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Introduction to the Economics of Atmospheric Carbon-Dioxide Control

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Introduction to the Economics of Controlling Atmospheric Carbon-Dioxide

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The objective of this paper is to provide an introduction to the economics of controlling the stock of carbon-dioxide in the atmosphere. The paper starts with a brief summary of the arguments against a wait-and-see strategy and in favour of controlling carbon emissions. It then provides a basic analysis of the effect of carbon tax on net-cash flow maximising agents' emissions and offers two possible ways for setting the tax rate. The first one computes an atmospheric carbon-dioxide stock-targeting tax rate with abstinence of some agents, whereas the second considers universal cooperation and computes a welfare-maximising carbon-tax rate. While these computations assume a fixed rate of depletion of the atmospheric stock of carbon dioxide, the last section takes the depletion rate to be dependent on the distribution of the usable land between plants and humans and the change in the usable land to be dependent on the change in the atmospheric carbon-dioxide stock. The usable land allocation required for achieving a target stock of atmospheric carbon dioxide is subsequently computed. (*JEL* Q52, Q54)

Keywords: Emissions; Carbon-Cycle Imbalance; Atmospheric Carbon Stock; Global Warming; Usable land: Control Measures; Carbon Tax; Plants-Humans Land Allocation

1. The Carbon-Cycle's Imbalance and Its Expected Implications

Carbon emissions are essential for life. In their absence, Earth would become an icy planet. However, excessive concentration of carbon-dioxide in the atmosphere would render Earth a hot, desolate planet. Global warming is a process where emissions of greenhouse gases (GHGs) create conditions that lead to a rise in the temperature of the surface of the planet and subsequently to climate change. The principal GHG is Carbon Dioxide. It is responsible for about eighty percents of the green-house effect. Hence, the accumulation of GHGs reflects mainly the imbalance in the atmospheric carbon cycle—the emissions of CO_2 by humans, animals and bacteria beyond the level absorbed by plants through photosynthesis. Per capita, the largest emitters of CO_2 are the rich industrialized countries (see Levy et al., 2011, for international comparison).

Monthly measurements of carbon-dioxide concentration in the troposphere have begun by Charles Keeling in 1958 at the astronomical observatory below the summit of Mt. Mauna Loa (4,169 m) in Hawaii. Taken far away from major source and sink sites of carbon-dioxide, those measurements provide a good assessment of the Earth's background level of atmospheric carbon-dioxide. They form *The Keeling Curve*, which is displayed in Figure 1. The oscillations around this curve reflect seasonal variations of the imbalance between humans, animals and bacteria aggregate carbon-dioxide emissions, on the one hand, and absorption of carbon dioxide by plants, on the other hand, in the land-wise larger and more populated northern hemisphere.

