2011

A Stock Targeting International Carbon-Tax Rule with Uncertainty and Diminishing Compliance

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University of Wollongong
Economics Working Paper Series 2011

http://www.uow.edu.au/commerce/econ/wpapers.html

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WP 11-09

November 2011

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This paper develops a rule for setting periodically and internationally a carbon-dioxide atmospheric stock limiting tax in a world inhabited by expected utility maximizing stakeholders facing diminishing mean and increasing variance of their output level due to climate change. The stakeholders are classified as poor, hence unable and/or unwilling to pay, countries and rich countries. Due to ideological and cultural differences, the rich countries' willingness to pay the tax is not identical. Consequently, the number of complying rich countries diminishes with the tax level.

JEL Classification: Q52, Q54

Keywords: Economics; Carbon-Dioxide Stock; Climate Change; Uncertainty; Carbon Tax; International Compliance

1. Introduction

The atmosphere is an indivisible open-access natural resource. In the absence of property rights, private formation of markets for the externalities created by greenhouse gasses' emissions, climate change in particular, is impossible. The policy measures for controlling these externalities are classified as quantity-based instruments and price-based instruments. Theoretical comparisons of these instruments have followed Weitzman's (1974) generic analysis of stock-based externalities, which linked their relative efficiency to the relative slopes of the marginal benefits and costs of control. In the context of greenhouse gasses, Pizer (2002), Hoel and Karp (2002), Newell and Pizer, (2003) and Fischer and Newell (2008) have provided arguments in favour of price-based instruments.

Carbon tax (or price) has been implemented in several Scandinavian and European countries, in Canada and New Zealand and, most recently, in Australia. An analysis of factors; such as free-riding, overstated expectations and guilt; affecting the efficiency of a unilateral carbon tax has been provided by Levy (2011). Since the atmosphere is indivisible and climate change is a global stock-based externality, an internationally broader cooperation and planning is desirable.

The objective of this paper is to provide an international planner a tax rule for limiting the atmospheric stock of the principal greenhouse gas, carbon-dioxide, in a world inhabited by stakeholders facing intensified economic uncertainty due to climate change. As in Levy (2011), the stakeholders are divided into poor, hence unable and/or unwilling to pay, countries and rich, but not necessarily willing to pay, countries. The construction of the rule is based on five premises. The first one is that the controllable carbon-dioxide emissions are by-product of fossil-fuel used in a broadly defined human productive activity. The second premise is that the accumulation of carbon-dioxide in the atmosphere affects climate and a deterioration of climate diminishes output mean and increases the output variance. The third premise is that stakeholders are risk-averse and maximize expected utilities from their random net profit. The fourth one is that the international planner takes into account the stakeholders expected utility maximizing emissions, but is bounded rational in the following sense. Due to compounded complexity and for practicality, the international planner does not maximize the sum of the discounted net benefits over a planning horizon. As is commonly practiced by central banks with regard to interest rate, the international planner sets the carbon tax at the beginning of each period at a flat rate required for achieving an atmospheric carbon-dioxide stock target. The fifth premise is that the stakeholders are sovereign countries in various stages of economic development. They have different ability and willingness to pay for carbon-dioxide emissions and cannot be forced to pay the carbon tax set by the international planner. Hence, the higher the tax rate, the smaller the number of complying countries.

Taking into account the first, second and third premises, section 2 derives the effect of carbon tax on the emissions of countries' representative agents. With the fourth and fifth premises in mind, section 3 incorporates the agents' expected utility maximizing carbon-dioxide emissions into the motion equation of the atmospheric carbon stock and formulates the periodical carbon-tax rate required for limiting the atmospheric stock to a predetermined level with an allowance for a negative relationship between the number of complying countries and the carbon-tax level. Section 4 concludes with a discussion of the properties of the constructed carbon-tax rule.

