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Boundary element formulations for fracture mechanics problems

Wei-Liang Wu

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BOUNDARY ELEMENT FORMULATIONS FOR
FRACTURE MECHANICS PROBLEMS

By
Wei-Liang Wu

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To My Parents with Love and Gratitude.
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