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Abstract

While a mind-altering-substance consumption alleviates current level of depression, it facilitates future depression. Our analysis incorporates this trade off and shows that the stationary state of a consistently overly ambitious sophisticated substance user is improved by impatience, and that this improvement is amplified by the ratio of the instantaneous depression-relief effect to the state-degradation effect of the substance. The analysis also shows that the existence of a supportive personal community leads to permanent cyclical substance consumption when the user is relatively patient.

Keywords: Substance abuse, ambition, consistency, sophistication, impatience, state-maturation, state-degradation, depression, relief, community support, cycles

JEL Classification: D91, I12

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1. Introduction

Consumption of mind-altering substances and depression are persistent and interrelated problems (cf., Aseltine *et al*., 1998; Swendsen and Merikangas, 2000; Sitharthan *et al*., 2001; Langbehn *et al*., 2006). Commonly based on the Becker and Murphy's (1988) optimal control model of rational addiction, the attempts of economists to provide a rationale to the persistent and significant consumption of mind-altering substances with forward-looking utility maximization have ignored the interrelationship between the consumption of these substances and depression and the role of these substances' consumption in providing instantaneous relief. In this paper we propose an optimal-control model of rational consumption of mind-altering substances that explicitly deals with these fundamental interrelationship and role. The proposed model does so by taking into account the intertemporal trade off associated with the use of mind-altering substances: the alleviation of the user's current depression, on the one hand, and the degradation of the user's state that intensifies his future level of depression, on the other hand. By assuming that depression stems from a discrepancy between the individual's actual state and desired state and that a greater discrepancy generates stronger depression, the proposed model also incorporates some of the features of Maslow's (1954) motivation theory.

In particular, we consider the case of people endowed with a long-lasting strong inclination to set their desired state relatively high and using mind-altering substances to moderate their level of depression in a calculated manner. We refer to them as COASSUs — Consistently Overly Ambitious Sophisticated Substance Users. We analyze the properties of the mind-altering-substance-consumption path that minimizes their lifetime depression. Our preliminary analysis suggests that a COASSU is impatient: his rate of time preference exceeds the rate of his state-maturation. Yet the COASSU's stationary state is improved by impatience and this improvement is amplified by the ratio of the instantaneous depression-relieving effect (the blessing) of the mind-altering substance to the state-eroding effect (the curse) of the substance.

Concern and compassion induce family members, friends and some community organizations to support users. The personal community support may take the form of encouragement to undergo a rehabilitating treatment, or the form of accommodation and improvement of the user's state by extending material and mental assistance and combating stigmatization and marginalization.[1] Empirical studies have stressed the significance of the first form of support (cf., Booth *et al*., 1992; Aseltine *et al*., 1998; Marshall *et al*., 2005). Since our paper deals with rational consumption, we extend the analysis to explore the possible implications of the second form of personal community support for the COASSU's consumption of mind-altering substances. In contrast to the preliminary analysis, a more complex consumption path of the mind-altering substance is possible in the extended framework. Intuitively, the improvement in the COASSU's state, following a supportive community effort, makes the COASSU reduce the consumption of the mind-altering substance for a while. However, this moderation of substance abuse lessens the community level of concern; which, in turn, lowers the supportive community effort, lowers the COASSU's state and worsens his depression. As the COASSU's depression intensifies, so does his consumption of the mind-altering substance, which, in turn, increases the community's levels of concern and support, improves the COASSU's

[1] It is also possible that other substance users provide the individual user material, mental and social support (cf., Levy *et al*., 2006).

state, and so force. Our investigation reveals that such cycles arise when the COASSU's rate of time preference is lower than his rate of state-maturation — a possible case in the extended framework while impossible in the preliminary model.

Our analysis is organized as follows. In section 2 we introduce a set of assumptions portraying the COASSU and his vicious circle. Using this set of assumptions, we derive in section 3 the consumption rule of a mind-altering substance for a lifetime depression-minimizing COASSU in the case where there is no external reaction to substance abuse. We compute the COASSU's stationary state and substance abuse in that case and displays their asymptotic stability properties. In section 4 we consider a supportive community reaction and spell the conditions that generate a permanent cyclical pattern in the COASSU's consumption of mind-altering substances.

2. The COASSU and his vicious circle

The description of the COASSU and his vicious circle employs the following notations:

t a continuous time index;

$x(t)*$ the COASSU's desired state at t - a combined index ($x* \in R_+$) of the COASSU's desired positions with regard to material wealth, relationship, family, status, and non-material wealth;

$x(t)$ the COASSU's actual state at t, $0 \leq x(t) \leq x*$;

$c(t)$ the COASSU's consumption of the mind-altering substance at t;

$s(t)$ the COASSU's level of depression at t;

$g(c(t))$ the depression-relief degree generated by the consumption of the mind-altering substance at t;

r the rate of gross improvement of the COASSU's state — state-maturation rate;[2]

[2] Our use of the term state-maturation reflects the widespread phenomenon that the individual's status and rank automatically rise with years of membership in the organization, or community, and with age.

