2018

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Publication Details

Sheppard, C. J. R. & Mehta, S. B. (2018). Partially coherent imaging in phase space. In C. Fournier, M. P. Georges & G. Popescu (Eds.), Unconventional Optical Imaging: Proceedings of SPIE Vol. 10677 (pp. 106770W-1-106770W-6). United States: S P I E - International Society for Optical Engineering.

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Abstract

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Colin J. R. Sheppard, Shalin B. Mehta, "Partially coherent imaging in phase space," Proc. SPIE 10677, Unconventional Optical Imaging, 106770W (24 May 2018); doi: 10.1117/12.2309824

SPIE.

Event: SPIE Photonics Europe, 2018, Strasbourg, France

Partially coherent imaging in phase space

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ABSTRACT

Conventional optical microscopes, such as brightfield, darkfield, phase contrast or differential interference contrast microscopes are partially coherent imaging systems. Imaging in a partially coherent system was first analyzed by Hopkins only in 1953. He propagated the mutual intensity through the optical system, but did not give an expression for the mutual intensity of the image itself. The mutual intensity is a four dimensional (4D) quantity that contains information about the modulus and phase of the image wave field, which depends on the object's complex refractive index in 3D. The mutual intensity is related to other representations such as the Wigner distribution function (WDF) and ambiguity function. Explicit expressions for different phase space representations of the image wave field are given. The expressions separate into system and object dependent parts. In addition, explicit relationships between the defocused partially coherent cross-coefficient and phase space representations in the image plane are derived.

Keywords: Microscope imaging; partially-coherent imaging; Wigner distribution function; phase space; phase imaging; transmission cross coefficient

1. INTRODUCTION

Conventional optical microscopes, such as brightfield, darkfield, phase contrast or differential interference contrast microscopes are partially coherent imaging systems. Although image formation in fully coherent or incoherent systems has been well understood from the time of Abbe and Rayleigh, imaging in a partially coherent system was first analyzed by Hopkins only in 1953.[1] He propagated the mutual intensity through the optical system, but did not give an expression for the mutual intensity of the image itself. The mutual intensity is a four dimensional (4D) quantity containing information about the modulus and phase of the image wave field, which depends on the object's complex refractive index in 3D. The mutual intensity is related to other representations in phase space, such as the Wigner distribution function (WDF) and the ambiguity function. These are measurable quantities, and many papers have described methods for their experimental determination.

In a series of recent papers, we described how a model for imaging in a partially-coherent, brightfield or phase contrast, optical microscope can be developed, based on filtering of the WDF of the object amplitude transmission: the phase-space imager model.[2-5] In the limit of a slowly varying object, as will be the case if the Rytov approximation is valid, the WDF has a simple physical interpretation. The model much simplifies to give a spatially varying intensity that depends on the local phase gradient of the sample, according to the value of the phase gradient transfer function (PGTF) of the imaging system, which gives the image intensity for a locally-constant phase gradient object. These considerations justify the consideration of image formation in terms of the WDF of the object.

Explicit expressions for the different phase space representations of the image wave field are presented. These expressions are separated into system and object dependent parts. In addition, explicit relationships between the defocused partially coherent cross-coefficient and phase space representations in the image plane are derived. The stochastic wave field in the image plane can be described in terms of different 6D system-dependent kernels, all Fourier transforms of the system mutual spectrum, the region of overlap of two displaced objective pupils and the source. The image intensity can be expressed in terms of a 4D kernel, the PSI-kernel,[2-5] which is the convolution in spatial frequency of the source and the WDF of the objective pupil. The PSI-kernel is given by a marginal (projection) of, or a section through, the different 6D kernels.

