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Abstract

A health-risking illegal personal service is transacted when the expected extra satisfaction rate exceeds the ratio of the expected extra cost to the legal service price. Its prevalence decreases with the costs of risk bearing for the providers and clients. Law-enforcement effort lowers (raises) the equilibrium price of the illegal and hazardous service when the ratio of the providers' and the clients' degrees of absolute risk aversion is greater (smaller) than the ratio of the law-enforcement elasticities of their cost bearing.

Key words: Unsafe service; Health risk; Legal risk; Law enforcement

JEL Classification: D4, D8, I1, K4

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1. Introduction

Due to the risk of contracting Human Immunodeficiency Virus, Hepatitis C Virus and/or other blood-borne diseases, engagement in unprotected personal service without adequate medical information on own and partner's health condition is similar to playing a Russian Roulette. For this reason, periodical tests of sex workers and the use of condom are mandatory in places where personal service is legal (for instance, in most of Australia's states and territories). Anecdotic evidence suggests that despite the health and legal risk clients and sex-workers in these seemingly legal markets of personal service are engaged in unsafe exchange for obtaining greater stimulation and higher payment. The confidentiality of this exchange is a mutual interest for the client and the sex-worker and is highly secured by the intimate nature of the service and privacy rights.

In view of the expected return and risk differentials between the safe and legal personal service (henceforth formal service) and the unsafe and illegal personal service (henceforth informal service) for clients and sex-workers, a mean-variance portfolio approach is proposed in this paper for analyzing the demand for, and the supply of, the informal service. The economic literature includes only a small number of studies of the personal-service industry. The most significant one is Moffatt's (2004) assessment of the determinants of sex-workers' service prices in the United Kingdom's underground industry with hedonic price regression equation. The prevalence of double-type personal service in a seemingly single-type legal market and a portfolio approach for explaining clients and sex-workers' choices of the personal service type have not yet been considered. A similar phenomenon, but in a non-commercial context, and a similar approach, but with a single representative agent rather than suppliers and clients, have been considered by Levy (2002) for explaining the prevalence of AIDS in a population of rational people.

Sex-workers may have different physical attributes, personality and interpersonal skills. Naturally, the personal-service market is segmented by quality into sub-markets of fairly homogeneous group of sex-workers and their budget-wise matching group of clients. The structure of such a sub-market is outlined in section 2. The supply of, and demand for, the informal service in a sub-market are derived in sections 3 and 4, respectively. The equilibrium in a sub market is described in section 5. The effects of the infection and legal risks on the informal-service market dynamics are discussed in section 6.

The analysis leads to the following conclusions. The rate of the expected extra satisfaction from the informal service must be greater than the ratio of the informal service expected extra cost to the formal service price for the informal service to be transacted. The prevalence of the informal service decreases with the costs of risk bearing for the providers and the clients. As long as the law-enforcement effort is fixed, the costs of risk-bearing rise from one week to another between two consecutive tests due to increasing infection risk. The equilibrium price of the informal service rises with the service providers' costs of risk bearing and falls with the clients' costs of risk bearing. When the ratio of the increase in the provider's infection risk to the increase in the client's infection risk is larger, equal to, or smaller than the ratio of their previous week's overall risk and when law-enforcement effort is fixed, the current week's price of the informal service is higher, equal to, or lower than its previous week's price, respectively. When law-enforcement effort is responsive to prevalence, the individual clients and suppliers' tendency to moderate their infection risks by concentrating the informal services in the early weeks of the period is counterbalanced by the legal risk stemming from a concerted law-enforcement effort. Spending on law enforcement effort lowers (raises) the equilibrium price of the informal service if the ratio of the providers' and the clients' degrees of absolute risk aversion is greater (smaller) than the ratio of the elasticities of their legal risks.

2. Sub-market structure

The clients and sex-workers in a sub-market incorporate into their decision-making process on the type of service the tradeoff between the expected return gains and the added costs of bearing health and legal risks stemming from the informal service. There are many small personal service providing agencies, and their operation is regulated and randomly inspected. The sex-workers are known, registered and periodically tested for sexually transmitted diseases. Formally, the sex-workers offer the legal safe service in a regulated fixed price. Informally, they solicit clients to have, or are induced by clients to provide, the more stimulating illegal unsafe service for an unreported (and hence untaxed) additional payment. The right of clients to see the sex-worker's medical certificate helps eliminating those who are positively tested at the beginning of the period. Yet the personal-service market is porous. Unlike the sex-workers, clients are neither subjected to periodical medical tests, nor obliged to present a medical clearance. It is possible that externally invisible sexually transmitted diseases are carried by some clients. Sex-workers engaged in informal service with these clients might be infected and subsequently might transmit pathogenic viruses to other clients until their exclusion from the market by the next mandatory test.