δ the COASSU's marginal state-degradation caused by the consumption of the mind-altering substance; and

ρ the COASSU's rate of time preference.

Assumption 1 (overly ambitious): A COASSU sets his desired state too high relatively to his ability and/or organizational and social constraints; i.e., $x(t)^* - x(t) > 0$ for every *t*.[3]

Assumption 2 (consistently ambitious): Despite recurrent failures a COASSU's desired state is not modified; i.e., $x(t)^* = x^*$ for any *t*.

Assumption 3 (depression and relief): A COASSU's level of instantaneous depression increases with the magnitude of his failure - the distance between his desired state and his actual state - and is eased by the numbing effect of his current consumption of the mind-altering substance. More specifically,

$$s(t) = g(c(t))[x^* - x(t)], \qquad (1)$$

where the depression-relief degree, g, is a convex function of c displaying $g' < 0$, $g'' > 0$, $g(c=0) = 1$ and $\lim_{c \to \infty} g(c) = 0$.[4] [5] [6]

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[3] The COASSU may be meritorious but his aspired state is extremely difficult to attain (e.g., having publications in the highest-ranked economic journals, in the case of an academic economist). He might also be a victim of marginalization, harassment and abuse based on factors such as race, ethnicity, gender, appearance, handicap, religion and eccentricity.

[4] We acknowledge that a broader specification may incorporate a relief-reducing (tolerance) effect of an addictive capital stock *A*: namely, $g(c; A)$ with $g_c < 0$ and $g_{cA} > 0$ (e.g., $g = A/c$). The inclusion of another state variable (*A*) and another state equation (e.g., $\dot{A}(t) = c - \gamma A$, $\gamma > 0$) enormously complicates the model; the analysis of the effect of the personal community support in section 4 in particular (three state variables - *x*, *A* and *E*); while the addictive aspect is not the focus of our paper.

[5] An alternative quadratic specification, $s(t) = g(c(t))[x^* - x(t)]^2$, has been considered. The quadratic specification satisfies the conditions of Mangasarian's theorem on the sufficiency of Pontryagin's maximum-principle. The consideration of the quadratic specification leads to results and conclusions similar to those obtained in section 3 with the linear specification. We use the linear specification to facilitate the analysis in section 4 of the effects of the supportive community reaction.

Assumption 4 (state-evolution): The instantaneous change in a COASSU's actual state is given by the difference between his state-maturation and his state-degradation:

$$\dot{x}(t) = rx(t) - \delta c(t) \text{.}^{7} \tag{2}$$

Assumption 5 (lifetime depression): A COASSU's lifetime depression function is additively separable and reflecting time-consistent preferences, $\int_0^\infty e^{-\rho t} g(c(t))[x^* - x(t)]dt$.[8]

Assumptions 1 to 5 complement one another. A COASSU is trapped in a vicious circle where failures lead to substance abuse (to relieve current depression) and substance abuse degrades his actual state and leads to future failures. That is, although substance abuse reduces the COASSU's instantaneous level of depression, it raises the potential level of depression for the rest of his life.[9] A COASSU cannot escape this vicious circle by leaving his present physical and social environment. The debilitating effect of substance abuse and the record of failures render him an incompetent and undesired candidate for immigration.

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6 Despite his repeated failures, the COASSU is not suicidal. He prefers consuming the mind-altering substance to suicide for relieving his depression. A suicidal person is too proud to bear the degradation caused by the consumption of mind-altering substances.

7 An alternative specification may suggest that the state-degradation is not independent of state but rather intensified by the current state: $\dot{x}(t) = [r - \delta c(t)]x(t)$.

8 An equivalent assumption of additively separable lifetime utility function and time-consistent preferences is used in the rational addiction literature. The case of uncertain life expectancy is analyzed by Levy (2000 and 2002).

9 A negative internality in Herrnstein *et al*.'s (1993) terminology. Time-inconsistent preferences reflecting a need for immediate gratification (O'Donoghue and Rabin, 2000) should exacerbate the COASSU's vicious circle.

3. Depression-minimizing substance consumption

A COASSU chooses a consumption path of the mind-altering substance so as to minimize his lifetime depression subject to the evolution of his actual state. As shown in Appendix A, the following consumption rule of the mind-altering substance is adopted by a COASSU:

$$\dot{c}(t) = \frac{(\rho - r)[x^* - x(t)] + \delta[g(c(t))/g'(c(t))] + \overbrace{[rx(t) - \delta c(t)]}^{\dot{x}(t)}}{[g''(c(t))/g'(c(t))][x^* - x(t)]} \tag{3}$$

or equivalently,

$$\dot{c}(t) = \frac{\overbrace{(\rho - r)[x^* - x(t)] + rx(t)}^{A(t)} - \overbrace{\delta c(t)[1 + 1/\xi(c(t))]}^{B(t)}}{[g''(c(t))/g'(c(t))][x^* - x(t)]} \tag{4}$$

where

$$\xi(c(t)) \equiv -g'(c(t))\frac{c(t)}{g(c(t))} \tag{5}$$

denotes the elasticity of the instantaneous depression-relief degree with respect to the consumption of the mind-altering substance.

