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MEDIAN INCOME VERSUS PER CAPITA INCOME: IMPLICATIONS FOR ASSESSING GROWTH AND CONVERGENCE

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MEDIAN INCOME VERSUS PER CAPITA INCOME: IMPLICATIONS FOR ASSESSING GROWTH AND CONVERGENCE

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ABSTRACT

When the distribution of income is skewed, median income is a better indicator of the earnings of the representative member of society than the commonly used per capita income. Median income is linked to per capita income and the disparity in income-generating assets' ownership. While ownership identity is only important in the determination of the size of the change in per capita income, it is crucial for determining both the direction and the magnitude of changes in median income and, thereby, the prospects of growth and convergence in median income-centred analyses.
* The author is grateful to Joan Rodgers, Don Lewis and Charles Harvie.
I. Introduction

Since the publication of Solow's (1956) and Swan's (1956) seminal articles, economic growth and international comparative studies have been centred on per capita income and per capita productive assets and have used them as the prominent indicators of the earnings, production potential and wealth of representative members of society. There are considerable analytical and measuring advantages in using per capita indicators. However, ownership of productive assets is unlikely to be equally distributed among people and the distribution of income is usually skewed, with per capita income considerably larger than the median. That is, a majority of the population earns income levels lower than per capita income. The greater the skewness of the distribution of income the less reliable the per capita income in assessing the economic well being of the representative agent and the greater the likelihood that the ordering of policies affecting the distribution of income by majority vote is closer to the preferences of the median income earners than to the preferences of the per capita income earners. It is therefore sensible to consider median income and its determinants and explore the implications of median income-centred models for growth and convergence of income across countries.

The objective of this paper is threefold. First, to offer a method for modelling the median income and to link median income to per capita income and the variances and covariances of the distributions of ownership of income-generating assets. Second, to compare conceptually the growth rates of median income and per capita income and to highlight the effects of asset accumulation and ownership identity on these rates and on the prospects of convergence, or divergence, of incomes across countries. Third, to compare between the golden rule of capital accumulation
advocated by per capita income-centred models and the golden rule stemming from a median income-centred model.

Correspondingly, the analysis is organised in three sections as follows. Section II develops the relationship between median income, per capita income and ownership of income-generating assets. This relationship is used in Section III for analysing the growth rate of median income and modifying the condition of cross-country income-convergence. It is also used in Section IV for modifying the neoclassical per capita income-centred golden rule of capital accumulation to the case where the representative income is taken as being equal to the median income. A summary of the main analytical findings is provided in Section V.

II. MEDIAN INCOME AND ITS RELATIONSHIP WITH PER CAPITA INCOME AND ASSET OWNERSHIP

Unlike the analysis of per capita income, the modelling of median income employs assumptions about the distribution of income and income-generating assets within a society rather than the aggregate production function. A sensible choice of a personal-income equation should be consistent with a probability density function that provides a reasonable approximation of observed income distributions. As argued by Adelman and Levy (1984) and Levy (1987) a personal-income equation which satisfies this requirement displays the income of the $i$-th person as a product of a positive scalar $\theta$ and an exponential dispersion factor

$$y_i = \theta \exp(\varepsilon_i)$$

where $\varepsilon$ is normally distributed among the members of the society with zero mean and a finite variance. This specification coincides with the assumption that the lognormal distribution provides a reasonable portrayal of income distribution.
For a person receiving the median income, $\epsilon$ is equal to zero and hence her, or his, income is equal to $\theta$. That is, $\theta$ can be interpreted as the median income ($y_{med}$) and the aforementioned personal-income equation can be rewritten as

$$y_i = y_{med} \exp(\epsilon_i).$$  \(2)\)

Furthermore, personal income can be linked to possession of income-generating assets (e.g., various types of human capital and physical capital, perishable inputs and access to technology, production systems and markets) by letting $\epsilon_i$ be a weighted sum of the deviations of person $i$'s endowments of these assets from their average stocks in the population

$$\epsilon_i = \Phi'(e_i - \mu_e).$$  \(3)\)

Here, $e_i$ is a Kx1 vector of person $i$'s endowments of the income-generating assets, $\mu_e$ is a Kx1 vector of the average endowments of these income-generating assets within the population, and $\Phi$ is a Kx1 vector of positive scalars (weights) indicating the rates of change of the income of person $i$, relative to the median income, induced by an infinitesimal growth in person $i$'s stocks of the income-generating assets.

