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Abstract

Golay complementary sequences, often referred to as Golay pairs, are characterised by the property that the sum of their aperiodic autocorrelation functions equals to zero, except for the zero shift. Because of this property, Golay complementary sequences can be utilised to construct Hadamard matrices defining sets of orthogonal spreading sequences for DS CDMA systems of the lengths not necessary being a power of 2. In the paper, we present an evaluation, from the viewpoint of DS CDMA applications of some sets of spreading sequences based on Golay complementary sequences. We then modify those sets of sequences to enhance their correlation properties for asynchronous operation and simulate a multi-user DS CDMA system utilising the modified sequences.

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Golay Sequences for DS CDMA Applications

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Abstract

Golay complementary sequences, often referred to as Golay pairs, are characterised by the property that the sum of their aperiodic autocorrelation functions equals to zero, except for the zero shift. Because of this property, Golay complementary sequences can be utilised to construct Hadamard matrices defining sets of orthogonal spreading sequences for DS CDMA systems of the lengths not necessary being a power of 2. In the paper, we present an evaluation, from the viewpoint of DS CDMA applications of some sets of spreading sequences based on Golay complementary sequences. We then modify those sets of sequences to enhance their correlation properties for asynchronous operation and simulate a multi-user DS CDMA system utilising the modified sequences.

1. Introduction

Orthogonal spreading sequences are used in direct sequence code division multiple access (DS CDMA) systems for channel separation and to provide a spreading gain, e.g. [1]. The most popular class of such spreading sequences are the sets of Walsh-Hadamard sequences [2], which are easy to generate. However, the cross-correlation between two Walsh-Hadamard sequences can rise considerably in magnitude if there is a non-zero delay shift between them. Unfortunately, this is very often the case for an up-link (mobile to base station) transmission, due to differences in the corresponding propagation delays. As a result, significant multi-access interference (MAI) [3] occurs which needs to be combated either by complicated multi-user detection algorithms [4], or reduction in bandwidth utilization. Moreover, due to their very regular structure, Walsh-Hadamard sequences are characterized with very poor auto-correlation properties. In real systems, this is alleviated by the use of scrambling codes on the top of Walsh-Hadamard sequences. These are normally very long codes having very distinctive peaks at zero in their auto-correlation functions. For example, in UMTS these cods are 2^{18} bits long [5]. In addition to improving synchronization properties, scrambling also helps in reducing MAI.

Another important drawback of using Walsh-Hadamard sequences or modified Walsh-Hadamard sequences [6] is the fact that sequence length must be equal to the integer power of 2. This is not the case if orthogonal sequences based on complementary Gold sequences are used instead of Walsh-Hadamard sequences.

Complementary pairs and sets of orthogonal spreading sequences defined on the base of Golay complementary sequences have been long studied for application in DS CDMA systems, [7], [8], [9]. In this paper, we want to extend those considerations to the sequences based on Golay-Hadamard matrices modified using the method introduced in [6]. As a result of modification, the new sets of orthogonal sequences are characterised with much lower peaks in aperiodic cross-correlation functions, and exhibit good autocorrelation properties, too. To illustrate usefulness of the modified sequences, we then simulate the multi channel DS CDMA system utilising those modified sequences and compare the results to those obtained in the case of unmodified sequences. Our considerations are not limited to bipolar sequences but we investigate quadri-phase sequences as well.

The paper is organised as follows. In the next section, we show some basic techniques to design Hadamard matrices based on complementary Golay sequences. In Section 3, we describe the sequence modification method, and show the correlation parameters for some sequence sets of lengths 26 and 32. The simulation of DS CDMA system utilising the developed sequences are presented in Section 4, and Section 5 concludes the paper.

2. Construction of Hadamard Matrices Using Golay Sequences

For a pair of Golay complementary sequences S_1, and S_2, the sum of their aperiodic autocorrelation functions equals to zero, except for the zero shift [10]:

$$c_{S_1}(\tau) + c_{S_2}(\tau) = 0 \qquad \tau \neq 0 \tag{1}$$

where $c_{S_1}(\tau)$ and $c_{S_2}(\tau)$ denote the aperiodic autocorrelation functions [10].

