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A three-level-similarity measuring method of participant opinions in multiple-criteria group decision supports

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Keywords
opinions, multiple, criteria, group, decision, supports, measuring, method, participant, three, level, similarity

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A three-level-similarity measuring method of participant opinions in multiple-criteria group decision supports

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Abstract

Measuring opinion similarity between participants (MOSP) is an important strategy to reduce the chance of making and applying inappropriate decisions in multi-criteria group decision making applications. Due to the small-sized opinion data and the varieties of opinion representations, measuring the similarity between opinions is difficult and has not been well-studied in developing decision support. Considering that the similarity changes with the number of concerned criteria, this paper develops a gradual aggregation algorithm (GAA) and establishes a three-level-similarity measuring (TLSM) method based on it to measure the opinion similarity at the assessment level, the criterion level and the problem level. Two applications of the TLSM method on social policy selection and energy policy evaluation are conducted. The study indicates that the TLSM method can effectively measure the similarity between opinions in small-size with possibly missing values and simulate the dynamic generation of a decision.

Keywords: multi-criteria group decision making, opinion similarity, measuring

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method, aggregation operator, opinion analysis

1. Introduction

Multiple-criteria group decision making (MCGDM) is recognised as an efficient strategy in many organisational decision problems [1, 2], where a final decision is made based on the opinions of individual participants. Overly similar opinions increase the chance of putting an inappropriate decision into effect. In practice, making an appropriate decision is already a time-consuming and costly task; however, tuning an inappropriate decision will cost even more. To reduce this risk, measuring opinion similarity between participants (MOSP) in advance is an important issue in developing decision support for essential decision problems.

Opinion similarity is used in many fields such as online recommender systems [3, 4]. However, the MOSP problem is still an unsolved and challenging issue. Difficulties in solving the MOSP problem include the effective processing of small-size opinion data and the varied opinion representations. Due to the restrictions on time, cost, private policies, and other issues, a decision is often made on small sized opinion data of a limited number of participants. Even though all participants would like to express their opinions thoroughly in an ideal situation, the small-size opinion data makes it is very hard to apply methods for large-size data to solving the MOSP problem. Varied opinion representation is another difficulty in solving the MOSP problem. Participants prefer to express their opinions in their own ways based on their understandings of and experiences in a given topic. However, this is bound to difficulties for measuring the similarity between their opinions. A strategy commonly used to regulate opinion representation is providing a fixed number of choices, for example, some predefined linguistic terms or a set of ordinal numbers

[2, 5, 6]. However, this cannot completely avoid varied opinion representations because the pre-defined choices may have different semantics for different persons and for different evaluation criteria.

Keeping the aforementioned difficulties in mind, this paper presents a three-level-similarity measuring (TLSM) method to solve the MOSP problem based on three assumptions: 1) Given a criterion, if the opinions of two participant are similar for the majority of options, then they are similar; 2) Given a set of criteria, if the opinions of two participants are similar for the majority of important criteria, then they are similar; and 3) Given a decision problem, if the opinions of two participants produce a similar decision, then they are similar.

The rest of the paper is organized as follows. Section 2 reviews related works in opinion analysis, similarity measurement and aggregation operations. Section 3 develops a gradual aggregation algorithm (GAA) which is used to generate an overall opinion similarity. In Section 4, we introduce the TLSM method in detail. Section 5 illustrates two case studies in social policy selection and energy policy evaluation problems. Section 6 summarises the main contributions of the work and future study plans.

2. Related works

Opinion analysis is extensively studied in social psychology fields [7]; recently, requirements for effectively extracting, summarizing, and segmenting opinions of general or specific users boosted the growing research on opinion mining and sentiment analysis [8–10]. Many opinion mining systems have been developed and applied [9, 11, 12] . However, these methods are not suitable for the MOSP problem because of the aforementioned difficulties. In the MCGDM field, study of opinion analysis is conducted in two main areas. Qualitative studies analyze and simulate the behaviour patterns of peoples based on their opinions of a considered affair [13, 14]. Quantitative research focuses on how to represent and process opinions in a computational framework [5, 15]. For instance, fuzzy sets and fuzzy logic are widely used as opinion representation and process facilities [16, 17] because they can effectively interpret and model the subjective information with uncertainties. These computation-based techniques provide support to develop solutions for the MOSP problem.

Similarity measurement is widely studied in human knowledge representation, behaviour analysis, and real-world problem solving [18–20]. Generally speaking, a similarity metric can be derived from a distance metric. The Euclidean metric, the absolute value metric, and the Tchebycheff metric are commonly used. Noting that the majority of existing similarity metrics will ultimately produce a crisp numeric value, which cannot sufficiently depict the fuzziness in real cases, Chakraborty and Chakraborty [21] defined a similarity metric whose value is a fuzzy set and implemented a clustering algorithm to solve a group decision making problem.

Using aggregation to integrate evaluations of individual participants is a crucial step to develop a solution for an MCGDM problem. According to whether or not an aggregation operator explicitly considers the relevant importance (weights) of the evaluation criteria, three main types of aggregation operators are used in MCGDM research. The first type treats all evaluation criteria equally. Typical examples include the arithmetic mean, the geometric mean, and the t-norms (or t-conorms) [22, 23]. The second type explicitly distinguishes the weights of the evaluation criteria either by their impacts on the decision problem, or by their processing order. The weighted mean and the ordered weighted aggregation (OWA)

[24], as well as their extensions [25, 26] belong to this type. A third type is defined by certain integral theories, such as the Segno and Choquet integrals [27–29]. Currently existing aggregation operators in MCGDM research often assume that the inputs are complete and simply ignore any missing values when generating an aggregation result. This assumption is not consistent with the realities of applications. How to process missing values is, therefore, a key concern when applying an aggregation operator; but this issue has not yet been solved. Although so many powerful aggregation operators have been presented, little is known about how to select an appropriate one in real applications. Beliakov [30] reported a solution by using the mathematical programming technique to adjust the parameters of a form-fixed aggregation operator.

3. A gradual aggregation algorithm

3.1. Motivations and implementations

Two practical issues are commonly faced in an MCGDM problem. The first one is how to handle missing values. The other issue is how to generate a decision dynamically which refers to the procedure of making a final decision from a sketched one based on a few numbers of criteria at the initial stage and then amending it in the following stages by considering more criteria added gradually. To solve these two issues, this section develops a gradual aggregation algorithm (GAA) which is implemented in two ways, i.e., the ordinary gradual aggregation (OGA) and the weighted gradual aggregation (WGA). The difference between them is that the OGA does not explicitly process the criteria weights but leaves it to the aggregation operator; while the WGA does.

