Optimal control of broadcasting spectrum

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A socially desirable number of royalties-paying users of state-owned broadcasting spectrum is derived within an optimal control framework that allows free entry and exit. The analysis takes into account the trade-off between the benefits from higher variety and royalties' revenues and the costs of the intensified interferences associated with entry. It also considers the opposing effects of broadcasts on aggregate income: information dissemination versus diversion of productive time. The steady-state of the broadcasting industry is derived for the case where these effects offset one another and for the case where the positive information-dissemination effect is dominant.

Key Words: Economics; Optimal Control; Spectrum; Variety; Interferences; Royalties
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1. INTRODUCTION

The socially efficient size of the broadcasting industry is a vexed theme in regulatory economics, particularly with respect to over-the-air (OTA) terrestrial services. The public good nature of wireless terrestrial broadcasts and their educational, cultural and political impacts as well as the presence of barriers to entry, such as scarce bandwidth and sunk costs, lend support to strict regulation of the broadcasting industry. Indeed, broadcasts (programs' and advertisements' contents, production and delivery) and entry rules (including license) for prospective OTA terrestrial broadcasters have been tightly regulated in all major OECD countries (cf. Webbink 1973), and until the late 1970s the television broadcasting industries in these countries comprised a handful of licensed and highly protected public and commercial players.

Since the 1980s alternative transmission techniques, such as satellite and cable, have created a more favourable environment for entry into the television broadcasting industry. In turn, this industry has started converging to monopolistically competition with a mix of free transmissions and pay TV services. Yet, incumbent OTA broadcasters' concentration of nation-wide audience shares have remained very high (Motta and Polo, 1997; Caves, 2006). In contrast, the radio broadcasting industry has been less concentrated and more localized. Yet entry to this industry has also remained tightly regulated due to sunk costs and tight spectrum constraint.

The recent adoption of digital transmission technologies has expanded the scope for program variety in both the television and radio broadcasting industries. Digital technology is spectral efficient and its adoption releases a significant amount of HF, VHF and UHF spaces as analogue signals are turned off. However, the gains in spectral efficiency have not relaxed the spectrum constraint for the television broadcasting industry. In the US, the digital dividend was mainly auctioned off to large telecommunications carriers in order to accommodate the future deployment of 4G mobile-phone networks. Similar allocation of the digital dividend is expected in most other OECD countries. The situation is more complex for radio broadcasting as only few countries have successfully adopted and rolled out digital platforms for radio transmissions and even fewer have clear digital switchover plans for analogue radio broadcasting. Still, buffer zones between broadcasters' bands have to be reduced in order to accommodate new entrants to the OTA broadcasting industry.

The spectral efficiency of digital technologies does not resolve the scarcity of broadcasting spectrum. Expansion of the broadcasting industry can therefore be expected to intensify the fundamental trade-off between the pursuit of the overall quality of programs through diversity and reception (quality of signals). If the scope for program diversity is moderate, the entry of additional broadcasters reduces the audience for the incumbents with little benefit to the consumers. Moreover, as entry entails significant (sunk) infrastructure costs, the net social benefits from entry might be negative. In contrast, if there is a scope for a significant added program-diversity from entry, it may well be the case that the incremental consumer benefits from the expanded set of choice exceed the negative welfare effects of entry barriers. However, even if this were to be the case, the transmission of the additional OTA terrestrial services might require tighter allocation of an already congested HF/VHF/UHF spectrum. As OTA terrestrial commercial broadcasters' income is derived from advertising, the scope for added program-diversity relies considerably on the profitability of diversity to advertisers (cf., Steiner, 1952; Spence and Owen, 1977; Mankiw and Whinston, 1986; Anderson and Coate, 2005).

The variety-reception trade off is likely to be most prominent under a deregulatory scheme that allows free entry and exit — open access. It has often been observed that opening the broadcasting industry to new entrants irremediably leads to consolidation and a return to monopolistic competition and resetting of entry barriers after an initial entry of small private operators. The evolution of the Italian industry after the 1980s open access reforms is a case in point (Noam 1992; Hazlet 2005). Another evidence is the high index of audience concentration amongst traditional OTA broadcasters in the United States, Germany and Japan despite a decade of reforms aimed at diversifying production and transmission through cable and satellite platforms (Motta and Polo 1997).

In view of deregulatory trends and the variety-reception trade off, our theoretical analysis explores the optimal steady-state number of OTA broadcasters and its stability under open access. We treat spectrum as a state-owned time-invariant scarce natural resource. As in the case of any other state-owned natural resource, governments are entitled to royalties on its use. Hence, in addition to the direct benefits from the service provided by the broadcasting industry there are substantial indirect benefits — the public services financed by the states' royalties on this natural resource. We portray a conceptual framework where the state's royalties are allowed to vary over time in order to maximize the stream of the discounted direct and indirect benefits stemming from the use of the broadcasting spectrum and where users enjoy free entry and exit. We derive the steady state of the royalties-based optimally controlled industry and identify conditions that allow for a stable path to the steady state along which the number of broadcasters gradually increases rather than reverting to the observed concentration and regulation in the aftermath of reforms.