The analysis of the demand for the informal service in a personal-service sub-market is based on the assumption that the clients are rational, risk averse and seemingly identical. A person showing symptoms of sexually transmitted disease cannot be a client – his demand for a formal, or informal, service is declined. Although the clients in a sub-market are seemingly identical, some of the clients carry sexually transmitted diseases. Due to lack of external symptoms they are not yet aware of their infection and rejected by the service providers. Consistently with cyclical needs and scheduling of personal activities, time is measured by weeks. Each client is perfectly informed about the weekly prices of the formal and informal services and decides on the numbers of these services for the week.

The analysis of the supply of the informal service in a sub-market is based on the following assumptions. All intended providers are simultaneously tested for sexually transmitted diseases every T (a positive integer) weeks. The actual service providers are those tested negative. They are rational, risk averse, identical and fully employed — each providing a fixed number of equal duration services per week. Believing that, due to the existence of identical service providers, the clients do not use her services exclusively, each sex-worker regards all the clients as having equal probability of carrying sexually transmitted diseases. Since the real nature and price of her service are not observed by the patrons, each sex-worker makes her own decision on formal and informal services. Being identical, the sex-workers' decisions on their expected-utility maximizing weekly combination of formal and informal services are uniform. Their optimal weekly combination is affected by the current prices and full costs of the formal service and the informal service.

The full cost of the informal service includes the cost of bearing the risk of contracting diseases. The infection risk increases from one week to another during the T-week period between tests for the sex-workers and their clients. Furthermore, the costs of risk bearing for sex-workers and clients engaged in informal service rise with the law-enforcement effort. Having been more intensively engaged in sexual activity, exposed to a larger number of partners, and more effectively targeted by law-enforcers, the sex-workers' gradients of the two types of risk are larger than those of the clients. Yet if endowed with a larger degree of risk aversion, a client's cost of risk-bearing might not be lower than that of a sex-worker.

3. Supply of the informal service

The analysis of the weekly supply of the informal service is based on the following assumptions. The number of sex-workers in the personal-service sub-market during the *T*-week period between two consecutive tests is large – *n*. This number is also fixed as symptoms of sexually transmitted diseases do not become visible during the T-week period. The durations of formal and informal services are fixed and identical and hence the sum of the number of formal service (N_F^S) and the number of informal services (N_{IF}^S) offered by any sex-worker (who is assumed to be fully employed) during any given week $t \in (1,T)$ is fixed and known – *N*.

The sex-worker's average variable cost of providing formal service is constant, c_F, and is mainly comprised of the agency's patronage fee. Her cost of an informal service, c_{IF}, is a normally distributed random variable with a time-invariant mean μ_c but with a weekly changing variance, $\sigma_c^2(t)$, that contains two components. The first component is the sex-worker's infection risk, $\sigma_{H_S}^2(t)$. It reflects the sex-worker's risk of contracting a disease during an informal service and its implications for her future health, employment and income. The sex-worker's infection risk rises with the number of weeks That have passed from the last test (i.e., $\sigma_{H_S}^2(t) > \sigma_{H_S}^2(t-1)\ \forall t \in (2,T)$). The second component, $\sigma_s^2(g(t))$, represents the sex-worker's risk stemming from random inspection of the nature of her service. It is an increasing function of the spending on law-enforcement effort, *g*, during the week ($d\sigma_S^2(g)/dg > 0$). The larger the spending on law-enforcement effort the higher the likelihood of inspection and adverse effects on an informal-service providing sex-worker's license and flow of future incomes. As these components are positive and additive,

$$\sigma_c^2(t) = \sigma_{H_S}^2(t) + \sigma_S^2(g(t)) > 0\,. \qquad (1)$$

The price of the formal service (P_F) is regulated and fixed. Due to the risk borne by the service providers, the price of the informal service (P_{IF} (t)) is higher than the price of the formal service. Since sex-workers are registered and it is a common knowledge that *N* services are given during a week by a fully employed sex-worker, every service is reported and a flat tax rate, $0 < \tau < 1$, is applied. The informal services are falsely reported as formal services.