Recalling assumptions 1 and 3, the denominator of the right-hand expression in equation (4) is negative. While $B(t)$ is positive, $A(t)$ might be negative when $\rho < r$. As demonstrated by Proposition 1, $\rho > r$ for a COASSU; in which case $\dot{c}(t) \begin{smallmatrix} > \\ = \\ < \end{smallmatrix} 0$ when $A(t) \begin{smallmatrix} < \\ = \\ > \end{smallmatrix} B(t)$. That is, the likelihood that the consumption of the mind-altering substance by a COASSU increases from one instance to another is diminishing in the difference between his desired state and current state, proportionally to the difference between his rate of time preference and his rate of state-maturation, as well as in the state-maturation level. This likelihood is, however, increasing in the current consumption of the mind-altering substance and its negative adverse effect on a COASSU's state in an intensity that is moderated by the elasticity of the instantaneous depression-relief degree of the substance.

For tractability, the COASSU's stationary consumption level of the mind-altering substance and state are computed for an instantaneous depression-relief degree displaying a constant elasticity μ and satisfying assumption 3:

$$g(t) = e^{-\mu c(t)}\text{.}^{10} \tag{6}$$

By substituting this specification into equation (3),

$$\dot{c}(t) = \frac{(\rho - r)[x^* - x(t)] + (\delta / \mu) + \overbrace{[rx(t) - \delta c(t)]}^{\dot{x}(t)}}{-\mu[x^* - x(t)]} \tag{7}$$

and by incorporating the steady-state condition ($\dot{x} = 0 = \dot{c}$) into equation (7) and equation (2), the isocline $\dot{c} = 0$ and $\dot{x} = 0$ are, respectively,

$$(\rho - r)[x^* - x_{SS}] - (\delta / \mu) = 0 \tag{8}$$

$$c = (r / \delta)x. \tag{9}$$

Subsequently, the COASSU's stationary state and substance consumption are

$$x_{ss} = x^* - \frac{\delta / \mu}{\rho - r} \tag{10}$$

$$c_{ss} = (r / \delta)x^* - [r / \mu(\rho - r)]. \tag{11}$$

As proven in Appendix B, these results and the associated phase-plane analysis lead to the following propositions.[11]

[10] An alternative specification, which is consistent with assumption 3 but displaying increasing elasticity in the substance consumption, is $g(t) = 1/[1 + \gamma c(t)]$, where γ is a positive scalar.

Proposition 1 (impatience): $\rho > r$. That is, the COASSU's rate of time preference exceeds his rate of state-maturation.

Proposition 2: The gap between the COASSU's desired state and stationary state is narrowed by the difference between his rate of time preference and his rate of state-maturation. This narrowing of the desired-stationary state-gap is strengthened by the ratio of the elasticity of the instantaneous depression-relief degree (the blessing) to the state-degradation effect (the curse) of the mind-altering substance.

Proposition 3: The COASSU's stationary consumption of the mind-altering substance is intensified by the difference between his rate of time preference and his rate of state-maturation and by the elasticity of the instantaneous depression-relief degree provided by the substance, but moderated by the state-degradation effect of the substance.

Proposition 4: The COASSU's stationary consumption of the mind-altering substance is moderated by his rate of state-maturation if $\frac{\delta\rho}{\mu(\rho - r)^2} > x^*$, but intensified by his rate of state-maturation if $\frac{\delta\rho}{\mu(\rho - r)^2} < x^*$.

This proposition suggests that when the COASSU's state-degradation coefficient is large (small) and his elasticity of the instantaneous depression-relief degree of the substance as well as his difference between the rate of time preference and rate of state-maturation are small (large), it is likely that a rise in the COASSU's rate of state-maturation induces him to moderate (increase) his consumption of the mind-altering substance.

[11] The deviations from these results when a quadratic instantaneous depression function is employed and their implications for the COASSU's state and substance consumption are highlighted in Appendix D.

Proposition 5: If the COASSU's rate of state-maturation is larger than half his rate of time preference (i.e., $r > 0.5\rho$), then his (x_{SS}, c_{SS}) is a saddle point and can only be approached along two convergent arms. Along the lower convergent arm the COASSU's state is improved, despite the increase in his consumption of the mind-altering substance; whereas along the upper convergent arm the COASSU's state is degraded despite the moderation of his consumption of the mind-altering substance.

Insert Figure 1 here

Proposition 6: If the COASSU's rate of state-maturation is smaller than half his rate of time preference (i.e., $r < 0.5\rho$), then his (x_{SS}, c_{SS}) is an asymptotically unstable spiral.

