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Managing cognitive load in the mathematics classroom

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Context

Contemporary debates on effective pedagogies for K–12 mathematics have called for shifts in the way teachers and teacher educators conceptualise mathematics as a subject and how it should be taught. This is reflected by changes in the curriculum including the inclusion of a strand called Working Mathematically within K–12 mathematics curriculum documents not only in New South Wales but also across Australia (New South Wales Board of Studies, 2002). This strand brings focus to mental processes that underpin students' ability to acquire mathematical principles, concepts, and conventions, and the use of this knowledge in the solution of problems.

The focus on cognitive processes that support mathematical learning and problem solving is a welcome change. However, there is a paucity of information about the nature of links that need to be made between process and mathematics content, and how students might be assisted to construct the links.

In this paper, we outline results of research about cognitive load that is associated with mental processes, the management of this load so that students can be better supported in the construction of connected mathematical information, and the interpretation of that information in making sense of worked examples. We attempt to show that worked examples can be effective in promoting useful and powerful mathematics schemas.

Cognitive processes that underlie mathematics problem solving

An understanding of the cognitive processes that drive mathematics learning and knowledge organisation is critical for the design of effective approaches to mathematics teaching. Figure 1 shows a model of human memory structures and the processing of information. This model is based on components of working memory advanced by Baddeley and Hitch (2000). The model identifies two key attributes about how students deal with math-

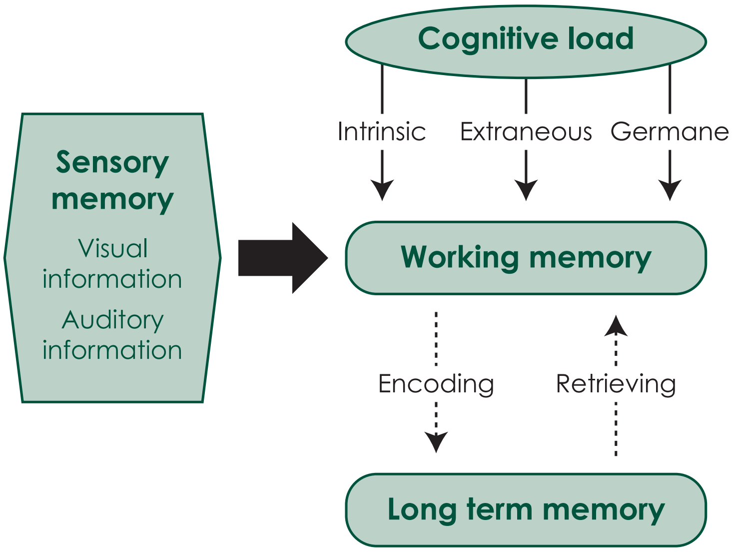


Figure 1. Model of human memory.

ematical information. Firstly, it shows connections between the processing of incoming mathematical information, reorganisation and the subsequent retrieval of that information for later use. Secondly, the model draws attention to the types of cognitive load that students could experience as they attempt to make sense of incoming mathematical information.

There are three types of memory: *sensory memory*, *working memory* (WM), and *long term memory* (LTM). In the following sections we look at both working and long term memory in some detail as these are more directly related to loads that can be exerted on the processing of incoming information.

Working memory (WM)

Working memory can be approximated to the idea of consciousness. If we are consciously aware of information then we are utilising working memory. A multitude of models of working memory has been proposed over the decades. Despite differences, all models tend to share two common basic characteristics about working memory: limitations in processing capacity and duration.

During a learning episode, new information from the environment is processed through WM. However, there are limitations to both the storage capacity of WM and the duration of time new information can be held and processed in WM. Although learners can process around seven separate items of information at any one time (Miller, 1956) this number significantly declines if items need to be compared or contrasted in some way (Kalyuga, 2006). For example, many young students starting at Stage 1 or 2 multiplication would have difficulty performing 87×28 as a mental operation as the task overloads the capacity of WM. This is so because the solution could involve two levels of processing. For example, students will usually have to compute 80×28 and 7×28 mentally, following which they have to add the resulting values. Both these computations, in turn, demand the use of further mental strategies.

Long-term memory (LTM)

In contrast to working memory, LTM is characterised by its limitless capacity for the storage of organised information. Mathematical knowledge and problem solving strategies are stored in the long-term memory. Information in LTM is also more robust and unlike WM, information in LTM is stored in a more or less permanent form (Newell & Simon, 1972).