It can be proven [11], [12] that if matrices **A** and **B** are the circulant matrices created from a pair of Golay complementary sequences S_1, and S_2 then the matrix

$$\mathbf{G} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B}^{\mathbf{T}} & -\mathbf{A}^{\mathbf{T}} \end{bmatrix} \qquad (2)$$

is a Hadamard matrix. The matrix **G** can be used to generate spreading sequences for DS CDMA applications.

Originally, Golay complementary sequences have been defined as bipolar sequences, and one can generate bipolar Golay sequences of all lengths N, such that

$$N = 2^a 10^b 26^c \qquad (3)$$

where a, b, c, are non-negative integers. Hence, using the construction prescribed by Eq. (2), the sets of orthogonal spreading sequences of length $2N$ can be designed.

Another technique, which can be employed to construct Hadamard matrices from bipolar complementary sequences, is based on the Goethals-Seidel array [13]. In [14], Craigen, Holzmann and Kharaghani have shown that if U and V are Golay complementary sequences of length l_1 and X, Y are Golay complementary sequences of length l_2 then

$$A = [U, X]$$
$$B = [U, -X]$$
$$C = [V, Y]$$
$$D = [V, -Y]$$

are four complementary sequences of length l_1+l_2, and that there is a Hadamard matrix of order

$$N = 4(l_1 + l_2)$$

constructed from Goethals-Seidel array. This technique produces many more Hadamard matrices than the construction given by Eq. (2).

However, much more Hadamard matrices can be produced if instead of bipolar Golay sequences, complementary quadri-phase Golay sequences are used [15]. The quadri-phase or complex Golay complementary sequences can be used to create quadri-phase Hadamard matrices and the resulting quadri-phase spreading sequences can be easily used in DS CDMA systems.

Holzmann and Kharaghani have published in [16] the results of a computer search for quadri-phase Golay sequences of lengths up to 13, and in [14] a methodology for designing of quadri-phase Golay sequences is given.

To show a usefulness of complementary Golay sequences for asynchronous DS CDMA systems, let us consider here the correlation properties of spreading sequence sets defined by Hadamard matrices derived from the complementary Golay sequences of length 13 and 16.

Example 1
In [16], two quadri-phase complementary Golay sequences of length 13 are presented. These are:

$$S_{1\text{-}13} = [\ 1\ 1\ 1\ j\ -1\ 1\ 1\ -j\ 1\ -1\ 1\ -j\ j\ ]$$

$$S_{2\text{-}13} = [\ 1\ j\ -1\ -1\ -1\ j\ -1\ 1\ 1\ -j\ -1\ 1\ -j\ ]$$

To construct the set of 26-chip spreading sequences, we first obtain two circulant matrices of order 13, **A** and **B,** and then substitute them into Eq. (2) to obtain the Hadamard matrix of order 26.

The resulting 26-chip Golay-Hadamard sequences are characterised by the following correlation parameters:

$$C_{\max} = 0.9615$$
$$R_{CC} = 0.9777$$
$$R_{AC} = 0.5574$$

where $C_{\max}$ denotes the maximum value of a peak in aperiodic cross-correlation functions between any pair of the sequences, R_{CC} denotes the mean square aperiodic cross-correlation for the set of sequences [17], and R_{AC} denotes the mean square aperiodic autocorrelation value for the sequence set.

Example 2
Another tested set of is obtained from two 16-chip bipolar Golay complementary sequences, $S_{1\text{-}16}$ and $S_{2\text{-}16}$, also given in [16]:

$S_{1\text{-}16}$ = [++- - -+-+++++-++-]

$S_{2\text{-}16}$ = [++- - -+-+- - - -+- -+]

The resulting set of 32 Golay-Hadamard sequences of length 32 has the following parrameters:

$$C_{\max} = 0.9688$$
$$R_{CC} = 0.9849$$
$$R_{AC} = 0.4688$$

From both examples it is clearly visible, due to the low values of R_{AC} that the resulting sets of spreading sequences have both very good synchronisation properties, much better than it is in the case of

Walsh-Hadamard sequences, where for the length of 32 we have:

$$C_{\max} = 0.9688$$
$$R_{CC} = 0.7873\,.$$
$$R_{AC} = 6.5938$$

3. Sequence Modification Method

Further improvement to the values of correlation parameters of the sequence sets based on Golay-Hadamard matrices, can be obtained using the method introduced in [6].