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2. THE PHASE SPACE IMAGER MODEL

The image intensity according to Hopkins is[1]

$$I(\mathbf{x}) = \iint T(\mathbf{m}_1)T^*(\mathbf{m}_2)C(\mathbf{m}_1,\mathbf{m}_2)\exp[2\pi i(\mathbf{m}_1-\mathbf{m}_2)\cdot\mathbf{x}]\,\mathrm{d}\,\mathbf{m}_1\,\mathrm{d}\,\mathbf{m}_2, \tag{1}$$

where $C(\mathbf{m}_1,\mathbf{m}_2)$ is the transmission cross-coefficient (TCC) and $T(\cdot)$ is the object spectrum. Here we have replaced the summations of Hopkins by integrals (Fourier transforms).[6] The image intensity has been separated into system and object dependent parts.

We introduce central and difference coordinates, $\mathbf{m}=\frac{1}{2}(\mathbf{m}_1+\mathbf{m}_2), \mathbf{m}'=\mathbf{m}_1-\mathbf{m}_2; \mathbf{x}=\frac{1}{2}(\mathbf{x}_1+\mathbf{x}_2), \mathbf{x}'=\mathbf{x}_1-\mathbf{x}_2$. The phase-space imager window (PSI-window) is then $C'(\mathbf{m},\mathbf{m}')$, a rotated version of the TCC. The phase space imager (PSI) is

$$\Psi(\mathbf{m},\mathbf{x}) = \int T\left(\mathbf{m}+\tfrac{1}{2}\mathbf{m}'\right)T^*\left(\mathbf{m}-\tfrac{1}{2}\mathbf{m}'\right)C'(\mathbf{m},\mathbf{m}')\exp(2\pi i\mathbf{m}'\cdot\mathbf{x})\mathrm{d}\mathbf{m}', \tag{2}$$

and the image intensity is then

$$I(\mathbf{x}) = \int \Psi(\mathbf{m},\mathbf{x})\,\mathrm{d}\,\mathbf{m}. \tag{3}$$

Introducing the Wigner distribution function (WDF) of the object $W_T(\mathbf{m},\mathbf{x})$, given by $W_T(\mathbf{m},\mathbf{x}) = \int T(\mathbf{m}+\frac{1}{2}\mathbf{m}')T^*(\mathbf{m}-\frac{1}{2}\mathbf{m}')\exp(2\pi i\mathbf{m}'\cdot\mathbf{x})\mathrm{d}\,\mathbf{m}'$, and the PSI imager kernel $K(\mathbf{m},\mathbf{x}) = \mathrm{F}_{m'}^{-1}[C'(\mathbf{m},\mathbf{m}')]$, then

$$\Psi(\mathbf{m},\mathbf{x}) = W_T(\mathbf{m},\mathbf{x}) \otimes_{\mathbf{x}} K(\mathbf{m},\mathbf{x}). \tag{4}$$

3. THE IMAGE IN PHASE SPACE

Different phase space quantities, the mutual intensity J , the mutual coherence function Γ , the WDF W , the ambiguity function A, the spectral correlation function (SCF) γ, and the mutual spectrum M , are related by Fourier transformations:

$$\begin{array}{ccc} J(\mathbf{x}_1,\mathbf{x}_2)=\Gamma(\mathbf{x}',\mathbf{x}) & \underset{\mathbf{x}\leftrightarrow\mathbf{m}'}{\Leftrightarrow} & A(\mathbf{x}',\mathbf{m}') \\ \Updownarrow_{\mathbf{x}'\leftrightarrow\mathbf{m}} & & \Updownarrow_{\mathbf{x}'\leftrightarrow\mathbf{m}} \\ W(\mathbf{m},\mathbf{x}) & \underset{\mathbf{x}\leftrightarrow\mathbf{m}'}{\Leftrightarrow} & \gamma(\mathbf{m},\mathbf{m}')=M(\mathbf{m}_1,\mathbf{m}_2). \end{array} \tag{5}$$

The ambiguity function was introduced into optics by Papoulis,[7] and the WDF by Bastiaans.[8] Several papers have considered partially coherent imaging in phase space.[9-14]