To set the stage and motivate the royalties-based optimal control of the OTA broadcasting industry, Section 2 presents the basic dynamics of the broadcasting industry and Section 3 computes and illustrates the industry's open access steady state under ad hoc fixed royalties and immediate adjustment. In constructing the optimal control model in Section 4, two opposing effects on the consumers' incomes are introduced: information dissemination vis-à-vis diversion of productive time. Section 5 derives the optimal royalties that take these effects into account and the steady-state size of the industry when these opposing effects offset one another (income neutrality). Section 6 considers the case of a dominant positive information-dissemination effect on aggregate income.

2. DYNAMICS OF THE OTA BROADCASTING INDUSTRY

Let $n(t)$ denote the number of suppliers (broadcasters) of OTA transmitted programs (broadcasts) at time t. At every instance t each supplier uses a single channel and delivers a single program. Let the suppliers be technologically and location-wise identical and paying royalties, $g(t)$, to the government for using a band at t. Also let the width of each band (channel) be technologically determined and fixed, ω,.and let the bands be evenly spread along a homogeneous spectrum available to the broadcasting industry $\hat{S}$. Then, the buffer zones between bands evenly diminish as the number of broadcasters increases and broadcasted programs are equally receivable by any consumer. For tractability, let us further assume that the consumers are located at an identical and physically unobstructed distance from the broadcasters (e.g., a circular residential area with broadcasters located at its centre). Then all broadcast programs are equally receivable by all. Finally, in our setting the programs' consumers are also users of broadcast time. Namely, they advertise their services during programs.

Broadcasters enter (exit) the industry as long as the above-normal profit (ANP) from broadcasting is positive (negative):

$$\dot{n}(t) = \phi ANP(t) \tag{1}$$

where ϕ is a positive scalar reflecting the speed of adjustment (i.e., ease of entry and exit).

With $\hat{S}$ denoting the spectrum available to the broadcasting industry, $0 \leq n(t) \leq \frac{\hat{S}}{\omega}$.

From the perspective of the consumers, the overall quality of the aerially transmitted programs, $Q(t)$, rises with variety and reception. While the variety of broadcasts rises with (and is equal to) the number of channels, interferences intensify as the buffer zone between the channels diminishes. Namely, reception is inversely related to the size of the unused spectrum (S), which is given by:

$$S(t) = \hat{S} - \omega n(t) \tag{2}$$

Consequently,

$$Q(t) = q(n(t), S(t)) \tag{3}$$

where $q(0, \hat{S}) = 0$. The direct, variety, effect of the number of channels on quality, is positive but not increasing: $q_n > 0$ and $q_{nn} < 0$. The indirect negative effect of the number of channels on quality, through deteriorating reception, is negative: -ωq_s, where $q_s > 0$, and (for simplicity) unchanged $q_{ss} = 0$. Up to a critical number of channels $\widetilde{n} < \frac{\hat{S}}{\omega}$, the positive variety effect dominates the negative interfernce effect: $(qn - \omega qs) \gtreqless 0$ for $n \gtreqless \widetilde{n}$.

The overall demand for broadcasts increases with quality. Consequently, the broadcasting industry's aggregate revenue from advertisements and subscription fees at any t is $R(q(t))$ with $R(0) = 0, R_q > 0$, and, for tractability, $R_{qq} = 0$. Assuming that the consumers do not have favourite channels, the industry's aggregate revenue is equally distributed. The instantaneous operational cost of each channel is, for simplicity, time-invariant, c, and so also the (foregone) normal profit attainable in other industries, π.

In sum, the change in the number of broadcasters (channels) is given by:

$$\dot{n}(t) = \phi \left[\frac{R(q(n(t), \hat{S} - \omega n(t)))}{n(t)} - (c + \pi + g(t)) \right] \tag{4}$$

The royalties charged on bands reduce the above normal profit and, subsequently, the number of broadcasters. In turn, the variety of broadcasts is reduced but the reception of each is improved. If the former (latter) effect dominates the latter (former), the industry's overall revenue decreases (increases), the number of broadcasters is diminished (increased), and so forth. As long as the broadcasting industry is not in the optimal steady state, time-invariant royalties are not optimal. In the ensuing sections we first demonstrate the role of fixed ad hoc royalties in the said process and then the determination of the optimal royalties.

3. AD HOC STEADY STATE WITH FIXED ROYALTIES AND IMMEDIATE ADJUSTMENT

In this scenario, $g(t)$ is equal to a positive time-invariant scalar, g, and the immediate adjustment ($\phi \to \infty$) of the number of broadcasters exhausts the broadcasting industry's above normal profit:

$$ANP(t) = \frac{R\left(q\left(n(t), (\hat{S} - \omega n(t)\right)\right)}{n(t)} - (c + \pi + g) = 0 \quad \forall t \tag{5}$$

Recalling our assumptions, $R(0) = 0$, and the slope of the industry's revenue curve is:

$$\frac{dR}{dn} = R_Q(q_n - \delta q_s) \gtreqless 0 \quad \forall n \gtreqless \tilde{n} \tag{6}$$

as depicted in Figure 1 by the inverted parabola. The industry cost function is linear in n, $C(t) = (c + \pi + g_0)n(t)$.. The interior steady state of the industry is in the intersection between the industry's revenue curve and cost line, which, as displayed by the arrows along the horizontal axis, is asymptotically stable.