Following the notations in [23], an aggregation operator $\mathcal{A}$ over a closed set X is denoted by $\mathcal{A} : \bigcup_{i\in\mathbb{N}^+}\{A_i : X^i \to X\}$ where A_i is called the i-ary aggregation operator in $\mathcal{A}$. For convenience, let X be a closed subset of $\mathbb{R}$.

Definition 3.1. *Let $\mathcal{A}$ and $\mathcal{B}$ be two aggregation operators. A mapping G_n from X^n to X is called an n-ary ordinary gradual aggregation (OGA) with respect to $\mathcal{A}$ and $\mathcal{B}$:*

$$G_n(x_1, \cdots, x_n) = B_n\Big(\Big\{A_i(x_1, \cdots, x_i), i = 1, \ldots, n\Big\}\Big).$$

Definition 3.2. *Let $\mathcal{A}$ and $\mathcal{B}$ be two aggregation operators; w_i the weight of input x_i, $i = 1, \ldots, n$. A mapping G_n from X^n to X is called an n-ary weighted gradual aggregation (WGA) with respect to $\mathcal{A}$ and $\mathcal{B}$:*

$$G_n(x_1, \cdots, x_n; w_1, \cdots, w_n) = B_n\Big(\Big\{A_i(x_1, \cdots, x_i; w_1, \cdots, w_i), i = 1, \ldots, n\Big\}\Big).$$

The OGA and the WGA inherit some properties of $\mathcal{A}$ and $\mathcal{B}$ which are given below. These properties indicate that the OGA and the WGA can be used to implement aggregation procedure.

Proposition 3.1. *If both $\mathcal{A}$ and $\mathcal{B}$ are idempotent, so do OGA and WGA.* □

Proposition 3.2. *If both $\mathcal{A}$ and $\mathcal{B}$ are monotonic, so do OGA and WGA.* □

Proposition 3.3. *If both $\mathcal{A}$ and $\mathcal{B}$ are bounded, So do OGA and WGA.* □

3.2. Weights assignment and adjustment

Although it does not explicitly process the weights of criteria, the OGA assigns implicitly a set of weights to its inputs based on their processing orders when both $\mathcal{A}$ and $\mathcal{B}$ are arithmetic means. Suppose the inputs x_1, ..., x_n are indexed by their processing orders, whose weights are not given. Then by the OGA, we have

$$A_i(x_1, x_2, \ldots, x_i) = \frac{x_1 + x_2 + \ldots + x_i}{i}, \quad i = 1, \ldots, n$$

and

$$G_n(x_1,\ldots,x_n) = \frac{\sum\limits_{i=1}^{n} A_i(x_1,\ldots,x_i)}{n} = \sum_{i=1}^{n} x_i \left(\frac{1}{n} \sum_{j=i}^{n} \frac{1}{j} \right). \tag{1}$$

Let β_i be the coefficient of x_i in Eq. (1), i.e., $\beta_i = \frac{1}{n}\sum_{j=i}^{n}\frac{1}{j}$, $i = 1,\ldots,n$. The sum of β_is is

$$\beta_1 + \beta_2 + \cdots + \beta_n = 1, \tag{2}$$

and the order of β_is is

$$\beta_1 > \beta_2 > \cdots > \beta_n > 0. \tag{3}$$

Eq. (2) shows that β_1, ..., β_n form a set of weights and are assigned to the inputs implicitly. Eq. (3) indicates that the an input processed earlier gains a larger weight. Intuitively, this weight assignment result is consistent with a real decision procedure where the most important criteria are often processed preferentially.

Furthermore, these assigned weights change their values with the number n of inputs. Figure 1 illustrates changes of the first five assigned weights when $n \leqslant 18$. It shows that each β_i is convergent with the increase of n. A conclusion is drawn from this observation that, given a larger n, the newly added inputs will exert little impact on a sketchy decision. Since the parameter n in a real problem cannot be too large, the impacts of the most important criteria corresponding to the inputs—which are processed preferentially—are therefore strengthened.

Compared with the OGA, the WGA can explicitly adjust the initially assigned weights of the inputs in its aggregation procedure. By replacing A_i with the weighted

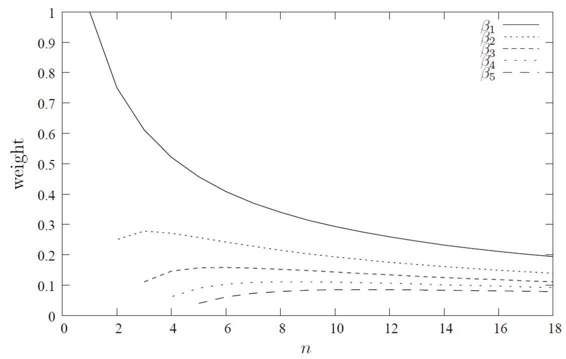


Figure 1: Changing weights with the number of inputs.

mean, and supposing the initial weight of input x_i is w_i, we have

$$A_i(x_1,\ldots,x_i;w_1,\ldots,w_i) = \frac{w_1}{\sum\limits_{j=1}^{i} w_j}x_1 + \cdots + \frac{w_i}{\sum\limits_{j=1}^{i} w_j}x_i, \quad i = 1,\ldots,n$$

and

$$G_n(x_1,\ldots,x_n;w_1,\ldots,w_n) = \frac{1}{n}\sum_{i=1}^{n} x_i w_i \left(\sum_{k=i}^{n} \frac{1}{\sum_{j=1}^{k} w_j}\right).$$

Let β_i be the coefficient of x_i, i.e., $\beta_i = \frac{w_i}{n}\sum_{k=i}^{n}\frac{1}{\sum_{j=1}^{k} w_j}$, $i = 1,\ldots,n$. Then we have

$$\beta_1 + \beta_2 + \cdots + \beta_n = 1, \tag{4}$$

i.e., β_1, β_2, ..., β_n form a set of weights and the inputs are re-weighted by them. Comparing β_i and w_i, we have a loose inequity that

$$\beta_i \geqslant \frac{n-(i-1)}{n} w_i, \quad i = 1,\ldots,n. \tag{5}$$

Further analysis indicates that $\beta_1 \geqslant w_1$ and if n is larger enough and i is smaller, the first several β_is are very near to, even greater than, the initial w_is. This means the impacts of the corresponding criteria are still preserved by the WGA.

The above algorithm and discussions indicate that the GAA can effectively maintain the impacts of important criteria that is very important feature for making decisions dynamically and processing missing values.

3.3. Dynamic decision and missing values

The processing order of the inputs emphasised in the GAA is closely related to the dynamic generation of a decision and process of missing values.