The system mutual spectrum G , and the system SCF G' , are given by

$$G(\mathbf{m}_1,\mathbf{m}_2,\xi) = P(\mathbf{m}_1+\xi)P^*(\mathbf{m}_2+\xi)S(\xi) = G'(\mathbf{m},\mathbf{m}',\xi). \tag{6}$$

The image mutual intensity is

$$J(\mathbf{x}_1,\mathbf{x}_2) = \iint K_J(\mathbf{m}_1,\mathbf{m}_2,\mathbf{x}_1-\mathbf{x}_2)T(\mathbf{m}_1)T^*(\mathbf{m}_2)\exp[2\pi i(\mathbf{m}_1\cdot\mathbf{x}_1-\mathbf{m}_2\cdot\mathbf{x}_2)]\,\mathrm{d}\,\mathbf{m}_1\,\mathrm{d}\,\mathbf{m}_2, \tag{7}$$

where the six dimensional system dependent kernel

$$K_J(\mathbf{m}_1,\mathbf{m}_2,\mathbf{x}') = \int G(\mathbf{m}_1,\mathbf{m}_2,\xi)\exp(2\pi i\xi\cdot\mathbf{x}')\,\mathrm{d}\,\xi. \tag{8}$$

The image mutual coherence function is

$$\Gamma(\mathbf{x}',\mathbf{x}) = \iint K_\Gamma(\mathbf{m},\mathbf{x}-\mathbf{x}'',\mathbf{x}')W_T(\mathbf{m},\mathbf{x}'')\exp(2\pi i\,\mathbf{m}\cdot\mathbf{x}')\,\mathrm{d}\,\mathbf{m}\,\mathrm{d}\,\mathbf{x}'', \tag{9}$$

where the kernel

$$K_\Gamma(\mathbf{m},\mathbf{x},\mathbf{x}') = \iint G'(\mathbf{m},\mathbf{m}',\xi)\exp[2\pi i(\mathbf{m}'\cdot\mathbf{x}+\xi\cdot\mathbf{x}')]\,\mathrm{d}\,\mathbf{m}'\,\mathrm{d}\,\xi = \int K_J\left(\mathbf{m}+\tfrac{1}{2}\mathbf{m}',\mathbf{m}-\tfrac{1}{2}\mathbf{m}',\mathbf{x}'\right)\exp(2\pi i\,\mathbf{m}'\cdot\mathbf{x})\,\mathrm{d}\,\mathbf{m}'. \tag{10}$$

The image ambiguity function is

$$A(\mathbf{x}',\mathbf{m}') = \iint K_A(\mathbf{m},\mathbf{m}',\mathbf{x}')W_T(\mathbf{m},\mathbf{x}'')\exp[2\pi i(\mathbf{m}\cdot\mathbf{x}' - \mathbf{m}'\cdot\mathbf{x}'')]\,\mathrm{d}\mathbf{m}\,\mathrm{d}\mathbf{x}'', \tag{11}$$

where the kernel

$$K_A(\mathbf{m},\mathbf{m}',\mathbf{x}') = \int G'(\mathbf{m},\mathbf{m}',\boldsymbol{\xi})\exp(2\pi i\boldsymbol{\xi}\cdot\mathbf{x}')\,\mathrm{d}\boldsymbol{\xi} = K_J(\mathbf{m}_1,\mathbf{m}_2,\mathbf{x}'). \tag{12}$$

The image WDF is

$$W(\mathbf{m},\mathbf{x}) = \iint K_W(\mathbf{m}'',\mathbf{x}-\mathbf{x}'',\mathbf{m}-\mathbf{m}'')W_T(\mathbf{m}'',\mathbf{x}'')\,\mathrm{d}\mathbf{m}''\,\mathrm{d}\mathbf{x}'', \tag{13}$$

where the kernel is

$$K_W(\mathbf{m},\mathbf{x},\boldsymbol{\xi}) = \int G'(\mathbf{m},\mathbf{m}',\boldsymbol{\xi})\exp(2\pi i\mathbf{m}'\cdot\mathbf{x})\,\mathrm{d}\mathbf{m}'. \tag{14}$$