The question of how information is stored in LTM and later retrieved for use has been the subject of discussion for many decades. A number of models has been advanced in discussion of this issue (Marshall, 1995). Barlett (1932) coined the term "problem schema" in his classic paper about general constructions that allow people to categorise information and the way it is used. Schemas are the structures in long-term memory that allow us, for example, to read the text on this page effortlessly, so that reading this sentence places little load on WM. For an individual without a relevant sophisticated schema base (a child, or perhaps an adult without the schemas that included academic jargon), this task would be effortful. In cases where word schemas are not developed, the task would be impossible. Thus, schemas in LTM allow us to negotiate effortlessly the world around us.

Understanding how a schema develops is important when devising appropriate teaching strategies.

Schema development

The quality of students' mathematical knowledge can exert a major influence on the deployment of that knowledge during learning and solution attempts. Quality of mathematical knowledge can be interpreted in terms of the degree of organisation of the different bits of mathematical information that constitute that knowledge. The framework of a schema helps to visualise connections that exist between core ideas and their components, and among the components. These comprise mathematical definitions and rules as well as knowledge about how to deal with a particular class of problems.

For instance, the understanding of the relations between parts and whole of a fraction, can be facilitated for a group of children by first involving them in an activity that embodies the notion of fractions, such as slicing a string into two equal parts. During this activity the children may be introduced to terms such as "half," "part" and "whole" in reference to the string that is being sliced. Thus, via the activity, the children have the opportunity to develop meaningful relations between the three terms. In a subsequent lesson, when the children are introduced to the term "fraction" with discussion about parts and whole, this word enters into the WM for a brief period, perhaps lasting a few seconds. During this period, children need to establish links to related prior concepts such as parts and wholes so that a meaningful schema about fractions is developed and stored in the LTM. That is, the child accommodates and assimilates concepts of fractions into an existing schema consisting of half, part and whole. Likewise, Chinnappan (1998) showed that geometric knowledge that was organised into meaningful and well-connected schemas played a critical role in fostering students' ability to use that knowledge appropriately during solution attempts.

Investigations conducted by Kirschner (2002) led to the conclusion that mathematical knowledge bases that are effectively organised in the form of schemas will facilitate more effective activation and use of knowledge during problem solving.

High levels of performance in mathematics rely heavily on schema acquisition. The development of schemas is an ongoing process involving cycles of modification and assimilation of incoming information. The establishment of schemas lessens the burden on the finite mental resources of working memory. Hence, mathematics instruction should attempt to promote activities that will facilitate schema development while being sensitive to the limited processing capacity of WM.

Types of cognitive load

The mental resources required of working memory to learn, perform or understand a task can vary quite dramatically between tasks. Some mathematics tasks may involve little cognitive load while others will be very complex and, therefore, heavy in cognitive load. If a mathematical task exceeds the mental resources available in working memory then cognitive *overload* will occur. There are at least three types of cognitive load that can be imposed on learners.

Intrinsic, extraneous and germane loads

The level of *intrinsic load* relates to the complexity of a task relative to a particular learner. Learning to memorise a mathematical formula, such as that for the area of a circle ($A = \pi r^2$), is a task that is low in complexity and would impose a low intrinsic load. To process this information, students simply need to consider the various elements of this information in isolation. Students would not need to process concurrently any other information, such as the formula for circumference, $A = 2\pi r$. Thus, recalling a simple formula could be taught with little or no interaction with other elements of information. It is a "low-element-interactivity task" (Sweller & Chandler, 1994).

However, applying the formula ($A = \pi r^2$) to a novel mathematical problem requires the learner to relate and compare parts of the formula (specifically radius, r, and area) with other learning elements in the problem. This is a task that is high in intrinsic load.

In mathematics, most tasks involve high intrinsic load (generated by high levels of element interactivity) because mathematics tasks demand that students draw upon multiple elements of information and integrate that information to solve a problem. Cognitive load becomes an issue (due to working memory limitations) when information is high in complexity, and thus high in intrinsic load.

In summary, mathematics tasks will range in complexity from low intrinsic load (low-element-interactivity) or high intrinsic load (high-element-interactivity). Intrinsic load and the degree of element interactivity will also be dependent on the learner. A task that is low in intrinsic load for an experienced mathematics teacher could be very high in intrinsic load for a student. Teachers need to be aware of the intrinsic load (natural complexity) associated with any mathematical task. In some cases, tasks may need to be segmented into sub-tasks in order to control intrinsic load (Mayer and Chandler, 2001).