When making a decision, there is a natural processing order in the considered criteria, i.e., the most important criteria are often considered preferentially, then the secondary important criteria, and finally the not so important criteria. Similarly, as shown in Section 3.2, the GAA implementations can assign (reassign) a set of decreasingly changed weights to the inputs according to their processing orders. In this sense, the GAA implementations are models of the generation of a dynamic decision.

Two intuitive strategies to handle missing values are: 1) completely discard them; or 2) try to impute them. The GAA implementations can partially combine these. When the parameter n in GAA is smaller than the total number of inputs, some inputs will not be considered naturally. Obviously, if missing values exist in the unprocessed inputs; these missing values have no effect on the obtained aggregation result. However, if the missing values exists for some key criteria; in this situation, the GAA repeatedly use the aggregation operator can partially impute the missing values through using $\mathcal{A}$ to calculate a set of candidate results by slightly assigning or adjusting the weights of those inputs and using the aggregation operator $\mathcal{B}$ to generate an aggregation. To illustrate this procedure, let us consider the example below.

Table 1: An example for processing a missing value.

		S1	S2		S3			
No.	Input	OGA	DM	OGA-DM	IM-0	OGA-0	IM-M	OGA-M
1	0.840	0.840	0.840	0.840	0.840	0.840	0.840	0.840
2	**0.783**	0.812	0.912	0.876	0.000	0.420	0.549	0.694
3	0.912	0.845	0.335	0.696	0.912	0.584	0.912	0.767
4	0.335	0.718	0.278	0.591	0.335	0.522	0.335	0.659
5	0.278	0.630	0.477	0.568	0.278	0.473	0.278	0.583
6	0.477	0.604	0.365	0.535	0.477	0.474	0.477	0.565
7	0.365	0.570	0.952	0.594	0.365	0.458	0.365	0.537
8	0.952	0.618	0.636	0.599	0.952	0.520	0.952	0.588
9	0.636	0.620	0.142	0.549	0.636	0.533	0.636	0.594
10	0.142	0.572			0.142	0.494	0.142	0.549
result	**0.572**	**0.683**	**0.549**	**0.650**	**0.494**	**0.532**	**0.549**	**0.638**

Example 3.1. For illustrative purpose, suppose 10 inputs are given (the second column in Table 1) and the aggregation algorithm used is the arithmetic mean. We compare three scenarios: (S1) no missing value; (S2) ignore missing value; and (S3) replace the missing value with 0 and the mean of the others.

For (S1), the aggregation result without using the OGA is 0.572 (column "Input"); while it is 0.683 (column "OGA") with the OGA, where $\mathcal{A}$ and $\mathcal{B}$ are both the arithmetic means, and the third column in Table 1 shows the intermediate results of using it. For (S2), the aggregation result without using the OGA is 0.549 (column "DM"); while it is 0.650 by using the OGA (column "OGA-DM"). For (S3), the aggregation results without using the OGA are 0.494 and 0.549 for replacing the missing value by 0 (column "IM-0") and the mean of the others (column "IM-M"), respectively; while they are 0.532 (column "OGA-0") and 0.638 (column “OGA-M”) by using the OGA, respectively.

If taking (S1) as benchmark, we noted that the OGA generates a result with bigger difference from the benchmark than the other methods. This fact indicates that the OGA pays more attention on the missing value.

4. A three-level-similarity measuring method for the MOSP problem

4.1. The MOSP problem

An MOSP problem is briefly addressed as follows: given an MCGDM problem with a set of candidate options, the participants evaluate them in terms of a set of evaluation criteria and everyone completes a report containing evaluations summarised in linguistic terms; after collecting these evaluation reports, a question arises: can we identify which two participants have similar opinions based on the collected evaluation reports.

For convenience of discussion, we use $O = \{o_i | i \in I\}$ for the candidate options, $C = \{c_j | j \in J\}$ for the evaluation criteria, and $E = \{e_k | k \in K\}$ for the participants. The evaluation report from participant e_k is denoted by a matrix $V_k = (v_{ij})_{I \times J}$, where v_{ij} is the evaluation (i.e., opinion) on option o_i about criterion c_j. v_{ij} is either an element in T_j which is the collected linguistic terms used for criterion c_j, or a blank for “not available” or “no answer”, or a question mark for “unclear. Without loss of generality, we suppose that each participant provides only one term for each option about each criterion.

4.2. Overview of the TLSM method

The outline of the TLSM method is shown in Table 2. By this method, the similarity of two participants opinions will be measured at three sequential levels, i.e., the assessment level, the criterion level, and the problem level.

Table 2: Outline of main processes in the TLSM method.

Process level	Main steps
Assessment	*Input: two experts' evaluation reports; evaluation term set* T_j
	Output: the similarity about criterion c_j
	1.1 determine a similarity matrix for evaluation terms for criterion c_j;
	1.2 determine a clustering algorithm;
	1.3 generate semantic-equal groups by the clustering algorithm;
	1.4 calculate similarity between two opinions for criterion.
Criterion	*Input: the similarity at the assessment level and weights of criteria*
	Output: similarity with respect to each criterion against the criteria set
	2.1 identify a similarity utility function u_j of each criterion c_j;
	2.2 calculate similarity with respect to criterion c_j by u_j.
Problem	*Input*: similarities obtained at the criterion level
	Output: similarity between two opinions
	3.1 construct the GAA from a pair of aggregation operators;
	3.2 calculate the similarity between opinions using the GAA.

At the assessment level, the evaluations of two participants are compared option by option in terms of a given criterion. The comparison is conducted based on the assumption that the more candidate options on which two participants have similar evaluations, the higher similarity of their opinions is. To determine whether two evaluations are similar or not, the TLSM method compares their semantics: two opinions are said to be similar (or have similar semantics) if they are represented by terms in the same semantic-equal group which is built through pair-wisely comparing semantics of all terms used. By the option-by-option comparison conducted on the two participants' evaluations, how similar of the two participants opinions is known on a given criterion. The similarity is proportional positively to the number of options with similar evaluations against the total number of options.

At the criterion level, the different impacts (weights) of evaluation criteria are further considered. The TLSM method defines for each criterion a similarity utility function (SUF) based on its weight against those of others. An SUF is proportional positively to similarity obtained at the assessment level and is proportional inversely to the weights of criteria. The SUF is used to emphases that similarity of preferential criterion is more important than non-preferential criterion. Based on these SUFs, we can measure to what extent the two participants have similar opinions on each given criterion against a set of criteria.