Insert Figure 2 here

4. The effect of personal community support

Concern and compassion induce family members, friends and other community members to accommodate substance use, to materially and mentally support users, and to combat stigmatization and marginalization of users. The presence of personal community support modifies equation 2, which displays the instantaneous change in the COASSU's state, as follows

$$\dot{x}(t) = r\,x(t) - \delta\,c(t) + \beta\,E(t)\,, \tag{12}$$

where *E(t)* denotes the effort invested by the community in improving the substance-user's state and β, a positive scalar, the average effect of the supportive community effort. Due to increasing potential private and public costs, the supportive community effort is assumed to intensify with the excessive consumption of the mind-altering substance:

$$\dot{E}(t) = \alpha[\, c(t) - \hat{c}\,]\,, \tag{13}$$

where α is a positive scalar indicating the community inclination to react and $\hat{c}$ is the maximum mind-altering-substance consumption perceived by the community as normal.[12]

Although the community care and support act to improve substance-users' state, they do not necessarily enhance the COASSU's state. A COASSU anticipates the community support and incorporates it into his selection process of the depression-relieving substance-consumption path: he chooses c so as to minimize $\int_0^\infty e^{-\rho t} g(c(t))[x^* - x(t)]dt$ subject to his actual state-evolution equation (12) and the supportive-community-effort-investment equation (13). The Hamiltonian corresponding to this optimal-control problem is:

$$H(t) = g(c(t))[x^* - x(t)] + \lambda(t)[rx(t) - \delta c(t) + \beta E(t)] + \theta(t)\,\alpha[c(t) - \hat{c}] \tag{14}$$

where the costate variable $\theta(t)$ indicates the shadow price of the supportive community reaction at *t*. The first-order conditions are:

$$\frac{\partial H}{\partial c} = g'(c(t))[x^* - x(t)] - \lambda(t)\delta + \theta(t)\,\alpha = 0 \tag{15}$$

$$\dot{\lambda}(t) - \rho\lambda(t) = -\frac{\partial H}{\partial x} = g(c(t)) - \lambda(t)r \tag{16}$$

$$\dot{\theta}(t) - \rho\theta(t) = -\frac{\partial H}{\partial E} = -\lambda(t)\beta \tag{17}$$

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[12] The consumption level of a mind-altering substance perceived as normal is culture-dependent. An acceptable level of alcohol consumption in one culture might be considered alcoholism in another (Vaillant, 1983).

and the tranversality conditions $\lim_{t\to\infty}\lambda(t)x(t) = 0 = \lim_{t\to\infty}\theta(t)E(t)$.

The necessary conditions in the present case allow a complex dynamics of the mind-altering-substance consumption such as persistent cyclical paths. Unlike the explosive oscillations indicated in Proposition 6, a possible long-run equilibrium in this community-reaction augmented model is not a single point but an invariant manifold – a limit cycle.[13] In order to examine the possibility of such a complex dynamics we use equation (15) to express the consumption of the mind-altering substance consumption as a function of λ , θ and *x*:

$$\{g'(c(t))[x^* - x(t)] = \lambda(t)\delta - \theta(t)\alpha\} \Rightarrow \{c(t) = c(x(t), \lambda(t), \theta(t))\} . \qquad (15')$$

By differentiation, we obtain the following properties of the COASSU's substance-consumption function:

$$c_x = \frac{g'(c(t))}{g''(c(t))[x^* - x(t)]} < 0, \quad c_\lambda = \frac{\delta}{g''(c(t))[x^* - x(t)]} > 0, \quad c_\theta = \frac{-\alpha}{g''(c(t))[x^* - x(t)]} < 0 . \qquad (18)$$

The substitution of equation (15') into equations (12), (13), (16), (17) yields:

$$\dot{x}(t) = r\,x(t) - \delta\,c(x(t), \lambda(t), \theta(t)) + \beta\,E(t) \qquad (12')$$

$$\dot{E}(t) = \alpha c(x(t), \lambda(t), \theta(t)) - \alpha\hat{c} \qquad (13')$$

$$\dot{\lambda}(t) = [\rho - r]\lambda(t) + g(c(x(t), \lambda(t), \theta(t))) \qquad (16')$$

$$\dot{\theta}(t) = \rho\theta(t) - \lambda(t)\beta . \qquad (17')$$

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[13] The economic literature has examined cycles in similar contexts. The theory of rational addiction is capable of explaining cyclical consumption paths expressed as damped or explosives waves or limit cycles (Dockner and Feichtinger, 1993). Furthermore, permanent cycles in alcohol consumption seem to be empirically significant. For instance, Kerr, Fillmore and Bostrom (2002) suggest the possible existence of subgroups of consumers that move between abstention and light drinking of alcoholic beverages and moderate and heavy drinking.

With this presentation of the first-order conditions, the condition for a limit cycle between the COASSU's state and the supportive community reaction; and, consequently (through equation (15')), for cyclical consumption of the mind-altering substance; can be summarized as follows. (See Appendix C for proof.)

Proposition 7: If the COASSU is sufficiently patient so that $0 < \rho < r$, there is a limit cycle between his state (*x*) and the supportive community effort (*E*).

Proposition 8: If the COASSU's state (*x*) displays cyclical behavior his mind-altering-substance consumption (*c*) is cyclical.