For example, suppose that:

$$Q(t) = n(t)\left[\hat{S} - \omega n(t)\right] \tag{7}$$

and:

$$R(t) = pQ(t) \tag{8}$$

where p is, for simplicity, a constant and time-invariant marginal return on quality (in terms of revenues from subscriptions fee and/or commercial advertisements). With these specifications in mind, the industry's revenue is:

$$R(t) = p\hat{S}n(t) - p\omega n(t)^2 \tag{9}$$

and the industry's above-normal profit is:

$$ANP(t) = p\hat{S} - p\omega n(t) - (c + \pi + g) \tag{10}$$

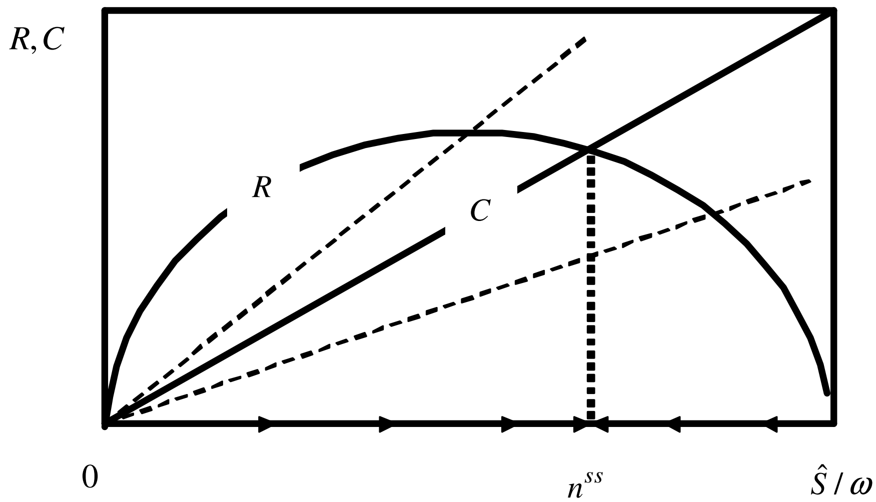

FIG. 1 Spectrum use under immediate adjustment and fixed royalties

By setting ANP to zero:

$$n^{ss} = \frac{1}{\omega}\left[\hat{S} - \frac{c+\pi+g}{p}\right] \tag{11}$$

As indicated by the dashed cost lines, the government can increase, or reduce, the steady-state number of spectrum users by lowering, or raising, royalties.

4. OPTIMAL CONTROL OF THE BROADCASTING INDUSTRY

Let us now analyze a socially optimal determination of royalties and adjustment of the number of broadcasters. Although the government allows free entry and exist, it indirectly control the number of broadcasters by choosing the trajectory of royalties $\{g\}$ per band that maximizes the consumers' lifetime utility. The royalties received by the government from the broadcasters are immediately directed to finance public services. Consumers (households, more likely) are infinitely lived and have an aggregate income, $Y(t)$, from which they pay $R(q(t))$ to broadcasters for advertisements and access to programs and the remainder, $Y(t) - R(q(t))$, for private goods. Broadcasts have two opposing effects on aggregate income. On the one hand, they disseminate information that enhances knowledge and opportunities for transactions. On the other hand, they divert time from work and active investment in human and social capitals. These opposing effects are intensified by the quality of the broadcasts and hence:

$$Y(t) = Y(Q(t)) = Y\left(q\left(n(t)\right), \left(\hat{S} - \omega n(t)\right)\right) \tag{12}$$

where Y_q is positive (negative) if the former effect dominates (is dominated by) the latter.

The consumers derive instantaneous utility from the quality of the broadcasts, $u_1(Q(t))$, from the private goods, $u_2\left(Y(Q(t) - R(Q(t))\right)$, and from the public services financed by the royalties, $u_3\left(n(t)g(t)\right)$.In recalling (3), the consumers' overall instantaneous utility is:

$$u(t) = u_1\left(q(n(t), \hat{S} - \omega n(t)\right) + u_2\left(Y(q(n(t), \hat{S} - \omega n(t)\right) - R\left(q(n(t), \hat{S} - \omega n(t)\right) + u_3\left(n(t)g(t)\right) \tag{13}$$

where $u_i' > 0$ for $i = 1, 2, 3$ and $u_1'' \leq 0$, $u_2'' \leq 0$ and $u_3'' < 0$.

