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RESCHEDULING A SOVEREIGN DEBT: A THEORETICAL ANALYSIS

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ABSTRACT

If the probability of default increases with the debt burden, the set of feasible rescheduling schemes of a sovereign debt to private creditors can be depicted by an inverted U-shaped curve in the annual repayment-repayment period plane. If both the creditor and the indebted sovereign are risk averse and maximise expected utility from debt repayments and unpaid debt, respectively, the Pareto efficient rescheduling schemes might be located on the upward sloping side of the feasible rescheduling curve, displaying a positive relationship between the renegotiated annual repayment and the renegotiated repayment period, rather than the conventional trade-off. (*JEL* F34)

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I Introduction

The heavy burden of external debts on the developing countries has led to a heightened concern about these countries' ability and commitment to repay their liabilities. This concern is rooted in the fact that, unlike a private debt, a sovereign debt is not subjected to laws of bankruptcy or to an enforcement of a collateral. Thus, when the potential penalties on default are not substantial, a rise in a country's level of indebtedness reduces that country's inclination to service its external liabilities. It is possible, however, that the probability of unilateral repudiation is moderated by a country's concern about the adverse effect of a default on its trustworthy reputation and, subsequently, on its access to foreign loans (Eaton and Gersovitz, 1981; Kletzer, 1984; and Grossman and Van Huyck, 1988) and by a threat of direct sanctions (Bulow and Rogoff, 1989).

In view of the developing countries' high level of indebtedness, it has been argued by Krugman (1989) that a country's financial obligations act like a high marginal tax rate which deters governments from taking painful measures to improve a country's economic performance and discourages capital formation. Thus, when an indebted country is on the downward sloping side of the 'debt relief Laffer curve', both debtor and creditor can benefit from a debt-reduction. Kenen (1990) and Sachs (1990) have asserted that the external debt's overhang is a primary cause for economic slow-down for many of the developing countries and have recommended the establishment of an international institution for organising debt relief and debt-rescheduling negotiations between private creditors and indebted countries. In contrast, Bulow and Rogoff (1990) have argued that the external debt problem is a symptom of poor economic management and growth, that the presence of official creditors in debt negotiations ossifies the bargaining position of private creditors, and that efficiency would be best served by having less official involvement.

The basic approach for solving the developing countries' debt problem has been rescheduling and concerted lending. Alternatively, market-based debt-reduction schemes, such as debt buy-backs, securitisation and debt-equity swaps, have been suggested. They have been expected to reduce both the exposure of banks and the external liabilities of countries without the need for a collective bargaining of new credit or debt forgiveness, which might be accompanied by a severe free-rider problem. However, as has been shown by Krugman (1989), these market-based debt-reduction schemes cannot serve as an alternative to the orthodox strategy of rescheduling and concerted lending.

Assuming that a country's inclination to repay its liabilities decreases with its debt burden, but that the costs of repudiation are substantial, and recognising that a short-term illiquidity does not necessarily lead to a long-term insolvency, this paper firstly analyses the set of debt-rescheduling schemes which are feasible for both the debtor and the creditor. Subsequently, the paper analyses the Pareto efficient rescheduling schemes for the creditor and the debtor under the assumption that they are risk averse and maximise expected utility from the sum of debt repayments and from the unpaid debt, respectively. The structure and the major findings of the paper are as follows.

Section II analyses the debt-repayment constraint and the set of the rescheduling schemes which are feasible for both the debtor and the creditor. It shows that this set can be depicted by an inverted U-shaped curve in the annual repayment-repayment period plane.

Section III describes the creditor's choice of a rescheduling scheme from the set of the feasible schemes. It postulates that the creditor's choice of the annual debt-repayment and the debt-repayment period maximises the creditor's expected utility from the perceived sum of annual debt-repayments subject to the debt-repayment constraint, and provides a geometrical presentation of the creditor's rescheduling choice. It is argued that the creditor's indifference curves in the annual repayment-repayment period plane can be upward sloping and hence the creditor's optimal rescheduling scheme can be on the upward sloping side of the feasible rescheduling curve. That is, it is possible that the creditor prefers to bear a lower risk of default and hence requires the smallest amount of annual repayment between the two feasible ones associated with his preferred length of the repayment period.

Section IV analyses the properties of the creditor's most preferred rescheduling scheme. In particular, it highlights the effects of the international interest rate, the country's external liabilities and potential output and the creditor's market share on the creditor's choice of a rescheduling scheme.

Section V analyses the debtor's preferences about rescheduling schemes and indicates that the debtor's indifference curves are downward sloping in the plane spanned by the annual repayment and the repayment period and that the debtor prefers rescheduling schemes characterised by a shorter repayment period but not necessarily a smaller annual repayment due to an increase in the probability of admissible default as the annual repayment rises.

Section VI considers both the creditor's and the debtor's preferences about the feasible rescheduling schemes and demonstrates that it is possible that the Pareto efficient set of rescheduling schemes is located on the upward sloping side of the feasible rescheduling curve below (and including) the creditor's choice and hence revealing a positive relationship between the length of the repayment period and the amount of the annual repayment rather than a trade-off.

Finally, section VII concludes the paper with a brief summary.

II. Debt-Repayment Constraint and the Feasible Rescheduling Curve

The following analysis considers a situation in which a country is liable to a single private creditor, or to a well-coordinated syndicate of private creditors. It is assumed that the country's external liabilities cannot be serviced in accordance with the originally contracted terms, and that both the indebted country and its creditor have a sufficient incentive for rescheduling the country's liabilities (D_0) over a T-year period in equal, in constant prices, annual repayments (M) of principal and interest. An underlying rationale for equal annual repayments can be a creditor's aversion and a debtor's aversion toward excessive oscillations in their business cycles that might stem from a variable annual repayment.