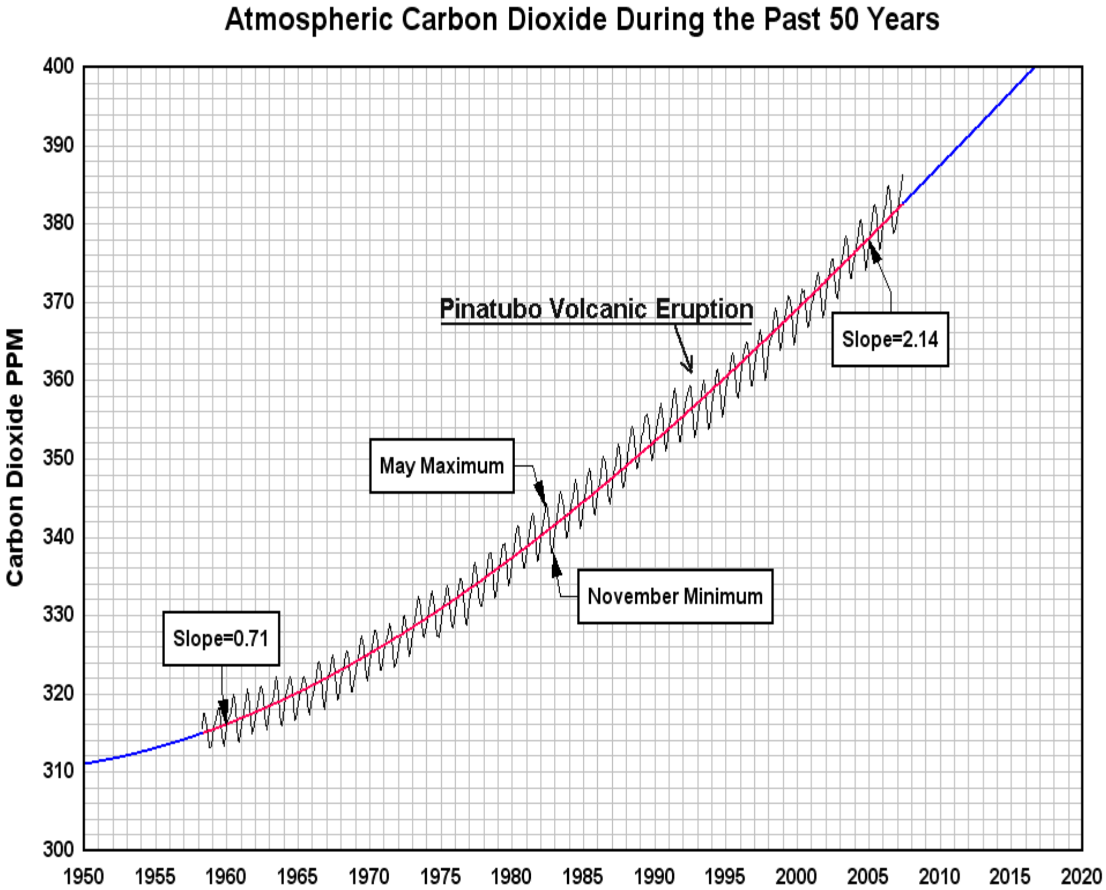

Figure 1. The Keeling Curve
Source: National Oceanic & Atmospheric Administration (NOAA), U.S. Department of Commerce, http://www.esrl.noaa.gov/gmd/ccgg/trends/

Analyses of bubbles trapped in ice-cores extracted from Antarctica have provided indications of concentrations of carbon dioxide in Earth's atmosphere during the earlier 600,000 years. As can be seen from Figure 2, when contrasted with the deep historical concentration levels, the Keeling Curve reveals unprecedented levels and rate of accumulation of atmospheric carbon dioxide from about 280 particles per million on the eve of the Industrial Revolution (1750 AD) to 380 particles per million in 2010. The industrialization and modernisation of the developing countries and the deforestation of tropical lands by logging and clearing for cash-crops intensify the imbalance in the carbon cycle and strengthen the aforesaid trend.

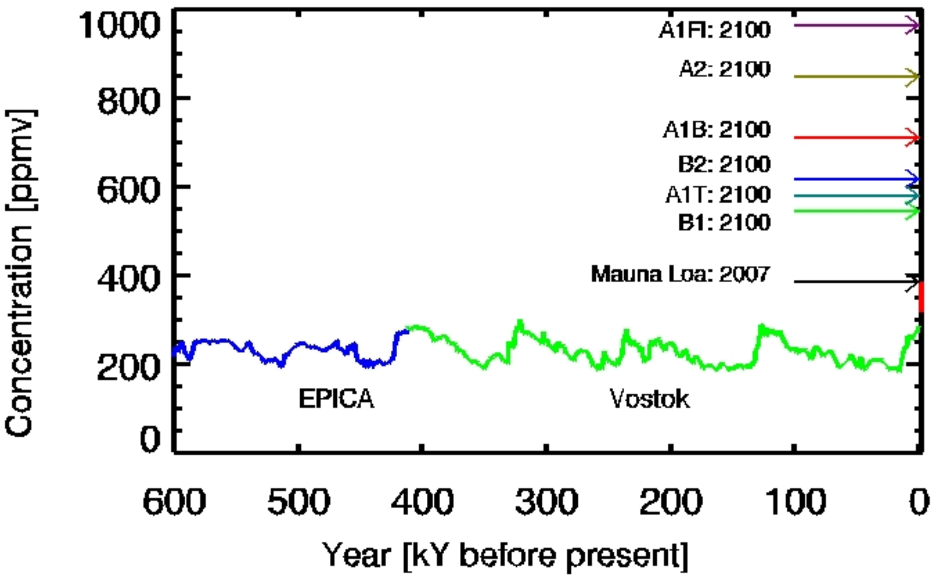

Figure 2. Historical carbon-dioxide concentrations derived from EPICA (in blue) and Vostok (in green) ice cores and The Keeling Curve (in red)
Source: http://planetforlife.com/co2history/index.html