Summing up, a sex-worker's weekly net profit ($\pi(t)$) is normally distributed with

$$E(\pi(t)) = (1-\tau)(P_F - c_F)N + [(P_{IF}(t) - P_F) - (\mu_c - c_F)]N_{IF}^S(t) \qquad (2)$$

and

$$\mathrm{var}(\pi(t)) = [\sigma_{H_S}^2(t) + \sigma_s^2(g(t))]N_{IF}^S(t)^2\,. \qquad (3)$$

The sex-worker's weekly utility from her weekly net profit is negatively exponential with a unit upper-bound, reflecting a constant degree of absolute risk aversion $R_s > 0$:

$$u_t^S = 1 - \exp(-R_S \pi(t)) . \qquad (4)$$

Maximizing this weekly expected utility $(1 - E(\exp(-R_S \pi(t))))$ with respect to N_{IF}^S is equivalent to maximizing the power term in the moment-generating function of the sex-worker's normally distributed weekly profit evaluated at $-R_s$.[1] Namely,

$$\max_{N_{IF}^S(t)} \{(1-\tau)(P_F - c_F)N + [(P_{IF}(t) - P_F) - (\mu_c - c_F)]N_{IF}^S(t) - 0.5R_S[\sigma_{H_S}^2(t) + \sigma_S^2(g(t))]N_{IF}^S(t)^2\}$$

Since $R_s[\sigma_{H_S}^2(t) + \sigma_s^2(g(t))] > 0$, the second-order condition for maximum is satisfied and an interior solution exists as long as the price differential $(P_{IF}(t) - P_F)$ exceeds the expected cost differential $(\mu_c - c_F)$. The necessary condition for maximum implies that any sex-worker's weekly supply of informal service is:

$$N_{IF}^S(t) = \frac{(P_{IF}(t) - P_F) - (\mu_c - c_F)}{R_s[\sigma_{H_S}^2(t) + \sigma_s^2(g(t))]} . \qquad (5)$$

Consequently, the aggregate weekly supply of the informal service ($\hat{N}_{IF}^S(t)$) is given by

$$\hat{N}_{IF}^s(t) = \frac{n[(P_{IF}(t) - P_F) - (\mu_c - c_F)]}{R_s[\sigma_{H_S}^2(t) + \sigma_s^2(g(t))]} . \qquad (6)$$

4. Demand for the informal service

The analysis of the weekly demand for the informal service is conducted within the following framework. The number of clients of the formal and informal services during a given week is large – $m(t)$. Some of the clients carry sexually transmitted diseases, but show no symptoms and are not aware of their condition. They are identical to the rest of the clients in any other respect. For each client, y is the weekly fixed income, N_F^D the number of formal services sought during the week, N_{IF}^D the number of informal services sought during the week, r_F the return on the risk-free formal service, and r_{IF} the normally distributed random return on the risky informal service with constant mean μ_{IF} and weekly variance $\sigma_{IF}^2(t)$.

Since the informal service is more satisfying than the formal service, $\mu_{IF} > r_F$. Let the scalar $\alpha > 0$ be the rate of the expected satisfaction differential between the informal and formal services, then

$$\mu_{IF} = (1+\alpha) r_F . \qquad (7)$$

[1] See Freund (1956) for the development of this procedure and Hammond (1974) for a discussion of its generality.

The variance of the return on the informal service represents the risk stemming from an unsafe service for the client's health – the client's infection-risk, $\sigma^2_{H_d}(t) > 0$. This risk rises with the number of weeks passed from the sex-workers' last test (i.e., $\sigma^2_{H_d}(t) > \sigma^2_{H_d}(t-1)\ \forall t \in (2,T)$). The variance of the return on the informal service also reflects the client's concerns about the inconvenience and loss of reputation accompanying a possible inspection by law-enforcers of the nature of the service, $\sigma^2_d(g(t)) > 0$ – an increasing function of the spending on law-enforcement effort during the week ($\partial\sigma^2_d(g(t))/\partial g > 0$). In sum,