That method is based on the fact that for a matrix **H** to be orthogonal, it must fulfil the condition

$$\mathbf{H}\mathbf{H}^T = N\mathbf{I} \tag{4}$$

where $\mathbf{H}^T$ is the transposed Hadamard matrix of order N, and $\mathbf{I}$ is the $N \times N$ unity matrix. The modification is achieved by taking another orthogonal $N \times N$ matrix $\mathbf{D}_N$, and the new set of sequences is based on a matrix $\mathbf{W}_N$, given by:

$$\mathbf{W}_N = \mathbf{H}\mathbf{D}_N \tag{5}$$

The matrix $\mathbf{W}_N$ is also orthogonal, since:

$$\mathbf{W}_N \mathbf{W}_N^T = \mathbf{H}\mathbf{D}_N (\mathbf{H}\mathbf{D}_N)^T = \mathbf{H}\mathbf{D}_N \mathbf{D}_N^T \mathbf{H}^T \tag{6}$$

and because of the orthogonality of matrix $\mathbf{D}_N$, we have

$$\mathbf{D}_N \mathbf{D}_N^T = k\mathbf{I} \tag{7}$$

where k is a real constant. Substituting (7) into (6) yields

$$\mathbf{W}_N \mathbf{W}_N^T = k\mathbf{H}\mathbf{H}^T = kN\mathbf{I}\,. \tag{8}$$

In addition, if $k = 1$, then the sequences defined by the matrix $\mathbf{W}_N$ are not only orthogonal, but possess the same normalization as the sequences defined by the original Hadamard matrix **H**. However, other correlation properties of the sequences defined by $\mathbf{W}_N$ can be significantly different to those of the original sequences.

From equation (5) it is not clear how to chose the matrix $\mathbf{D}_N$ to achieve the desired properties of the sequences defined by the $\mathbf{W}_N$. In addition, there are only a few known methods to construct the orthogonal matrices. However, a simple class of orthogonal matrices of any order are diagonal matrices with their elements $d_{i,j}$ fulfilling the condition:

$$\left|d_{m,n}\right| = \begin{cases} 0 \text{ for } m \neq n \\ k \text{ for } m = n \end{cases}; \quad m, n = 1, \ldots, N \tag{9}$$

To preserve the normalization of the sequences, the elements of $\mathbf{D}_N$, being in general complex numbers, must be of the form:

$$d_{m,n} = \begin{cases} 0 & \text{for } m \neq n \\ \exp(j\phi_m) & \text{for } m = n \end{cases}; \quad m, n = 1, \ldots, N \tag{10}$$

where the phase coefficients ϕ_m; m = 1, 2, …, N, are real numbers taking their values from the interval [0, 2π). The values of ϕ_m; m = 1, 2, …, N, can be optimised to achieve the desired correlation and/or spectral properties, e.g. minimum out-off-phase autocorrelation or minimal value of peaks in aperiodic cross-correlation functions.

From the application point of view, the most important classes of spreading sequences are bipolar sequences and quadri-phase sequences. In the first case, the values of ϕ_m; m = 1, 2, …, N, are limited to {0, π}, and in the second case the coefficients ϕ_m can take values from the set {0, 0.5π, π, 1.5π}.

We now show the results of applying the diagonal modification method to improve correlation properties of sequence sets constructed in the previous section. As an exhaustive search, in both cases, would require too much of processing time, we have performed a random search of 1000 trials, and searched for the diagonal matrix producing the lowest value of $C_{\max}$ for the modified sequence set.

Example 3
In this example we show the results of minimizing $C_{\max}$ for the 26-chip sequence set developed in Example 1. After 10000 trials, the lowest value of $C_{\max}$ has been achieved for the matrix D_{26} with the following quadri-phase symbols on the diagonal:

[11j –1–j –j –jj1–1jj11–1–1–1–1–1111j–1–11].

The modified sequences have the following correlation parameters:

$$C_{\max} = 0.4300$$
$$R_{CC} = 0.9599$$
$$R_{AC} = 1.0016$$

In Fig.1 and Fig.2, we present the plots of the maximum peak magnitudes for the cross-correlation functions between any possible pair of the modified sequence set, and the maximum peaks in the auto-correlation functions for all the modified sequences, respectively. For the comparison, we also present there the corresponding plots for the unmodified sequence set.