Dealing with a sovereign country and assuming that the debtor's asset holdings abroad are negligible, the creditor is aware of the fact that his control over the rescheduled repayment is fairly limited. Nevertheless, it is assumed that the probability of default (p) perceived by the creditor is smaller than one due to the substantial costs that can be inflicted directly by retaliatory sanctions, such as a ban on trade and credit, and indirectly through a loss of trustworthy reputation. It is assumed further that the perceived probability of default: 1. increases with the burden of servicing the debt, which is measured by the ratio of the *renegotiated* annual repayment (M) to the indebted country's level of gross national product (or export) in constant prices (Y); and 2. decreases with the creditor's ability to retaliate by limiting the country access to the international credit market proportional to the creditor's market share (s). These assumptions are incorporated into the following linear expression:

$$p = \alpha(M/Y) - \lambda s. \tag{1}$$

Here, α is a positive scalar indicating the marginal effect of the debt burden on the country's inclination to default. Following Berg and Sachs (1988), it can be argued that α is affected by the indebted country's social, political and economic structure. Similarly, λ is a positive scalar indicating the deterrent effect of the creditor's power in the international credit market.

It is also assumed that the indebted country's investment is adversely affected by the annual debt repayments through a decline in the governmental budget for investment and through an (expected and actual) increase in tax rates which, in the presence of capital mobility, discourages capital formation and repatriation of flight capital (Helpman, 1989). Since the country's GNP is directly related to investment, this assumption implies that the indebted country's GNP is adversely affected by the annual debt repayment as indicated, for convenience, by the following linear equation:

$$Y = \bar{Y} - \delta M, \delta \geq 1 \tag{2}$$

where $\bar{Y}$ is the highest level of GNP attainable had the annual debt-repayments been nil, and δ is a positive scalar indicating the potentially marginal and adverse effect of the renegotiated annual debt-repayment on the country's GNP. Equation 2 implies that the renegotiated debt-repayment cannot exceed $\bar{Y}/\delta$.

The substitution of equation 2 into equation 1 for Y implies that the probability of default perceived by the creditor is given by

$$p = \alpha M / (\bar{Y}-\delta M) - \lambda s. \tag{3}$$

Since $0 < p < 1$ and increases with the renegotiated annual debt repayment, M should be restricted further to lie within the open interval (M_{min}, M_{max}), where $p(M_{min}) = 0$ and $p(M_{max}) = 1$. The substitution of these boundary conditions into equation 3 implies:

$$M_{min} = \left(\frac{\lambda s}{\alpha+\delta\lambda s}\right)\bar{Y} \tag{4}$$

and

$$M_{max} = \left(\frac{1+\lambda s}{\alpha + \delta(1+\lambda s)}\right)\bar{Y}. \tag{5}$$

Note further that $M_{max} < \bar{Y}/\delta$.

The *actual* annual debt-repayment (Q) is perceived by the creditor to be distributed as follows:

$$Q = \begin{cases} M & 1-\alpha M/(\bar{Y}-\delta M) + \lambda s \\ 0 & \alpha M/(\bar{Y}-\delta M) - \lambda s. \end{cases} \tag{6}$$

The nature of the analysis will not be substantially changed by replacing the assumption of a complete default by a partial default, i.e., an actual annual repayment of γM dollars ($0<\gamma<1$) with probability p.

The analysis of the *debt-repayment constraint* is conducted under the simplifying assumption that *a-priori* the distribution of the actual annual debt-repayment is perceived by the creditor to be stable over the repayment period. That is, the parameters α, δ and λ, as well as the creditor's market share and the indebted country's potential output, remain the same over the T years of the repayment period. Alternatively, one may incorporate, for example, an anticipation of economic growth, or economic slowdown, into the analysis by multiplying $(\bar{Y} - \delta M)$ by a shift factor g^t, where g is a positive scalar (equal to one plus the anticipated growth rate). In view of equation 4, the sum of the expected debt-repayments (SEDR) discounted by the international annual (real) interest rate (r), which is assumed to be constant, over the T-year period is given by:

$$\text{SEDR} = \sum_{t=0}^{T} [1-\alpha M/(\bar{Y}-\delta M)+\lambda s]M/(1+r)^t$$

$$= [1-\alpha M/(\bar{Y}-\delta M)+\lambda s]M[\beta(1-\beta^T)/(1-\beta)] \quad (7)$$

where

$$\beta = 1/(1+r). \quad (8)$$

Assuming that the debtor and the creditor have identical assessments of the probability of default (i.e., they both use equation 3 and assign identical values to its parameters α, δ, λ, s and $\bar{Y}$) and that the creditor believes that a short-term, or current, illiquidity does not necessarily lead to a long-term insolvency and prefers to tolerate the uncertainty about future repayments and to renegotiate debt-repayment's terms with an allowance for episodes of temporary illiquidity rather than retaliate, the rescheduling schemes (M, T) which are feasible for both the debtor and the creditor should obey the following debt-repayment constraint:

$$[1 - \alpha M/(\bar{Y}-\delta M)+\lambda s]M[\beta(1-\beta^T)/(1-\beta)] = D_0. \tag{9}$$

This constraint can be equivalently rendered as

$$T = \{\log\{\beta-(1-\beta)D_0/[1-\alpha M/(\bar{Y}-\delta M)+\lambda s]M\}/\log\beta\} - 1. \tag{10}$$

The set of all the feasible rescheduling schemes can be depicted by a curve in the M-T plane, which is referred hereafter as the *feasible rescheduling curve*. By differentiating equation 10 with respect to M and by recalling the assumption that the probability of default rises with M, it can be shown that, unlike the case of a private debt where a collateral can be enforced, along the feasible rescheduling curve of a sovereign debt the period of repayment is not necessarily shortened by an increase in the annual repayment. In fact, it can be shown that up to a critical level of annual debt-repayment, the rise in the probability of default stemming from an increase in M is dominant and hence the creditor should be compensated by a longer repayment period. However, beyond that critical level, the mere burden of the annual debt-repayment dominates the probability of default and consequently the debtor has to be compensated by a shorter repayment-period. This relationship between the length of the repayment-period and the annual repayment along the feasible rescheduling curve is described in greater detail by Proposition 1 and Figure 1.