At the problem level, the similarity is measured using the GAA. The GAA takes the similarities obtained at the criterion level as inputs and re-orders them according to the decreasing-ordered weights of the corresponding criteria. The aggregation algorithm will generate a set of candidate values of the overall similarity of two participants' opinions at the first stage, and then derives the overall similarity from them at the second stage. The overall similarity obtained indicates to what extent the two participants have similar opinions on a decision problem.

The details of the TLSM method are described in the following sections.

4.3. Measuring similarity at the assessment level

To measure similarity at the assessment level, we need to divide the term set T_j for criterion c_j into several semantic-equal groups. To do so, a similarity matrix of the terms in T_j is built by pair-wise comparison based on their semantics; then a clustering algorithm is used, such as the Hierarchical Clustering for Fuzzy Similarity Matrix (HCFSM) [31], to generate semantic-equal groups. We use pair-wise comparison for some practical considerations. Firstly, the semantic interpretation of linguistic terms varies person to person and case by case. Pair-wise comparison can avoid difficulties in defining a commonly-acceptable semantic of a term for all persons and for all cases. Secondly, some linguistic terms are incomparable. Hence it is hard to define an appropriate and rational similarity measurement for those terms. Thirdly, similarity between terms may be changeable. Two terms may be distinguishable in one context but indistinguishable in the other. Pair-wise comparison has been proved an effective strategy to analyse relationships between factors; for instance, the Analytic Hierarchy Process (AHP) technique extensively uses pair-wise comparison to obtain local-priority and global-priority. Using it can better fit an application's specific setting and avoid potential heavy and complicated calculations. Nonetheless, we do not reject other methods to determine the semantic similarity matrix.

For a given criterion c_j, the similarity matrix S_j is denoted by $S_j = (s_{pr})_{p_j \times p_j}$, where s_{pr} is the semantic similarity of terms t_p and t_r, and $s_{pr} \in [0, 1]$, $s_{rr} = 1$, $s_{pr} = s_{rp}$ for any $p, r \in \{1, \ldots, p_j\}$.

After obtaining the similarity matrix, the TLSM method will segment the term set by a clustering algorithm. Noting that the total number of terms in the term set is often between 5 and 9, the TLSM method uses the HCFSM as an example to illustrate the segmenting:

- derive the transitive closure $\hat{S}_j$ from S_j by $\hat{S}_j = S_j \bigcup S_j^2 \bigcup S_j^4 \bigcup \cdots$, where S_j^{2k} is the max-min composition of S_j^k;
- decompose $\hat{S}_j$ into a set of α-level equivalence class $(\hat{S}_j)_\alpha$; and
- terms in T_j whose similarities belong to the same $(\hat{S}_j)_\alpha$ form a semantic-equal term group TG_j^α and are treated with similar semantic.

Based on the segmentation of T_j, a similarity at the assessment level is defined according to the number of candidate options (nsp_j), on which the two opinions are similar, and the total number of candidate options n. As a simple illustrative example, the TLSM let the similarity be the ratio of them.

4.4. Measuring similarity at the criterion level

The main task in this step is to identify an appropriate SUF for each criterion which needs to satisfy two requirements: 1) it is proportional to similarity at the assessment level (PSA); and 2) it is proportional inversely to the weight of a criterion (PRW). Formally, an SUF is defined below.

Definition 4.1. *An SUF $u(nsp, w)$ of a given criterion c is a mapping from $\mathbb{N} \times W$ to $[0, 1]$ if u satisfies the PSA and PRW requirements, where $\mathbb{N}$ is the set of natural numbers and W is the range of weights.*

Functions satisfying Definition 4.1 are numerous. For simplicity, this study uses the following monotone and continuous function for illustrating purpose:

$$u_j(nsp_j, wc_j) = \left(\frac{nsp_j}{n}\right)^{f(wc_j)} \tag{6}$$

where nsp_j/n is the similarity at the assessment level and $f(wc_j)$ is a parameter determined by wc_j. Because the weight wc_j could be a numeric value or a linguistic term, we will consider these two forms accordingly.

4.4.1. Weights are non-negative real numbers

Suppose $wc_1 \geqslant wc_2 \geqslant \cdots \geqslant wc_m$ is a set of normalised numeric weights and $wc_j \geqslant 0$, $\sum_{j=1}^{m} wc_j = 1$, $m = |C|$. In this situation, we can determine the parameter $f(wc_j)$ as follows: 1) determine a reference value wc_{j_0} and set $f(wc_{j_0}) = 1$; and 2) for each wc_j, set $f(wc_j) = wc_j/wc_{j_0}$. To find a wc_{j_0} from $wc_1, \ldots, wc_m$, the following illustrative method is used: if m is odd, then set $wc_{j_0} = wc_{(m+1)/2}$; if m is even, then set $wc_{j_0} = (wc_{m/2} + wc_{m/2+1})/2$. Based on this wc_{j_0}, all wc_js are then mapped to $[0, \infty)$ by

$$f(wc_{j_0}) = 1, \quad f(wc_j) = \frac{wc_j}{wc_{j_0}}, \quad j = 1, \ldots, m. \tag{7}$$

4.4.2. Weights are linguistic terms

Linguistic weights are often represented by fuzzy numbers (or fuzzy sets). Specific numeric features of a fuzzy number (set), such as its centre of gravity (COG) or its generalized integral, can be used to determine the parameter $f(wc_j)$. A brief outline for determining this parameter is given as: 1) select a numeric feature NF of fuzzy numbers and calculate NF_j of the linguistic weight wc_j; 2) determine $f(NF_j)$ following steps for $f(wc_j)$ in Section 4.4.1 and set $f(wc_j) = f(NF_j)$.

Following this outline, suppose the linguistic weights are "Very high (VH)", "Fairly high (FH)", "Medium (M)", "Rather low (RL)", and "Very low (VL)" and their corresponding fuzzy numbers are shown in Figure 2(b). Let the selected numeric feature be the horizontal coordinate of COG of a fuzzy number, i.e.,

$$NF_j = \frac{\int x\mu(x)dx}{\int \mu(x)dx} \tag{8}$$

where $\mu(x)$ is the membership function of the fuzzy number. By Eq. (8) and following steps in Section 4.4.1, the $f(NF_j)$ is calculated and shown in Table 3. Replacing the $f(wc_j)$ in Eq. (7) by $f(NF_j)$, we obtain the SUFs for the five linguistic weights, which can then be applied to calculate the similarity at the criterion level.