The existence of a permanent cyclical consumption path of the mind-altering substance is conditioned on the COASSU's rate of time preference being lower than the COASSU's rate of state-maturation. In contrast, Proposition 1 (the "impatience" proposition) precludes this case due to a violation of Assumption 1 (the "overly ambitious" assumption). As stressed earlier, the inclusion of a supportive community reaction alters the analysis substantially. In the extended analysis, Proposition 1 is no longer valid and the condition for cyclical consumption ($0 < \rho < r$) does not violate Assumption 1. This is due to the steady-state level of *x* in the extended analysis being given by

$$\bar{x} = x^* - [\frac{\alpha\beta - \delta\rho}{\rho(\rho - r)}], \tag{19}$$

which is different from the steady-state level displayed by equation (10). In addition, there is no overshooting when $\alpha\beta < \delta\rho$ or, equivalently, as long as the COASSU's rate of time preference exceeds the ratio of the reactive-support effect ($\alpha\beta$) to the state-degradation effect of the mind-altering substance (i.e., $\rho > \alpha\beta / \delta$).

5. Concluding remarks

This paper analyzes the vicious circle of depression and substance abuse in the case of a consistently overly ambitious sophisticated user who selects a mind-altering-substance-consumption path to minimize the sum of the discounted instantaneous levels of depression stemming from the gap between his actual and desired states over the rest of his lifespan. The vicious circle reflects the negative internality associated with substance abuse: present consumption of the mind-altering substance alleviates the present level of depression but leads to future failures and, consequently, depression. The preliminary analysis shows that the COASSU's stationary state is improved by the difference between his rate of time preference and his rate of state-maturation, and that this improvement is amplified by the ratio of the effectiveness of the substance in reducing instantaneous depression (the blessing) to the state-degradation effect of the substance (the curse). However, when a supportive personal community reaction to excessive use is taken into account the COASSU's rate of state-maturation can be greater than his rate of time preference, in which case his mind-altering-substance consumption displays permanent cyclical paths.

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Appendix A: Solution of the optimal-control problem

The present-value Hamiltonian corresponding to the constrained minimization problem described in section 3 is

$$H(t) = e^{-\rho t} g(c(t))[x^* - x(t)] + \lambda(t)[rx(t) - \delta c(t)] \tag{A1}$$

where the costate variable $\lambda(t)$ indicates the shadow price of the user's state at t. Since $g(c)$ is convex and $x^* - x(t) > 0$, H is convex in the control variable (c). Recalling assumption 3, *H* is linearly decreasing in *x*. In addition to the state equation (equation (2)), the conditions for minimum lifetime depression are:

$$\dot{\lambda}(t) = -\frac{\partial H}{\partial x} = e^{-\rho t} g(c(t)) - \lambda(t) r \tag{A2}$$

$$\frac{\partial H}{\partial c} = e^{-\rho t} g'(c(t))[x^* - x(t)] - \lambda(t)\delta = 0 \tag{A3}$$

and the tranversality condition $\lim_{t \to \infty} \lambda(t) x(t) = 0$. Equation (3) is obtained by differentiating equation (A3) with respect to time, substituting the right-hand sides of equation (A2) and equation (A3) for $\dot{\lambda}$ and λ, collecting terms and multiplying both sides of the resultant equation by $[1/g'(c(t))]$.

Appendix B: Proofs of propositions 1-6

Proof of Proposition 1: By virtue of equation (10), $x_{SS} \geq x^*$ when $\rho \leq r$. However, neither accurate nor over shooting is compatible with assumption 1 and with minimizing lifetime depression.

Proof of Proposition 2: Straightforward from equation (10).

Proof of Proposition 3: Straightforward from equation (11).

Proof of Proposition 4: By differentiating equation (11) with respect to r.

Proof of Proposition 5: By differentiating equation (2), $\frac{d\dot{x}}{dc} = -\delta < 0$, and therefore the horizontal arrows above (below) the isocline $\dot{x} = 0$ are leftward (rightward) directed. By differentiating equation (7) with respect to x

$\frac{d\dot{c}}{dx} = \frac{\delta\mu c + \delta - r\mu x^*}{\mu^2 (x^* - x)^2} = \frac{\delta\mu[c + (1/\mu) - (r/\delta)x^*]}{\mu^2 (x^* - x)^2}$. Hence, $\frac{d\dot{c}}{dx} < 0$ at the vicinity of the stationary point if $c_{SS} < (r/\delta)x^* - (1/\mu)$. Recalling equation (11), $c_{SS} = \left(\frac{r}{\delta}\right)x^* - [r/\mu(\rho - r)] < \left(\frac{r}{\delta}\right)x^* - 1/\mu$ if $[r/\mu(\rho - r)] > 1/\mu$ or, equivalently, if $[r/(\rho - r)] > 1$. This in turn implies that $\frac{d\dot{c}}{dx} < 0$ at the vicinity of the stationary point if $r > 0.5\rho$. In this case, the vertical arrows are downward (upward) directed in the region on the right (left) hand side of the isocline $\dot{c} = 0$. These directions of the horizontal and vertical arrows at the vicinity of the steady state implies that (x_{SS}, c_{SS}) is a saddle point and can be approached along the two convergence arms as displayed by Figure 1.