Figure 3 reveals a high level of positive correlation between temperature variation from the present level and carbon-dioxide concentration in the atmosphere. The equilibrium climate sensitivity—the global average surface warming of Earth following a doubling of the concentration of CO_2 in the atmosphere—is assessed to be most likely about three degrees Celsius. The subsequent adverse effect of this equilibrium climate sensitivity on the level of global output in the twenty-second century is assessed to be only a few percents, hence negligible in present value. This assessment might lend support to a wait-and-see strategy. However, due to the compounded uncertainty embedded in the assembly of the components of the used benefit-cost models, the upper-tail of the probability density function of the Earth's surface-temperature change might be fat (Weitzman, 2009, 2011). An increase of six degrees Celsius, rather than the expected three, in the Earth's surface average temperature will deprive massive populations of the river-water supply that has been essential for their existence. A six-degree Celsius rise will also dilute major ocean conveyers. Another argument against a wait-and-see strategy is irreversibility. Since carbon-dioxide emissions remain in the atmosphere for many years, the implications of decisions on current emissions for the stock of GHGs are difficult-to-reverse. Moreover, the warming of the oceans causes acidification of their waters, hence reduces their absorptive capacity of carbon-dioxide and increases the possibility of a release of another slowly depleted GHG, methane hydrate, from the continent shelves to the atmosphere.

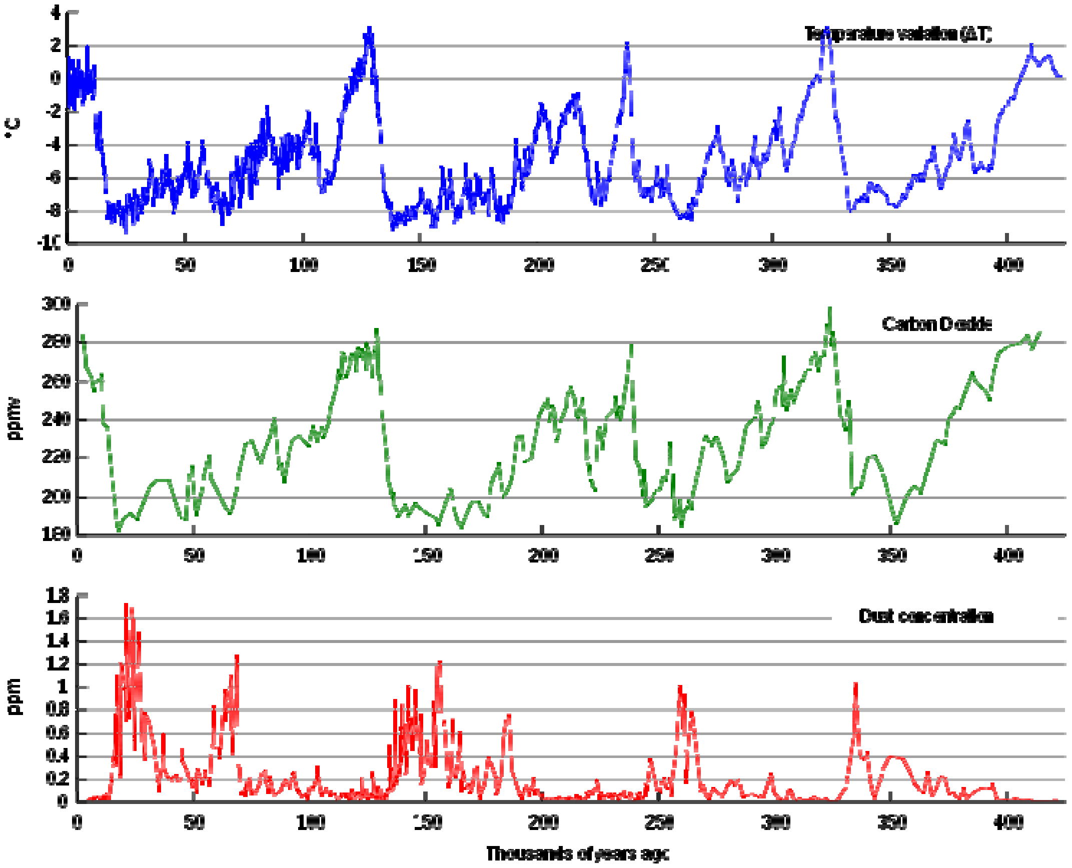

Figure 3. Carbon dioxide and dust concentration and surface temperature variation
Source: Vostok_Petit_data.svg file

2. Market-Based Control of Carbon Emissions

Intergenerational ethics and, subsequently, sense of responsibility, ensures assignment of significant weights to future benefits and costs by the present generation. In the absence of such ethics and sense of responsibility current emissions of carbon-dioxide into the atmosphere by one agent are excessive and impose negative external effects on other agents and future generations by aggravating the imbalance in the carbon cycle and increasing the stock of atmospheric green-house gasses. Failure of private initiatives to set markets for negative external effects justifies public intervention. The Coase Theorem implies that, as long as transaction costs are sufficiently low, some negative externalities can be moderated by assignment of property rights. However, the atmosphere is indivisible: it belongs to all and no one. Moreover, emissions spread and the sources of their atmospheric concentration in any given location are numerous. Thus, transaction costs, if assignment of property rights were possible, are very high.