$$\sigma^2_{IF}(t) = \sigma^2_{H_d}(t) + \sigma^2_d(g(t)) > 0\,. \tag{8}$$

For simplicity, it is assumed that a fixed share, ε, of the client's weekly income is spent on formal and informal services, and the client's periodical utility from formal and informal services is independent of his utility from spending on other goods, $\hat{u}^d((1-\varepsilon)y)$.[2] The client's utility from his weekly personal services is given by a negative exponential function displaying a unit upper bound and a constant degree of absolute risk aversion, $R_d > 0$. Hence, the client's overall weekly utility is expressed as:

$$u^d_t = \hat{u}^d((1-\varepsilon)y) + \{1 - \exp\{-R_d[r_F N^D_F(t) + r_{IF} N^D_{IF}(t)]\}\}\,. \tag{9}$$

Since a fixed share of the client's weekly income is spent on formal and informal services and the prices of the formal and informal services are exogenously given, the client's number of formal services during the week is given by:

$$N^D_F(t) = \frac{\varepsilon}{P_F} y - \frac{P_{IF}}{P_F} N^D_{IF}(t) \tag{10}$$

and his weekly utility can be rewritten as:

$$u^d_t = \hat{u}^d((1-\varepsilon)y) + 1 - \exp\{-R_d[r_F(\frac{\varepsilon y}{P_F} - \frac{P_{IF}(t)}{P_F} N^D_{IF}(t)) + r_{IF} N^D_{IF}(t)]\}\,. \tag{11}$$

The client chooses a weekly combination of numbers of formal services and informal services that maximizes his weekly expected utility. Recalling the definition of the moment-generating function and that r_{IF} is normally distributed, maximizing $E(u^d(\cdot))$ with respect to N^D_{IF} is equivalent to maximizing

$$v(t) = r_F(\frac{\varepsilon y}{P_F} - \frac{P_{IF}(t)}{P_F} N^D_{IF}(t)) + (1+\alpha) r_F N^D_{IF}(t) - 0.5 R_d[\sigma^2_{H_d}(t) + \sigma^2_d(g(t))] N^D_{IF}(t)^2$$

.(12)

Since $R_d[\sigma^2_{H_d}(t) + \sigma^2_d(g(t))] > 0$, the second-order condition for maximum is satisfied and, as long as the relative expected return on the formal service $(1+\alpha)$ exceeds the relative price ($P_{IF}(t)/P_F$), there exists an interior solution. Each client's weekly demand for the illegal service is:

[2] This assumption can be justified by habitual sexual activity.

$$N_{IF}^{D}(t)=\frac{[1+\alpha-\frac{P_{IF}(t)}{P_F}]r_F}{R_d[\sigma_{H_d}^2(t)+\sigma_d^2(g(t))]} \quad (13)$$

and the aggregate weekly demand for the informal service ($\widehat{N}_{IF}^{d}(t)$) is:

$$\widehat{N}_{IF}^{d}(t)=\frac{m(t)[1+\alpha-\frac{P_{IF}(t)}{P_F}]r_F}{R_d[\sigma_{H_d}^2(t)+\sigma_d^2(g(t))]} \ . \quad (14)$$

5. Informal-service equilibrium price and prevalence

In view of the aggregate supply and demand equations (6) and (14), the informal service's equilibrium price in week t is:

$$P_{IF}^{*}(t)=\frac{m(t)(1+\alpha)r_F R_s[\sigma_{H_s}^2(t)+\sigma_s^2(g(t))]+n(P_F+\mu_c-c_F)R_d[\sigma_{H_d}^2(t)+\sigma_d^2(g(t))]}{m(t)\frac{r_F}{P_F}R_s[\sigma_{H_s}^2(t)+\sigma_s^2(g(t))]+nR_d[\sigma_{H_d}^2(t)+\sigma_d^2(g(t))]}$$

(15)

and the prevalence of the informal service (i.e., the equilibrium number of informal services transacted) in week t is:

$$\widehat{N}_{IF}^{*}(t)=\frac{\alpha-[(\mu_c-c_F)/P_F]}{(1/nP_F)R_s[\sigma_{H_s}^2(t)+\sigma_s^2(g(t))]+(1/m(t)r_F)R_d[\sigma_{H_d}^2(t)+\sigma_d^2(g(t))]} \quad .(16)$$

(See detailed derivation in the Appendix.)

