominal interest rate, $R(t)$, and the expected inflation rate, $p^e(t)$, for January 1953 to July 1971 period in Table 4 indicate that the expected inflation rate has a large and significant effect on the composite predictors of the rate of inflation. Similar results are obtained for both the periods from January 1953 to December 1982 ($T = 360$)
Table 1  Maximum Likelihood Estimates of Parameters of the 
Ex Post Real Rate

<table>
<thead>
<tr>
<th>Period</th>
<th>T</th>
<th>Model+</th>
<th>$\sigma_y$</th>
<th>$\sigma_e$</th>
<th>$\phi$</th>
<th>-2Ln Like</th>
<th>Q-Stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/1953</td>
<td>222</td>
<td>(a)</td>
<td>0.164</td>
<td>2.310</td>
<td>0.985</td>
<td>614.11</td>
<td>16.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.07)</td>
<td>(0.12)</td>
<td>(0.01)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7/1971</td>
<td></td>
<td>(b)</td>
<td>0.121</td>
<td>2.325</td>
<td>-</td>
<td>615.34</td>
<td>15.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.07)</td>
<td>(0.12)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/1953</td>
<td>360</td>
<td>(a)</td>
<td>0.660</td>
<td>2.628</td>
<td>0.966</td>
<td>1138.49</td>
<td>33.4*</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.16)</td>
<td>(0.13)</td>
<td>(0.02)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12/1982</td>
<td></td>
<td>(b)</td>
<td>0.524</td>
<td>2.670</td>
<td>-</td>
<td>1138.36</td>
<td>32.7*</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.10)</td>
<td>(0.12)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1953,1Q</td>
<td>120</td>
<td>(a)</td>
<td>0.387</td>
<td>1.278</td>
<td>0.911</td>
<td>257.70</td>
<td>36.7*</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.19)</td>
<td>(0.15)</td>
<td>(0.06)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1982,4Q</td>
<td></td>
<td>(b)</td>
<td>0.768</td>
<td>1.348</td>
<td>-</td>
<td>260.37</td>
<td>34.1*</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.15)</td>
<td>(0.13)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: Standard errors or coefficients are shown in parentheses.
+ indicates with EARR assumes to follow
(a) AR(1) process, (b) a random walk process.
Q means quarter.

Table 2  Specification of the ARIMA Model for the Ex Post Real Rate.

<table>
<thead>
<tr>
<th>Period</th>
<th>ARIMA Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/1953 – 7/1971 (T = 120)</td>
<td>[(1-L)r_t = (1 - 0.77L) \epsilon_t ]</td>
</tr>
<tr>
<td></td>
<td>Innovations variance ( $\sigma_e^2$ ) = 6.40</td>
</tr>
<tr>
<td></td>
<td>-2 ln likelihood = 630.94</td>
</tr>
</tbody>
</table>

and the quarterly data of 1953-1Q to 1982-4Q (T = 120). Since there is evidence that the rate of inflation, $p(t)$, is not stationary, we also estimate regressions on the change in the rate of inflation, $p(t) - p(t-1)$, using predictors of the change: $R(t) - p(t-1)$ and $p(t) - p(t-1)$. Again, the coefficient of the filtered predictor, $p(t) - p(t-1)$, is large and significant as for level of the rate of inflation. These regression results suggested that the market did draw additional information from the past inflation rate in forming its expectations about future inflation. It is interesting to
note that the coefficient of the nominal interest rate is relatively small when compared with that of the expected inflation rate. This may be due to the fact that the Kalman filter operates in a 'predict-correct' fashion which combines the system's uncertainty with its noisy measurement in correcting the available estimates. The filtered estimates, thus, reflect the long term movements of the underlying series. In this case, the seasonality (or the seasonal autocorrelation) underlying the consumer price index series, so is the inflation rate series, is exploited by the model. This increases the predictive power of the filtered estimates of the expected inflation rate. Therefore it is not surprising that the weight given to the expected inflation rate in the composite predictions is large and that to the nominal interest rate is small.