The control instruments of carbon emissions (and GHGs in general) at the disposal of local, national and international policy makers are classified as standards or market-based instruments. The set of market-based instruments includes emission trading schemes, emission taxes and abatement subsidies. Emissions trading schemes are based on two principles: a cap on aggregate emissions and tradeable emission-permits that sum up to the cap. The imposition of a cap and allocation of permits reduce uncertainty about total emission and its sources. The cap may be changed over time so as to meet domestic and/or international targets of emission reductions. The assumption underlying the implementation of a carbon-trading scheme is that efficiency will be achieved through market-based redistribution of permits. However, the redistribution of carbon-emission permits depends on the initial allocation of such rights, which might be bias in favour of certain industries and consumers. For example, the Australian Federal Government's *Green Paper on Emission Trading* of July 2008, indicates that firms in carbon-intensive industries such as aluminium production and electrical power generation would initially receive free permits and other compensations in order to maintain their operation and prevent them from moving off-shore. The agricultural sector would not initially be restricted. Other small polluters with large aggregate political influence would receive a cent-to-cent compensation on any rise in petrol price stemming from the scheme through excise tax reduction on petrol. Furthermore, the market of carbon emission permits is unlikely to be perfectly competitive. Some traders will be large, better informed and more sophisticated and hence will possess a significant market power. The implementation of a carbon-emission-trading scheme involves a huge monitoring and enforcement effort and is not necessarily the most efficient method.

An alternative method that does not require huge monitoring and enforcement effort is based on application of a uniform carbon-tax rate on the purchasing of inputs such as coal and petrol. The carbon-tax rate can be changed over time to meet environmental targets. However, the notion of tax is unpopular among consumers and governments are influenced by public sentiments. Furthermore, the inclusion of a uniform carbon tax on certain inputs does not provide a perfectly adequate signal, hence incentive, to users. For example, the carbon emissions by vehicles vary with make, vintage, maintenance, traffic conditions, load and driver's behaviour. Optimally, the carbon tax rate should vary in accordance with these factors. There are concerns that carbon-tax might be regressive as the share of spending on utilities, electricity in particular, is higher for low-income earners. These concerns serve as an argument in favour of subsidy of greener technologies and use of arable land and other natural resources. Carbon tax on utilities such as electricity can be made progressive for households: no tax up to a certain essential level, and thereafter rising along a step-diagram. The carbon-tax rate can be varied over time to meet environmental targets. The implementation of carbon tax is perceived to involve greater uncertainty about emissions, but lower monitoring and enforcement costs, than trading schemes. The effects of carbon-tax on emission-reduction, consumer-goods' prices and welfare depend on the elasticity of demand to goods whose production is carbon-fuel intensive. Comparisons of efficiency of price-incentive instruments and quota instruments have been provided by Pizer (2002), Hoel and Karp (2002), Newell and Pizer (2003) and Fischer and Newell (2008).

In addition to environmental and economic aspects, the choice of market-based instruments depends on social and cultural aspects. There is a fear that unilateral implementation of emission-tax, in particular, would reduce disposable income, worsen terms of trade through price-inflation, and increase unemployment. In many countries the introduction of new taxes is very unpopular. Unpopularity leads to compromised implementation. It seems that North Europeans, Scandinavians in particular, are more tolerant toward taxes, hence paying for pollution, than Americans (cf. Berck and Helfand, 2010). Most notably, carbon tax has amounted to about 3% of Sweden's GDP (*vis-à-vis* about 1% in the US) and lowered Sweden's aggregate carbon emissions to a level below the target set in the *Kyoto Protocol*. Carbon tax has also been implemented in Canada and New Zealand. Since 2008 a cap on the aggregate carbon emissions of electrical power producers in a region comprising ten north and central eastern states of the US has been set and tradable emission-permits were initially auctioned at a clearing price of about 3 dollars per ton.