Proof of Proposition 6: As in the proof of proposition 5, but given that $r < 0.5\rho$ then $\frac{d\dot{c}}{dx} < 0$. In this case, the vertical arrows are upward (downward) directed in the region on the right (left) hand side of the isocline $\dot{c} = 0$. These directions of the horizontal and vertical arrows at the vicinity of the steady state implies that (x_{SS}, c_{SS}) is either a spiral or a center. The linearization of the differential equation system consisting of equation (2) and equation (7) reveals that the trace of the state-transition matrix is $r + \delta/(x^* - x)$. Recalling assumptions 1 and 4, $r + \delta/(x^* - x) > 0$. That is, the real part of the conjugate-complex characteristic roots of the state-transition matrix is positive and hence (x_{SS}, c_{SS}) is an asymptotically unstable spiral.

Appendix C: Proofs of Propositions 7 and 8

Proof of Proposition 7: Following Feichtinger et al. (1994), it is necessary to show, for proving the existence of a limit cycle, that the determinant of the Jacobean of the system of equations (12'), (13'), (16') and (17')

$$|J| = \begin{vmatrix} \frac{\partial \dot{x}}{\partial x} & \frac{\partial \dot{x}}{\partial E} & \frac{\partial \dot{x}}{\partial \lambda} & \frac{\partial \dot{x}}{\partial \theta} \\ \frac{\partial \dot{E}}{\partial x} & \frac{\partial \dot{E}}{\partial E} & \frac{\partial \dot{E}}{\partial \lambda} & \frac{\partial \dot{E}}{\partial \theta} \\ \frac{\partial \dot{\lambda}}{\partial x} & \frac{\partial \dot{\lambda}}{\partial E} & \frac{\partial \dot{\lambda}}{\partial \lambda} & \frac{\partial \dot{\lambda}}{\partial \theta} \\ \frac{\partial \dot{\theta}}{\partial x} & \frac{\partial \dot{\theta}}{\partial E} & \frac{\partial \dot{\theta}}{\partial \lambda} & \frac{\partial \dot{\theta}}{\partial \theta} \end{vmatrix} \qquad \text{(C1)}$$

and the term

$$\mathrm{M} = \begin{vmatrix} \frac{\partial \dot{x}}{\partial x} & \frac{\partial \dot{x}}{\partial \lambda} \\ \frac{\partial \dot{\lambda}}{\partial x} & \frac{\partial \dot{\lambda}}{\partial \lambda} \end{vmatrix} + \begin{vmatrix} \frac{\partial \dot{E}}{\partial E} & \frac{\partial \dot{E}}{\partial \theta} \\ \frac{\partial \dot{\theta}}{\partial E} & \frac{\partial \dot{\theta}}{\partial \theta} \end{vmatrix} + 2 \begin{vmatrix} \frac{\partial \dot{x}}{\partial E} & \frac{\partial \dot{x}}{\partial \theta} \\ \frac{\partial \dot{\lambda}}{\partial E} & \frac{\partial \dot{\lambda}}{\partial \theta} \end{vmatrix} \qquad \text{(C2)}$$

are positive when calculated with the steady-state levels ($\bar{x}$, $\bar{E}$, $\bar{\lambda}$, $\bar{\theta}$).[14] In addition, the value of the bifurcation parameter (ρ) given by the following condition

$$|J| = \left(\frac{M}{2}\right)^2 + \rho^2 \left(\frac{M}{2}\right) \qquad \text{(C3)}$$

must be positive as well.[15]

Note that

$$|J| = -\beta[\alpha\, c_x\, \rho(\rho - r)\, g' c_\lambda - \rho\, \alpha\, g' c_\lambda\, c_x] > 0 \Leftrightarrow r > \rho \qquad \text{(C4)}$$

---

[14] The steady state equilibrium is found when $\dot{\lambda} = \dot{\theta} = \dot{x} = \dot{E} = 0$.

[15] These are the conditions for the existence of a limit cycle according to the Hopfbifurcation theorem (e.g., Guckenheimer and Holmes, 1990).

and

$$M = (r - \delta c_x)(\rho - r) g' c_\lambda + g' c_x c_\lambda \delta + 2\beta g' c_\theta > 0 \Leftrightarrow r > \rho . \quad \text{(C5)}$$

From C4, C5 and C3, the necessary and sufficient condition for the existence of a limit cycle between *x* and *E* is $0 < \rho < r$.

Proof of Proposition 8: The cyclical behavior of *c* is implied by equation (15’) when *x* displays cyclical behavior.