The weighted sum specified in equation 3 has zero mean and a variance $\Phi'\Sigma\Phi$, where $\Sigma$ is a KxK matrix of the variances and covariances of the personal endowments of income-generating assets within the population. It is worth noting that if the degree of aggregation of the income-generating assets is sufficiently low as to accommodate for a very large number (K) of assets, each independently distributed (i.e., $\Sigma$ is diagonal) and having a high probability of making a small contribution to the weighted sum, then the central limit theorem lends support to the assumption that $\epsilon$ is normally distributed.

By taking the mean of equation 2, the per capita income ($y_{pc}$) can be rendered as the product of the median income and the
moment-generating function of ε. Recalling further that $\varepsilon \sim N(0, \Phi'\Sigma\Phi)$ and rearranging terms

$$y_{med} = \frac{y_{pc}}{\exp\{0.5\Phi'\Sigma\Phi\}}. \quad (4)$$

Equation 4 suggests that the relationship between median income and per capita income depends on the weighted sum of the variances and covariances of the distribution of income-generating assets within the population under consideration, where the weights correspond to the effects of these assets on personal earning. More specifically, the median income is obtained by "discounting" the per capita income in accordance with the overall degree of ownership inequality in the distribution of income-generating assets among people.

III. GROWTH RATE AND CONVERGENCE IN A MEDIAN INCOME-CENTRED MODEL

Equation 4 can be equivalently rendered as

$$\ln\left(\frac{y_{pc}}{y_{med}}\right) = 0.5\Phi'\Sigma\Phi. \quad (5)$$

which by differentiating with respect to time implies that the rate of change of the median income is equal to the rate of change of per capita income minus: 1. the weighted sum of the changes in the variance and covariances of income-generating assets' ownership, and 2. the weighted sum of the changes in the contribution of these assets to personal income:

$$\frac{\dot{y}_{med}}{y_{med}} = \frac{\dot{y}_{pc}}{y_{pc}} - (\Phi' \Phi + \Phi' \Sigma \Phi). \quad (6)$$
Equation 6, and equation 4, reveal the fundamental difference between per capita income and median income and their response to the accumulation of income-generating assets. An increase in the stock of any of the income-generating assets, as long as it is employed and all other things remaining the same, raises the aggregate output and thereby per capita income regardless of the identity of the owners of the new stock. Recalling equations 1 and 3, ownership identity is only important in the determination of the size, but not the direction, of the change in aggregate output and per capita income – the larger the stocks of all other income-generating factors owned by an individual, the higher the rise of the aggregate output and per capita income induced by an additional unit of an income-generating factor accumulated by that person. In contrast, ownership identity is crucial for determining both the direction and the size of the change in the median income because the contributions of the new capital stock or market-access to the aggregate income is discounted by its effect on the variance of ownership of that asset and the covariances of ownerships between that asset and the rest of the income-generating assets. When the increase in the stock of an income-generating asset is accompanied by increased overall equality in the distribution of assets’ ownership (i.e., when it reduces the denominator of the right-hand-side term in equation 4) median income rises more than per capita income. However, when the increase in the stock of an income-generating asset raises the overall asset-ownership inequality, median income rises at a lower rate than per capita income. Moreover, if the increase in the overall asset-ownership inequality dominates the contribution of the new stock of production asset to the aggregate income-generation process, median income declines.

Equation 6 also has interesting implications for the prospects of convergence of incomes across countries. Let $A$ and $B$ denote two countries, or groups of countries, with $A$ having initially a
higher per capita income than B. In this case, the per capita income-centred economic growth literature considers

\[
\frac{\dot{y}_{pc}^B - \dot{y}_{pc}^A}{\frac{\dot{y}_{pc}^B}{\dot{y}_{pc}^A}} > 0
\]