Table 3: Linguistic weight, numeric feature, parameter of SUF of criteria.

wc_j	VH	FH	M	RL	VL
NF	0.9	0.767	0.5	0.233	0.1
$f(NF)$	1.800	1.534	1	0.466	0.200

After determining the SUF for each given criterion, we apply them to measure the similarity of the opinions of two participants at the criterion level. Suppose a referential criterion is weighted "FH" and the evaluations of two participants are treated similarly for seven out of nine candidate options, then the similarity of the opinions with respect to this criterion is 0.680 (= $(7/9)^{1.534}$).

4.5. Measuring similarity at the problem level

The similarity of two opinions about each individual criterion provides a single perspective by which we observe the similarity of two opinions. While a set of criteria is considered, we need to integrate those observations to form a comprehensive one. The GAA developed in Section 3 is used for this task. The following two examples illustrate how to use it. Suppose the similarities about 10 criteria are obtained at the criterion level as shown in the second column of Table 1.

Example 4.1. This example illustrates the usage of OGA. Assume that both $\mathcal{A}$ and $\mathcal{B}$ are the arithmetic means. For the 10 inputs, the OGA firstly generates 10 candidate similarities for the final one $\bar{s}$ by A_i ($i = 1, \ldots, 10$) and they are: 0.840 ($\bar{s}_1$), 0.812 ($\bar{s}_2$), 0.845 ($\bar{s}_3$), 0.718 ($\bar{s}_4$), 0.630 ($\bar{s}_5$), 0.604 ($\bar{s}_6$), 0.570 ($\bar{s}_7$), 0.617 ($\bar{s}_8$), 0.619 ($\bar{s}_9$), 0.572 ($\bar{s}_{10}$). Then the GAA applies B_{10} to $\bar{s}_1, \ldots, \bar{s}_{10}$ and produces $\bar{s} = 0.683$, i.e., the similarity of the two experts' opinions is 0.683.

Example 4.2. This example illustrates the usage of WGA. Assume that $\mathcal{A}$ is the OWA aggregation [24] and $\mathcal{B}$ is the arithmetic mean. Because an OWA aggregation needs the weights of inputs, we randomly generate 10 unnormalised weights for them as: 0.394 (w_1), 0.798 (w_2), 0.198 (w_3), 0.768 (w_4), 0.554 (w_5), 0.629 (w_6), 0.513(w_7), 0.916 (w_8), 0.717 (w_9), 0.607 (w_{10}). Then, the WGA calculates the candidate values of $\bar{s}_i$s following OWA: 0.952 ($\bar{s}_1$), 0.925 ($\bar{s}_2$), 0.913 ($\bar{s}_3$), 0.866 ($\bar{s}_4$), 0.819 ($\bar{s}_5$), 0.755 ($\bar{s}_6$), 0.703 ($\bar{s}_7$), 0.632 ($\bar{s}_8$), 0.586 ($\bar{s}_9$), 0.541 ($\bar{s}_{10}$). Finally, the WGA applies the B_{10} to $\bar{s}_1, \ldots, \bar{s}_{10}$ to get the overall similarity, which is 0.769.

Based on the similarity measurement at the three levels, an overall similarity between the opinions of two participants is generated, which can be used as the answer of the MOSP problem.

5. Applications in policy selection and evaluation

This section applies the TLSM method to an social policy selection application and an energy policy evaluation application.

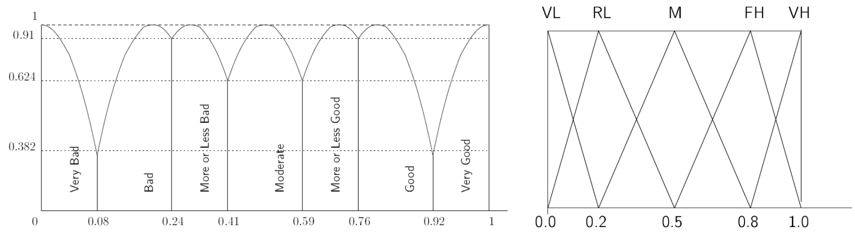
5.1. Case 1: Do similarities exist between social actors?

This example is quoted from [31]. In a social policy selection problem, six social actors (i.e., participants) have presented their assessments for seven possible policies (i.e., options). The social impact matrix (i.e., evaluation report) is given in Table 4 and the semantics of the used linguistic terms are given in Figure 2(a). The problem is to answer whether or not similarities exist between these social actors.

Table 4: An illustrative example of social impact matrix

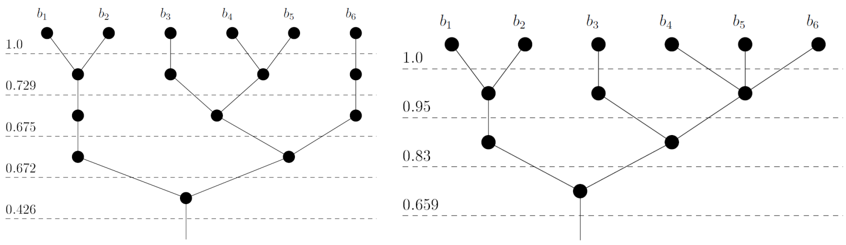
Social	Policy options						
actors	a_1	a_2	a_3	a_4	a_5	a_6	a_7
b_1	Very good	Good	Moderate	bad	Fairly good	Fairly bad	Very bad
b_2	Very good	Good	Moderate	Bad	Fairly good	Very bad	Very bad
b_3	Very bad	Fairly bad	Moderate	Good	Very good	Good	Moderate
b_4	Very bad	Fairly bad	Fairly bad	Good	Fairly good	Good	Very good
b_5	Very bad	Bad	Fairly bad	Moderate	Fairly good	Good	Very good
b_6	Very bad	Good	Bad	Good	Good	Good	Very good

Firstly, we recited the solution in [31] as a comparison with the TLSM method. The Munda's method includes three main steps.



(a) Linguistic assessments in Munda's method. (b) Linguistic weights in Case 2.

Figure 2: Semantic of linguistic terms.



(a) Result by Munda's method. (b) Result by the TLSM method.

Figure 3: Dendrogram of similarities between experts.

- Generate a similarity matrix between the social actors by a similarity measurement $s(b_i, b_j)$: $\left(1 + \left[\sum_{k=1}^{7} \left(\iint_{x,y} |x - y| f_i(x) g_j(y) \mathrm{d}y \mathrm{d}x\right)^2\right]^{1/2}\right)^{-1}$, where $\iint_{x,y} |x-y| f(x) g(y) \mathrm{d}y \mathrm{d}x$ is the semantic distance between two linguistic terms x and y. The obtained similarity matrix S is shown in Table 5.

- Generate hierarchical clustering by the HCFSM algorithm (Figure 3(a)).

- Analyze clustering result: the social actors b_1 and b_2 have higher similarity.