2. Production impeding climate change and agents' emissions

Consider a world where the carbon-dioxide emissions of the representative agent of each country $i = 1, 2, 3, ..., N$ at time t, x_{it}, are proportional to his fossil-fuel consumption, e_{it}. That is,

$$x_{it} = \alpha_i e_{it} \tag{1}$$

where the positive scalar α_i reflects the emission-intensity of country i's production process' fossil-fuel consumption. Suppose, further, that country i's representative agent's output at t, y_{it}, is proportional to his fossil-fuel consumption, but uncertain:

$$y_{it} = (a_{it} + \varepsilon_{it})e_{it}. \tag{2}$$

The positive scalar a_{it} is the expected marginal product of fossil fuel and dependent on the climate at period t. The additional factor, ε_{it}, is a zero-mean normally distributed random deviation from this expected marginal product caused by random disturbances in the representative agent's production environment. That is, $\varepsilon_{it} \sim \mathcal{N}(0, \sigma_{it}^2)$. It is assumed that the variance of this random deviation intensifies with the deterioration of the climate caused by an increasing divergence from the climate-wise ideal atmospheric carbon-dioxide stock. As countries have different geographical conditions, the climate-wise ideal atmospheric stock is country-specific. It is defined, henceforth, as the stock level associated with both the smallest marginal product's variance, σ_i^2, and the largest expected marginal product, $a_i > 0$, of fossil fuel for the representative agent of country i. With S_{t-1} denoting the atmospheric stock of carbon-dioxide at the beginning of period t, S_i^o the ideal stock, and $\beta_i > 0$ the sensitivity of country i's marginal product's variance to climate change, the variance of ε_{it} and of the agent's marginal product at t is expressed as:

$$\sigma_{it}^2 = \sigma_i^2[1+\beta_i(S_{t-1}-S_i^o)^2]. \tag{3}$$

As also assumed, the representative agent's expected marginal product of fossil fuel decreases from the maximal level a_i with the divergence of the actual atmospheric carbon-dioxide stock from the climate-wise ideal stock. This assumption is formally represented by:

$$a_{it} = a_i / [1+\gamma_i(S_{t-1}-S_i^o)^2] \tag{4}$$

where $\gamma_i > 0$ reflects the sensitivity of the agent's expected marginal product to climate change.

Since $x_{it} = \alpha_i e_{it}$ implies that $e_{it} = x_{it} / \alpha_i$, country i's representative agent's production function can be portrayed as:

$$y_{it} = \left[\frac{a_i}{\alpha_i[1+\gamma_i(S_{t-1}-S_i^o)^2]} + (\varepsilon_{it}/\alpha_i)\right]x_{it}. \tag{5}$$

With the price of fossil fuel for country i's representative agent at t being equal to q_{it}, the imputed price of carbon-dioxide emission before tax is q_{it}/α_i. With the price of the product at t being equal to p_{it} and the carbon-tax rate $\tau_{it} \geq 0$, the net profit of country i's representative agent at t is:

$$\pi_{it} = p_{it}\left[\frac{a_i}{\alpha_i[1+\gamma_i(S_{t-1}-S_i^o)^2]} + (\varepsilon_{it}/\alpha_i)\right]x_{it} - [(q_{it}/\alpha_i)+\tau_{it}]x_{it}. \tag{6}$$

This net profit is normally distributed with

$$E(\pi_{it}) = \left[\frac{p_{it}a_i}{\alpha_i[1+\gamma_i(S_{t-1}-S_i^o)^2]} - (q_{it}/\alpha_i) - \tau_{it}\right]x_{it} \tag{7}$$

and

$$VAR(\pi_{it}) = \{p_{it}^2\sigma_i^2[1+\beta_i(S_{t-1}-S_i^o)^2]/\alpha_i^2\}x_{it}^2 . \tag{8}$$

The representative agent derives utility from net profit. As suggested for tractability by Freund (1956), his utility function is taken to be negative exponential:

$$u_{it} = 1-\exp(-R_i\pi_{it}) \tag{9}$$

where $R_i > 0$ represents the agent's degree of absolute risk aversion. Since net profit is random, and in agreement with Von Neumann-Morgenstern utility theorem, each country's representative agent chooses an emission level that maximizes his expected utility. As demonstrated by Freund (1956) with a normally distributed utility generating variable,

$$E(u_{it}) = 1-\exp\{-0.5R_i[E(\pi_{it})-0.5R_iVAR(\pi_{it})]\} . \tag{10}$$

Consequently, country i's representative agent's decision problem can be expressed as:

$$\max_{x_{it}}\left\{\left[\frac{p_{it}a_i}{\alpha_i[1+\gamma_i(S_{t-1}-S_i^o)^2]}-(q_{it}/\alpha_i)-\tau_{it}\right]x_{it}-0.5R_i\{p_{it}^2\sigma_i^2[1+\beta_i(S_{t-1}-S_i^o)^2]/\alpha_i^2\}x_{it}^2\right\}$$
.(11)

