Magnetoplasmon emission versus Landau-level scattering in resonant tunneling through double-barrier structures

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Magnetoplasmon emission versus Landau-level scattering in resonant tunneling through double-barrier structures

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We consider electron tunneling in a parallel magnetic field taking into account the electron-electron interaction. It is shown that a self-consistent treatment of the dynamical electron-electron interaction in resonant tunneling can lead to the proposed plasmon assisted resonant tunneling. Such magnetoplasmon assisted resonant tunneling gives rise to satellite peaks or shoulders in the tunneling current. At low temperatures, only magnetoplasmon emission processes contribute and satellites only appear on the high bias side of the main resonance. The mechanism proposed here may be used to study the magnetotunneling in high mobility systems where disorder is at minimum.

I. INTRODUCTION

Double-barrier resonant tunneling structures (DBRTS's) represent interesting physical systems with which several fundamental properties related to quantum transport in small structures can be studied. Some well known phenomena in DBRTS's are negative differential resistance,\textsuperscript{1,2} intrinsic bistability,\textsuperscript{3,4} and tristabilities.\textsuperscript{5,6} These phenomena, together with the fast response time in DBRTS's, make these structures a promising class of materials for the next generation electronic devices. Recently, an electron tunneling mechanism in DBRTS's was observed and reported.\textsuperscript{7} When an asymmetric DBRTS is biased in such a way that large charge accumulation occurs in the resonant well, an additional tunneling channel can become available, which is classified as plasmon assisted resonant tunneling (PAT). PAT occurs when, near the resonant tunneling, an incoming electron lowers its energy to the resonant level by emitting a quantum-well plasmon. Due to the nature of quantum-well plasmon dispersion, such processes can occur near the main resonance, as long as the difference between the incoming energy and the resonant level is smaller than the maximum plasmon energy. As a result, a satellite is observed in the tunneling $I$-$V$ characteristics. If one further includes the static level shift and treats both level shift and PAT self-consistently (via $n_{2D} \sim I$, where $I$ is the tunneling current), the resulting $I$-$V$ curve exhibits multiple instabilities.

When a magnetic field parallel to the tunneling current is applied to the system, more interesting tunneling phenomena can occur.\textsuperscript{8} On the high bias side of the main resonance peak, a series of side resonances will appear. The spacing between the successive resonance is roughly the same for fixed magnetic field and increases approximately linearly with the magnetic field strength. This phenomenon is commonly understood within the picture of inter-Landau-level elastic scattering. This elastic scattering process can be described as

\[ \varepsilon_s + \varepsilon_n = \varepsilon_s' + \varepsilon_n'. \]
toplasmon dispersion, the spacing is always nonlinear. In
this work, we shall assume that the charge accumulation in the
well is negligible (such as the case of a symmetric
DBRTS or an asymmetric DBRTS of thicker emitter
barrier) and shall only consider plasmon excitation in the
emitter. The tunneling rate presented below is also valid
for the case where plasmon excitation occurs in the cen-
ter well. However, in case of a charged well, one has to
include the dynamical resonance level shift, which causes
the tipover of the resonant curve.8,10

II. CALCULATION OF THE TUNNELING RATE

Let us consider a typical DBRTS (symmetric or asym-
metric). The model Hamiltonian can be written as

\[ H = H_0 + H_1, \]

where \( H_0 \) is the one-body Hamiltonian describing the
single particle energy, including elastic coupling with the
static barriers,

\[
H_0 = \sum_{n, k_y} \epsilon_n^{e} c_n^{\dagger} c_n + \sum_{p, \nu} \epsilon_{p}^{e} a_{p, \nu}^{\dagger} a_{p, \nu}
+ \sum_{p, \nu} V_{p, \nu} (a_{p, \nu}^{\dagger} c_{n, k_y} + c_{n, k_y}^{\dagger} a_{p, \nu}),
\]

In the above equation, \( p = (n, k_y, k_z) \), \( \epsilon_n^{e} = \epsilon_c + (n + 1/2)\hbar \omega_c \), with \( \epsilon_c \) the energy of quasibound states in the
well and \( n \) the Landau level (LL) index. \( c_{n, k_y}^{\dagger} (c_{n, k_y}) \) is
the creation (annihilation) operator for electrons in the
resonant well with the LL index \( n \) and center coordinate
\( x_0 = k_y/eB \). Similarly, \( \epsilon_{p}^{e} = \epsilon_c + \epsilon_{p}^{v} \), with \( \nu = l(r) \) for
the left lead (right lead). \( a_{p, \nu}^{\dagger} (a_{p, \nu}) \) is the creation (anni-
hilation) operator for electrons in the leads. \( V_{p, \nu} \) is the elastic
coupling between the electrons and the static barrier.