We now measure the similarity between the social actors b_1 and b_4 as an illustration of the TLSM method procedure. Because the problem setting does not mention evaluation criteria, we assume that only one criterion is considered.

Step 1: Measuring similarity at the assessment level. Firstly, we define a dis-

Table 5: Similarity matrix between six social actors.

	b_1	b_2	b_3	b_4	b_5	b_6
b_1	1	0.729	0.426	0.399	0.403	0.403
b_2	0.729	1	0.410	0.386	0.390	0.390
b_3	0.426	0.410	1	0.675	0.584	0.569
b_4	0.399	0.386	0.675	1	0.729	0.672
b_5	0.403	0.390	0.584	0.729	1	0.595
b_6	0.403	0.390	0.569	0.672	0.595	1

tance measure $d(t_i, t_j) = |x_i - x_j|$ between two terms t_i and t_j whose membership functions are fuzzy numbers and $\mu_{t_i}(x_i) = 1$, $\mu_{t_j}(x_j) = 1$. Correspondingly, the similarity between t_i and t_j is defined by $s_{ij} = 1 - d(t_i, t_j)$ and the similarity matrix obtained is shown in Table 6. The dendrogram for the seven evaluation terms by the HCFSM algorithm is presented in Figure 4.

Table 6: Similarity matrix for linguistic assessments.

Term	Very bad	Bad	Fairly bad	Moderate	Fairly good	good	Very good
Very bad	1.0	0.8	0.7	0.5	0.3	0.2	0.0
Bad	0.8	1.0	0.9	0.7	0.5	0.4	0.2
Fairly bad	0.7	0.9	1.0	0.8	0.6	0.5	0.3
Moderate	0.5	0.7	0.8	1.0	0.8	0.7	0.5
Fairly good	0.3	0.5	0.6	0.8	1.0	0.9	0.7
good	0.2	0.4	0.5	0.7	0.9	1.0	0.8
Very good	0.0	0.2	0.3	0.5	0.7	0.8	1.0

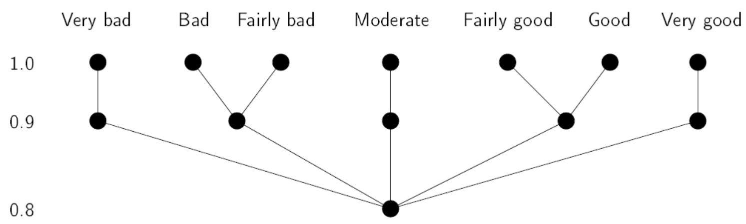


Figure 4: Dengrogram of linguistic assessments (terms).

Secondly, we take the 0.9-level equivalence-class in Figure 4 to compare the evaluations of actors b_1 and b_4. It is noted that these two social actors have a similar opinion on policy a_5 only. Table 7 lists the number of similar options of each pair of social actors.

Table 7: Number of options with similar opinions by pairwise comparison.

nsp	b_1	b_2	b_3	b_4	b_5	b_6
b_1	7	6	1	1	1	2
b_2	6	7	1	1	1	2
b_3	1	1	7	4	3	3
b_4	1	1	4	7	6	6
b_5	1	1	3	6	7	5
b_6	2	2	3	6	5	7

Step 2: Measuring similarity at the criterion level. Based on the one criterion assumption, we need only to determine a unique parameter $f(wc)$ for the SUF. Suppose the SUF is of the form in Eq. (6). Noticing that setting $f(wc)$ to be less than, equal to, or greater than 1.0 gives three typical utilities of a criterion, we discuss them below respectively.

The first situation is setting $f(wc) = 1$. The SUF is a linear function, by which the similarity between b_1 and b_4 is 0.143. Table 8 illustrates the pair-wise similarity of all actors under this setting.

Table 8: Pair-wise comparison of similarity at the criterion level ($f(wc) = 1$).

	b_1	b_2	b_3	b_4	b_5	b_6
b_1	1	0.857	0.143	0.143	0.143	0.286
b_2	0.857	1	0.143	0.143	0.143	0.286
b_3	0.143	0.143	1	0.571	0.429	0.429
b_4	0.143	0.143	0.571	1	0.857	0.857
b_5	0.143	0.143	0.429	0.857	1	0.714
b_6	0.286	0.286	0.429	0.857	0.714	1

The second situation is setting $f(wc) > 1$. The obtained SUF increases slowly with a smaller similarity at the assessment level and then increases quickly with a larger one. Suppose $f(wc) = 2$, then the pair-wise similarities of the six actors are shown in Table 9.

Table 9: Pairwise comparison of similarity at the criterion level ($f(wc) = 2$).

	b_1	b_2	b_3	b_4	b_5	b_6
b_1	1	0.735	0.020	0.020	0.020	0.082
b_2	0.735	1	0.020	0.020	0.020	0.082
b_3	0.020	0.020	1	0.327	0.184	0.184
b_4	0.020	0.020	0.327	1	0.735	0.735
b_5	0.020	0.020	0.184	0.735	1	0.510
b_6	0.082	0.082	0.184	0.735	0.510	1

The third situation is $f(wc) < 1$. The obtained SUF increases quickly with a smaller similarity at the assessment level and then increases slowly with a bigger one. When setting $f(wc) = 1/3$, the pair-wise similarities are shown in Table 10.

Table 10: Pairwise comparison of similarity at the criterion level ($f(wc) = 1/3$).

	b_1	b_2	b_3	b_4	b_5	b_6
b_1	1	0.950	0.523	0.523	0.523	0.659
b_2	0.950	1	0.523	0.523	0.523	0.659
b_3	0.523	0.523	1	0.830	0.754	0.754
b_4	0.523	0.523	0.830	1	0.950	0.950
b_5	0.523	0.523	0.754	0.950	1	0.894
b_6	0.659	0.659	0.754	0.950	0.894	1

Based on the identified SUF, the similarity between b_1 and b_4 is obtained at the criterion level.

Step 3: Measuring similarity at the problem level. Because only one criterion is considered, no aggregation is needed; therefore, the similarity at the problem level is that at the criterion level, i.e., the similarity between b_1 and b_4 is 0.020.

Based on the similarity matrix in Table 10, we can use the HCFSM to obtain a similar dendrogram (Figure 3(b)). Comparing the two dendrograms in Figure 3, we recognized two minor differences: 1) social actor b_6 will join the group of b_4

and b_5 earlier than social actor b_3; and 2) the parameter α is slightly different.