PROPOSITION 1: *Along the feasible rescheduling curve*

$$\frac{dT}{dM}\begin{cases} >0 \text{ for} & M_{min} < M < \bar{Y}/\delta - \phi \\ =0 \text{ for} & M = \bar{Y}/\delta - \phi \\ <0 \text{ for} & \bar{Y}/\delta - \phi < M \leq M_{max} \end{cases} \tag{11}$$

where

$$\phi = \frac{\bar{Y}}{\delta}\sqrt{1 - \frac{(1+\lambda s)}{[(1+\lambda s)\delta + \alpha]}}. \tag{12}$$

The proof of this proposition is provided in the Appendix.

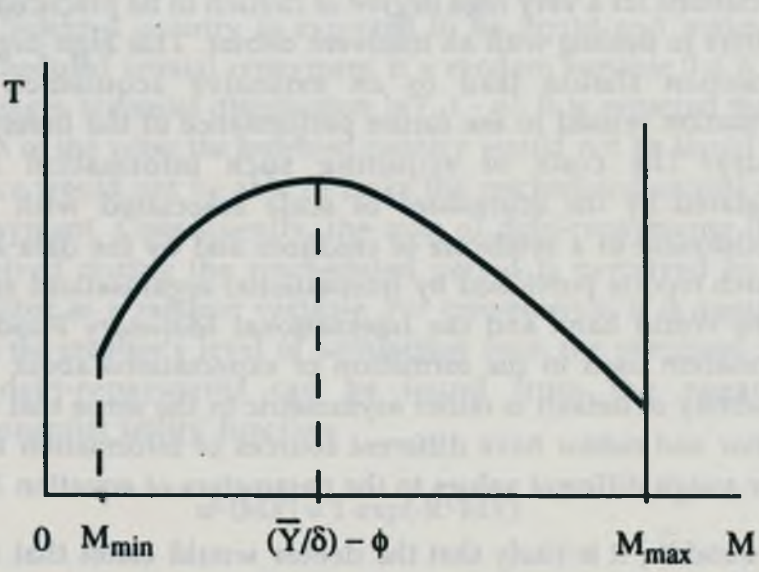

Figure 1. The Feasible Rescheduling Curve

Figure 1 illustrates the above-mentioned proposition by displaying the set of all the rescheduling schemes which are feasible for both the debtor and the creditor,

$$\Omega = \{(M,T)\in \mathcal{R}^2_+ : [1-\alpha M/(\bar{Y}-\delta M)+\lambda s]M[\beta(1-\beta^T)/(1-\beta)]=D_0\},$$

as an inverted U-shaped curve in the plane spanned by M and T. Equation 9 indicates further that an increase in the debtor's initial liabilities and an increase in the international interest rate shift the feasible rescheduling curve upward, whereas an increase in the country's potential GNP level and an increase in the creditor's market share in the international credit market shift the curve downward by reducing the probability of default.

The above analysis of the feasible rescheduling curve has been conducted under the assumption that the debtor's state is common knowledge. This assumption can be justified by the expectations for a very high degree of caution to be practiced by creditors in dealing with an insolvent debtor. This high degree of caution should lead to an extensive acquisition of information related to the future performance of the indebted country. The costs of acquiring such information are moderated by the economies of scale associated with the establishment of a syndicate of creditors and by the data and research reports publicised by international organisations such as The World Bank and the International Monetary Fund. If information used in the formation of expectations about the probability of default is rather asymmetric in the sense that the creditor and debtor have different sources of information and hence assign different values to the parameters of equation 3 α, δ, λ, s and $\bar{Y}$, it is likely that the debtor would claim that the probability of default is lower than that perceived by the creditor, for any given amount of annual debt-repayment, in order to obtain a shorter repayment period than that required by the creditor. (Cf. Kletzer, 1989.) That is, under the alternative assumption of asymmetric information about the probability of default, it is likely that the feasible rescheduling curve considered by the debtor lies below the feasible rescheduling curve considered by the creditor.

The following sections describe the creditor's and the debtor's choices of a rescheduling scheme from the feasible set.

III. Creditor's Choice of a Feasible Rescheduling Scheme

It is postulated that the creditor's choice of an annual debt-repayment and a debt-repayment period is a feasible combination (M^*, T^*) which maximises his expected utility from the perceived sum of the actual debt-repayments subject to the debt-repayment constraint described in the previous section. By virtue of equation 6, the number of years in which the indebted country is expected to be liquid and make the rescheduled annual repayment is a random variable $0 \leq X \leq T$ having a binomial distribution $b(T, 1 - p)$. It is expected that in $T - X$ of the years the indebted country would not be liquid and hence would not be able to make the rescheduled annual debt repayment. Consequently, the sum of debt-repayments (MX) received during the rescheduled period is perceived by the creditor as a random variable. For convenience, it is assumed that the creditor's level of satisfaction from the perceived sum of debt-repayments can be found from the negative-exponential utility function:

$$u^c(MX) = 1\text{-}\exp(-R^c MX) \tag{13}$$

where R^c is a positive scalar indicating the creditor's degree of absolute risk-aversion. (For a discussion of the properties and generality of the negative-exponential utility function see Hammond, 1974.)

By taking the expectation of both sides of equation 13, the creditor's expected utility from the accumulated debt-repayments can be expressed as

$$E[u^C(MX)] = 1 - E[\exp(-R^C MX)]. \tag{14}$$

Recalling that X has a binomial distribution, $E[\exp(-R^C MX)]$ can be interpreted as the moment-generating function of the binomial distribution evaluated at $-R^C M$. Therefore,

$$E[u^C(MX)] = 1 - [p+(1-p)\exp(-R^C M)]^T \tag{15}$$

and the creditor's rescheduling problem can be regarded equivalently as choosing $M_{min}<M<M_{max}$ and T as to maximise the right-hand side of equation 15 subject to the expected debt-repayment constraint 10 and the perceived probability of default displayed by equation 3. An outline of the mathematical solution to this problem is given in the Appendix.