COROLLARY 1: The informal service is traded if, and only if, the rate of the expected extra satisfaction from the informal service ($\alpha=\mu_{IF}/r_F-1$) is greater than the ratio of the informal service's expected extra cost to the formal service's price ($(\mu_c-c_F)/P_F$).

This corollary is straightforwardly obtained from equation (16). Its underlying rationale is as follows. From equation (14), the demand for the informal service is positive only when $\alpha>\frac{P_{IF}^{*}-P_F}{P_F}$. From equation (6), the supply of the informal service is positive only when $P_{IF}^{*}-P_F>\mu_c-c_F$. Consequently, if a positive quantity of the informal service is traded then it must be that $\alpha>\frac{P_{IF}^{*}-P_F}{P_F}>\frac{\mu_c-c}{P_F}$.

Henceforth, it is assumed that $\alpha>\frac{\mu_c-c}{P_F}$.

The equilibrium in the informal-service market also reflects the following expected properties.

COROLLARY 2: The prevalence of the informal service decreases with the costs of risk bearing for the providers and the clients. (Straightforward from Eq. (16).)

COROLLARY 3: The equilibrium price of the informal service rises with the service providers' costs of risk bearing ($R_s[\sigma^2_{H_s} + \sigma^2_s(g)]$) and decreases with the clients' costs of risk bearing ($R_d[\sigma^2_{H_d} + \sigma^2_d(g)]$). (See Appendix for proof.)

6. Informal-service market dynamics

In view of the increase in the infection risk with the passage of time from the last test, there is an incentive for clients and providers to be more intensively engaged in informal services during the early phase of a period defined by two consecutive tests than in later weeks. In turn, there may be readiness for implementing relatively large law-enforcement effort in the early part of the period. The potentially high risk of service-disruption, loss of reputation for clients, and loss of employment and income for suppliers during the early phase may be taken into account by the service providers and clients. Common knowledge of these incentive and law-enforcement potential response moderates the concentration of informal services and, consequently, law-enforcement effort in the early phase.

In the special case of time-invariant law enforcement effort, the informal-service market's response to the rise in the infection risk between two consecutive tests can be displayed by a shift upward of the weekly aggregate supply curve and a shift downward of the weekly aggregate demand curve of the informal service. In agreement with corollaries 2 and 3, both shifts reduce the number of informal services transacted from one week to another but have opposite effects on the informal service price: the upward supply shift inflates the informal service price, whereas the downward demand shift deflates. This market dynamics is summarized by the following propositions.

PROPOSITION 1: If the law-enforcement effort is time-invariant, then the prevalence of the informal service declines from one week to another between two consecutive tests. (See Appendix for proof.)

PROPOSITION 2: If the law-enforcement effort is time-invariant and the ratio of the current weekly rise in the provider's infection risk to the current weekly rise in the client's infection risk ($[\sigma^2_{H_s}(t) - \sigma^2_{H_s}(t-1)]/[\sigma^2_{H_d}(t) - \sigma^2_{H_d}(t-1])$) is larger than, equal to, or smaller than, the ratio of their previous week's overall risk ($[\sigma^2_{H_s}(t-1) + \sigma^2_s(g)]/[\sigma^2_{H_d}(t-1) + \sigma^2_d(g)]$), then the current week's price of the informal service is higher than, equal to, or lower than, the previous week's price, respectively. (See Appendix for proof.)

Recalling Corollary 2 and that $\partial\sigma^2_s(g)/\partial g > 0$ and $\partial\sigma^2_d(g)/\partial g > 0$, a rise in the spending on law-enforcement increases the costs of risk bearing for the informal service's providers and clients and, in turn, lowers the prevalence of the informal service. The net effect of the additional spending on law-enforcement effort on the price of the informal service depends on the ratio of the providers' and the clients' degrees of absolute risk aversion and on the ratio of the providers' and the clients' elasticities of risk-bearing, which are defined (with the omission of the time index for convenience) as:

$$\xi_s \equiv \frac{\partial\{R_s[\sigma_{H_s}^2 + \sigma_s^2(g)]\}}{\partial g} \frac{g}{R_s[\sigma_{H_s}^2 + \sigma_s^2(g)]} = \frac{(\partial\sigma_s^2(g)/\partial g)g}{[\sigma_{H_s}^2 + \sigma_s^2(g)]} \tag{17}$$

and

$$\xi_d \equiv \frac{\partial\{R_d[\sigma_{H_d}^2 + \sigma_d^2(g)]\}}{\partial g} \frac{g}{R_d[\sigma_{H_d}^2 + \sigma_d^2(g)]} = \frac{(\partial\sigma_d^2(g)/\partial g)g}{[\sigma_{H_d}^2 + \sigma_d^2(g)]}. \tag{18}$$

PROPOSITION 3: If $\frac{\xi_s}{\xi_d} > \frac{R_s}{R_d}$, then the price of the informal service rises with law-enforcement effort. If $\frac{\xi_s}{\xi_d} < \frac{R_s}{R_d}$, then the price of the informal service falls with law-enforcement effort. (See proof in the Appendix.)