When considering the electron-electron interaction, we
shall consider that (i) the only important interaction is
the interaction between the tunneling electron (after it
leaves the emitter) with those electrons in the emitter
and the charge concentration in the emitter is taken to be constant. (ii) The interaction among electrons in the emitter is constant if the electron concentration in the emitter is considered to be constant. Such a constant energy shift is unimportant in the present model. (iii) There is no charge accumulation in the well. Now the interaction Hamiltonian is given as

\[
H_I = \sum_{n, k_y, k_z, q} v_q C_{nn', q}^{(n')} (q) c_{n, k_y, k_z}^{\dagger} c_{n', k_y, k_z}^{\dagger} q_{-q} a_{n, k_y} a_{n, k_z},
\]

where \( v_q \) is the Fourier coefficient of Coulomb interaction
between the tunneling electron in the well and electrons in the emitter. The form factor is given as

\[
C_{nn', q} (q) = (\tau' / n!)(X^{n-n'} e^{-X} \left[ L_{n'}^{n-n'} (X) \right] ^2),
\]

where \( X = q^2 / 2eB \) and \( L_n \) is an associated Laguerre polynomial.

The calculation of transmission through a DBRTS is
carried out within the so-called \( S \)-matrix formalism.11–14
Let \( T(p, p') \) be the transmission matrix describing the
transmission probability per unit final longitudinal momentum \( k_z' \),
that a particle incident with energy \( \epsilon_p \) will be transmitted
with energy \( \epsilon_{p'} \). The total transmission probability \( T_1 (p) \)
with initial momentum \( p \) can be written as a sum (or integral) of the transmission matrix \( T(p, p') \) over the
final momentum \( p' \),

\[
T_1 (p) = \int dp' T(p, p').
\]

The transmission matrix can be written as

\[
T(p, p') = \Gamma_1 (p) \Gamma_1 (p') \left[ \partial \epsilon_{p'} / \partial k_z' \right] \int d\tau d\tau' \exp \left[ i(\epsilon_{p'} - \epsilon_p) \tau + \epsilon_{p' \tau} - \epsilon_{p \tau} / \hbar) G_{\alpha \alpha'} (\tau, s, t) \right].
\]

Here, \( p(p') \) is for the left (right) lead, \( \alpha = (n, k_y) \) and
\( \int dp \to \sum_n \int d k_y \int d k_y \). The elastic resonant width is

\[
\Gamma_1 (p) = 2 \pi \sum_{p'} \left| V_{p', \nu} \right|^2 \delta (\alpha, \alpha') \delta (\epsilon_{p'} - \epsilon_{p}),
\]

with a similar expression for \( \Gamma_1 (p') \). The interacting two-
body Green's function is defined as

\[
G_{\alpha \alpha'} (\tau, s, t) = \theta (s) \theta (t) (c_{\alpha} (\tau - s) c_{\alpha'}^{\dagger} (\tau) c_{\alpha'} (t) c_{\alpha}^{\dagger} (0)).
\]

The leading terms in a perturbation expansion of \( G_{\alpha \alpha'} \)
are given by those up to the second order of the electro-
nelectron interaction. The one-body Green's function is
simply \( G_1^{(1)} (t) = -i \theta (t) \exp [-i (\epsilon_n + \Gamma / 2) t / \hbar] \). In the
interaction picture, the two-body Green's function is given as

\[
G_1^{(2)} (t) = -i \theta (t) \exp [-i (\epsilon_n + \Gamma / 2) t / \hbar].
\]
\begin{align*}
G_{\alpha,\alpha'}(\tau, s, t) &= \theta(s) \theta(t) \left\langle \epsilon_{\alpha}(\tau - s) U^\dagger(t) \right.
\left. - s) \epsilon_{\alpha'}(t) U(t, 0) \epsilon_{\alpha'}(0) \right\rangle,
\end{align*}

with \( U(t, t_0) \) the time evolution operator,

\[ U(t, t_0) = \sum_{m=0}^{\infty} T \left[ -i \int_{t_0}^{t} dt' \hat{H}_{\text{int}}(t') \right]^m / m! \]  