Due to the computational complexity of the problem, it is useful to consider the geometrical presentation of the creditor's preferences and rescheduling choice in the M-T plane. A creditor's indifference curve can be defined as the locus of all combinations of M and T which yield the same level of expected utility from the sum of debt repayments.

PROPOSITION 2: *The creditor's indifference curves are upward sloping in the M-T plane for all*

$$M > \frac{1}{R^c} \ln\left[\frac{p' + (1-p)R^c}{p'}\right].$$

The proof of this proposition is given in the Appendix.

Since the effect of an increase in the annual repayment on the probability of default is positive and increases with M (i.e., $p'>0$ and $p''>0$, as indicated by equation 3, $(1/R^c)\ln\{[p'+(1-p)R^c] / p'\}$ diminishes as M increases. Thus, the larger the value of M the greater the likelihood that the rescheduling schemes constituted by M are lying on upward sloping indifference curves. Recalling that $p(M_{max}) = 1$, the right-hand side of the inequality indicated in proposition 2 is equal to zero and hance the creditor's indifference curves are upward sloping in the vicinity of M_{max}. Moreover, equation 3 indicates that the larger α and δ and the smaller $\bar{Y}$, the smaller the value of $(1-p)/p'$ and, consequently, the larger the range of M for which the creditor's indifference curves are upward sloping. These arguments and the fact that $p(M_{min}) = 0$ imply that proposition 2 can be extended as follows.

PROPOSITION 2': *If* $M_{min} \geq \frac{1}{R^c} \ln[1 + R^c / p'(M_{min})]$, *the creditor's indifference curves are upward sloping in the M-T plane for all* $M_{min} < M < M_{max}$. .

The analysis of the creditor's choice of a rescheduling scheme proceeds under the assumption that the intrinsic value of p' is considerably large so that the condition indicated in proposition 2′ is satisfied and, consequently, the creditor's indifference curves are upward sloping in the M-T plane for all $M_{min}<M<M_{max}$.

The differentiation of equation A.17 with respect to T implies that as long as the creditor's indifference curves are upward sloping

$$\frac{d^2T}{dMdT} \bigg| E(u^c) = \text{const.} > 0. \tag{16}$$

Thus, the longer the repayment period the steeper the indifference curve. This, in turn, implies that the creditor's indifference curves are convex in the M-T plane and that as one moves upward along a perpendicular line in the M-T plane the slope of the creditor's indifference curves increases.

By virtue of equation 15 and the fact that $0<p+(1-p)\exp(-R^cM)<1$, the creditor's expected utility increases with T for any given level of M. In view of this property and that the creditor's indifference curves are upward sloping, the creditor's expected utility from the perceived sum of the actual debt-repayments declines with the annual debt-repayment for any given period of repayment. The underlying rationale is that a rise in the annual payment, *ceteris paribus*, increases the probability of default and, consequently, lowers the expectations for debt-repayment and also raises the costs of risk bearing.

Since the creditor's expected utility rises with T for any given value of M and the creditor's indifference curves are, under the aforementioned assumption, upward sloping and convex, there exists a unique and interior solution to the creditor's rescheduling problem — the creditor's optimal rescheduling scheme is the tangency point $A=(M^*, T^*)$ between the upward sloping side of the feasible rescheduling curve and the highest possible indifference curve as depicted in Figure 2. Since this tangency point is located on the upward sloping segment of the feasible rescheduling curve, the creditor's choice of the annual repayment lies in the open interval $[M_{min}, (\bar{Y}/\delta - \phi)]$. That is, given the debt-repayment period T^*, the creditor prefers to bear a lower risk of default and rather selects M^* than the higher feasible level of annual debt-repayment M_1.

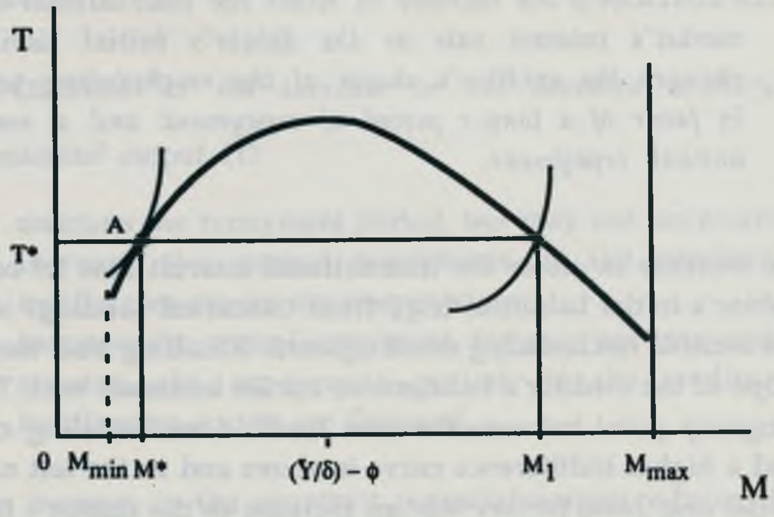

Figure 2. The Creditor's Choice of a Feasible Rescheduling Scheme

IV. Comparative Statics of the Creditor's Choice of a Feasible Rescheduling Scheme

The analysis of the properties of the creditor's choice of the debt rescheduling scheme described above is summarised by propositions 3-5. Each proposition is followed by a brief discussion that outlines the proof and the underlying rationale.