In July 2011 the Federal Government of Australia proposed the highest carbon price—an initial price of 23 Australian dollars per ton of emissions generated by about 500 largest polluters—for implementation on 1 July 2012. Households would not be charged directly, but can expect higher prices of utilities. The less inelastic the demand for utilities, the greater the expected price hike. To moderate the regressive effect of the carbon price, all households with annual income lower than 80,000 dollars would enjoy some reduction in income tax. Those with annual income lower than 18,300 dollars would be released from paying income tax and lodging a tax return. In addition to the aforementioned general concerns, the following problems are embedded in the Australian Federal Government’s carbon-pricing proposal. First, the proposal allows the vast majority of direct emitters of carbon dioxide to free-ride, though indirect moderating effect is expected. Second, some of the domestic major industrial polluters and ninety percents of the households would be compensated. The compensation would weaken their incentive to reduce emissions and would come at the expense of alternative use of tax revenues. Third, the tax would not be applied on the huge export of coal. Carbon emissions from burning Australian coal in the importing countries are not negligible and affect all. Fourth, while some countries and states price carbon emissions, most do not and free-ride. Fifth, although the Australian Federal Government’s proposed carbon price is about four times higher than the effective globally average, it is almost three-times lower than Nordhaus’ (2010) estimate of 64 USD per ton already required in 2010 for limiting global warming to two degrees Celsius.

The implementation of carbon tax by some affluent countries might not lead to a reduction in global emissions. Levy (2011) has considered an interaction between tax-collecting rich countries, abstaining rich countries and abstaining poor countries. In his setting, the abstaining countries can lose reputation and suffer from guilt and might overstate the tax’s emission-moderating effect. The computed equilibrium reveals that even with loss of reputation and guilt, taxing emissions and directing the revenues to green investment would not necessarily reduce global emissions, nor the tax-collecting rich countries’ emissions. Nevertheless, a unilateral implementation of the tax can be viewed as a moral obligation of rich countries to lead the way in addressing the problem of global warming.

3. How is a Carbon Tax expected to work?

A basic model (which ignores, for simplicity, issues such as market-power, uncertainty, risk aversion and time-preferences) is constructed to demonstrate the effect of carbon tax on agents’ carbon-dioxide emissions and, subsequently, to compute the desirable carbon-tax rate for achieving a predetermined atmospheric stock, or a global welfare level. In that basic model, output increases concavely with the use of energy extracted from burning fossil fuel. The carbon-dioxide level emitted by each agent (household or firm) i=1,2,3,...,N is proportional to the quantity of fossil fuel used by the agent and hence production can be represented as a concave function of the agent’s level of carbon-dioxide emissions. The model employs the following notations and basic specifications. The carbon-based energy used by agent i at time t is denoted by E_{it}. The carbon emissions of agent i at time t, x_{it}, are given by:

$$x_{it} = \alpha_i E_{it} \tag{1}$$

where $\alpha_i > 0$ reflects the emission-intensity of the agent’s production process’ fuel consumption.

Agent i’s output at t is denoted by y_{it} and is given by:

$$y_{it} = a_i E_{it} - b_i E_{it}^2, \quad a_i >> 2b_i > 0. \tag{2}$$

The price of energy for agent i at t is q_{it}. The price of agent i’s product at t is p_{it}. The carbon-tax rate at t is flat and equal to $\tau_t > 0$.

Each agent i chooses her emission level at t to maximise her net cash-flow. Noting that $x_{it} = \alpha_i E_{it}$ implies $E_{it} = x_{it} / \alpha_i$, each agent’s production function can be expressed as:

$$y_{it} = (a_i / \alpha_i) x_{it} - (a_i / \alpha_i^2) x_{it}^2 \tag{3}$$

and her imputed price of carbon before tax is q_{it} / α_i. Hence, the decision problem of agent i is expressed as:

$$\max_{x_i} \{ p_{it} [(a_i / \alpha_i) x_{it} - (b_i / \alpha_i^2) x_{it}^2] - [(q_{it} / \alpha_i) + \tau_t] x_{it} \}. \tag{4}$$

The necessary and sufficient condition for maximum implies equality between the marginal return on emissions and the full price of emissions—the sum of their imputed price and tax rate:

$$p_{it} [(a_i / \alpha_i) - 2(b_i / \alpha_i^2) x_{it}^*] = (q_{it} / \alpha_i) + \tau_t. \tag{5}$$

The net cash-flow maximising carbon-emission level at t for agent i is:

$$x_{it}^* = \left(\frac{a_i}{2b_i / \alpha_i} \right) - \left(\frac{1}{2(b_i / \alpha_i^2) p_{it}} \right) [(q_{it} / \alpha_i) + \tau_t]. \tag{6}$$

The effect of the emission tax on agent i's abatement is weakened by the marginal net revenue and by the rate in which the emissions' marginal product diminishes. Consequently, the stock (S) of carbon-dioxide in the atmosphere at the end of period t is:

$$S_t = \sum_{i=1}^{N} x_{it}^{*} - \delta S_{t-1} \tag{7}$$

where $0 < \delta < 1$ represents the depletion rate of atmospheric carbon-dioxide stock through photosynthesis, sinking and dissemination to space.