Table 3 Maximum Likelihood Estimates of Parameters of the Inflation Rate

<table>
<thead>
<tr>
<th>Period</th>
<th>T</th>
<th>σ_w</th>
<th>σ_e</th>
<th>-2 Ln Like</th>
<th>Q-Stat.\textsuperscript{a}</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/1953 - 7/1971</td>
<td>222</td>
<td>0.309*</td>
<td>2.284*</td>
<td>620.47</td>
<td>11.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.09)</td>
<td>(0.12)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/1953 - 12/1982</td>
<td>360</td>
<td>0.847*</td>
<td>2.596*</td>
<td>1164.80</td>
<td>34.3*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.15)</td>
<td>(0.13)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1953,1Q - 1982,4Q</td>
<td>120</td>
<td>0.616*</td>
<td>1.052*</td>
<td>223.25</td>
<td>27.0*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.12)</td>
<td>(0.10)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: Standard errors of coefficients are shown in parenthesis.

\textsuperscript{a} Box-Ljung Q(12) Statistic: asterisks in this column indicate a significant value at 5% level.

I have concluded that Fama's test is not powerful enough to reject the joint hypotheses that the real interest rate is constant and that Treasury bill market is efficient. The low autocorrelation observed in the \textit{ex post} real rate is probably accounted for by the relative stability of the U.S. economy over the short sample period (the 1950s and the 1960s).
Table 4  Regression Tests of Market Efficiency

<table>
<thead>
<tr>
<th>Period</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\gamma$</th>
<th>$\sigma_e$</th>
<th>$R^2$</th>
<th>D.W.</th>
<th>$\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$1/53-7/71$</td>
<td>-0.737</td>
<td>0.958</td>
<td>-</td>
<td>2.396</td>
<td>0.292</td>
<td>1.78</td>
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Notes: The prediction equations are:

A: $P_t = \alpha + \beta R_t + \gamma P^e_t + \epsilon_t$

B: $(P_t - P_{t-1}) = \alpha + \beta (R_t - P_{t-1}) + \gamma (P^e_t - P^e_{t-1}) + \epsilon_t$

Standard errors in parentheses.
6 CONCLUSION

The results of this study have three important implications. First, it illustrates that the Kalman filtering method is a powerful means of capturing the unobservable components of economic time series. Second, it provides useful information on the nature of the underlying series. Third, it demonstrates that the state space model is a promising method for analysing and modeling time series.

Conditional on the assumption that the \textit{ex ante} real interest rate is constant, Fama used the sample autocorrelation function of the \textit{ex post} real interest rate and the regression relationship of rates of inflation on the market interest rates and the prior rate of inflation to test market efficiency. These tests, however, may not be powerful enough to lead to a rejection of the joint hypothesis of market efficiency and constancy of the \textit{ex ante} real rate. By using the signal extraction framework of the state space model, we have shown that the variance of \textit{ex ante} real rate is significantly greater than zero. It is relatively small when compared with the variance of the forecast errors in the expectations of inflation. Our empirical results show that the \textit{ex ante} real rate may even be a nonstationary stochastic process. This indicates that there is substantial variability of the monthly \textit{ex ante} real rate and leads to rejection of Fama's hypothesis that it is constant.

When the observed inflation rate series is further decomposed into the expected inflation rate, and the market's forecasting error of inflation, the estimated variances of the expected inflation rate and that of forecasting error are significantly different from zero. Regression results based on the filtered estimates of the expected inflation rate show a large and significant effect on the composite predictions of the rate of inflation. This strongly suggests that the market draws on more information from the past inflation rates, and individual past inflation rates contain little information about future rates of inflation. We can therefore conclude that Fama's test is not powerful enough to reject the joint hypothesis.

In fact, the state space model provides us with a method for constructing an optimal predictor of inflation based on the past history of inflation rates. Our analysis supports the hypothesis that expectations of inflation have accounted for most of the variation in short term
interest rates during the post war period, and that those expectations embody significant information beyond that contained in past inflation rates alone.
FOOTNOTES

1 It has been shown that the conditional expectation is optimal in the sense that it minimises the expected loss if (1) the loss function is symmetric and nondecreasing for positive arguments and (2) the conditional distribution of $X(t)$ given $Y(t-1)$ has a unimodal density function that is symmetric about $X(t)$. Under the assumption that $U(t)$ and $V(t)$ are uncorrelated, serially independent multivariate normal processes and the loss function is symmetric, the conditional expectation is the optimal predictor.