PROPOSITION 3: *An increase in either the international credit market's interest rate or the debtor's initial liabilities changes the creditor's choice of the rescheduling scheme in favor of a longer period of repayment and a smaller annual repayment.*

An increase in either the international interest rate (r) or the debtor's initial liabilities (e.g., from concerted lending) shifts the feasible rescheduling curve upward. Recalling also that the slope of the creditor's indifference curves increases with T, the tangency point between the new feasible rescheduling curve and a higher indifference curve is above and to the left of the initial one. Note further that an increase in the debtor's initial liabilities might also affect the slope of the creditor's indifference curves by reducing the creditor's tangible wealth and consequently changing his attitude toward risk. However, the differentiation of equation A.17 with respect to R^C does not provide a clear indication of the direction of that change. If an increase in the creditor's degree of risk aversion moderates the slope of the indifference curves, it reduces, or even offsets, the decline in M imposed by the upward shift of the feasible rescheduling curve. Thus, if the creditor's degree of absolute risk aversion rises with a reduction in his tangible wealth, the increase in the debtor's initial liabilities might also increase the annual debt repayment required by the debtor. In any other circumstances, the annual repayment required by the creditor decreases as the debtor's initial liabilities increase. This proposition also implies that a debt-reduction shortens the repayment period. However, a debt-reduction does not affect the creditor's tangible wealth and (subsequently) the creditor's degree of risk aversion and indifference curves. Thus, a debt-reduction raises the annual debt repayment required by the creditor.

PROPOSITION 4: *An increase in the indebted country's potential output* $(\bar{Y})$

i. shortens the repayment period, but may not necessarily increase the annual repayment, if the creditor's indifference curves are steepened; or
ii. increases the annual repayment, but may not necessarily shorten the repayment period, if the creditor's indifference curves are flattened.

An increase in the country's potential output reduces the probability of default perceived by the creditor and debtor for any given amount of annual debt-repayment and hence shifts the feasible rescheduling curve downward. By virtue of equation 3, the greater the marginal effect of the debt burden on the country's inclination to default (i.e., α), the larger the downward shift of the feasible rescheduling curve for a given increase in the country's potential output. Moreover, equation A.17 indicates that an increase in the country's potential output may also change the slope of the creditor's indifference curves since p, p′ and θ are affected by $\bar{Y}$. However, the differentiation of equation A.17 with respect to $\bar{Y}$ indicates that the change in the slope of the creditor's indifference curves is not clear *a-priori*. If the increase in $\bar{Y}$ steepens the creditor's indifference curves, the tangency point between the lower rescheduling curve and a steeper indifference curve should reveal a shorter repayment period but not necessarily a larger annual repayment. The steeper the (new) indifference curve and the larger the downward shift of the feasible rescheduling curve (i.e., the larger the α), the smaller the likelihood of a larger annual repayment. In contrast, if the increase in $\bar{Y}$ flattens the creditor's indifference curves, the (new) tangency point should reveal a larger annual repayment but not necessarily a shorter repayment period. The flatter the (new) indifference curve and the smaller the downward shift of the feasible rescheduling curve (i.e., the smaller the α), the smaller the likelihood of a shorter repayment period.

PROPOSITION 5: *An increase in the creditor's market share*

i. shortens the repayment period, but may not necessarily increase the annual repayment, if the creditor's indifference curves are steepened; or
ii. increases the annual repayment, but may not necessarily shorten the repayment period, if the creditor's indifference curves are flattened.

Private creditors can substantially increase their control over the international credit market by increasing their market share through mergers and syndication. Equation 3 indicates that an increase in the creditors' market share raises the potential costs of default for the debtor and hence reduces the probability of default for any given amount of annual debt repayment proportional to λ. In terms of Figure 1, the feasible rescheduling curve shifts downward. Furthermore, equation A.17 indicates that an increase in the creditor's control over the international credit market may also change the slope of the creditor's indifference curves since p, p' and θ are affected by s. However, the differentiation of equation A.17 with respect to s indicates that the change in the slope of the creditor's indifference curves is not clear *a-priori*. Similarly to the discussion of proposition 4, if the increase in the creditor's market share steepens his indifference curves, the tangency point between the lower rescheduling curve and a steeper indifference curve should reveal a shorter repayment period but not necessarily a larger annual repayment. The steeper the (new) indifference curve and the larger the downward shift of the feasible rescheduling curve (i.e., the larger the λ), the smaller the likelihood of a larger annual repayment. In contrast, if the increase in the creditor's control over the international credit market flattens his indifference curves, the (new) tangency point should reveal a larger annual repayment but not necessarily a shorter repayment period. The flatter the (new) indifference curve and the smaller the downward shift of the feasible rescheduling curve (i.e., the smaller the λ), the smaller the likelihood of a shorter repayment period.

V. Debtor's Choice of a Feasible Rescheduling Scheme

In analogy to the creditor's problem, the debtor's objective is postulated as maximising expected utility from actually repaying as little as possible, subject to the expected debt-repayment constraint portrayed by the feasible rescheduling curve. That is, the larger the amount of actually unpaid debt (D_0 - MX) the higher the debtor's level of satisfaction.

Similarly to the analysis of the creditor's preferences, it is assumed that the debtor's preferences can be represented by the following negative exponential utility function

$$u^d = 1\text{-}\exp\{-R^d(D_0\text{-}MX)\} \tag{17}$$

where R^d is a positive scalar indicating the debtor's degree of absolute risk aversion. In which case,

$$E(u^d) = 1\text{-}\exp(-R^dD_0)E[\exp(R^dMX)]. \tag{18}$$

Since X is a random variable having a binomial distribution b(T,1-p), $E[\exp(R^dMX)]$ is the moment-generating function of the binomial distribution evaluated at R^dM and consequently the debtor's expected utility function can be expressed as:

$$E[u^d(D_0\text{-}MX)] = 1\text{-}\exp(-R^d D_0)[p + (1\text{-}p)\exp(R^dM)]^T. \tag{19}$$

PROPOSITION 6: *The debtor's indifference curves are downward sloping in the M-T plane.*

The proof of this proposition is given in the Appendix.