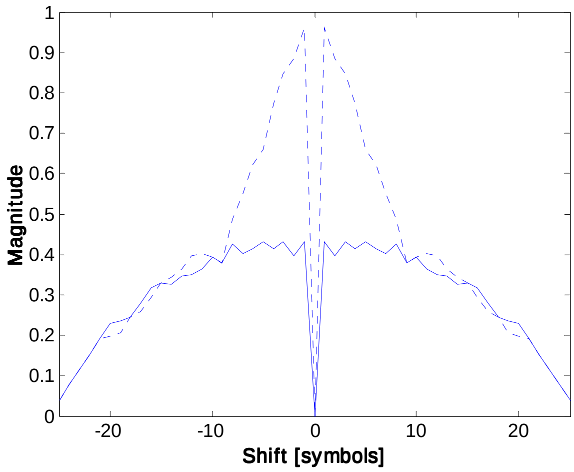

Figure 1: Peak magnitude of aperiodic cross-correlation functions for the families of spreading sequences; dotted line – sequences defined by original Golay-Hadamard matrix, solid line - modified sequences.

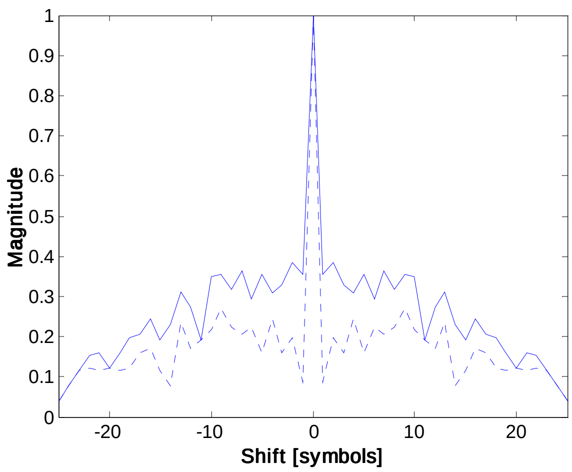

Figure 2: Peak magnitude of aperiodic auto-correlation functions for the families of spreading sequences; dotted line – sequences defined by original Golay-Hadamard matrix, solid line - modified sequences.

Example 4

In this example we show the results of minimizing $C_{\max}$ for the 32-chip sequence set developed in Example 1. Because the original sequences are bipolar, we first searched for the modification matrix D_{32} with the bipolar diagonal elements. After 10000 trials, the lowest value of $C_{\max}$ has been achieved in 9 different cases. The corresponding diagonals and the resulting correlation parameters are listed in Table 1.

In Fig. 3 and 4, we present the plots of the maximum peak magnitudes for the cross-correlation functions between any possible pair of the modified sequence set, and the maximum peaks in the auto-correlation functions for all the modified sequences, respectively. These modified sequences have been obtained using D_{32} with the diagonal:

[--+-+----+-++-+--+-+-++++-+-+-++]

Table 1: The diagonals of D_{32} and the corresponding correlation parameters.

Diagonal elements	$C_{\max}$	R_{CC}	R_{AC}
+++---++-----++++-+--+++--+++-+-	0.4688	0.9638	0.1211
++--++---+++-++---+---+-+--+++++	0.4688	0.9724	0.8555
+++--++-----++----++--+-+++--+-+	0.4688	0.9735	0.8203
--+++++-+-++++-+-+--+--+-++-+---	0.4688	0.9711	0.8945
++--+--+++++++-----+-++++---++++	0.4688	0.9676	1.0039
-+++--+--+++-+--+---+--+----++-+	0.4688	0.9681	0.9883
-+---+-++-++--+-+-+--++-++-+++-+	0.4688	0.9676	1.0039
-+-+-+++--+++-+--++-+++-++-+--++	0.4688	0.9675	0.0078
--+-+----+-++-+--+-+-++++-+-+-++	0.4688	0.9594	0.2578

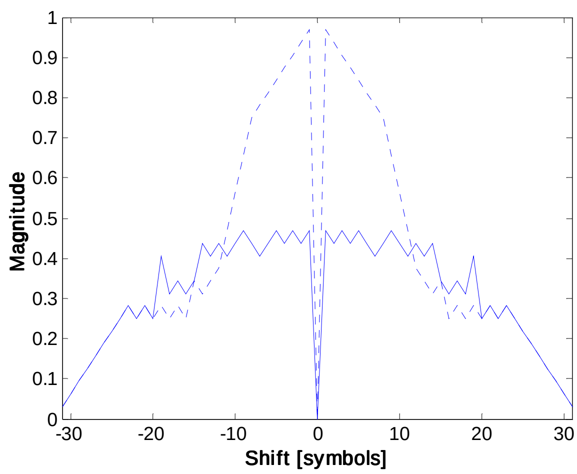

Figure 3: Peak magnitude of aperiodic cross-correlation functions for the families of spreading sequences; dotted line – sequences defined by original Golay-Hadamard matrix, solid line - modified bipolar sequences.