7. Concluding remarks

The behavior of rational providers and clients in an apparently legal personal-service market, where a health-risking service is illegally traded alongside a safe and legal alternative, was analyzed. The analysis assessed the effects of the costs of the risk borne by expected-utility-maximizing providers and clients on the informal service equilibrium price and prevalence and their dynamics. The development and introduction of medical tests that provide instantaneous result and can be inexpensively and mutually applied by the service providers and their clients before engagement in unprotected personal service will eliminate the risk of contracting diseases for the participants in the personal service market and for the general public, increase the clients', providers' and public welfare, and facilitates the transformation of the currently hazardous service to non-hazardous one.

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APPENDIX

The informal service equilibrium price and prevalence: From equation (6) and (14), the informal service market-clearing condition is

$$\frac{m(1+\alpha-\frac{P_{IF}^{*}}{P_F})r_F}{R_d[\sigma_{H_d}^2+\sigma_d^2(g)]}=\frac{n[(P_{IF}^{*}-P_F)-(\mu_c-c_F)]}{R_s[\sigma_{H_s}^2+\sigma_s^2(g)]} \quad \text{(A.1)}$$

which implies that the informal service equilibrium price is

$$P_{IF}^{*}=\frac{m(1+\alpha)r_F R_s[\sigma_{H_s}^2+\sigma_s^2(g)]+n(P_F+\mu_c-c_F)R_d[\sigma_{H_d}^2+\sigma_d^2(g)]}{m\frac{r_F}{P_F}R_s[\sigma_{H_s}^2+\sigma_s^2(g)]+nR_d[\sigma_{H_d}^2+\sigma_d^2(g)]}. \quad \text{(A.2)}$$

Equation (14) also implies

$$P_{IF}^{*}=(1+\alpha)P_F-\frac{P_F R_d[\sigma_{H_d}^2+\sigma_d^2(g)]}{mr_F}\widehat{N}_{IF}^{d}. \quad \text{(A.3)}$$

Equation (6) implies,

$$P_{IF}^{*}=P_F+\mu_c-c_F+\frac{R_s[\sigma_{H_s}^2+\sigma_s^2(g)]}{n}\widehat{N}_{IF}^{s}. \quad \text{(A.4)}$$

In equilibrium,

$$(1+\alpha)P_F-\frac{P_F R_d[\sigma_{H_d}^2+\sigma_d^2(g)]}{mr_F}\widehat{N}_{IF}^{*}=P_F+\mu_c-c_F+\frac{R_s[\sigma_{H_s}^2+\sigma_s^2(g)]}{n}\widehat{N}_{IF}^{*} \quad \text{(A.5)}$$

implying

$$\widehat{N}_{IF}^{*}=\frac{\alpha-(\mu_c-c_F)/P_F}{(1/nP_F)R_s[\sigma_{H_s}^2+\sigma_s^2(g)]+(1/mr_F)R_d[\sigma_{H_d}^2+\sigma_d^2(g)]}. \quad \text{(A.6)}$$

Proof of Corollary 3: By differentiating equation (15) with respect to the provider's cost of risk bearing,

$$\frac{\partial P_{IF}^{*}}{\partial R_s[\sigma_{H_s}^2+\sigma_s^2(g)]}=\frac{m(1+\alpha)r_F\{m\frac{r_F}{P_F}R_s[\sigma_{H_s}^2+\sigma_s^2(g)]+nR_d[\sigma_{H_d}^2+\sigma_d^2(g)]\}}{\{m\frac{r_F}{P_F}R_s[\sigma_{H_s}^2+\sigma_s^2(g)]+nR_d[\sigma_{H_d}^2+\sigma_d^2(g)]\}^2}$$