Appendix D: Depression-minimization with a quadratic instantaneous-depression function

The alternative quadratic specification of the instantaneous-depression function indicated in footnote 5 renders the Hamiltonian convex in the state variable *x*

$$H(t) = e^{-\rho t} g(c(t))[x^* - x(t)]^2 + \lambda(t)[rx(t) - \delta c(t)] \quad \text{(D1)}$$

In this case, the conditions of Mangasarian’s theorem on the sufficiency of Pontryagin’s maximum-principle are satisfied. In addition to the state equation (2), the conditions for minimum lifetime suffering are:

$$\dot{\lambda}(t) = -\frac{\partial H}{\partial x} = 2e^{-\rho t} g(c(t))[x^* - x(t)] - \lambda(t) r \quad \text{(D2)}$$

$$\frac{\partial H}{\partial c} = e^{-\rho t} g'(c(t))[x^* - x(t)]^2 - \lambda(t)\delta = 0 \quad \text{(D3)}$$

and the tranversality condition $\lim_{t \to \infty} \lambda(t) x(t) = 0.$

By differentiating equation (D3) with respect to time, substituting the right-hand sides of Equation (D2) and equation (D3) for $\dot{\lambda}$ and λ, collecting terms and multiplying both sides of the resultant equation by $[1/ g'(c(t))]$ we obtain

$$\dot{c}(t) = \frac{(\rho - r)[x^* - x(t)] + 2\delta[g(c(t))/g'(c(t))] + 2\overbrace{[rx(t) - \delta c(t)]}^{\dot{x}(t)}}{[g''(c(t))/g'(c(t))][x^* - x(t)]}. \quad \text{(D4)}$$

Recalling the constant-elasticity specification of the instantaneous depression-relieving function, then

$$\dot{c}(t) = \frac{(\rho - r)[x^* - x(t)] - 2(\delta/\mu) + 2\overbrace{[rx(t) - \delta c(t)]}^{\dot{x}(t)}}{-\mu[x^* - x(t)]}. \quad \text{(D5)}$$

By substituting the steady-state condition ($\dot{x} = 0 = \dot{c}$) into equation (D5) the isocline $\dot{c} = 0$ is given by

$$(\rho - r)[x^* - x_{SS}] - 2(\delta/\mu) = 0 \quad \text{(D6)}$$

and the stationary state and substance consumption are

$$x_{SS} = x^* - 2\left[\frac{\delta}{\mu(\rho - r)}\right] \quad \text{(D7)}$$

and

$$c_{SS} = \left(\frac{r}{\delta}\right)x^* - 2\left[\frac{r}{\mu(\rho - r)}\right]. \quad \text{(D8)}$$

These results lead to the following changes in propositions 4-6.

Proposition 4: The stationary consumption of the mind-altering substance declines with the state-maturation rate if $\frac{2\delta\rho}{\mu(\rho - r)^2} > x^*$, but rises with the state-maturation rate if $\frac{2\delta\rho}{\mu(\rho - r)^2} < x^*$.

Proof: By differentiating equation (D8) with respect to r.

Proposition 5: If $r > \rho/(1+0.5\mu)$ then (x_{ss}, c_{ss}) is a saddle point and can (only) be approached along two convergent arms as displayed by Figure 1. Along the lower convergent arm the COASSU's state is improved despite the increase in his consumption of the substance, whereas along the upper convergent arm the COASSU's state is degraded despite the decline in his consumption of the substance.

Proof: By differentiating equation (2), $\frac{d\dot{x}}{dc} = -\delta < 0$, and therefore the horizontal arrows above (below) the isocline $\dot{x} = 0$ are leftward (rightward) directed. By differentiating equation (D5) with respect to x, $\frac{d\dot{c}}{dx} = \frac{-2r\mu(x^*-x)+2\delta-2\mu\dot{x}}{\mu^2(x^*-x)^2}$. Hence, $\frac{d\dot{c}}{dx} < 0$ at the vicinity of the stationary point if $-r\mu(x^*-x_{ss})+\delta < 0$. Substituting the right-hand-side of equation (D7) for x_{ss} and rearranging terms, $\frac{d\dot{c}}{dx} < 0$ at the vicinity of the stationary point if $r > \rho/(1+0.5\mu)$. In this case, the vertical arrows are downward (upward) directed in the region on the right (left) hand side of the isocline $\dot{c} = 0$. These directions of the horizontal and vertical arrows at the vicinity of the steady state implies that (x_{ss}, c_{ss}) is a saddle point and can be approached along the two convergence arms as displayed in Figure 1.

Proposition 6: If $r < \rho/(1+0.5\mu)$ then (x_{ss}, c_{ss}) is an asymptotically unstable spiral.

Proof: As in the proof of proposition 5, but given that $r < \rho/(1+0.5\mu)$ then $\frac{d\dot{c}}{dx} > 0$. In this case, the vertical arrows are upward (downward) directed in the region on the right (left) hand side of the isocline $\dot{c} = 0$. These directions of the horizontal and vertical arrows at the vicinity of the steady state implies that (x_{SS}, c_{SS}) is either a spiral or a center. The linearization of the differential-equation system consisting of equation (2) and equation (D5) reveals that the trace of the state-transition matrix is $r + 2\delta/(x^* - x)$. Recalling our assumptions, $r + 2\delta/(x^* - x) > 0$. That is, the real part of the conjugate-complex characteristic roots of the state-transition matrix is positive and hence (x_{SS}, c_{SS}) is an asymptotically unstable spiral.