With an additively separable lifetime utility and time-consistent preferences represented by a fixed positive rate of time preference, ρ, the public planner's decision-problem is formally displayed (with the time index omitted for compactness) as:

$$\max_{\{g\}} \int_0^\infty e^{-\rho t}\left[u_1 q(n, \hat{S} - \omega n) + u_2\left(Y(q(n, \hat{S} - \omega n\right) - R\left(q(n, \hat{S} - \omega n\right) + u_3\left(ng\right)\right] dt \tag{14}$$

subject to the broadcasters' motion-equation (4). The Hamiltonian associated with this problem is concave in the control variable g. However, its concavity in the state variable n is not verifiable, $Q_n = (q_n - \omega q_s) \gtreqless 0 \quad as n \gtreqless \tilde{n}$, and $Y(Q) - R(Q)$ can be concave, convex, or linear in Q. In this general framework, the maximum-principle conditions might not be sufficient.

Our analysis of the optimal control of the broadcasting spectrum continues with a special and analytically simpler case that does not entail the said problem. We adopt the explicit specifications (7) and (9) of the industry's quality of service and revenue used in Section 3 and assume that:

$$u_1 = \alpha(\hat{S}_n - \omega n^2) \tag{15}$$

and:

$$u_2 = \beta\left[Y - p(\hat{S}_n - \omega n^2)\right] \tag{16}$$

where α and β are positive scalars.

We further assume that the consumers are not aware of the role of the spectrum-generated royalties in financing public services and hence equate only the utility gained from an infinitesimal improvement in the quality of broadcasts to their loss of utility from the foregone consumption of private goods. This privately perceived equality between the benefit and opportunity costs implies:

$$\alpha = \beta p \tag{17}$$

With these features in mind, the combined utility from broadcasts and private goods for the consumers, who are both viewers and advertisers, is proportional to the broadcast-quality influenced aggregate income:

$$u_1 + u_2 = \beta Y(Q) \tag{18}$$

Consequently, the planner's decision-problem can be now portrayed as :

$$\max_{\{g\}} \int_0^\infty e^{-\rho t} \left[\beta Y(Q) + u_3(ng)\right] dt \text{ subject to: } \dot{n} = \phi \left[p\left(\hat{S} - \omega n\right) - (c + \pi + g)\right] \tag{19}$$

and where Q is given by expression (9).

The present-value Hamiltonian associated with this problem is:

$$H = e^{-\rho t} e^{-\rho t} \left[\beta Y(Q) + u_3(ng)\right] + \lambda \phi \left[p\left(\hat{S} - \omega n\right) - (c + \pi + g)\right] \tag{20}$$

The co-state variable, λ, reflects the public planner's shadow value (in utiles) of broadcasts and, consequently, of unused spectrum. This present-value Hamiltonian is concave in the control variable and, as long as $Y_Q \geq 0$, is also concave in the state variable. In which case, the following Pontryagin's maximum-principle conditions are (by the Mangasarian theorem) sufficient:

$$\dot{\lambda} = -e^{-\rho t} \left[\beta Y_Q Q_n + u_3'(ng)\, g\right] + \lambda \phi p \omega \quad \text{(the adjoint equation)} \tag{21}$$

$$\frac{\partial H}{\partial g} = e^{-\rho t} u_3'(ng)\, n - \lambda \phi = 0 \quad \text{(the optimality equation)} \tag{22}$$

$$\dot{n} = \phi\left[p\left(\hat{S} - \omega n\right) - (c + \pi + g)\right] \qquad \text{(the state equation)} \qquad (23)$$

$$\lim_{t\to\infty} H(t) = 0 \qquad \text{(the transversality equation)} \qquad (24)$$

By differentiating the optimality condition with respect to time and subsequently substituting the adjoint equation for $\dot{\lambda}$ and the optimality condition for $\phi\lambda$ and rearranging terms, the following Euler equation describes the intertemporal change in royalties per band:

$$\dot{g} = \frac{\left[\frac{\phi}{n}g - (\rho + \phi p\omega)\right] + \left[\frac{\phi\beta Y_Q Q_n}{u_3' n}\right]}{-n(\frac{u_3''}{u_3'})} - \frac{\dot{n}}{n}\left\{g + \frac{1}{n(\frac{u_3''}{u_3'})}\right\} \qquad (25)$$

For tractability, let the consumers' instantaneous utility from the public services financed by the royalties on the used spectrum be isoelastic:

$$u_3 = (ng)^{\gamma} \qquad 0 < \gamma < 1 \qquad (26)$$

and recall that $Q_n = \hat{S} - 2\omega n$. Then, the band-royalties' optimal rate of change is[2]:

$$\frac{\dot{g}}{g} = \frac{\left[\frac{\phi}{n}g - (\rho + \phi p\omega)\right] + \left[\frac{\phi\beta Y_Q(\hat{S} - 2\omega n)g^{1-\gamma}}{\gamma n^{\gamma}}\right]}{1-\gamma} + \frac{\gamma}{(1-\gamma)}\frac{\dot{n}}{n} \qquad (27)$$

This more specified Euler equation suggests that the socially optimal rate of change in royalties is moderated by the planner's rate of time preference and the broadcasters' marginal return on the quality of their programs. If $Y_Q > 0$, the rate of change in royalties rises with the contribution of the programs to the aggregate income proportionally to the marginal utility from consumption of private goods, to the available broadcast spectrum and to the entry-exit speed.