The PSI-kernel is $K(\mathbf{m},\mathbf{x}) = \int K_\Gamma(\mathbf{m},\mathbf{x},\mathbf{x}')\,\mathrm{d}\mathbf{x}' = \int K_W(\mathbf{m},\mathbf{x},\boldsymbol{\xi})\,\mathrm{d}\boldsymbol{\xi}$. The kernels are related by the Fourier transformations:

$$\begin{array}{ccccc} K_\Gamma(\mathbf{m},\mathbf{x},\mathbf{x}') & \underset{\mathbf{x}\leftrightarrow\mathbf{m}'}{\Leftrightarrow} & K_A(\mathbf{m},\mathbf{m}',\mathbf{x}') & = & K_J(\mathbf{m}_1,\mathbf{m}_2,\mathbf{x}') \\ \mathrm{C}_{\mathbf{x}'\leftrightarrow\boldsymbol{\xi}} & & \mathrm{C}_{\mathbf{x}'\leftrightarrow\boldsymbol{\xi}} & & \mathrm{C}_{\mathbf{x}'\leftrightarrow\boldsymbol{\xi}} \\ K_W(\mathbf{m},\mathbf{x},\boldsymbol{\xi}) & \underset{\mathbf{x}\leftrightarrow\mathbf{m}'}{\Leftrightarrow} & G'(\mathbf{m},\mathbf{m}',\boldsymbol{\xi}) & = & G(\mathbf{m}_1,\mathbf{m}_2,\boldsymbol{\xi}). \end{array} \tag{15}$$

For a slowly varying object, separating into modulus and phase, $t(\mathbf{x}) = \sqrt{I_T(\mathbf{x})}\exp(i\phi_T) = \sqrt{I_T(\mathbf{x})}\exp\left[2\pi i\mathbf{m}_T(\mathbf{x})\cdot\mathbf{x}\right]$, where $\mathbf{m}_T(\mathbf{x})$ is the instantaneous frequency (local phase gradient), the object WDF is

$$W_T(\mathbf{m},\mathbf{x}) = I_T(\mathbf{x})\delta\left[\mathbf{m}-\mathbf{m}_T(\mathbf{x})\right]. \tag{16}$$

An object that satisfies the Rytov condition is a slowly varying object. The image WDF is

$$W(\mathbf{m},\mathbf{x}) \approx I_T(\mathbf{x})\delta\left[\mathbf{m}-\mathbf{m}_T(\mathbf{x})\right]PGTF(\mathbf{m}), \tag{17}$$

where the phase gradient transfer function $PGTF$, which gives the image intensity for a locally constant phase gradient object, is given by

$$PGTF(\mathbf{m}) = \iint K_W(\mathbf{m},\mathbf{x},\boldsymbol{\xi})\,\mathrm{d}\boldsymbol{\xi}\,\mathrm{d}\mathbf{x} = C(\mathbf{m},\mathbf{m}) = C'(\mathbf{m},\mathbf{0}) = \int S(\boldsymbol{\xi})\left|P(\boldsymbol{\xi}+\mathbf{m})\right|^2\mathrm{d}\boldsymbol{\xi}. \tag{18}$$

The image intensity is then simply

$$I(\mathbf{x}) = I_T(\mathbf{x})PGTF\left[\mathbf{m}_T(\mathbf{x})\right]. \tag{19}$$

$I_T(\mathbf{x})$ can be recovered from a measured $I(\mathbf{x})$ for a known PGTF. The PGTF has been investigated for different optical systems, including brightfield,[15] stereoscopic,[16] confocal,[15] differential interference contrast (DIC),[17, 18] and differential phase contrast (DPC), based on asymmetric detection[17, 19] or asymmetric illumination.[20] The PGTF is symmetric for a brightfield microscope. It is antisymmetric and linear for DPC, approximately linear for phase-shifting DIC,[21] but is nonlinear and not antisymmetric for conventional DIC. The very simple forms of Eqs. 17 and 19 are our justification for introducing the WDF of the object, and for calculating the WDF of the image.