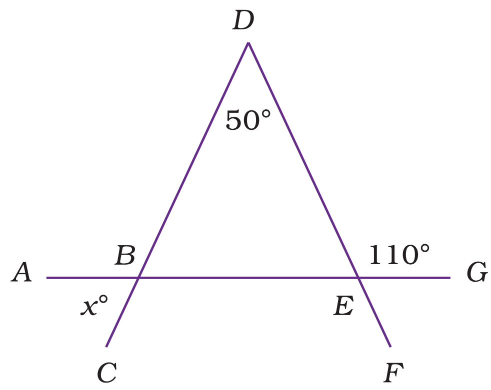
Extraneous load is imposed solely by the *instructional format* that is used during the course of teaching. Instructional format refers to the organisation of texts and visuals used by teachers to help learners understand a given concept or problem context. Mathematics can be taught in a variety of ways and each format of instruction can be expected to generate its own extraneous cognitive load. In certain formats, students are given written texts only, while other formats may involve an amalgam of written texts (scripts) and visual texts (diagrams/animations). Research shows that a *split-attention effect* tends to be induced where written texts are not integrated with visual texts.

For example, practical work, demonstrations, problem-solving, and studying worked examples will introduce different levels of extraneous load. Sweller (1994) investigated extraneous cognitive load that could be imposed by the format of worked problems in the domain of geometry. Figure 2 shows the format of a conventional geometry problem and its solution, whereas Figure 3 shows a worked example for the same problem in an integrated format.

In the conventional worked example, we have a diagram located above the text which outlines the solution steps. Seen separately, neither the diagram nor the text below the diagram give the student much meaningful information. In order to comprehend the problem, the solver has to integrate the diagram and the solution steps. The processing of the diagram and attempts to connect it with the information presented in the solution steps requires the solver to draw on considerable cognitive resources, thus introducing extraneous cognitive load. This load can be attributed solely to the format of the worked example. The search by the solver to map the diagram with solution steps reduces available cognitive resources that can be used for schema development and automation.

The integrated worked example (Figure 3), on the other hand, releases the solvers' mental capacity such that he or she can focus more on the relational dimensions of the problem, thus developing useful schema for problems of the type presented in the worked example.

Germane Loads refers to activities that involve cognitive load and effort that directly relate and contribute to schema development and automation. Germane activities may include self-explanations (Chi, Bassock, Lewis, Reimann & Glaser 1989), mental imagery (Cooper, Tindall-Ford, Chandler & Sweller, 2001) and study of rich worked examples. Learning activities that are germane in nature bring about meaningful learning (van Cog, Pass & van Merrienboer, 2006). For example, a student's attempt to justify a solution or "self-explain" the difficulty in solving a problem contributes to germane load. Such processing activities demand that students search their LTM and construct chains of reasoning. In so doing students are encouraged to extend existing schemas that would help them learn or solve problems in a meaningful manner. Because students are encouraged to engage in multidirectional knowledge search, germane activities are effective in helping students construct powerful domain-specific schemas. From a mathematics teaching perspective, it is important that teachers explore ways of fostering germane load. One approach could be by asking students to present alternative ways of solving a problem and establishing similarities and differences between the different approaches.



Procedure for finding x:

Angle DBE	=	Angle DEG – Angle BDE (External angles of triangle equal the sum of the opposite internal angles)
	=	110° – 50°
	=	60°
Angle x	=	Angle DBE (Vertically opposite angles)
	=	60°

Figure 2. Conventional geometry problem and solution (Sweller, 1994, p. 300).

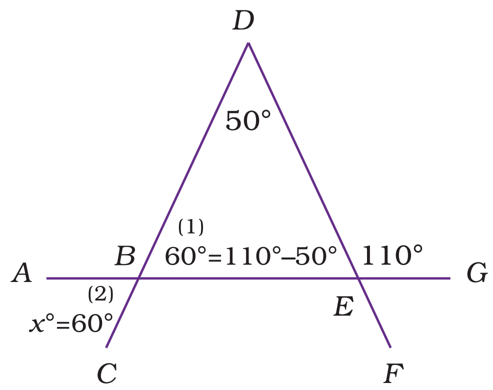


Figure 3. Integrated geometry problem and solution (Sweller, 1994, p. 302).

Worked examples and schema development

Worked examples provide a step-by-step demonstration of how to solve a given problem. Although students could be given directions in problem-solving processes through a series of instructions, exercises and feedback, it is usually the case that students benefit from examples to understand concepts and procedures. Such behaviour is manifested in their explicit mentioning of examples when they solve problems (Chi, et al, 1989). Students who studied worked examples were, in general, found to be better problem-solvers compared to those who engage in conventional problem solving. Zhu and Simon (1987) found that the use of worked examples could act as an effective alternative to conventional classroom instruction.

When students are asked to solve problems, a great deal of their mental effort is directed towards understanding the new problem which involves high levels of extraneous load. From a cognitive processing perspective, problem-solving consumes a high proportion of the limited working memory capacity leaving few resources for constructing schemas.