2 $Y(t) = X(t) + v(t)$
$X(t) = X(t-1) + u(t)$

Then $[1-L]Y(t) = u(t) + v(t) - v(t-1)$

This is an ARIMA (0, 1, 1) process of the form $[1-L]Y(t) = \eta(t) - \theta \eta(t-1)$ where $\eta(t)$ is a white noise.

3 It is easy to show that the forecast of the $p(t)$ given only the past inflation rates is

$p^*(t) = \sum \beta_i p(t-i)$

$= (1-\theta) p(t-1) + \theta (1-\theta) p(t-2) + \theta^2 (1-\theta) p(t-3) + \ldots.$

$p^*(t)$ will have a larger prediction error variance, (i.e., $\text{var}[p(t) - p^*(t)]$) than that of $p^o(t)$ (i.e., $\text{var}[p(t) - p^o(t)]$).

4 This is due to Wold’s decomposition theorem.

5 I am grateful to Professor Charles Nelson for providing me with this data.
REFERENCES


APPENDIX A

Time Series Solution of the Maximum Likelihood Estimates of the State Space Model.

Short Term Interest Rates:

(A.1a) \[ r(t) = i(t) - \varepsilon(t) \quad \varepsilon(t) \sim \text{NID}(0, \sigma_e^2) \]

and

(A.1b) \[ i(t) = i(t-1) + V(t) \quad V(t) \sim \text{NID}(0, \sigma_v^2) \]

Thus

\[ (1-L)r(t) = V(t) - \varepsilon(t) + \varepsilon(t-1) \]

\[ \text{Var}[\Delta r(t)] = \gamma_0 = \sigma_v^2 + 2\sigma_e^2 \]

\[ R_{\Delta r}(1) = \rho_1 = -\sigma_e^2 \]

This model is equivalent to an ARIMA (0,1,1) model.

(A.2) \[ \Delta Y(t) = a(t) + \theta a(t-1) \quad a(t) \sim \text{NID}(0, \sigma_a^2) \]

\[ \text{Var}[\Delta Y(t)] = \gamma_0 = (1 + \theta^2) \sigma_a^2 \]

\[ R_{\Delta Y}(1) = \rho_1 = \theta \sigma_a^2 \]

By equating the two models,

\[ \sigma_v^2 + 2\sigma_e^2 = (1 + \theta^2) \sigma_a^2 \]

\[ -\sigma_e^2 = \theta \sigma_a^2 \]

This implies:

\[ \sigma_e^2 = -\theta \sigma_a^2 \]

\[ \sigma_v^2 = (1 + \theta)^2 \sigma_a^2 \]

In model (A.1), a relatively low value of \( \sigma_v^2 \) corresponds to a value of \( \theta \) close to -1 in (A.2). In these circumstances, ML estimates of zero will not be uncommon for \( \sigma_v^2 \).

Similarly, if \[ i(t) = \phi i(t-1) + V(t) \quad V(t) \sim \text{NID}(0, \sigma_v^2) \]

Then \[ r(t) = \phi r(t-1) + V(t) - \varepsilon(t) + \varepsilon(t-1) \]

This model is equivalent to ARIMA (1,0,1) model,

\[ Z(t) = \phi_1 Z(t-1) + a(t) + \theta_1 a(t-1) \]

This implies:

\[ \phi = \phi_1 \]

\[ \sigma_e^2 = (-\theta_1/\phi) \sigma_a^2 \]

\[ \sigma_v^2 = \left(\frac{1 + \phi \theta_1}{\phi + \theta_1}\right) \sigma_a^2 \]

If \( \phi_1 \) approaches to 1, then \( \sigma_e^2 \) and \( \sigma_v^2 \) will be the same as in the random walk case as before.
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