$$-\frac{m\frac{r_F}{P_F}\{m(1+\alpha)r_F R_s[\sigma_{H_s}^2+\sigma_s^2(g)]+n(P_F+\mu_c-c_F)R_d[\sigma_{H_d}^2+\sigma_d^2(g)]\}}{\{m\frac{r_F}{P_F}R_s[\sigma_{H_s}^2+\sigma_s^2(g)]+nR_d[\sigma_{H_d}^2+\sigma_d^2(g)]\}^2}$$

$$=\frac{[\alpha-(\mu_c-c_F)/P_F]mnr_F R_d[\sigma_{H_d}^2+\sigma_d^2(g)]}{\{m\frac{r_F}{P_F}R_s[\sigma_{H_s}^2+\sigma_s^2(g)]+nR_d[\sigma_{H_d}^2+\sigma_d^2(g)]\}^2} \quad \text{.(A.7)}$$

Recalling corollary 1,

$$\frac{\partial P_{IF}^*}{\partial R_s[\sigma_{H_s}^2+\sigma_s^2(g)]}=\frac{[\alpha-(\mu_c-c_F)/P_F]mnr_F R_d[\sigma_{H_d}^2+\sigma_d^2(g)]}{\{m\frac{r_F}{P_F}R_s[\sigma_{H_s}^2+\sigma_s^2(g)]+nR_d[\sigma_{H_d}^2+\sigma_d^2(g)]\}^2}>0\,. \qquad \text{(A.8)}$$

By differentiating equation (15) with respect to the client's cost of risk bearing,

$$\frac{\partial P_{IF}^*}{\partial R_d[\sigma_{H_d}^2+\sigma_d^2(g)]}=\frac{n(P_F+\mu_c-c_F)\{m\frac{r_F}{P_F}R_s[\sigma_{H_s}^2+\sigma_s^2(g)]+nR_d[\sigma_{H_d}^2+\sigma_d^2(g)]\}}{\{m\frac{r_F}{P_F}R_s[\sigma_{H_s}^2+\sigma_s^2(g)]+nR_d[\sigma_{H_d}^2+\sigma_d^2(g)]\}^2}$$

$$-\frac{n\{m(1+\alpha)r_F R_s[\sigma_{H_s}^2+\sigma_s^2(g)]+n(P_F+\mu_c-c_F)R_d[\sigma_{H_d}^2+\sigma_d^2(g)]\}}{\{m\frac{r_F}{P_F}R_s[\sigma_{H_s}^2+\sigma_s^2(g)]+nR_d[\sigma_{H_d}^2+\sigma_d^2(g)]\}^2}$$

$$=\frac{[(P_F+\mu_c-c_F)/P_F-(1+\alpha)]nmr_F R_s[\sigma_{H_s}^2+\sigma_s^2(g)]}{\{m\frac{r_F}{P_F}R_s[\sigma_{H_s}^2+\sigma_s^2(g)]+nR_d[\sigma_{H_d}^2+\sigma_d^2(g)]\}^2}$$

$$=-\frac{[\alpha-(\mu_c-c_F)/P_F]mnr_F R_s[\sigma_{H_s}^2+\sigma_s^2(g)]}{\{m\frac{r_F}{P_F}R_s[\sigma_{H_s}^2+\sigma_s^2(g)]+nR_d[\sigma_{H_d}^2+\sigma_d^2(g)]\}^2}$$

.(A.9)

Recalling corollary 1,

$$\frac{\partial P_{IF}^*}{\partial R_d[\sigma_{H_d}^2+\sigma_d^2(g)]}=-\frac{[\alpha-(\mu_c-c_F)/P_F]mnr_F R_s[\sigma_{H_s}^2+\sigma_s^2(g)]}{\{m\frac{r_F}{P_F}R_s[\sigma_{H_s}^2+\sigma_s^2(g)]+nR_d[\sigma_{H_d}^2+\sigma_d^2(g)]\}^2}<0\,. \qquad \text{(A.10)}$$

Proof of Proposition 1: Since $\sigma_s^2(g)$ and $\sigma_d^2(g)$ are fixed and $\sigma_{H_s}^2(t)>\sigma_{H_s}^2(t-1)$ and $\sigma_{H_d}^2(t)>\sigma_{H_d}^2(t-1)\ \forall t\in(2,T)$, the providers' and clients' costs of risk bearing increase from one week to another between two consecutive tests. As can be seen from Eq. (16), the prevalence of the informal service decreases with the costs of risk bearing for the providers and the clients.