(9)

To the second order, it can be written as

\[ G_{\alpha,\alpha'}^{(2)}(\tau, s, t) = \theta(s) \theta(t) \sum_{n_1, n_2, k_1, k_2, q_1, q_2} \left\langle \epsilon_{n_1, k_1}(\tau - s) \int_{t_0}^{t} dt' \hat{H}_{\text{int}}(t') \epsilon_{n_1, k_1}(t) \right. \left. -i \int_{t_0}^{t} dt' \hat{H}_{\text{int}}(t') \epsilon_{n_1, k_1}(0) \right\rangle 
\]

\[ \times \left\langle \epsilon_{n_2, k_2}(\tau - s) \int_{t_0}^{t} dt' \hat{H}_{\text{int}}(t') \epsilon_{n_2, k_2}(t) \right. \left. -i \int_{t_0}^{t} dt' \hat{H}_{\text{int}}(t') \epsilon_{n_2, k_2}(0) \right\rangle 
\]

\[ \times \left\langle \epsilon_{n_1, k_1}(\tau - s) \epsilon_{n_1, k_1}(t) \epsilon_{n_2, k_2}(t) \epsilon_{n_2, k_2}(0) \right\rangle 
\]

\[ \times \left\langle \epsilon_{n_1, k_1}(\tau - s) \epsilon_{n_1, k_1}(t) \epsilon_{n_2, k_2}(t) \epsilon_{n_2, k_2}(0) \right\rangle 
\]

\[ \times \left\langle \epsilon_{n_1, k_1}(\tau - s) \epsilon_{n_1, k_1}(t) \epsilon_{n_1, k_1}(t) \epsilon_{n_1, k_1}(0) \right\rangle 
\]

\[ \times \left\langle \epsilon_{n_2, k_2}(\tau - s) \epsilon_{n_2, k_2}(t) \epsilon_{n_2, k_2}(t) \epsilon_{n_2, k_2}(0) \right\rangle 
\]

\[ \times \left\langle \epsilon_{n_1, k_1}(\tau - s) \epsilon_{n_1, k_1}(t) \epsilon_{n_1, k_1}(t) \epsilon_{n_1, k_1}(0) \right\rangle 
\]

\[ \times \left\langle \epsilon_{n_2, k_2}(\tau - s) \epsilon_{n_2, k_2}(t) \epsilon_{n_2, k_2}(t) \epsilon_{n_2, k_2}(0) \right\rangle 
\]

(10)

In the above equation, we have used the following definition for the density operator:

\[ \rho(q, \tau) = \sum_{n, k} F_{nn\tau}(q) \epsilon_{n, k}(\tau) \epsilon_{n, k}(\tau), \]

where \( F_{nn\tau}(q) \) is related to \( C_{nn\tau}(q) \) via \( C_{nn\tau}(q) = | F_{nn\tau}(q) |^2 \). In the following, we shall assume that the medium is also a pure two-dimensional system. In this case \( v_q = (2\pi e^2 / q) e^{-qd} \), where \( d \) is the separation between the emitter and the resonant well.