(7)
as the condition for convergence. In contrast, the median income-centred growth model portrayed by equation 6 suggests that when the median income in country A is also higher than the median income in country B the condition for convergence is rather

\[
\frac{\dot{y}_{pc}^B - \dot{y}_{pc}^A}{\frac{\dot{y}_{pc}^B}{\dot{y}_{pc}^A}} > (\Phi_B' \Sigma_B \Phi_B + \Phi_A' \Sigma_B \Phi_B) -
\]

\[
(\Phi_A' \Sigma_A \Phi_A + \Phi_A' \Sigma_A \Phi_A)
\]

(8)

That is, convergence can occur even when the per capita growth rate differential between B and A is negative, so long that this differential exceeds the change over time in the difference between the variances of the income dispersion factor ε between B and A. Conversely, divergence occurs if the change over time in the difference between the variances of ε between B and A exceeds the per capita growth rate differential between B and A even when this growth rate differential is positive.

Moreover, when the distribution of income-generating assets in country B is sufficiently more equal than in country A, it is possible that the median income in country B exceeds the median income in country A despite the positive per capita income differential in favour of A. In this case, convergence of these countries' median incomes occurs if
IV. The Golden Rule Stemming from a Median Income-Centred Model

Using a simple version of the neoclassical per capita income-centred growth model (cf., Phelps, 1966) as a benchmark, this section modifies the golden rule of capital accumulation to the case where the representative agent's income is equal to the median income. It is assumed, for simplicity, that capital and labour are the only production inputs and that they are fully employed in a Cobb-Douglas production process generating a single homogeneous good devoted either to consumption or investment. Capital ownership is assumed to be unequally distributed among people with a finite instantaneous variance $\sigma^2(t)$. Labor is taken, for convenience, as an homogenous and equally distributed input. The extension of the analysis to a broader and endogenous growth framework may consider the effects of human capital on production and satisfaction from leisure and the issue of time allocation along the lines of Ortigueira and Santos (1997), for example.

Consequently, and in view of equation 4, the instantaneous median income accruing to the representative household is

$$y_{med}(t) = f(k(t))\exp\{-0.5\phi(t)\sigma^2(t)\}$$

where $k$ is the capital-labour ratio, $f(k)$ is a concave function measuring the per capita output generated by $k$, and $\phi$ is a positive weight associated with the deviation of capital stock from the mean value in constructing $\varepsilon$ (or the rate of change of the
income accruing to person $i$, relative to the median income, induced by an infinitesimal increase in person $i$'s stock of capital).

A convenient conceptualisation of the social planner's inter-temporal decision problem, which also allows a straightforward comparison with the golden rule of capital accumulation stemming from the conventional, neoclassical and per capita income-centred model, treats the instantaneous saving rate, $s(t)$, as the control variable, assumes that $s$ is common to all members of society, and takes the planning horizon to be infinite. More formally, the trajectory of $s$ is chosen so as to maximise

$$\int_{0}^{\infty} e^{-\rho t} u((1 - s(t)) y_{med}(t)) dt$$

subject to the median income equation 10 and the conventional law of motion of the capital-labour ratio

$$k(t) = s(t) f(k(t)) - (\delta + n) k(t)$$

(11)

where $u$ is the instantaneous utility derived from consumption with $u' > 0$ and $u'' < 0$, $\rho$, $\delta$ and $n$ are fixed rates of time preference, capital depreciation and population growth, respectively.

The necessary conditions and the associated singular control imply that, in addition to the capital-motion equation 11, the instantaneous change in the consumption level of the median-income household, $c_{med}$, must obey at all times the following no-arbitrage rule (where the time index $t$ is omitted for compactness)

$$c_{med} = \left[ \frac{f'(k) - (\rho + \delta + n)}{-u''(c_{med})/u'(c_{med})} \right] - \left[ \frac{0.5(1 - s) \phi^2 \sigma_k f(k) + \phi \sigma^2 \phi + 0.5 \phi^2 \sigma}{-u''(c_{med})/u'(c_{med})} \right]$$

(12)
The first term on the right-hand side of equation 12 is the familiar golden rule obtained by the conventional per capita income-centred model. It states that the instantaneous change in consumption corresponds to the difference between the marginal product and the user cost of capital but discounted by the degree of concavity of the instantaneous utility function. The second term modifies this conventional golden rule to the case where the median income is taken as the representative household's income by altering the marginal product and user cost differential in accordance with the changes in the variance of the distribution of capital within the population. The underlying rationale for this modification is that, as indicated in equation 4, the variance of capital stock affects adversely the median income.