In comparison to problem-solving, studying an appropriately structured worked example of a problem is less demanding and involves less extraneous load. As a consequence, limited working memory resources can be directed towards germane load activities such as understanding the structure of the problem. During future attempts to solve an analogous problem, students are able to understand the problems rapidly and allocate more working memory to the more difficult aspects of a given problem, and transferring knowledge to new situations.

The use of worked examples in the course of a lesson is not uncommon in Australian mathematics classrooms. What we are suggesting here is that both students and teachers need to understand better the cognitive complexities involved during these activities so that students' involvement becomes more purposeful and meaningful.

To summarise, mathematics educators need to be mindful about how they approach teaching when information is high in complexity. During teaching, teachers need to ensure that they engage students in activities that are germane in nature that will lead to schema development and automation. Extraneous load activities that are not directly related to learning need to be minimised or eliminated during the learning process. The use of properly structured, worked examples should be supported as this instructional strategy assists students to make more efficient use of their working memory and develop powerful schemas. Instruction needs to be supported by activities that utilise worked examples that would constrain students' attention to aspects of the solution in which available cognitive resources assist them to deconstruct and reconstruct more refined and powerful representations of problems.

However, when students' experience with a particular topic of mathematics increases, they develop a rich body of domain-specific schemas in that topic, and thus, for this group of students, the use of worked examples as an instructional strategy may be counterproductive. Processing a worked example that fully describes the solution path does not improve the existing schemas in any significant manner. Thus, once learners develop a degree of expertise then conventional problem-solving activity becomes a more powerful form of mathematical instruction.

References

Baddeley, A. D. & Hitch, G. J. (2000). Development of working memory: Should the Pascual-Leone and the Baddeley and Hitch models be merged? *Journal of Experimental Child Psychology, 77*, 128–137.

Barlett, F. (1932). *Remembering: A study in experimental and social psychology*. New York: Cambridge University Press.

Board of Studies. (2002). *Mathematics K–6, 7–12*. Sydney: Board of Studies NSW.

Chi, M. T. H., Bassock, M., Lewis, R., Reimann, P., & Glaser, R. (1989). Self explanations: How students study and use examples in learning to solve problems. *Cognitive Science, 13*, 145–182.

Chinnappan, M. (1998). Schemas and mental models in geometry problem solving. *Educational Studies in Mathematics, 36*(3), 201–217.

Cooper, G., Tindall-Ford, S., Chandler, P., & Sweller, J. (2001). Learning by imagining procedures and concepts. *Journal of Experimental Psychology: Applied, 7*, 68–82.

Kalyuga, S. (2006). *Instructing and testing advanced learners: A cognitive load approach*. New York: Nova Science Publishers.

Kirschner, P. A. (2002). Cognitive load theory: implications of cognitive load theory on the design of learning. *Learning and Instruction, 12*(1), 1–10.

Marshall, S. P. (1995). *Schemas in problem solving*. NY: Cambridge University Press.

Mayer, R. E. & Chandler, P. (2001). When learning is just a click away: Does simple user interaction foster deeper understanding of multimedia messages? *Journal of Educational Psychology, 93*, 390–397.

Miller, G. A, (1956). The magical number seven, plus or minus two: Some limits on our capacity for processing. *Psychological Review, 63*, 81–97.

Newell, A. & Simon, H. (1972). *Human problem solving*. New Jersey: Prentice-Hall.

Sweller, J. (1994). Cognitive load theory, learning difficulty and instructional design. *Learning and Instruction, 4*, 295–312.

van Gog, T., Paas, F., & van Merrienboer, J. J. G. (2006). Effects of process-oriented worked examples on troubleshooting transfer performance. *Learning and Instruction, 16*,154–164.

From Helen Prochazka's

Scrapbook

Negative numbers

Mathematical objects are creations of the human mind—abstractions of the world.

Fibonacci was tackling a financial problem when he realised that he needed a new type of number to be able to solve it. He wrote: "This problem I have shown to be insoluble unless it is conceded that the first man had a debt."

Five centuries earlier, Indian book keepers thought along similar lines when they used negative numbers to denote debts. To the Indians, negative numbers made perfect sense. It may not be possible to remove 10 cows from 6 cows but there is nothing to stop 10 being subtracted from 6. And so with the invention of negative numbers we can write

$$6 - 10 = -4$$

However, it took a long time for these numbers to be accepted, just as it did for the Hindu-Arabic numerals. Most mathematicians continued to view them with disbelief for several centuries. Zero was abstract enough but at least some physical meaning could be attached to 0 cows; but what did −4 cows or −5 florins mean?

The colourful German monk, Michael Stifel went as far as to call them "numeri absurdi" or "stupid numbers"! Blaise Pascal was convinced they could not exist. So strong was the resistance that they did not come into widespread use until the 18th century, more than a thousand years after their initial use.