While it is impossible to generally identify the steady states of the system (27) and (23), in the two special cases stipulated in the ensuing sections they are identifiable. In both cases, Y_Q

[2] By differentiating the optimality condition with respect to time and subsequently substituting the adjoint equation for $\dot{\lambda}$ and the optimality condition for $\phi\lambda$ into the resultant singular-control equation:

$$-\left(\frac{u_3''}{u_3'}\right)n\dot{g} = \left[\frac{\phi}{n}g - (\rho + \phi p\omega\right] + \left(\frac{u_3''}{u_3'}\right)\dot{n}g + \frac{\dot{n}}{n} + \left[\frac{\phi\beta Y_Q Q_n}{u_3' n}\right]$$

$$\Rightarrow \dot{g} = \frac{\left[\frac{\phi}{n}g - (\rho + \phi p\omega\right] + \left(\frac{u_3''}{u_3'}\right)\dot{n}g + \frac{\dot{n}}{n} + \left[\frac{\phi\beta Y_Q Q_n}{u_3' n}\right]}{-n(\frac{u_3''}{u_3'})} = \frac{\left[\frac{\phi}{n}g - (\rho + \phi p\omega\right] + \left[\frac{\phi\beta Y_Q Q_n}{u_3' n}\right]}{-n(\frac{u_3''}{u_3'})} - \left\{g + \frac{1}{n(\frac{u_3''}{u_3'})}\right\}\frac{\dot{n}}{n}$$

As $u_3 = (ng)^{\gamma}, 0 < \gamma < 1, u_3' = \gamma(ng)^{\gamma-1}, u_3'' = -\gamma(1-\gamma)(ng)^{\gamma-2} - \frac{u_3''}{u_3'} = \frac{(1-\gamma)}{ng}$

By substitution and rearrangement of terms, we obtain equation (27).

is constant. Since the direction of the net effect of the quality of broadcasts on the aggregate income is not clear, we firstly consider the case of income neutrality.

5. STEADY STATE WITH INCOME-NEUTRALITY

With the positive information effect of broadcasts being exactly offset by the negative time-diversion effect,$Y_Q = 0$, and consequently:

$$\frac{\dot{g}}{g} = \frac{\left[\frac{\phi}{n}g - (\rho + \phi p\omega\right]}{1-\gamma} + \frac{\gamma}{(1-\gamma)}\frac{\dot{n}}{n} \tag{28}$$

or, equivalently:

$$\dot{g} = \frac{\phi g^2 - n(\rho + \phi p\varpi)g}{(1-\gamma)n} + \frac{\gamma}{(1-\gamma)}\frac{\dot{n}}{n}g \tag{29}$$

In steady state:

$$\phi g^* - (\rho + \phi p\omega)n^* = 0 \tag{30}$$

Recalling (23), the steady-state combination (n*,g*) should also satisfy:

$$\left[p\left(\hat{S} - \omega n^*\right) - (c + \pi + g) = 0\right] \Rightarrow \left[g^* = p\left(\hat{S} - \omega n^*\right) - (c + \pi)\right] \tag{31}$$

By substituting expression (31) into expression (30) for g^*,

$$\phi\left[p\left(\hat{S} - \omega n^*\right) - (c + \pi)\right] - (\rho + \phi p\omega)n^* = 0 \tag{32}$$

and the steady-state number of broadcasters is:

$$n^* = \frac{\phi\left[p\hat{S} - c - \pi\right]}{(\rho + \phi p\omega)} \tag{33}$$

In recalling equation (31), the steady-state optimal royalties on bands are:

$$g^* = \left[1 - \frac{\phi p\omega}{\rho + 2\phi p\omega}\right]\left(p\hat{S} - c - \pi\right) \tag{34}$$

In the case of income-neutrality, the steady-state number of broadcasters increases with the ease of entry and exit (ϕ), with the size of the available spectrum ($\hat{S}$) and with the broadcasters' marginal return on the quality of their service (p). It decreases with the planner's rate of time preference, with the bandwidth, with the broadcasting operational cost and with the normal profit in other industries. The steady-state royalties are a fraction $\frac{1-\phi p\omega}{\rho+2\phi p\omega}$ of the difference between the potential return on the entire spectrum and the band user's operational and opportunity costs $\left[p\hat{S}-c-\pi\right]$. This fraction is positively related to the public planner's rate of time preference and negatively to the speed of adjustment, to the bandwidth and to the marginal return on the quality of the broadcasts. Noting that $\frac{dg^*}{dn^*}=-p\omega$, the direction of the full effect of each of the said model parameters on $g*$ is opposite to its effect on $n*$