Now we note that \( \langle \rho(q, s') \rho(-q, t') \rangle \) depends only on the difference between \( s' \) and \( t' \), that is \( \langle \rho(q, s') \rho(-q, t') \rangle = \langle \rho(q, s' - t') \rho(-q, 0) \rangle \). The density-density correlation function can be related to the dielectric function of the electron gas via the fluctuation and dissipation theorem.\(^{15}\)

\[ (T_u \rho(q, \omega) \rho(-q, 0)) = \frac{-q}{2\pi e^2} \int_{-\infty}^{\infty} d\omega e^{-i\omega t} \frac{1}{1 - e^{-\beta\omega}} \text{Im} \left[ \frac{1}{\epsilon(q, \omega)} \right]. \]

(10)

We note that \( 1 / (1 - e^{-\beta\omega}) = N(-\omega) = 1 + N(\omega) \), where \( N(\omega) \) is the boson distribution function. The second order Green’s function can now be written as

\[ G_{\alpha,\alpha'}^{(2)}(\tau, s, t) = \frac{\theta(s) \theta(t)}{\pi} \sum_{\alpha_1, \alpha_2, \alpha_3, \alpha_4} \frac{2 \pi e^2}{q} e^{-2q\omega} F_{n_1 n_2}(q_1) F_{n_2 n_2}(q_2) \int_{-\infty}^{\infty} d\omega [1 + N(\omega)] \text{Im} \left[ \frac{1}{\epsilon(q, \omega)} \right] \]

\[ \times \int_{t_0}^{t} dt' \left\langle \epsilon_{\alpha_1}(\tau - s) \epsilon_{\alpha_2}(s') \epsilon_{\alpha_3}(s') \epsilon_{\alpha_4}(t) \epsilon_{\alpha_4}(t) \epsilon_{\alpha_3}(t) \epsilon_{\alpha_2}(0) \right\rangle e^{-\omega(t - t')}, \]

(11)
where $\alpha_1 = (n_1, k_{p1})$, $\alpha'_1 = (n_1, k_{p1} + q_p)$, $\alpha_2 = (n_2, k_{p2})$, $\alpha'_2 = (n_2, k_{p2} - q_p)$. The last factor in Eq. (12) including the $s'$ and the $t'$ integration can be evaluated. The $\langle s'|s \rangle$ term can be arranged into three combinations of four Green's functions [given by Figs. 2(b)–2(d)]. The integration is then completed after expanding these Green's functions in the wide band limit.

\[
G^0_\alpha(t, s, t) = \frac{G^0_\alpha(t, s, t)}{(\omega - \Delta_{nn'})^2} e^{i\Delta_{nn'}(t-s)} e^{-i\omega_{nn'}(t-s)} (e^{-i(\omega_{nn'})s} - 1)(e^{i(\omega_{nn'})s} - 1),
\]

\[
G^0_\alpha(t, s, t) = \frac{G^0_\alpha(t, s, t)}{\omega - \Delta_{nn'}} \left\{ \frac{\omega + \Delta_{nn'}}{(\omega + \Delta_{nn'})^2} - 1 \right\},
\]

\[
G^0_\alpha(t, s, t) = \frac{G^0_\alpha(t, s, t)}{\omega - \Delta_{nn'}} \left\{ \frac{-i\omega + \Delta_{nn'}}{(\omega + \Delta_{nn'})^2} - 1 \right\},
\]

where $\Delta_{nn'} = (n-n')\hbar\omega_c$. Now we can apply exponential resummation $G_\alpha(t, s, t) = G^0_\alpha(t, s, t) \exp[(G^1 + G^2 + G^3)/G^0]$ to obtain

\[
T_{tot}(\varepsilon_n) = \frac{\Gamma_L \Gamma_R}{\Gamma} \int_{-\infty}^{\infty} d\sigma \exp \left[ -\frac{|\sigma|}{2} + i(\varepsilon_{k_r} - \varepsilon_c)\sigma + \sum_{q,n',m'} 2\frac{\xi^2}{q} e^{-2q\xi C_{nn'}(q)} \right] \times \int_{-\infty}^{\infty} d\omega \left[ 1 + N(\omega) \right] \text{Im} \left[ \frac{1}{\varepsilon(q, \omega)} \right] \left( \frac{1\sigma}{\omega - \Delta_{nn'}} + \frac{e^{-i(\omega_{nn'})\sigma} - 1}{(\omega - \Delta_{nn'})^2} \right). \]

If the dynamics of the electron-electron interaction is ignored, Im$(1/\varepsilon(q, \omega)) \equiv 0$, we immediately recover the well-known resonant tunneling result,

\[
T_{tot} = \frac{\Gamma_L \Gamma_R}{(\varepsilon_s - \varepsilon_c)^2 + \Gamma^2/4}.
\]