4. Some Possible Ways of Setting Carbon-Tax Rate

This section describes two possible approaches to carbon-tax setting. The first one computes an atmospheric carbon-dioxide stock-targeting tax rate with abstinence of some of the agents, whereas the second considers universal cooperation and computes a welfare-maximising carbon-tax rate.

4.1. Atmospheric stock-targeting carbon-tax rate with abstinence

Let us assume that the world's N agents can be classified into a group of N_1 identical agents, with a_1, b_1, p_{1t}, q_{1t}, who cooperate and pay a carbon-tax $\tau_{1t} > 0$ that limits the stock of atmospheric carbon dioxide at the end of period t to a targeted level $\hat{S}_t$, and $N-N_1$ identical agents, with a_2, b_2, p_{2t}, q_{2t}, who abstain ($\tau_{2t} = 0$). In this scenario, the carbon tax-rate set by the group of the willing and cooperating agents satisfies the following equality:

$$\hat{S}_t = N_1 x_{1t}^{*} + (N - N_1) x_{2t}^{*} - \delta S_{t-1}. \tag{8}$$

Recalling (6),

$$x_{1t}^{*} = \left(\frac{a_1}{2b_1 / \alpha_1} \right) - \left(\frac{1}{2(b_1 / \alpha_1{}^2) p_{1t}} \right) [(q_{1t} / \alpha_1) + \tau_{1t}] \tag{9}$$

and

$$x_{2t}^{*} = \left(\frac{a_2}{2b_2 / \alpha_2} \right) - \left(\frac{1}{2(b_2 / \alpha_2{}^2) p_{2t}} \right) [(q_{2t} / \alpha_2) \tag{10}$$

and

$$\begin{aligned}\hat{S}_t &= N_1\left(\frac{a_1}{2b_1/\alpha_1}\right) - N_1\left(\frac{1}{2(b_1/\alpha_1{}^2)p_{1t}}\right)[(q_{1t}/\alpha_1)+\tau^*_{1t}] \\ &\quad + (N-N_1)\left(\frac{a_2}{2b_2/\alpha_2} - \frac{q_{2t}/\alpha_2}{2(b_2/\alpha_2{}^2)p_{2t}}\right) - \delta S_{t-1}.\end{aligned} \tag{11}$$

Consequently, the carbon-tax rate paid by the group of the willing and cooperating members is:

$$\tau^*_{1t} = \frac{N_1\left(\frac{a_1}{2b_1/\alpha_1}\right) + (N-N_1)\left(\frac{a_2}{2b_2/\alpha_2} - \frac{q_{2t}/\alpha_2}{2(b_2/\alpha_2{}^2)p_{2t}}\right) - \delta S_{t-1}}{2(b_1/\alpha_1{}^2)p_{1t}/N_1} - (q_{1t}/\alpha_1)\,. \tag{12}$$

By substituting this tax rate into equation (9), the emissions abated by a willing and cooperating member are:

$$\begin{aligned}\Delta x^*_{1t} &= \left(\frac{1}{2(b_1/\alpha_1{}^2)p_{1t}}\right)\tau^*_{1t} \\ &= \frac{N_1\alpha_1{}^4}{4b_1{}^2p_{1t}{}^2}\left[\frac{N_1a_1\alpha_1}{2b_1} + \frac{(N-N_1)\alpha_2[a_2-(q_{2t}/p_{2t})]}{2b_2} - \delta S_{t-1} - \frac{2q_{1t}b_1p_{1t}}{\alpha_1{}^3N_1}\right].\end{aligned} \tag{13}$$

When all of the N agents are identical (hence the agent-type index can be omitted) willing and cooperating,

$$\tau^*_t = \frac{N\left(\frac{a}{2b/\alpha}\right) - \delta S_{t-1}}{2(b/\alpha^2)p_t/N} - (q_t/\alpha)\,. \tag{14}$$

and the emissions abated by each agent are, of course, smaller:

$$\Delta x^*_{1t} = \left(\frac{1}{2(b/\alpha^2)p_t}\right)\tau^*_t = \left(\frac{N\alpha^4}{(2bp_t)^2}\right)\left[\frac{Na\alpha}{2b} - \delta S_{t-1} - \frac{2q_tbp_t}{\alpha^3N}\right]. \tag{15}$$