4. EFFECT OF DEFOCUS

In phase space tomography, measurement of the defocused intensity distribution allows the wavefield, including the phase of the mutual coherence function, to be determined.[22-25] For 2D fields, astigmatic components can be used to determine the 4D phase space representation.[23, 26-28] For the corresponding 2D problem for light propagating in a plane, the vectors become scalars. Now both the defocused transmission cross-coefficient and the mutual intensity are 3D quantities, so that measurement of the defocused intensity in the plane of propagation allows the mutual intensity to be recovered.[22] Indeed, Larkin and Sheppard showed that, for the coherent case, measurement of the intensity in the plane of propagation allows the phase to be retrieved using a direct calculation based on the 2D (x-z) generalized optical transfer function (GOTF). [29-31] This retrieval is valid even in the nonparaxial regime.

The image intensity weighted phase derivative is given as the first frequency moment of the WDF of the image.[32, 33] Thus we obtain

$$\begin{aligned} I(\mathbf{x})\nabla_{\mathbf{x}}\phi(\mathbf{x}) &= 2\pi\int W(\mathbf{m},\mathbf{x})\,\mathbf{m}\,\mathrm{d}\mathbf{m} = 2\pi\iiint W_T(\mathbf{m},\mathbf{x})K_W(\mathbf{m},\mathbf{x}-\mathbf{x}'',\xi)\,\mathbf{m}\,\mathrm{d}\mathbf{m}\,\mathrm{d}\xi\,\mathrm{d}\mathbf{x}'' \\ &= 2\pi\iint W_T(\mathbf{m},\mathbf{x})K(\mathbf{m},\mathbf{x}-\mathbf{x}'')\,\mathbf{m}\,\mathrm{d}\mathbf{m}\,\mathrm{d}\mathbf{x}'' = 2\pi\int \Psi(\mathbf{m},\mathbf{x})\,\mathbf{m}\,\mathrm{d}\mathbf{m}, \end{aligned} \quad (20)$$

so the intensity weighted phase derivative is given by the first frequency moment of the PSI, as well as of the image WDF, and the phase of the image wave field can be extracted from the PSI if this can be measured.

In imaging using the transport of intensity equation (TIE),[34-39] the complete phase space representation of the image field is not measured, but an estimate of the intensity axial derivative only is made, by subtracting two images at small equal and opposite values of defocus. The Wigner kernel for the axial intensity derivative is

$$K_W^{\Delta}(\mathbf{m},\mathbf{x},\xi''-\mathbf{m}) = -iS(\xi''-\mathbf{m})\xi''\cdot\int P(\xi''+\tfrac{1}{2}\mathbf{m}')P^*(\xi''-\tfrac{1}{2}\mathbf{m}')\exp(2\pi i\mathbf{m}'\cdot\mathbf{x})\mathbf{m}'\,\mathrm{d}\mathbf{m}', \quad (21)$$

or in terms of the corresponding system without defocus,

$$K_W^{\Delta}(\mathbf{m},\mathbf{x},\xi''-\mathbf{m}) = \frac{1}{\pi}\xi''\cdot\left[\nabla_{\mathbf{x}}K_W(\mathbf{m},\mathbf{x},\xi''-\mathbf{m})\right]. \quad (22)$$

Similarly for the PSI-kernel,

$$\begin{aligned} K^{\Delta}(\mathbf{m},\mathbf{x}) &= -i\iint P(\xi''+\tfrac{1}{2}\mathbf{m}')P^*(\xi''-\tfrac{1}{2}\mathbf{m}')S(\xi''-\mathbf{m})(\xi''\cdot\mathbf{m}')\exp(2\pi i\mathbf{m}'\cdot\mathbf{x})\,\mathrm{d}\mathbf{m}'d\xi'' \\ &= \frac{1}{\pi}\int \xi''\cdot\left[\nabla_{\mathbf{x}}K_W(\mathbf{m},\mathbf{x},\xi''-\mathbf{m})\right]\mathrm{d}\xi''. \end{aligned} \quad (23)$$