Furthermore, the differentiation of equation 19 with respect to T implies

$$\frac{dE[u^d(D_0\text{-}MX)]}{dT} = -\psi^T \log\psi < 0 \qquad (20)$$

for every M>0 since

$$\psi \equiv p(M) + (1\text{-}p(M)) \exp(R^d M) > 1. \qquad (21)$$

Thus, the debtor's indifference curves which are closer to the origin represent higher levels of expected utility. That is, the debtor prefers feasible rescheduling schemes characterised by an annual repayment smaller than M^* and a repayment period shorter than T^* as well as feasible rescheduling schemes characterised by an annual repayment larger than $\tilde{M}$ and a repayment period shorter than $\tilde{T}$ to the creditor's choice A = (M^*, T^*) as depicted by sections AA' and BB' of the feasible rescheduling curve in Figure 3 below. In the context of Figure 3, the debtor's most preferred rescheduling scheme is A', which is associated with the minimum annual debt repayment. More generally, the solution to the debtor's expected utility maximisation problem (by repaying as little as possible) is a corner one — either the feasible scheme associated with the minimum amount of annual repayment or the feasible scheme associated with the maximum annual repayment. In the latter case, the high probability of admissible default compensates for the large renegotiated annual repayment.

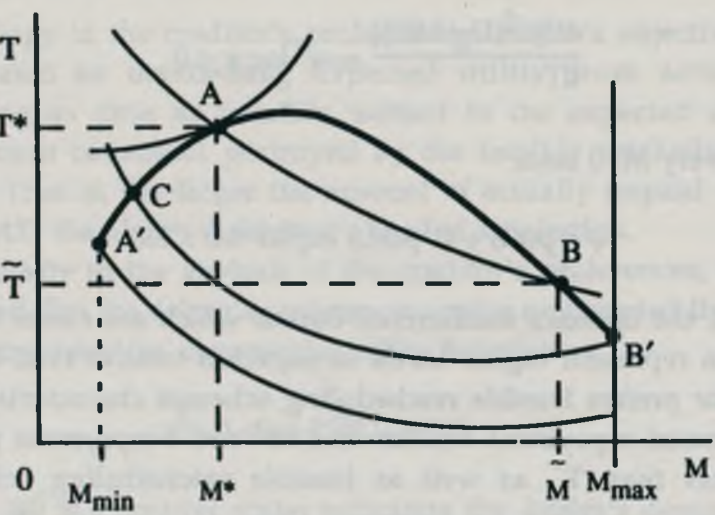

Figure 3. The Debtor's Choice of a Rescheduling Scheme

VI. Pareto Efficient Rescheduling Schemes

The *Pareto efficient rescheduling schemes* constitute a set of all those feasible schemes which are superior to the rest in the sense that either the creditor or debtor is better off while the other is not worse off. The particular combination of the creditor's and debtor's preferences displayed in Figure 4 implies that the set of the Pareto efficient rescheduling schemes might be entirely located on the upward sloping section A′A of the feasible rescheduling curve which is bounded from above by the creditor's most preferred scheme A and from below by the debtor's most preferred scheme A'. The underlying rationale is that by having a choice from section AB the debtor is worse off while the creditor is not better off. Moreover, by having a choice from section A'A the creditor is better off than by having a choice from section BB' while the debtor is not worse off and even better off in section A'C.

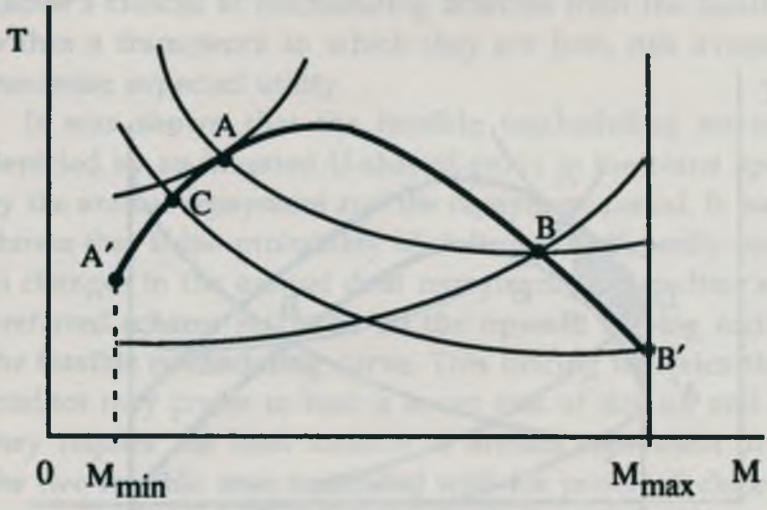

Figure 4. The Pareto Efficient Set of Rescheduling Schemes under Symmetric Information

Section A'A defines the Pareto efficient set of rescheduling schemes from which the rescheduling scheme is likely to be obtained within a bargaining process characterised by symmetric information. Obviously, the greater the creditor's (debtor's) relative bargaining power, the closer the negotiated rescheduling scheme to point A (A'). However, if information is asymmetric, it is likely that the feasible rescheduling curve considered by the debtor lies below the feasible rescheduling curve considered by the creditor, as is argued in section II. In which case, the Pareto efficient rescheduling schemes are located in the shaded area A'DAF as indicated in Figure 5. Since the creditor prefers A to E, while the debtor remains indifferent, E and, by a similar argument, the combinations of M and T in its close vicinity, are not Pareto efficient rescheduling schemes.

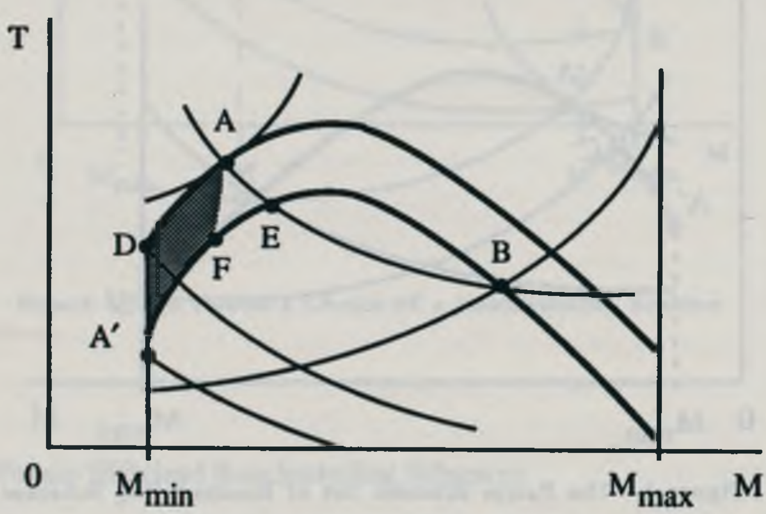

Figure 5. The Pareto Efficient Set of Rescheduling Schemes under Asymmetric Information

VII. Conclusion

Assuming that a country's inclination to default increases with its external debt burden, but decreases with the potential loss of access to the international credit market, and recognising that a short-term illiquidity does not necessarily lead to a long-term insolvency, this paper analysed the set of feasible rescheduling schemes and described the creditor's and debtor's choices of rescheduling schemes from the feasible set within a framework in which they are both risk averse and maximise expected utility.