Even better results can be obtained if the modifying matrix D_{32} has the quadri-phase diagonal elements. After 10000 random trials, the diagonal leading to the lowest value of $C_{\max}$ has been found to be:

[-1-1j-jj-j-j-jj11-11-1j-1-11j1-1j-1-1-j11-j-j-j-j1],

The following parameters of the modified sequence set have been achieved:

$$C_{\max} = 0.3763$$
$$R_{CC} = 0.9681$$
$$R_{AC} = 0.9902$$

In Fig. 5 and 6, we present the plots of the maximum peak magnitudes for the cross-correlation functions between any possible pair of the modified sequence set, and the maximum peaks in the auto-correlation functions for all the modified sequences, respectively.

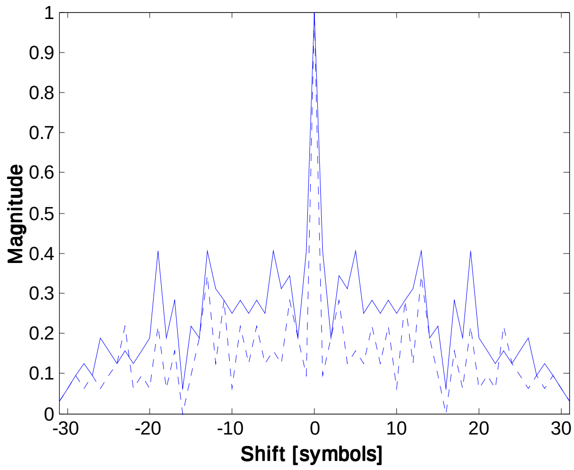

Figure 4: Peak magnitude of aperiodic auto-correlation functions for the families of spreading sequences; dotted line – sequences defined by original Golay-Hadamard matrix, solid line - modified bipolar sequences.

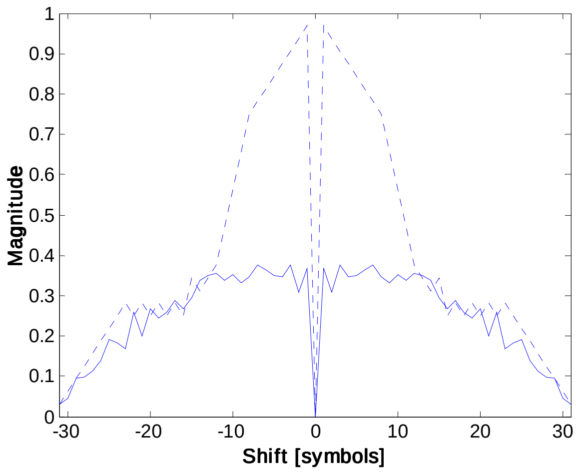

Figure 5: Peak magnitude of aperiodic cross-correlation functions for the families of spreading sequences; dotted line – sequences defined by original Golay-Hadamard matrix, solid line - modified quadri-phase sequences.

4. System Simulation Results

To better check the usefulness of the designed sequence sets for DS CDMA applications, we have simulated a multi channel DS CDMA system utilizing BPSK signals spread by these signatures. We transmitted 500 frames of 524 bits per frame in every channel with 8 randomly selected channels simultaneously active.

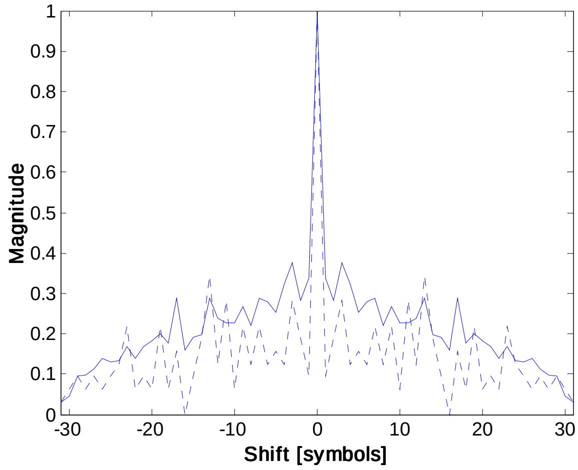

Figure 6: Peak magnitude of aperiodic auto-correlation functions for the families of spreading sequences; dotted line – sequences defined by original Golay-Hadamard matrix, solid line - modified quadri-phase sequences.