In identifying the nature of the steady-state combination indicated by equations (33) and (34) we note that the eigenvalues of the Jacobian (J*) of the system (23) and (29) in steady state are:

$$\mu_{1,2}=0.5\left[\dot{n}_n^*+\dot{g}_g^*\pm\sqrt{\left(\dot{n}_n^*+\dot{g}_g^*\right)^2-4\left(\dot{n}_n^*\dot{g}_g^*-\dot{n}_g^*\dot{g}_n^*\right)}\right] \tag{35}$$

where:

$$\dot{n}_n^*=-\phi p\omega<0 \tag{36}$$

$$\dot{n}_g^*=-\phi<0 \tag{37}$$

$$\dot{g}_n^*=\left[-\frac{\phi g^*}{n^*}+2\rho+(2+\gamma)\phi p\omega\right]\frac{\left(\frac{g^*}{n^*}\right)}{(1-\gamma)}<0 \tag{38}$$

and:

$$\dot{g}_n^*=\left[-\frac{\rho+\phi p\omega}{(1-\gamma)}+\frac{\left(\frac{g^*}{n^*}\right)\gamma\phi}{(1-\gamma)}\right]<0 \tag{39}$$

Hence,

$$trJ^*=-\left[\frac{1}{(1-\gamma)}\right]\left[\rho+(2-\gamma)\phi p\omega+\gamma\phi\left(\frac{g^*}{n^*}\right)\right]<0 \tag{40}$$

and:

$$\det J^*=-\frac{\phi^2}{(1-\gamma)}\left[\left(\frac{g^*}{n^*}\right)^2+\left(\frac{\rho}{\phi}+p\omega\right)\left(2\frac{g^*}{n^*}-p\omega\right)\right]\gtreqless 0 \tag{41}$$

as

$$\frac{g^*}{n^*} \gtreqless -\left[\frac{\rho}{\phi + p\omega}\right] + \sqrt{\left(\frac{\rho}{\phi + p\omega}\right)^2 + p\omega\left(\frac{\rho}{\phi + p\omega}\right)} \tag{42}$$

Recalling (33) and (34):

$$\frac{g^*}{n^*} = \frac{\left[p\hat{S} - c - \pi\right]\left[\frac{1-\phi p\omega}{\rho + 2\phi p\omega}\right]}{\phi\frac{(p\hat{S}-c-\pi)}{\rho+2\phi p\omega}} = \frac{\rho}{\phi + p\omega} \tag{43}$$

Consequently, $\det J^* < 0$ as $2\frac{\rho}{\phi+p\omega} > \sqrt{\left(\frac{\rho}{\phi+p\omega}\right)^2 + p\omega\left(\frac{\rho}{\phi+p\omega}\right)}$. In turn, $\sqrt{(trJ^*)^2 - 4\det J^*} > |trJ^*|$. As $\mu_1 > 0$ and $\mu_2 < 0$, the steady state (n^*, g^*) is a saddle point.

The steady state of combination of broadcaster' number and band royalties and its asymptotic properties can be displayed in the $n - g$ plane as follows. The isocline $\dot{n} = 0$ is given by $g = p(\hat{S} - \omega n) - c - \pi$ and is depicted by the downwardly sloped line in Figure 2. As indicated by expression (37), $\dot{n}_g < 0$. Consequently, the horizontal arrows in the phases above (below) the isocline $\dot{n} = 0$ are pointed leftward (rightward). By substituting equation (23) into equation. (29) for $\dot{n}$ and rearranging terms, the isocline $\dot{g} = 0$ is given by $g = -\left[\frac{\gamma(p\hat{S})-c-\pi}{1-\gamma}\right] + \left[\frac{\rho+(1+\gamma)\phi p\omega}{\phi(1-\gamma)n}\right]$ and depicted by the upward sloping line. As indicated by expression (38), $\dot{g}_n < 0$. Hence, the vertical arrows in the phases on the right (left) hand side of the isocline $\dot{g} = 0$ are pointed downward (upward). The stable manifold is depicted by the solid arrows and the prototype unstable courses by the dashed arrows. Along the south-western convergent arm, which in view of the initial high level of concentration of the industry is the more relevant one, the number of broadcasters rises, despite the increase of royalties on bands. Along the north-eastern convergent arm the number of channels decreases, despite the lowering of the band-royalties by the state. These counterintuitive optimal courses of the broadcasting industry are explained by the trade-off between variety and reception in forming the appeal of broadcasts for the viewers-advertisers and, subsequently, the industry's revenues and above-normal profits. Along the south-western convergent arm the positive effect of increasing variety in a less congested spectrum is dominant, whereas along the north-eastern convergent arm the interference effect in the highly congested spectrum is dominant.

6. STEADY STATE WITHOUT INCOME NEUTRALITY AND WITH CONSTANT MARGINAL UTILITY

We now attempt to analyze the effect of income sensitivity to broacasts'quality ($Y_Q \neq 0$) on the steady state of the industry's under some simplifying assumptions. Assuming that $Y_{QQ} < 0$