It was shown that the feasible rescheduling set can be depicted by an inverted U-shaped curve in the plane spanned by the annual repayment and the repayment period. It was also shown that if the probability of default is sufficiently sensitive to changes in the annual debt repayment, the creditor's most preferred scheme might be on the upward sloping section of the feasible rescheduling curve. This finding indicates that the creditor may prefer to bear a lower risk of default and hence may require the least amount of annual repayment between the two feasible ones associated with his preferred choice of a repayment period. The analysis also indicated the effects of the international interest rate, the country's external debt and potential output and the creditor's market share on the creditor's choice of a rescheduling scheme.

The analysis of the debtor's preferences indicated that if the debtor has an aversion toward risk, his indifference curves are downward sloping in the plane spanned by the annual repayment and the repayment period. In which case, the debtor's problem has a corner solution. The debtor prefers feasible rescheduling schemes characterised by a shorter repayment period, but not necessarily by a smaller annual repayment due to an increase in the probability of an admissible default as the annual repayment rises.

Finally, the paper demonstrated that the set of the Pareto efficient rescheduling schemes might be located on the upward sloping side of the feasible rescheduling curve below (and including) the creditor's choice and hence reflects that in the case of a sovereign debt there can be a positive relationship between the length of the repayment period and the amount of the annual repayment rather than the conventional trade-off which is a characteristic of a private debt-repayment.

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APPENDIX

Proof of Proposition 1: Recall that

$$T = \frac{\log\left\{\beta - \dfrac{(1-\beta)D_0}{[1-\alpha M/(\bar{Y}-\delta M)+\lambda s]M}\right\}}{\log\beta} - 1. \tag{A.1}$$

The slope of the feasible rescheduling curve is found by differentiating equation A.1 w.r.t. M:

$$\frac{dT}{dM} = \frac{\overbrace{(1\text{-}b)D_0}^{\overset{0}{\wedge}}\left\{\delta[(1+\lambda s)\delta+\alpha]M^2 - 2[(1+\lambda s)\delta+\alpha]\bar{Y}M+(1+\lambda s)\bar{Y}^2\right\}}{\underbrace{\left[\beta - \dfrac{(1-\beta)D_0}{[1-\alpha M/(\bar{Y}-\delta M)+\lambda s]M}\right]}_{\underset{0}{\wedge}} \underbrace{(\log\beta)}_{\underset{0}{\wedge}} \underbrace{\{[1-\alpha M/(\bar{Y}-\delta M)+\lambda s]M\}^2}_{\underset{0}{\vee}} \underbrace{(\bar{Y}-\delta M)^2}_{\underset{0}{\vee}}}. \tag{A.2}$$

Note that as long as $r>0$, $\beta<1$ and hence $\log\beta<0$. Recalling that $T>0$, it is required, therefore, that the numerator of the first term on the r.h.s. of equation A.1 would be negative:

$$\log\left\{\beta - \frac{(1-\beta)D_0}{[1-\alpha M/(\bar{Y}-\delta M)+\lambda s]M}\right\} < 0. \tag{A.3}$$

Consequently, the denominator of equation A.2 is positive, and recalling that $\beta<1$, we obtain that

$$\frac{dT}{dM} \gtreqless 0 \text{ as } V \equiv M^2 - \frac{2\bar{Y}}{\delta}M + \frac{(1+\lambda s)\bar{Y}^2}{\delta[(1+\lambda s)\delta+\alpha]} \gtreqless 0.$$

Note that

$$\frac{dV}{dM} = 2M - \frac{2\bar{Y}}{\delta} \tag{A.4}$$

and that

$$\frac{d^2V}{dM^2} = 2 > 0. \tag{A.5}$$

That is, V is a convex function of M. Moreover, equation 2 indicates that the feasible range of annual debt repayment is $0 \leq M < \bar{Y}/\delta$. In this range

$$\frac{dV}{dM} = 2M - \frac{2\bar{Y}}{\delta} < 0 \tag{A.6}$$

for every $M<\frac{\bar{Y}}{\delta}$ (including M_{max}) as depicted in Figure A1.