4.2. Global cooperation and a welfare-maximising carbon-tax rate

In this case of a cooperative world, the world's regulator substitutes x^*_{it} into her objective function, a global welfare function (W); say, the sum of all the agents' net cash-flows, plus the carbon-tax revenues (redistributed through public services), and minus the damage inflicted by the stock of carbon-dioxide in the atmosphere. Assuming that the benefit from the carbon-tax revenues generated through redistribution and provision of public goods and investment by the world's regulator are equal to the forgone privately generated ones, the carbon tax payments and revenues can be omitted from W. The world's regulator computes the carbon-tax rate that maximises the global welfare function:

$$\max_{\tau}\left\{W_t=\sum_{i=1}^{N}p_{it}[(a_i/\alpha_i)x_{it}^{*}-(b_i/\alpha_i^{2})x_{it}^{*2}]-(q_{it}/\alpha_i)x_{it}^{*}-D(S_t)\right\}$$

subject to the aforementioned atmospheric carbon-dioxide stock constraint. Here, $D(S_t)$ represents the damage caused by the atmospheric carbon-dioxide stock via global warming. The damage is assumed to increase convexly in S (i.e., $D', D'' > 0$) as represented by the following second-order polynomial:

$$D_t=\varphi_1 S_t+\varphi_2 S_t^{2},\quad \varphi_1,\varphi_2>0\,. \tag{14}$$

By substituting the carbon-dioxide stock equation into the damage function and the latter into W_t:

$$W_t=\sum_{i=1}^{N}p_{it}[((a_i-q_{it})/\alpha_i)x_{it}^{*}-(b_i/\alpha_i^{2})x_{it}^{*2}]$$
$$-\varphi_1\left[\sum_{i=1}^{N}x_{it}^{*}-\delta S_{t-1}\right]-\varphi_2\left[\sum_{i=1}^{N}x_{it}^{*}-\delta S_{t-1}\right]^{2}. \tag{15}$$

As the second-order condition is satisfied, the global welfare maximising carbon-dioxide tax can be computed from the following first-order condition:

$$\frac{dW(t)}{d\tau}=\sum_{i=1}^{N}p_i[((a_i-q_{it})/\alpha_i)-2(b_i/\alpha_i^{2})x_{it}^{*}]\frac{dx_{it}^{*}}{d\tau}$$
$$-\varphi_1\sum_{i=1}^{N}\frac{dx_{it}^{*}}{d\tau}-2\varphi_2\left[\sum_{i=1}^{N}x_{it}^{*}-\delta S_{t-1}\right]\sum_{i=1}^{N}\frac{dx_{it}^{*}}{d\tau}=0. \tag{16}$$

Recalling (6),

$$\frac{dx_{it}^{*}}{d\tau}=-\frac{1}{2(b_i/\alpha_i^{2})p_{it}} \tag{17}$$

and the necessary condition for maximum global welfare can be expressed as:

$$\sum_{i=1}^{N}\frac{q_{it}}{2(b_i/\alpha_i)}-(1+2\varphi_2)\sum_{i=1}^{N}\left(\frac{q_{it}}{2(b_i/\alpha_i)p_{it}}\right)+(\varphi_1-2\varphi_2\delta S_{t-1})\sum_{i=1}^{N}\frac{1}{2(b_i/\alpha_i^{2})p_{it}}$$
$$+2\varphi_2\sum_{i=1}^{N}\frac{a_i\alpha_i}{4(b_i/\alpha_i)^{2}p_{it}}=\tau_t^{o}(1+2\varphi_2)\sum_{i=1}^{N}\left(\frac{1}{2(b_i/\alpha_i^{2})p_{it}}\right) \qquad .(18)$$

Hence,

$$\tau_t^o = \frac{(\varphi_1 - 2\varphi_2\delta S_{t-1})}{(1+2\varphi_2)} - \frac{\sum_{i=1}^{N}\left(\frac{q_{it}}{2(b_i/\alpha_i)p_{it}}\right)}{\sum_{i=1}^{N}\left(\frac{1}{2(b_i/\alpha_i{}^2)p_{it}}\right)}$$

$$+\frac{\sum_{i=1}^{N}\frac{q_{it}}{2(b_i/\alpha_i)} + 2\varphi_2\sum_{i=1}^{N}\frac{a_i\alpha_i}{4(b_i/\alpha_i)^2 p_{it}}}{(1+2\varphi_2)\sum_{i=1}^{N}\left(\frac{1}{2(b_i/\alpha_i{}^2)p_{it}}\right)}. \quad .(19)$$