As the second-order condition for maximum is satisfied, it is obtained from the first-order condition that the expected utility maximising carbon-dioxide emission level at t for country i's representative agent is equal to the ratio of his expected marginal net profit to his marginal cost of risk-bearing:

$$x_{it}^* = \frac{\dfrac{p_{it}a_i}{\alpha_i[1+\gamma_i(S_{t-1}-S_i^o)^2]}-(q_{it}/\alpha_i)-\tau_{it}}{R_i\{p_{it}^2\sigma_i^2[1+\beta_i(S_{t-1}-S_i^o)^2]/\alpha_i^2\}} . \tag{12}$$

3. Stock targeting carbon-tax with abstinence

The accumulation of carbon-dioxide reflects the imbalance in the atmospheric carbon cycle; namely, the carbon-dioxide emissions of humans, animals, plants and bacteria beyond the sum of the molecules photosynthesised by plants, sunk into the Earth's surface and disseminated into space. With L_{it-1} denoting the population of country i at the beginning of period t, the change in the atmospheric stock of carbon-dioxide during period t is:

$$S_t - S_{t-1} = \sum_{i=1}^{N} L_{it-1}x_{it}^* + z_t - \delta S_{t-1} . \tag{13}$$

The variable z_t indicates the net (of photosynthesised carbon-dioxide) aggregate emission of wildlife at period t, and is assumed, for simplicity, to be accurately expected by the international planner at the beginning of the period. The parameter $0<\delta<1$ denotes the natural depletion (through sinking into the Earth's surface and dissemination into space) rate of atmospheric carbon-dioxide.

Let us assume that the world has N_r rich countries, which can pay carbon tax. The rest, $N-N_r$, are poor and stress their low per capita income and relatively low past and present emissions as reasons for abstinence from an internationally coordinated emission-control scheme. In the following simple illustration, the rich countries are taken to be identical with respect to technology, climate, fossil-fuel price, product price and population size, and so are also the poor countries. However, due to ideological and cultural differences, the rich countries do not have identical willingness to pay carbon tax. We let the number of rich countries willing to pay an internationally set carbon-tax, $N_{rc} \le N_r$, diminish with the carbon-tax rate, $\tau_t > 0$, in accordance with the following formula:

$$N_{rc}(t) = \frac{N_r}{1+\theta\tau_t} \tag{14}$$

where the parameter $\theta > 0$ is the average effect of the carbon-tax rate on the ratio of non-complying to complying rich countries; henceforth, the rich countries' non-compliance coefficient. We assume that the non-complying countries do not implement domestic emission-control schemes.

With this in mind, the international planner sets the periodical carbon-tax rate so as to limit the stock of atmospheric carbon dioxide at the end of period t to a targeted level $\hat{S}_t$. That is, the international planner sets the periodical carbon-tax rate to satisfy the following equality:

$$\hat{S}_t = N_{rc}(t)L_r x^*_{rc} + [N_r - N_{rc}(t)]L_r x^*_{ra} - (N-N_r)L_p x^*_p + z_t + (1-\delta)S_{t-1}. \tag{15}$$

Recalling (12), the emissions of the representative agent of a rich cooperative (rc) country are

$$x^*_{rct} = \frac{\dfrac{p_{rt}a_r}{\alpha_r[1+\gamma_r(S_{t-1}-S^o_r)^2]} - (q_{rt}/\alpha_r) - \tau_t}{R_r\{p^2_{rt}\sigma^2_r[1+\beta_r(S_{t-1}-S^o_r)^2]/\alpha^2_r\}} \tag{16}$$

the emissions of the representative agent of a rich abstaining (ra) country are

$$x_{rat}^{*}=\frac{\dfrac{p_{rt}a_r}{\alpha_r[1+\gamma_r(S_{t-1}-S_r^o)^2]}-(q_{rt}/\alpha_r)}{R_r\{p_{rt}^2\sigma_r^2[1+\beta_r(S_{t-1}-S_r^o)^2]/\alpha_r^2\}} \tag{17}$$

and the emissions of the representative agent of a poor country are

$$x_{pt}^{*}=\frac{\dfrac{p_{pt}a_p}{\alpha_p[1+\gamma_p(S_{t-1}-S_p^o)^2]}-(q_{pt}/\alpha_p)}{R_p\{p_{pt}^2\sigma_p^2[1+\beta_p(S_{t-1}-S_p^o)^2]/\alpha_p^2\}}. \tag{18}$$