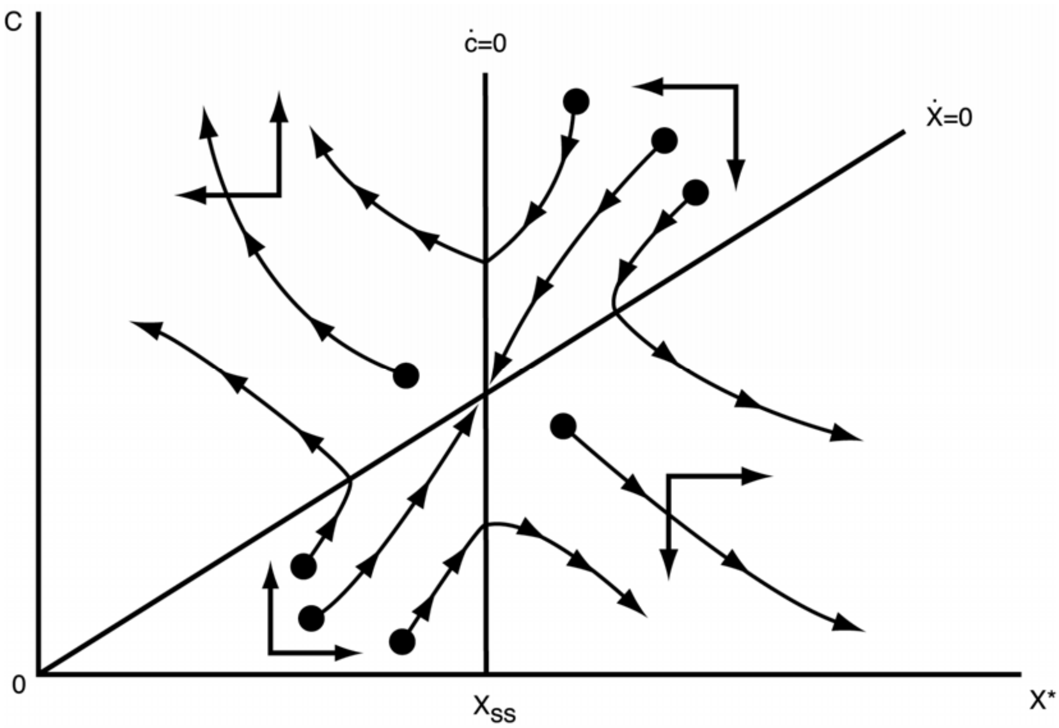

Figure 1. State and mind-altering-substance consumption when $r > 0.5\rho$

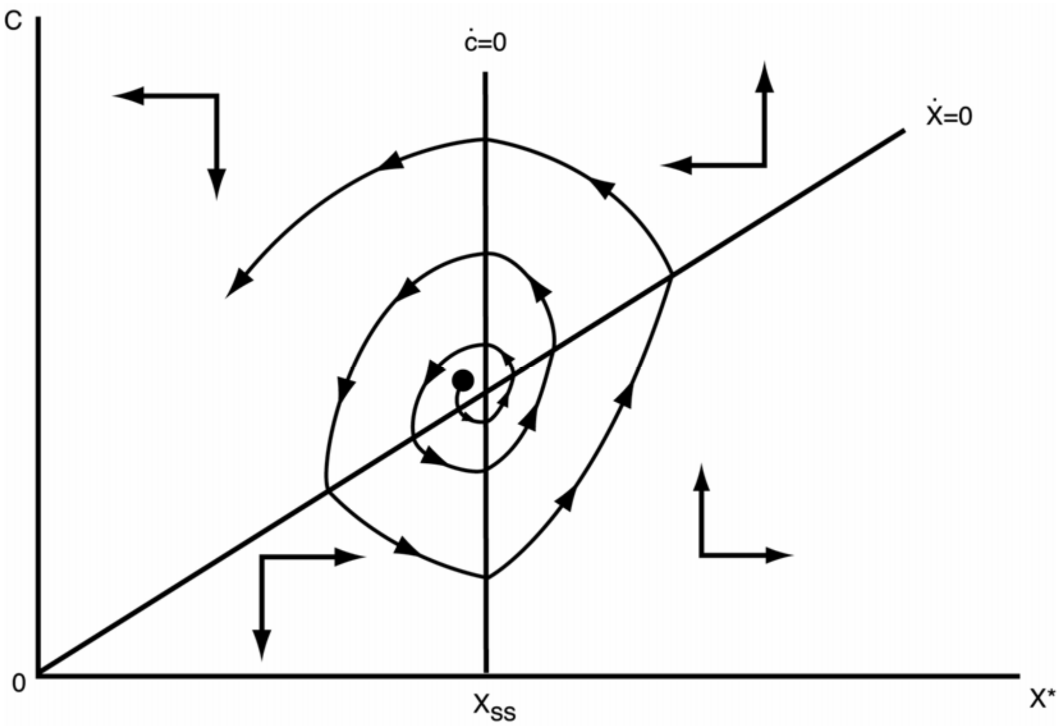

Figure 2. State and mind-altering-substance consumption when $r < 0.5\rho$