The term proportional to $e^3\text{Im}(1/\varepsilon(q, \omega))$ contains all contributions to the total tunneling rate, due to electronic excitations of the system. In the plasmon pole approximation, Im$(1/\varepsilon(q, \omega)) \propto \delta(\omega \pm \omega_p)$, where $\omega_p$ is the plasma frequency, we recover our previous result of plasmon assisted tunneling. The effect of plasmon excitation on resonant tunneling was treated in Ref. 8 by introducing an additional set of boson coordinates. The derivation presented here shows that such electronic excitation assisted tunneling can be studied within the framework of an elementary electron-electron interaction. The boson coordinates describing the collective excitation are completely redundant.

The dielectric function appearing in the above equation depends on the magnetic field. For an infinitely sharp LL, Im$[1/\varepsilon]$ only contains contributions from magnetoplasmons, i.e.,

\[
\text{Im} \frac{1}{\varepsilon(q, \omega)} = \pi \delta(\text{Re} \varepsilon(q, \omega)) = \pi \delta \left( 1 - \sum_m \frac{b_m \omega_m^2}{\omega^2 - m^2 \omega_c^2} \right)
\]

and

\[
b_m = 4mr_x \frac{k_F}{q} X^{m} e^{-X} \sum_{m'} \frac{m!}{(m + m')!} \left[ I_m^m(X) \right]^2 \left[ f_{m'} - f_{m+m'} \right],
\]

where $X = (q\xi)^2/(2\pi\xi B)$, and $r_x = me^2/k_F$ is the plasma parameter and $f_m$ is the Fermi distribution function. In general, there are countable infinite collective modes. However, the coupling to the higher mode decreases as $1/m$ and we shall only include the first few modes in our model calculation below. Furthermore, we shall assume that the mixing of different modes is negligible, i.e., the $m$th mode is given as

\[
\omega_{m}^{(m \pm)} = \pm \omega_c \sqrt{b_m + m^2}.
\]

Therefore, the total tunneling rate can be written as

\[
T_{tot}(\varepsilon_n) = \frac{\Gamma_L \Gamma_R}{\Gamma} \int_{-\infty}^{\infty} d\sigma \exp \left[ -\frac{|\sigma|}{2} + i(\varepsilon_{k_r} - \varepsilon_c)\sigma + \sum_{q,n',m} 2\frac{2\pi e^2}{q} C_{nn'}(q) \frac{b_m \omega_m^2}{\omega_p^2} e^{-2q^2} \right] \times \left[ 1 + N(\omega_{m}^{(m \pm)}) \right] F(q,\omega_p^{(m \pm)}),
\]

\[\left( 1 + N(\omega_p^{(m \pm)}) + N(\omega_p^{(m \pm)}) F(q,\omega_p^{(m \pm)}) \right),
\]
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FIG. 2. A class of diagrams, which contribute to the second order two-body Green’s functions. Here, a straight line with index α represents the single particle Green’s function \( G_\alpha \) and a wavy line represents the electron-electron interaction \( V_{\alpha \alpha'} = \nu_\alpha G_{\alpha \alpha'}(q) \), a bubble represents the density-density response.

\[
F(q, \omega_p^{(m+1)}) = \frac{\sigma}{(\omega_p^{(m+1)} - \Delta_{nn'})} + e^{-\sqrt{(\omega_p^{(m+1)} - \Delta_{nn'})^2}} \left( \frac{\omega_p^{(m+1)} - \Delta_{nn'}}{\omega_p^{(m+1)} - \Delta_{nn'}} \right)^2.
\]

In Eq. (17), two terms inside the curly bracket have direct physical meaning. The first term, which is proportional to \([1 + N(\omega)]\), represents the tunneling process assisted by emission of magnetoplasmons, while the second term, which is proportional to \(N(\omega)\), represents the tunneling process assisted by magnetoplasmon absorption. At low temperature, \( N \ll 1 \) and the absorption process can be neglected.