By substituting (14), (16), (17) and (18) into (15), the stock-targeting carbon-tax rate should satisfy:

$$\begin{aligned}\hat{S}_t=&\frac{N_r}{1+\theta\tau_t^*}L_r\frac{\dfrac{p_{rt}a_r}{\alpha_r[1+\gamma_r(S_{t-1}-S_r^o)^2]}-(q_{rt}/\alpha_r)-\tau_t^*}{R_r\{p_{rt}^2\sigma_r^2[1+\beta_r(S_{t-1}-S_r^o)^2]/\alpha_r^2\}}\\ &+\left[N_r-\frac{N_r}{1+\theta\tau_t^*}\right]L_r\frac{\dfrac{p_{rt}a_r}{\alpha_r[1+\gamma_r(S_{t-1}-S_r^o)^2]}-(q_{rt}/\alpha_r)}{R_r\{p_{rt}^2\sigma_r^2[1+\beta_r(S_{t-1}-S_r^o)^2]/\alpha_r^2\}}\\ &+(N-N_r)L_p\frac{\dfrac{p_{pt}a_p}{\alpha_p[1+\gamma_p(S_{t-1}-S_p^o)^2]}-(q_{pt}/\alpha_p)}{R_p\{p_{pt}^2\sigma_p^2[1+\beta_p(S_{t-1}-S_p^o)^2]/\alpha_p^2\}}+z_t+(1-\delta)S_{t-1}.\end{aligned} \tag{19}$$

By multiplying both sides of this equality by $(1+\theta\tau_t^*)$ and rearranging terms, the international planner's atmospheric stock targeting carbon-tax rate rule is:

$$\tau_t^*=\frac{\left[\dfrac{N_rL_r\alpha_r\left(\dfrac{p_{rt}a_r}{[1+\gamma_r(S_{t-1}-S_r^o)^2]}-q_{rt}\right)}{R_rp_{rt}^2\sigma_r^2[1+\beta_r(S_{t-1}-S_r^o)^2]}\right]+\left[\dfrac{(N-N_r)L_p\alpha_p\left(\dfrac{p_{pt}a_p}{[1+\gamma_p(S_{t-1}-S_p^o)^2]}-q_{pt}\right)}{R_pp_{pt}^2\sigma_p^2[1+\beta_p(S_{t-1}-S_p^o)^2]}\right]-\hat{S}_t+z_t+(1-\delta)S_{t-1}}{\theta[\hat{S}_t-z_t-(1-\delta)S_{t-1}]+\left[\dfrac{N_rL_r\alpha_r\left[\alpha_r-\theta\left(\dfrac{p_{rt}a_r}{[1+\gamma_r(S_{t-1}-S_r^o)^2]}-q_{rt}\right)\right]}{R_rp_{rt}^2\sigma_r^2[1+\beta_r(S_{t-1}-S_r^o)^2]}\right]-\left[\dfrac{(N-N_r)L_p\theta\alpha_p\left(\dfrac{p_{pt}a_p}{[1+\gamma_p(S_{t-1}-S_p^o)^2]}-q_{pt}\right)}{R_pp_{pt}^2\sigma_p^2[1+\beta_p(S_{t-1}-S_p^o)^2]}\right]} \tag{20}$$

4. Conclusion

This paper provide an international planner a rule for setting a flat carbon tax that limits greenhouse gasses' stock to a predetermined level in a world where each stakeholder faces, due to climate change, diminishing mean and increasing variance of the marginal product of his fossil-fuel consumption and maximizes expected utility from the uncertain net profit. The construction of the rule took into account that some of the stakeholders are poor countries, hence unable and/or unwilling to pay the carbon tax. It also took into account that, due to variation in willingness to pay, the number of the cooperating rich countries diminishes with the tax rate. For illustration, the rule was constructed for the tractable case where the poor countries are identical and so are also the rich countries, but with the aforesaid exception of non-identical willingness to pay carbon tax. For numerical simulations and application, the rule can be reformulated for the real case, where all the countries are different, with non-identical parameters.