5.2. Case 2: Energy policy selection with missing assessments

A governmental consultant committee has designed three national energy policies (O_1, O_2, O_3) for sustainable development and sent them to six domain experts (e_1, ..., e_6) for evaluation in terms of 16 primary and secondary criteria (c_1, ..., c_16). An expert's evaluation report includes two components: 1) the assessments on the importance of all criteria; and 2) the assessments on the impacts of the three alternative policies on sustainable development according to all criteria. All assessments are expressed by a term selected from a set of provided linguistic terms, or left blank for "unavailable", or with a question mark for "uncertain assessments (unknown or unsure)". After collecting the evaluation reports (Table 11) from those experts, the committee wants to know which two experts have similar opinions.

This study assumes that the linguistic terms used for weights of criteria and evaluations on policies are triangular normal fuzzy numbers as summarised in Table 12 and in Figure 2(b).

Based on the problem settings, the detailed steps are illustrated below.

Step 1: Measuring similarity at the assessment level. The similarity matrix S for assessment terms is obtained by using the same method in case 1 and it is

s_{ij}	AC	VL	L	UL	HUL
AC	1.0	0.8	0.5	0.2	0.0
VL	0.8	1.0	0.7	0.4	0.2
L	0.5	0.7	1.0	0.7	0.5
UL	0.2	0.4	0.7	1.0	0.8
HUL	0.0	0.2	0.5	0.8	1.0

By applying the HCSFM algorithm to S, we obtain three possible segments :

Table 11: Evaluation reports of six experts

c_i	w_i	O_1	O_2	O_3	O_1	O_2	O_3	O_1	O_2	O_3
		Expert 1			Expert 2			Expert 3		
1	VH	UL	L	AC	VL	VL	L	HUL	L	VL
2	FH	L	L	AC	UL	L	L	UL	UL	L
3	FH	UL	L	VL	UL	HUL	L	HUL	L	VL
4	FH	HUL	VL	AC	UL	UL	L	HUL	UL	HUL
5	FH	L	L	VL	L	VL	L	UL	VL	VL
6	FH	AC	VL	AC	VL	VL	UL	L	VL	AC
7	FH	L	UL	VL	UL	HUL	L	HUL	L	
8	FH	VL	L	VL	AC	AC	AC	UL	VL	VL
9	FH	AC	VL	L	AC	AC	AC	UL	VL	AC
10	FH	L	UL	L	VL	L	L	VL	VL	UL
11	FH	UL	UL	?	L	L	VL	VL	VL	HUL
12	FH	HUL	UL	L	HUL	HUL	VL	AC	AC	L
13	VH							UL	VL	UL
14	VH	VL	VL	VL	VL	VL	VL		VL	UL
15	FH	UL	HUL	VL	HUL	HUL	UL	L	HUL	HUL
16	FH	UL	UL	L	HUL	HUL	L	L	VL	L
		Expert 4			Expert 5			Expert 6		
1	VH	VL	L	UL	VL	UL	HUL	L	UL	HUL
2	FH	VL	L	VL	VL	UL	HUL	VL	L	UL
3	FH	AC	UL		VL	UL	HUL	L	HUL	HUL
4	FH	L	L	HUL	L	HUL	HUL	UL	UL	HUL
5	FH	AC	L	UL	AC	L	HUL	VL	VL	L
6	FH	UL	UL	HUL	AC	UL	HUL	L	UL	HUL
7	FH	UL	HUL	HUL	UL	L	HUL	HUL	HUL	HUL
8	FH	AC	VL	L	AC	UL	HUL	AC	AC	AC
9	FH	VL	AC	L	AC	UL	HUL	AC	AC	AC
10	FH	VL	L	AC	VL	UL	HUL	HUL	HUL	HUL
11	FH	HUL	HUL	L	L	UL	HUL			
12	FH	AC	AC	VL	VL	UL	HUL	L	UL	HUL
13	VH	VL	L	UL	AC	L	UL	L	HUL	HUL
14	VH	VL	VL	UL	VL	L	UL			
15	FH			UL	VL	UL	HUL	VL	UL	HUL
16	FH	UL	UL	HUL	VL	L	UL	L	UL	HUL

Table 12: Abbreviations and semantics of linguistic terms used in evaluation reports.

Abbreviation.	Names	Semantics
VH (AC)	Very high (Almost certain)	$(0.7, 1.0, 1.0)$
FH (VL)	Fairly high (Very likely)	$(0.5, 0.8, 1.0)$
M (L)	Medium (Likely)	$(0.2, 0.5, 0.8)$
RL (UL)	Rather low (Unlikely)	$(0.0, 0.2, 0.5)$
VL (HUL)	Very low (Highly Unlikely)	$(0.0, 0.0, 0.3)$
NA	No answer	

segment level	Segments
1.0	{AC}, {VL}, {L}, {UL}, {HUL}
0.8	{AC, VL}, {L}, {UL, HUL}
0.7	{AC, VL, L, UL, HUL}

Note that only two weights ("VH" and "FH") are used for the 16 criteria, and "VH" and "FH" are with same fuzzy membership functions of "AC" and "VL", this study uses the segments with 1.0-level for criteria with weight "VH" and the segments with 0.8-level for criteria with weight "FH". (The segments with 0.7-level will not be used in this study because it lacks capability to distinguish different terms.) Therefore, we can compare experts' opinions at the assessments level. Let us take experts e_1 and e_2 for example.

For criterion c_1: Because the weight of c_1 is "VH", two assessments are similar if and only if they are identical. Hence, the number of assessments with similar semantics between (UL, L, AC) (of e_1) and (VL, VL, L) (of e_2) about this criterion is 0.

For criterion c_2: Because the weight of c_2 is "FH", the assessment "AC" is treated the same as "VL"; so do "UL" and "HUL". Hence, the number of assessments with similar semantics between (L, L, AC) (of e_1) and (UL, L, L) (of e_2)) about this criterion is 1 because the two opinions have the same assessment on policy O_2 only.

Similarly, we can compare these two experts on the remaining 14 criteria one by one. Table 13 lists the number of options with similar opinion for all 16 criteria. Among the 16 criteria, criteria c_{11} and c_{13} are different from others due to the missing or uncertain assessments. This study treats them as dissimilar.

Table 13: Number of options with similar opinion for 16 criteria with respect to e_1 and e_2.

c_i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
no. of similar ass.	0	1	1	1	1	2	1	2	2	1	0	2	0	3	2	3

Step 2: Measuring similarity at the criterion level. This study uses the SUF defined in Eq. (6). The parameter $f(wc_j)$ is determined by the same method as used

in case 1. The numeric feature of these five linguistic terms are: $NF_{VH} = 0.9$, $NF_{FH} = 0.767$, $NF_M = 0.5$, $NF_{RL} = 0.233$, $NF_{VL} = 0.1$. The study sets $f(NF_M) = 1.0$ and calculates the parameters for the other four weights accordingly: $f(NF_{VH}) = 1.8$, $f(NF_{FH}) = 1.534$, $f(NF_{RL}) = 0.466$, $f(NF_{VL}) = 0.2$.