Proof of proposition 2:

$$\dot{P}_{IF}^*=\frac{\partial P_{IF}^*}{\partial R_s[\sigma_{H_s}^2+\sigma_s^2(g)]}R_s\dot{\sigma}_{H_s}^2+\frac{\partial P_{IF}^*}{\partial R_d[\sigma_{H_d}^2+\sigma_d^2(g)]}R_d\dot{\sigma}_{H_d}^2$$

Recalling Corollary 3 (i.e., Eq. (A8) and Eq. (A10)),

$$\dot{P}_{IF}^*=\frac{[\alpha-(\mu_c-c_F)/P_F]mnr_F R_d R_s\{[\sigma_{H_d}^2+\sigma_d^2(g)]\dot{\sigma}_{H_s}^2-[\sigma_{H_s}^2+\sigma_s^2(g)]\dot{\sigma}_{H_d}^2\}}{\{m\frac{r_F}{P_F}R_s[\sigma_{H_s}^2+\sigma_s^2(g)]+nR_d[\sigma_{H_d}^2+\sigma_d^2(g)]\}^2}\,.$$

Recalling Corollary 1, $\dot{P}_{IF}^{*} \overset{>}{\underset{<}{=}} 0 \quad as \quad [\sigma_{H_d}^2 + \sigma_d^2(g)]\dot{\sigma}_{H_s}^2 \overset{>}{\underset{<}{=}} [\sigma_{H_s}^2 + \sigma_s^2(g)]\dot{\sigma}_{H_d}^2$, or equivalently,

$$\dot{P}_{IF}^{*} \overset{>}{\underset{<}{=}} 0 \quad as \quad \left[\frac{\dot{\sigma}_{H_s}^2}{\dot{\sigma}_{H_d}^2}\right] \overset{>}{\underset{<}{=}} \left[\frac{\sigma_{H_s}^2 + \sigma_s^2(g)}{\sigma_{H_d}^2 + \sigma_d^2(g)}\right] .$$

Proof of Proposition 3:

$$\frac{\partial P_{IF}^*}{\partial g} = \frac{\partial P_{IF}^*}{\partial R_s[\sigma_{H_s}^2 + \sigma_s^2(g)]}\frac{\partial \sigma_s^2(g)}{\partial g} + \frac{\partial P_{IF}^*}{\partial R_d[\sigma_{H_d}^2 + \sigma_d^2(g)]}\frac{\partial \sigma_d^2(g)}{\partial g} \tag{A.11}$$

and in recalling Corollary 3 (i.e., Eq. (A.8) and Eq. (A.10),

$$\frac{\partial P_{IF}^*}{\partial g} = \frac{mnr_F[\alpha - (\mu_c - c_F)/P_F]\{R_d[\sigma_{H_d}^2 + \sigma_d^2(g)]\frac{\partial \sigma_s^2(g)}{\partial g} - R_s[\sigma_{H_s}^2 + \sigma_s^2(g)]\frac{\partial \sigma_d^2(g)}{\partial g}\}}{\{m\frac{r_F}{P_F}R_s[\sigma_{H_s}^2 + \sigma_s^2(g)] + nR_d[\sigma_{H_d}^2 + \sigma_d^2(g)]\}^2}$$

.(A.12)

As the denominator of this expression is positive, and as $\alpha > (\mu_c - c_F)/P_F$ (Corollary 1), $\frac{\partial P_{IF}^*}{\partial g} > 0$ when $R_d[\sigma_{H_d}^2 + \sigma_d^2(g)]\frac{\partial \sigma_s^2(g)}{\partial g} > R_s[\sigma_{H_s}^2 + \sigma_s^2(g)]\frac{\partial \sigma_d^2(g)}{\partial g}$ or $\frac{\partial P_{IF}^*}{\partial g} > 0$ when $R_d[\sigma_{H_d}^2 + \sigma_d^2(g)]\frac{\partial \sigma_s^2(g)}{\partial g} < R_s[\sigma_{H_s}^2 + \sigma_s^2(g)]\frac{\partial \sigma_d^2(g)}{\partial g}$.

By rearranging terms and recalling equations (17) and (18), these conditions can be equivalently expressed as displayed in the proposition.