An inspection of the constructed rule reveals that the international planner's carbon-tax rate decreases with the greenhouse gasses' stock-target, $\hat{S}_t$, with the natural depletion rate of greenhouse gasses, δ, and with the prices of fossil fuel in the rich countries and in the poor countries, q_{rt} and q_{pt}. The international planner's carbon-tax rate rises with the expected marginal product of energy in the rich and the poor countries attainable under ideal climate, a_r and a_p, with the emission-intensities of the poor countries' production process' fossil-fuel consumption, α_p and α_r, with the poor countries' population, $(N - N_r)L_p$ and with the expected periodical net aggregate carbon-dioxide emissions of wildlife, z_t. Despite the tax-increasing effect of the fossil-fuel consumption's emission-intensities, the carbon tax does not necessarily rise with the rich countries' population, $N_r L_r$. It can decrease with the population of the rich countries when the value of the rich countries' non-compliance coefficient, θ, is sufficiently low, as the burden is carried by a sufficiently large number of cooperating rich countries.

The inspection also reveals that the international planner's periodical carbon-tax rate decreases with the poor countries' representative agents' risk-bearing-cost coefficient, $R_p p_{pt}^2 \sigma_p^2 [1 + \beta_p (S_{t-1} - S_p^o)^2]$. More particularly, the carbon tax decreases with the poor countries' degree of absolute risk aversion, R_p, variance of marginal output under ideal climate, σ_p^2, and sensitivity of the marginal product's variance to climate change, β_p. The effect of the rich countries' representative agents' risk-bearing-cost coefficient, $R_r p_{rt}^2 \sigma_r^2 [1+\beta_r (S_{t-1} - S_r^o)^2]$, on the carbon tax set by the international planner is not clear.

The inspection further reveals that the international planner's periodical carbon tax rises with both the rich and poor countries' representative agents' expected marginal profit coefficients $\{p_{rt} a_r / [(1+\gamma_r (S_{t-1} - S_r^o)^2)] - q_{rt}\}$ and $\{p_{pt} a_p / [1+\gamma_p (S_{t-1} - S_p^o)^2)] - q_{pt}\}$, respectively, and in a rate intensified by the rich countries' non-compliance coefficient, θ. This property indicates that the international planner's periodical carbon tax rate decreases with the rich and poor countries' expected marginal product' sensitivity to climate change, γ_r and γ_p. In conjunction with the previously identified moderating effect of the poor countries' risk-bearing-cost coefficient, this property also indicates that the tax rate decreases with the deviation of the atmospheric carbon-dioxide stock from the climate-wise ideal level for the poor countries, $(S_{t-1} - S_p^o)^2$. The direction of the overall effect of the deviation of the atmospheric carbon-dioxide stock from the climate-wise ideal level for the rich countries, $(S_{t-1} - S_p^o)^2$, on the international planner's periodical carbon tax is not clear.

Finally, as long as the carbon-dioxide stock target is set below the actual initial stock minus its natural depletion and plus the expected net aggregate emissions of wildlife during the period (i.e., $\hat{S}_t < z_t + (1-\delta) S_{t-1}$), the international planner's periodical carbon tax rises with the rich countries' non-compliance coefficient, θ; whereas for $\hat{S}_t >> z_t + (1-\delta) S_{t-1}$, the carbon tax can decrease with θ.

References

Fischer, Carolyn and Richard G. Newell (2008), "Environmental and technology policies for climate mitigation", *Journal of Environmental Economics and Management* 55 (2), pp. 142–162.

Freund, R.J. (1956), "An introduction of risk into a risk programming model", *Econometrica* 24, 253-63.

Hoel, Michael and Larry Karp (2002), "Taxes versus quotas for a stock pollutant", *Resource and Energy Economics* 24 (4), pp. 367–384.

Levy, Amnon (2011), "Abstinence with Reputation Loss, Understating Expectations and Guilt and the Effectiveness of Emission Tax", University of Wollongong Economics Working Paper 11-01.

Newell, Richard G. and William A. Pizer (2003), "Regulating stock externalities under uncertainty", *Journal of Environmental Economics and Management* 45 (2), pp. 416–432.

Pizer, William A. (2002), "Combining price and quantity controls to mitigate global climate change", *Journal of Public Economics* 85 (3), pp. 409–434.

Weitzman, Martin L. (1974) "Prices vs. quantities", *Review of Economic Studies* 41 (4), 477–491.