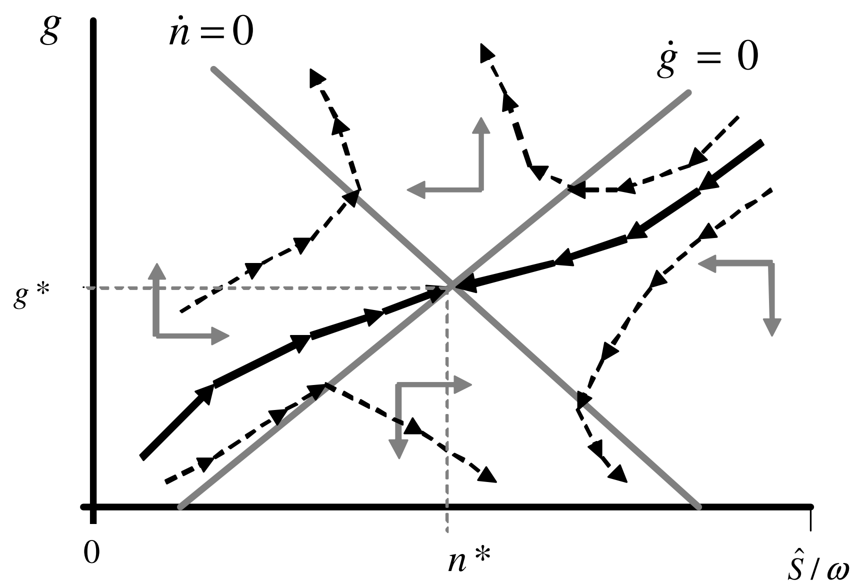

FIG. 2 Broadcasters-royalties phase-plane diagram under income-neutrality

and recalling equation (27), in steady state:

$$\phi \frac{g^*}{n^*} + \frac{\phi \beta Y_Q (\hat{S} - 2\omega n^*) g^{*(1-\gamma)}}{\gamma n^{*\gamma}} - (\rho + \phi p \omega) = 0 \tag{44}$$

By substituting equation (31) into equation (44):

$$\phi \left(p\hat{S} - c - \pi \right) + \frac{\phi}{\gamma} \beta Y_Q (\hat{S} - 2\omega n^*) \left[p\hat{S} - c - \pi - p\omega n^* \right]^{(1-\gamma)} n^{*(1-\gamma)} - (\rho + 2\phi p \omega) n^* = 0 \tag{45}$$

Equation (45) reveals that as long as $0 < \gamma < 1$ there are multiple steady states, which due to the complexity involved cannot be computed or analyzed. Note, however, that in the limiting case of the agents' utility from the public goods financed by the royalties, $\gamma \to 1$, there exists a unique, computable steady state with:

$$n^* = \frac{\phi \left(p\hat{S} - c - \pi \right) + \frac{\phi}{\gamma} \beta Y_Q \hat{S}}{\rho + 2\phi p \omega + 2\omega \frac{\phi}{\gamma} \beta Y_Q} \tag{46}$$

and:

$$g^* = \left(p\hat{S} - c - \pi \right) - p\omega n^* = \left(p\hat{S} - c - \pi \right) - p\omega \left[\frac{\phi \left(p\hat{S} - c - \pi \right) + \frac{\phi}{\gamma} \beta Y_Q \hat{S}}{\rho + 2\phi p \omega + 2\omega \frac{\phi}{\gamma} \beta Y_Q} \right] \tag{47}$$

Note further that the steady state computed in the previous section under income neutrality and indicated by Eq. (33) and Eq. (34) can be also obtained by substituting $Y_Q = 0$ into equations (46) and (47).

Clearly, in the case of income-sensitivity to the overall quality of programs, a broadcasting industry can only prevail in steady state (i.e. $n^* > 0$) if, and only if $Y_Q > 0$; namely, if, and only if, the information dissemination effect of the broadcasted programs dominates their adverse effect on time allocated to work (and less passive activities such as investment in human and the social capitals). Yet the effects of a positive income-sensitivity to broadcasts' quality on the steady-state size of the broadcasting industry and royalties are not straight forward[3]:

$$\frac{\partial n^*}{\partial Y_Q} \gtreqless 0 \text{ and } \frac{\partial g^*}{\partial Y_Q} \gtreqless 0 \text{ as } \left(\frac{\frac{\hat{S}}{\omega}}{\phi}\right) \gtreqless 2\left(\frac{c+\pi}{\rho}\right) \tag{48}$$

Namely, if the ratio of the maximal number of broadcasters to the entry-exit speed is larger (smaller) than twice the ratio of the operational and opportunity costs to the public planner's rate of time preference, then the steady-state number of broadcasters increases with the sensitivity of the aggregate income to the broadcasts' quality, whereas the steady-state royalties decreases.

Although a phase plane diagram cannot be constructed in this case, the local stability of the aforesaid steady-state combination of broadcasters' number and band-royalties can be assessed. In assessing the local stability of the said steady state note that in the present case Eq. (27) can be expressed as:

$$\dot{g} = \left\{ \frac{\left[\frac{\phi}{n} g - (\rho + \phi p\omega)\right] + \left[\frac{\phi}{\gamma}\beta Y_Q \left(\frac{\hat{S}}{n-2\omega}\right)\right]}{(1-\gamma)} + \frac{\gamma}{(1-\gamma)}\frac{\dot{n}}{n} \right\} g \tag{49}$$