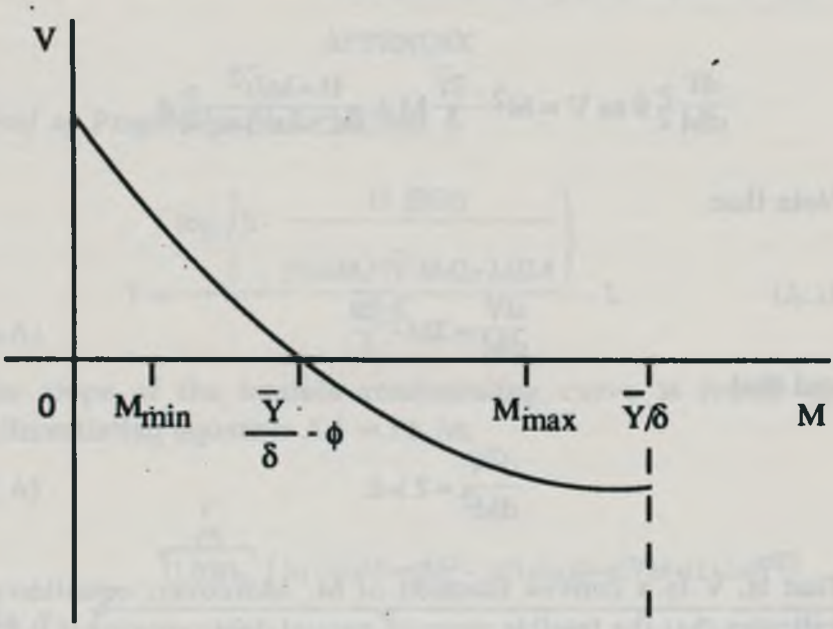

Figure A.1

The roots of V are

$$M_{1,2} = \bar{Y}/\delta \pm \phi \tag{A.7}$$

where

$$\phi = \frac{\bar{Y}}{\delta}\sqrt{1 - \frac{(1+\lambda s)}{[(1+\lambda s)\delta+\alpha]}}. \tag{A.8}$$

Since it is assumed that $\delta \geq 1$ and λ and α are positive, then $\phi > 0$ and $\bar{Y}/\delta + \phi$ is not in the feasible range. Thus,

$$V \begin{cases} > 0 \text{ for } 0 < M < \bar{Y}/\delta - \phi \\ = 0 \text{ for } M = \bar{Y}/\delta - \phi \\ < 0 \text{ for } \bar{Y}/\delta - \phi < M \leq \bar{Y}/\delta \end{cases} \tag{A.9}$$

and consequently,

$$\frac{dT}{dM} \begin{cases} > 0 \text{ for } M_{min} < M < \bar{Y}/\delta - \phi \\ = 0 \text{ for } M = \bar{Y}/\delta - \phi \\ < 0 \text{ for } \bar{Y}/\delta - \phi < M \leq M_{max}. \blacksquare \end{cases} \tag{A.10}$$

Mathematical presentation of the creditor's choice: By substituting the constraint 10 into equation 15 for T and then differentiating with respect to M, the necessary condition for maximum expected utility for the creditor from the perceived sum of actual debt-repayments is:

$$\frac{dE[u(MX)]}{dM} = -\theta(M^*)^{T(M^*)}[T(M^*)\theta'(M^*)/\theta(M^*) + T'(M^*)\log\theta(M^*)] = 0 \tag{A.11}$$

where

$$\theta(M^*) = p(M^*) + [1-p(M^*)]\exp(-R^c M^*) \tag{A.12}$$

$$T(M^*) = \{\log\{\beta-(1-\beta)D_0/[1-\alpha M^*/(\bar{Y}-\delta M^*)+\lambda s]M^*\}/\log\beta\} -1 \quad (A.13)$$

$$p(M^*) = \alpha M^*/(\bar{Y}-\delta M^*) - \lambda s \quad (A.14)$$

and $\theta'(M^*)$ and $T'(M^*)$ are the derivatives of $\theta(M^*)$ and $T(M^*)$, respectively, with respect to M evaluated at the optimal level M^*. Given that the second-order condition for maximum is satisfied, the necessary condition A.11 yields the creditor's optimal level of renegotiated annual debt-repayment as a function of the model's parameters:

$$M^* = f(D_0, \bar{Y}, r, \alpha, \delta, \lambda, s, R^C) \quad (A.15)$$

and the substitution of $f(D_0, \bar{Y}, r, \alpha, \delta, \lambda, s, R^C)$ into equation A.13 for M^* gives the optimal repayment period of the rescheduled debt:

$$T^* = g(D_0, \bar{Y}, r, \alpha, \delta, \lambda, s, R^C). \quad (A.16)$$

Proof of Proposition 2: The total differential of equation 15 implies that the slope of such an indifference curve is given by:

$$\frac{dT}{dM}\bigg|_{E(u^C)=\text{const.}} = \frac{T\{\exp(-R^C M)[p'+R^C(1-p)] - p'\}}{\theta\log\theta}. \quad (A.17)$$

where

$$\theta = p(M) + [1-p(M)]\exp(-R^C M). \quad (A.18)$$

Equation A.18 implies that for a risk averse creditor $0<\theta<1$ and hence the denominator in the above expression is negative. Consequently,

$$\frac{dT}{dM}\Big|_{E(u^c)=\text{const.}} \gtreqless 0 \text{ as } M \gtreqless \frac{1}{R^c}\ln\left[\frac{p'+(1-p)R^c}{p'}\right]. \blacksquare \qquad \text{(A.19)}$$

Proof of Proposition 6: By totally differentiating equation 19, we obtain that the slope of the debtor's indifference curve in the M-T plane is

$$\frac{dT}{dM}\Big|_{E(u^d)=\text{const.}} = \frac{-T\{\exp(R^dM)[R^d(1-p)-p']+p'\}}{\psi\log\psi} \qquad \text{(A.20)}$$

where

$$\psi \equiv p(M) + (1-p(M))\exp(R^dM). \qquad \text{(A.21)}$$

Since $\psi>1$ for every $M>0$, $\psi\log\psi$ is always positive and

$$\frac{dT}{dM}\Big|_{E(u^d)=\text{const.}} \lesseqgtr 0 \qquad \text{(A.22)}$$

as $\exp(R^dM)[R^d(1-p)-p']+p' \gtreqless 0$ or, equivalently, as $M \gtreqless \frac{1}{R^d}\ln\left[\frac{-p'}{R^d(1-p)-p'}\right]$.

This implies

$$\frac{dT}{dM}\Big|_{E(u^d)=\text{const.}} < 0 \qquad \text{(A.23)}$$

for all M>0 if

$$\ln\left[\frac{-p'}{R^d(1-p)-p'}\right] \le 0 \tag{A.24}$$

or, equivalently, if

$$\frac{-p'}{R^d(1-p)-p'} \le 1 \tag{A.25}$$

which, in turn, requires that

$$R^d(1-p) \ge 0. \tag{A.26}$$

Since it is assumed that $R^d>0$ and $0<p<1$, the above condition is satisfied. ■

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