As $I(\mathbf{x}) = \int W(\mathbf{m},\mathbf{x})\,\mathrm{d}\mathbf{m} = \int \Psi(\mathbf{m},\mathbf{x})\,\mathrm{d}\mathbf{m}$,

$$\frac{\partial I(\mathbf{x})}{\partial z} = \int \frac{\partial W(\mathbf{m},\mathbf{x})}{\partial z}\mathrm{d}\mathbf{m} = \int \frac{\partial \Psi(\mathbf{m},\mathbf{x})}{\partial z}\mathrm{d}\mathbf{m}. \quad (24)$$

Eq. 20, together with Eq. 22 or Eq. 23, is consistent with the transport of intensity equation,

$$\frac{\partial I(\mathbf{x})}{\partial z} = -\frac{1}{k}\nabla_{\mathbf{x}}\cdot\left[I\nabla_{\mathbf{x}}\phi(\mathbf{x})\right]. \quad (25)$$

For a slowly varying object,

$$\begin{aligned} I(\mathbf{x})\nabla_{\mathbf{x}}\phi(\mathbf{x}) &= 2\pi I_T(\mathbf{x})\int S\left[\xi''-\mathbf{m}_T(\mathbf{x})\right]|P(\xi'')|^2\,\xi''\,\mathrm{d}\xi'' \\ &= 2\pi I_T(\mathbf{x})C_{FM}\left[\mathbf{m}_T(\mathbf{x})\right] \\ \nabla_{\mathbf{x}}\phi(\mathbf{x}) &= 2\pi C_{CG}\left[\mathbf{m}_T(\mathbf{x})\right], \end{aligned} \quad (26)$$

where $C_{FM}(\mathbf{m})$ is the first moment, and $C_{CG}(\mathbf{m})$ is the centroid, respectively, of the region of overlap of the squared modulus of the pupil and the effective source:

$$\begin{aligned} C_{FM}(\mathbf{m}) &= \int S(\xi''-\mathbf{m})|P(\xi'')|^2\,\xi''\,\mathrm{d}\xi'', \\ C_{CG}(\mathbf{m}) &= \frac{C_{FM}(\mathbf{m})}{PGTF(\mathbf{m})}. \end{aligned} \quad (27)$$

From the TIE, we then have

$$\frac{\partial I(\mathbf{x})}{\partial z} = -\lambda \nabla_{\mathbf{x}} \cdot \left\{ I_T(\mathbf{x}) \int S\left[\xi'' - \mathbf{m}_T(\mathbf{x})\right] \left|P(\xi'')\right|^2 \xi'' \, \mathrm{d}\xi'' \right\} \tag{28}$$

or

$$\frac{\partial I(\mathbf{x})}{\partial z} = -\lambda \nabla_{\mathbf{x}} \cdot \left\{ I_T(\mathbf{x}) C_{FM}\left[\mathbf{m}_T(\mathbf{x})\right] \right\} = -\lambda \nabla_{\mathbf{x}} \cdot \left\{ I(\mathbf{x}) C_{CG}\left[\mathbf{m}_T(\mathbf{x})\right] \right\}. \tag{29}$$

Eq. 29 is of the same form as the TIE, which can be solved for $C_{CG}[\mathbf{m}_T(\mathbf{x})]$ by the usual methods.[37] Then $\mathbf{m}_T(\mathbf{x})$ can be found using a look-up-table, and ϕ_T recovered. The effect of $C_{CG}(\mathbf{m})$ is thus to filter the image of the object phase.

The case of a slowly varying object should be contrasted with that of a weak object, where interference of scattered light with scattered light is neglected, and the phase can be recovered using the weak object transfer function $C(\mathbf{m}, \mathbf{0})$.[40, 41] An object that satisfies the Born approximation is a weak object, but not all weak objects satisfy the Born approximation.

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