The current is then given as

\[
I = e \sum_n T_{tot}(\epsilon_n) g_n[f_e(\epsilon_p) - f_e(\epsilon_p + eV)],
\]

where \( g_n \) is the level degeneracy and \( f_e(\epsilon_p) \) is the Fermi distribution function of the emitter (collector).

III. RESULTS AND DISCUSSIONS

We have calculated the tunneling current, with the use of Eqs. (16) and (18). The parameters used in our calculation are \( E_F = 5 \text{ meV} \), \( d = 100 \text{ Å} \), \( \Gamma = 1 \text{ meV} \), and \( \epsilon_\alpha - \epsilon_{\text{bias}} = 15 \text{ meV} \). The result is plotted in Fig. 3. Only the first two magnetoplasmon modes are included. The resonant tunneling with plasmon emission can be clearly seen. For \( B = 1.2 \text{ T} \), the first satellite at 16.7 meV corresponds to the emission of a \( \omega^{(1)} \) plasmon, the next satellite at 19.2 meV is due to emission of a \( \omega^{(2)} \) plasmon. The emission of two \( \omega^{(1)} \) plasmons can also be seen at around 22.1 meV. All higher order processes are too weak to see. As magnetic field increases, the magnetoplasmon satellites move towards the higher energy.

In the meantime, the main resonances become stronger and the satellites become weaker as the field strength is increased. Therefore, applying a parallel magnetic field will have two competing effects. On the one hand, the plasmon frequency is enhanced and the satellites become better resolved; on the other hand, the real strength of the satellites is reduced. The spacing between the main resonance and the subsequent satellites is not proportional to \( B \), due to the inclusion of coherence breaking processes, and more importantly, due to the \( q \) dependence of \( \omega_p \). If one neglects these coherence breaking processes \( (n \neq n') \), the spacing will be close to linear in \( B \), especially at large \( B \). For a system with large emitter-well separation or large electron concentration in the emitter, the main contribution to the \( q \)-dependent part of \( \omega_p \) is from the small \( q \) regime. In this case, the spacing will also show close to linear behavior. Figure 4 shows the same calculation, but with a thicker emitter barrier, \( d = 200 \text{ Å} \) and \( \Gamma = 0.2 \text{ meV} \). As \( d \) is increased, the electron-plasmon coupling becomes weaker. The magnetoplasmon assisted tunneling, which gives rise to additional satellites in Fig. 3, now only shows some weak shoulder structure in this case. The ratio of the main resonance to the first plasmon satellite in Fig. 3 is about 4 and in Fig. 4 is about 15. This rapid increase is mainly because the main resonance becomes much sharper as \( \Gamma \) decreases. The electron-electron interaction is also reduced. However, this reduction is less important, because the dominant contribution comes from the small-\( q \) regime and the main effect of \( e^{-\sqrt{q}} \) is to cut off the contribution from the large-\( q \) regime.

Now we compare briefly the magnetoplasmon emission mechanism with the elastic scattering mechanism. From this work, it is evident that due to the electron-electron
interaction, magnetoplasmon excitation can open an additional channel for electron resonant tunneling through a DBRTS. However, unlike the case of inter-Landau-level elastic scattering, where a series of almost equal strength satellites were expected and observed, here only the single plasmon emission gives rise to a distinct satellite resonance (if the electron-plasmon coupling is strong enough as in the case of Fig. 3) and all higher order plasmon excitations only give rise to weak shoulders to the tunneling current. This is mainly due to the fact that the magnetoplasmon energy is strongly dependent on the wave vector $q$ and magnetoplasmon assisted tunneling contains all contributions of different $q$ processes. Therefore, for samples with realistic disorder concentration, such as those studied in Ref. 9, the elastic scattering should still be the plausible mechanism for understanding the magnetoresonant tunneling (one does need to bear in mind that no low bias satellite having been observed experimentally requires further explanation). The magnetoplasmon assisted tunneling may become a plausible mechanism in a system where scatterers are negligible. We, therefore, propose that magnetotunneling measurements be performed in high mobility and low scattering samples.

In conclusion, we have studied a resonant tunneling phenomenon in a parallel magnetic field. It is found that due to electron magnetoplasmon coupling, side resonances or shoulders structures can appear on the high bias side of the main resonance at low temperature.

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