Once the SUFs of all evaluation criteria are finalized, they can be used to obtain similarity at the criterion level. For instance, consider the criteria c_1 and c_6. The weight of c_1 is "VH" and $f(NF_{VH}) = 1.8$; hence the similarity with respect to c_1 is 0.000. Because the weight of c_6 is "FH" and the $f(NF_{FH}) = 1.534$, then the similarity with respect to c_6 is 0.537. For the other 14 criteria, the calculation is similar. The similarities at the criterion level between e_1 and e_2 are: $s_1 = 0.000$, $s_2 = 0.185$, $s_3 = 0.185$, $s_4 = 0.185$, $s_5 = 0.185$, $s_6 = 0.537$, $s_7 = 0.185$, $s_8 = 0.537$, $s_9 = 0.537$, $s_{10} = 0.185$, $s_{11} = 0.000$, $s_{12} = 0.537$, $s_{13} = 0.000$, $s_{14} = 1$, $s_{15} = 0.537$, $s_{16} = 1$.

Step 3: Measuring similarity at the problem level. The GAA is implemented as follows: 1) re-order the criteria by their weights in descending order; 2) set A_i to be the arithmetic mean, $i = 1, \ldots, 16$; and 3) set B_{16} to be the t-conorm maximum max.

To re-order the criteria, this study uses the NF values. Then following the order of criteria, the i-ary aggregation operator A_i is applied to those similarities at the criterion level to obtain candidate similarities between the two experts: 0.000, 0.000, 0.333, 0.296, 0.274, 0.259, 0.249, 0.285, 0.274, 0.300, 0.322, 0.310, 0.304, 0.320, 0.362, 0.362. From them the biggest is selected by B_{16}, which is 0.362. Therefore, the similarity between the experts e_1 and e_2 is 0.362.

Table 14 gives the pair-wise similarity of the six experts. Based on the pair-wise similarity measurement, the experts can be grouped again based on a clustering

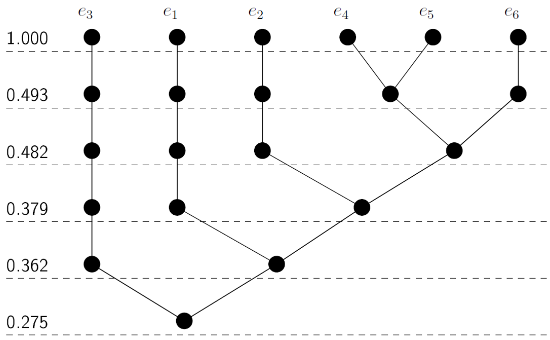


Figure 5: Deprogram of experts using the HCFSM.

method. For instance, Figure 5 is the dendrogram that uses the HCFSM algorithm. Further observation indicates that experts e_4, e_5, and e_6 have higher similarities in their opinions.

Table 14: Pair-wise similarities of all six experts.

	e_1	e_2	e_3	e_4	e_5	e_6
e_1	1	0.362	0.273	0.289	0.108	0.151
e_2	0.362	1	0.275	0.277	0.189	0.379
e_3	0.273	0.275	1	0.253	0.199	0.239
e_4	0.289	0.277	0.253	1	0.493	0.337
e_5	0.108	0.189	0.199	0.493	1	0.482
e_6	0.151	0.379	0.239	0.337	0.482	1

6. Conclusions and future works

MCGDM is an efficient strategy to support decision making in many applications. However, overly similar opinions of participants may lead to an inappropriate decision. To reduce the potential risk of putting an inappropriate decision into practice, measuring opinion similarity between participants (MOSP) is an important issue, which has not been solved. To solve the MOSP problem, our research develops a gradual aggregation algorithm to model the dynamic generation of a decision and to process the missing value. Based on the gradual aggregation algorithm, a three-level-similarity measuring (TLSM) method for the MOSP problem is presented which measures the similarity between two opinions at the assessment level, the criterion level, and the problem level. Applying the TLSM method, two applications in social policy selection and energy policy evaluation are conducted.

The main contributions of this research are summarised below. Firstly, the TLSM method provides a workable processing framework for the MOSP problem. The MOSP problem is a significant but easily neglected practical topic in many applications. Existing opinion similarity measuring methods can tackle a part of the MOSP problem; however, they do not present a whole solution for it. Secondly, the small size of relevant opinion samples is a primary obstacle that prevents existing statistical learning techniques from being applied to the MOSP problem. The TLSM method can resolve these problem partially. Moreover, the TLSM method combines an opinion with its provider in its entire processing. This helps to develop more effective opinion similarity measuring and analysis techniques to overcome difficulties resulting from separation of opinions and their providers in real applications. Finally, the experiments indicate that the TLSM method effectively handle missing data, uncertain information, and linguistic assessments by adjusting the developed gradual aggregation algorithm. Highly satisfactory results have been obtained from the experiments.

Based on the two case studies, some issues will be further studied. Firstly, the GAA is a novel technique to integrate information according to a group of inputs. The processing order of the inputs has special meaning and impact on the final result. This study rearranges the inputs according to the descent order of the weights of criteria and a satisfactory result is obtained; however, the GAA is still need to amend. Secondly, missing data and unclear answers are very common in real applications. The TSLM method treats them as distinct without distinguishing their real meanings and utilities further. This is an intuitive and simple processing strategy. Whether there is a better strategy is a further area requiring investigation. Moreover, we will pay more attention on how to select a clustering algorithm for the TSLM method. For simplicity and illustrating purpose, this paper mainly used the HCFSM method. Although the experiment results are consistent with our expectation, it is by no means that the HCFSM is the best one. We recognised that selecting an appropriate clustering method should base on real applications. Thirdly, the MOSP problem is a special case of the user opinion analysis and behaviour modelling problem. Due to a variety in the natures of different application contexts, effective techniques for solving the user opinion analysis and behaviour modelling problem have not yet been found. Our next step is to extend the TLSM method and develop new techniques to provide applicable solutions for both the MOSP problem and the user opinion analysis and behaviour modelling problem. Finally, the application of the proposed TSLM method involves heavy computational burden for large size decision making problems, which requires to develop a corresponding decision support system. We currently implemented the presented method using the C++ and Java programming languages in a Linux distribution. We aim to amend and integrate the method into a decision support system which is being designed and developed.

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