The terms of the Jacobian of the differential equation system (49) and (23) in steady state are:

$$\dot{g}_g^* = \left\{ \underbrace{\frac{\left[\frac{\phi}{n^*} g^* - (\rho + \phi p\omega)\right] + \left[\frac{\phi}{\gamma}\beta Y_Q \left(\frac{\hat{S}}{n^*-2\omega}\right)\right]}{(1-\gamma)}}_{\text{zero in steady state}} + \frac{\gamma}{(1-\gamma)}\underbrace{\frac{\dot{n}}{n^*}}_{=\text{zero}} + \frac{\phi + \overbrace{\dot{n}_g}^{=\text{-}\phi}}{(1-\gamma)n^*} \right\} = 0 \tag{50}$$

$$\dot{g}_n^* = -\frac{1}{(1-\gamma)} \left\{ \frac{\left[\phi g^* + \frac{\phi}{\gamma}\beta Y_Q \hat{S}\right]}{n^{*2}} + \gamma \frac{\phi p\omega n^* + \dot{n}}{n^{*2}} \right\} = \frac{\left[\phi g^* + \frac{\phi}{\gamma}\beta Y_Q \hat{S} + \gamma\phi p\omega n^*\right]}{(1-\gamma)n^{*2}} < 0 \tag{51}$$

[3] Note that:
$\frac{\partial n^*}{\partial Y_Q} = \frac{\frac{\phi}{\gamma}\beta\left[\rho\hat{S} - 2\omega\phi(c+\pi)\right]}{\left(\rho + 2\phi p\omega + 2\varpi\frac{\phi}{\gamma}\beta Y_Q\right)^2} \gtreqless 0$ as $\rho\hat{S} \gtreqless 2\omega\phi(c+\pi)$ and $\frac{\partial g^*}{\partial Y_Q} = -p\omega\frac{\partial n^*}{\partial Y_Q}$

$$\dot{n}_g^* = -\phi < 0 \tag{52}$$

$$\dot{n}_n^* = -\phi p\omega < 0 \tag{53}$$

Hence, the eigenvalues of this Jacobian are:

$$\mu_{1,2} = 0.5 \left[-\phi p\omega \pm \sqrt{(\phi p\omega)^2 + \frac{4\phi \left[\phi g + \frac{\phi}{\gamma}\beta Y_Q \hat{S} + \gamma\phi p\omega n \right]}{\left[(1-\gamma)\, n^2 \right]}} \right] \tag{54}$$

Since the discriminant is positive and larger than $\phi p\omega$, $\mu_1 > 0$ and $\mu_2 < 0$ and the steady state depicted by equations. (46) and (47) is a saddle point.

7. CONCLUSION

The advent of digital transmission technologies has done little to relieve constraints on the amount of spectrum allocated to the broadcasting industry. The fundamental, perennial trade-off between variety and reception still prevails. This trade off is likely to be most prominent under a deregulatory scheme that allows free entry. Since spectrum is a state-owned time-invariant scarce natural resource, we argued that, as in the case of any other state-owned natural resource, governments are entitled to royalties on its use. Therefore, in addition to the direct benefits from the service provided by the broadcasting industry, the indirect benefits to consumers from the public services financed by the royalties on this natural resource should be taken into account in the determination of the socially optimal allocation of bands to broadcasters.

We proposed an optimal control model that takes into account the aforesaid aspects, as well as the possible positive and negative effects of broadcasts on aggregate income, in setting the state's royalties on spectrum. We found that as long as the broadcasting industry is not in steady state, time-invariant royalties are not optimal. We demonstrated that under a privately perceived equality between the benefits and costs of broadcasting, the rate of change of the optimal royalties is moderated by the planner's rate of time preference and the broadcasters' marginal return on the quality of programs. We also found that the optimal royalties' rate of change rises with the contribution of the programs to the aggregate income, proportionally to the consumers' marginal utility from private goods, to the available broadcasting spectrum and to the entry-exit ease.

We then computed and analysed the steady state for two special cases. We found that, if the net effect of the opposing aspects of broadcasts on the aggregate income is nill, the steady-state number of broadcasters is unique and increases with the entry-exit ease, with the size of the available spectrum and with the broadcasters' marginal return on the quality of programs, but decreases with the planner's rate of time preference, with the bandwidth, with the broadcasting operational cost and with the normal profit in other industries. We also found that the steady-state royalties are a fraction of the difference between the potential return on the entire spectrum and the user's operational and opportunity costs, and that the fraction is positively related to the public planner's rate of time preference and negatively to the entry-exit ease, to the bandwidth and to the broadcasters' marginal return on the quality of the programs. In contrast to the observed consolidation and return to concentration in the aftermath of deregulatory reforms, at least in this special case of royalties-based optimal control of the broadcasting industry there is a singular arm along which the number of broadcasters gradually increases, but decelerated by rising royalties, and converges to a steady state level.

A unique, saddle-point steady state is also found when the marginal effect of the quality of broadcasts on the aggregate income and the consumers' marginal utility from the public goods financed by the spectrum royalties are positive and constant. In this case, the steady-state size of the broadcasting industry increases with the sensitivity of the aggregate income to the broadcasts' quality only if the ratio of the maximal number of broadcasters to the entry-exit speed is at least twice as large as the ratio of the operational and opportunity costs to the public planner's rate of time preference.

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