In the special case where all the N agents are identical,

$$\tau_t^o = \frac{(\varphi_1 - 2\varphi_2\delta S_{t-1}) - (1+2\varphi_2 - p_{it})q_{it}/\alpha_i + \varphi_2 a_i\alpha_i/b_i}{(1+2\varphi_2)}$$

If the damage function is linear (i.e., $\varphi_2 = 0$), then in the case of non-identical agents

$$\tau_t^o = \varphi_1 - \sum_{i=1}^{N}\left(\frac{q_{it}(1-p_{it})}{2(b_i/\alpha_i)p_{it}}\right)\Bigg/\sum_{i=1}^{N}\left(\frac{1}{2(b_i/\alpha_i{}^2)p_{it}}\right)$$

or

$$\tau_t^o = \varphi_1 + (p_t - 1)(q_t/\alpha)$$

in the case of identical agents. In the latter case, the implementation of the tax leads each agent to reduce her emissions by:

$$\Delta x_t^* = \frac{\varphi_1 + (p_t - 1)(q_t/\alpha)}{2(b/\alpha^2)p_t}.$$

5. Photosynthesis versus Emissions: Time-Variant Usable Land and Allocation

In the previous two sections a fixed rate of depletion of carbon-dioxide, δ, was assumed. In the real world, the carbon-dioxide's depletion rate depends on the intensity of photosynthesis and hence on the allocation of land between plants and humans. Humans and plants compete on the Earth's useable land. With L_t denoting the Earth's total usable (for simplicity, uniform) land (in acres) and L_t^h the land occupied by humans at period t, then $L_t - L_t^h$ is the land occupied by plants. Surface warming changes the Earth's usable land. Since surface warming is a function of the stock of carbon dioxide in the atmosphere, the change in the Earth's acreage of usable land is:

$$L_t = L_{t-1} - \mu(S_t - S_{t-1})\,. \tag{21}$$

In already warm regions surface warming diminishes usable land, whereas in cold regions it increases the size of usable land. The scalar μ is positive (negative) if the overall effect of surface warming on Earth’s usable land is negative (positive).

The change in the stock of carbon-dioxide in the atmosphere at t reflects the imbalance between humans’ emissions and plants’ photosynthesis. With linearity in land assumed (for simplicity) this change is:

$$S_t - S_{t-1} = \alpha[F(N_t)L_t^h] - \beta(L_t - L_t^h) \tag{22}$$

where $F(N_t)L_t^h$ is the aggregate human production function (with F is a concave in N), $\alpha > 0$ is emissions per unit of human output, and $\beta > 0$ is photosynthesis per acre. By substituting the usable land equation into the carbon-dioxide stock equation,

$$(1-\beta\mu)(S_t - S_{t-1}) = [\alpha F(N_t) + \beta]L_t^h - \beta L_{t-1}\,. \tag{23}$$

Let us reconsider the analytically simple case of targeting the atmospheric stock of carbon dioxide. The above equality implies that in order to achieve a target level of $\hat{S}_t$ (lower than S_{t-1}) units of carbon dioxide in the atmosphere the land occupied by humans at t should not exceed:

$$L_t^h = \frac{\beta L_{t-1} - (1-\beta\mu)(S_{t-1} - \hat{S}_t)}{\alpha F(N_t) + \beta} \tag{24}$$

and the land occupied by plants at t should be at least:

$$L_t^p = L_t - \frac{\beta L_{t-1} - (1-\beta\mu)(S_{t-1} - \hat{S}_t)}{\alpha F(N_t) + \beta}\,. \tag{25}$$

The land allocated to plants increases with the difference between the actual atmospheric carbon stock at the end of the previous period and the target level but at a rate that is moderated by the rates of land loss and photosynthesis. However, the total effect of the photosynthesis rate is not clear *a-priori*:

$$\frac{\partial L_t^p}{\partial \beta} = \left[\frac{\beta L_{t-1} - (1-\beta\mu)(S_{t-1} - \hat{S}_t) - [\mu(S_{t-1} - \hat{S}_t) + \beta][\alpha F(N_t) + \beta]}{[\alpha F(N_t) + \beta]^2}\right] \overset{>}{\underset{<}{=}} 0 \tag{26}$$

as

$$L_{t-1} \overset{>}{\underset{<}{=}} \left[\frac{(\beta\mu - 1)(S_{t-1} - \hat{S}_t) - [\mu(S_{t-1} - \hat{S}_t) + \beta][\alpha F(N_t) + \beta]}{\beta}\right]. \tag{27}$$

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