The modified golden rule 12 indicates further that the change in the variance of the distribution of capital induced by the accumulation of capital makes the difference between the stationary capital-labour ratios in the median income-centred model and the conventional per capita income-centred model. In the latter case, the stationary level of the capital-labour ratio is given by

$$ C_{ss}^{\text{conventional}} = f^{-1}(\rho + \delta + n) $$

(13)

whereas in the former it should satisfy

$$ f'(k_{ss}) = \rho + \delta + n + 0.5(1 - s)f(k_{ss})\phi^2 \frac{\partial \sigma^2}{\partial k}. $$

(14)

Recalling that $f$ is concave, the stationary capital-labour ratio in the median income-centred model is smaller than, equal to, or larger than the stationary capital-labour ratio in the per capita income-centred model if a rise in capital stock is accompanied by an increase, no change, or decline, in the variance of the distribution of capital ownership among people, respectively. Moreover, while the steady state in the conventional per capita
income-centred model is a saddle point, it is not necessarily so in the median income-centred model. Information about the relationship between capital-ownership inequality and capital accumulation, and the second derivative of \( \sigma \) with respect to capital in particular, is essential for assessing the asymptotic properties of the modified system's stationary point.

V. SUMMARY AND CONCLUSION

When the distribution of income is skewed, median income indicates the earnings of the representative member of society better than the commonly used per capita income, in the sense that the income levels accruing to a majority of the population are closer to the median level than to the per capita level. A method for modelling median income and its determinants was offered. Median income was related to per capita income and to the variances and covariances of the distribution of ownership of income-generating-factors. Differences between the effects of income-generating factors on median income and per capita income were highlighted. The difference between the rates of change of per capita income and median income was expressed as the weighted sum of the changes in the variance and covariances of the distribution of the ownership of income-generating factors within the population plus the weighted sum of the changes in the contribution of these factors to personal income. It was shown that while ownership identity is only important in the determination of the size of the change in per capita income, ownership identity is crucial for determining both the direction and the size of the change in the median income. It was consequently shown that a positive per capita income growth rate differential between a relatively low-income country and a relatively high-income country is neither a sufficient condition for convergence of the incomes accruing to their representative
agents, nor a necessary one. In addition, the conventional neoclassical golden rule of capital accumulation was modified by using a median income-centred growth model, and the role of changes in the variance of capital ownership in the determination of the socially optimal trajectory of consumption and capital stock was highlighted.

REFERENCES


HELPFUL NOTES

The Hamiltonian associated with the intertemporal decision problem presented in section 5 is
\[ H(t) = e^{-\rho t} u((1-s(t))e^{-0.5\phi(t)^2\sigma^2(t)f(k(t))) + \lambda(t)[s(t)f(k(t))}
\]
\[-(\delta + n)k(t)]

where \( \lambda \) is a co-state variable.

The evolution of \( \lambda \) is given by the adjoint equation through differentiating \(-H\) with respect to \( k \)

\[
\dot{\lambda} = -e^{-\rho t} u'(c_{med})(1-s)[f'(k)e^{-0.5\phi^2\sigma^2} - 0.5\phi^2 \sigma_k^2 e^{-0.5\phi^2\sigma^2} f(k)]
\]

\[-\lambda[sf'(k) - (\delta + n)]

where \( \sigma_k^2 \) denotes the derivative of the variance of \( k(\sigma^2) \) with respect to \( k \).

The optimality condition is given by

\[
\frac{\partial H}{\partial s} = -e^{-\rho t} u'(c_{med})e^{-0.5\phi^2\sigma^2} f(k) + \lambda f(k) = 0
\]

and equivalently,

\[
\lambda = e^{-(\rho t + 0.5\phi^2\sigma^2)} u'(c_{med}).
\]

The no-arbitrage rule 12 is obtained by differentiating (A3) with respect to \( t \), substituting A2 for \( \dot{\lambda} \) and A4 for \( \lambda \), dividing both sides of the equality by the right hand side of A4, and isolating \( c_{med} \).
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