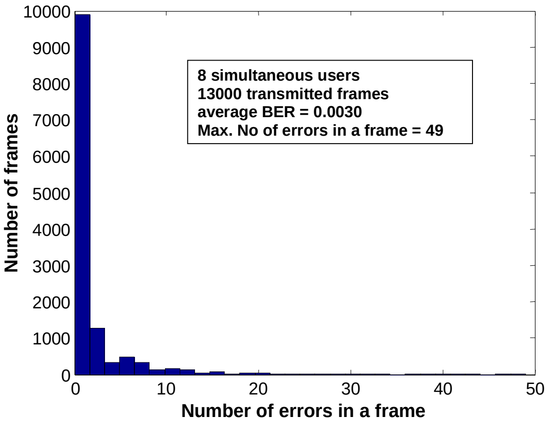

Figure 7: Histogram of a number of errors in transmitted frames for the DS CDMA system utilizing modified quadri-phase Golay-Hadamard spreading sequences of length 26.

The simulation has been performed for the set of 26-chip modified quadri-phase sequences, for 32-chip bipolar modified sequences, and for the set of 32-chip quadri-phase sequences. The achieved average bit error rates (BER) are 0.0030, and 0.0011, respectively, and the histograms of errors distributions are given in Fig. 7, 8, and 9.

5. Conclusions

In the paper, we considered an application of complementary Gold sequences to create orthogonal spreading sequences for asynchronous DS CDMA applications. We presented the technique of creating the original Golay-Hadamard matrices, and later a method to modify their correlation characteristics to achieve better performance in case of asynchronous operation.

In the considered cases, i.e. bipolar and quadri-phase sequences, there are only limited numbers of modifications possible for a given sequence length. Unfortunately, examining all of them becomes impractical even for a modest sequence length, e.g. N = 26 or 32. In the paper, we used a random search to find the appropriate modifications. However, other, more advanced search methods may produce even better designs.

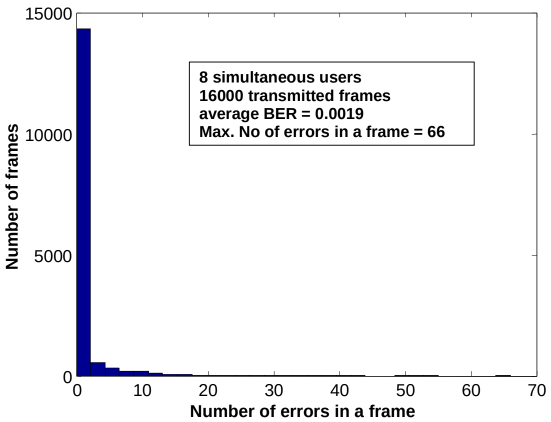

Figure 8: Histogram of a number of errors in transmitted frames for the DS CDMA system utilizing modified bipolar Golay-Hadamard spreading sequences of length 32.

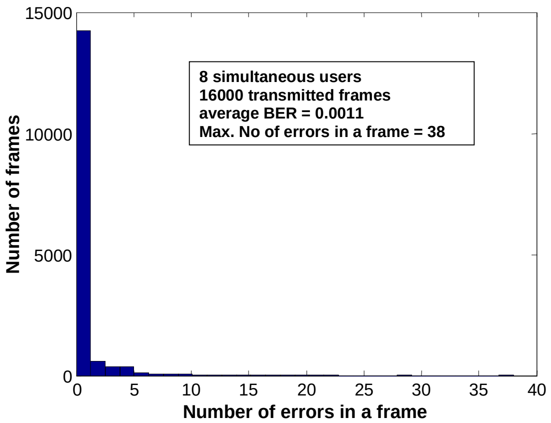

Figure 9: Histogram of a number of errors in transmitted frames for the DS CDMA system utilizing modified quadri-phase Golay-Hadamard spreading sequences of length 32.

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