Virgin magnetization of a magnetically shielded superconductor wire: Theory and experiment

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Virgin magnetization of a magnetically shielded superconductor wire: Theory and experiment

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On the basis of exact solutions to the London equation, the magnetic moment of a type II superconductor filament surrounded by a soft-magnet environment is calculated and the procedure of extracting the superconductor contribution from magnetic measurements is suggested. A comparison of theoretical results with experiments on MgB2/Fe wires allows the estimation of the value of critical current for the first magnetic flux penetration.

Recently hybrid systems composed of superconducting and soft-magnetic materials attracted much attention in view of possibilities to improve superconductor performance by shielding out an external field as well as a transport current self-field. Very intense investigations were carried out on superconducting MgB2 wires sheathed in iron, which became ideal objects to explore the magnetic shielding effect due to the simplicity of their fabrication. As was observed in recent experiments, such structures exhibit enhanced superconducting critical currents over a wide range of the external magnetic field.

The commonly used technique for the estimation of the critical current value is the measurement of the superconductor magnetization versus applied magnetic field. The total magnetization of MgB2/Fe wire is measured in the superconducting state (below \( T_c \)) and in the normal state (above \( T_c \)). After that the magnetization of superconducting core is determined by subtraction of the latter results from the former ones (because in the normal state, only the magnetic sheath is magnetized). The magnetization of the superconductor allows one to estimate the critical current value which is proportional to the height of the hysteretic magnetic loop. It is assumed in this procedure that the magnetization of iron sheath does not depend on the presence of the superconductor and, hence, is identical above and below \( T_c \). However, it is intuitively clear that this assumption may be somewhat incorrect. Indeed, due to the Meissner effect below \( T_c \), the superconductor expels the magnetic flux into the sheath. This expulsion does not happen in the normal state where the magnetic field is homogeneous in the cylindrical magnetically shielded cavity. Therefore, the magnetic field distribution in the magnet sheath as well as its magnetization can be different, depending on whether the core is in the superconducting state or in the normal one. Recently, this scenario has been supported by the magneto-optical visualization of local flux distributions within the iron sheath of a MgB2 superconducting wire.

In the present letter, we calculate exactly the distribution of magnetic field inside and outside a superconducting filament sheathed by a magnet layer, as well as the magnetization of such a structure in the region of reversible magnetic behavior, i.e., for the flux-free (Meissner) state of the superconductor and well below the saturation field of the magnet. Comparing theoretical results with experiment, we verify the above described procedure of the superconducting critical current estimation.

Let us consider an infinite cylindrical superconductor filament of radius \( R \) enveloped in a coaxial cylindrical magnetic sheath of thickness \( d \) with relative permeability \( \mu \) and exposed to the external magnetic field \( H_0 \) perpendicular to the cylinder axis (Fig. 1).

We start from the London equation for the magnetic induction \( B_{SC} \) in the superconducting area

\[
B_{SC} + \lambda^2 \text{curl} \text{ curl} B_{SC} = 0,
\]

with the London penetration depth \( \lambda \). The field outside the superconductor denoted by \( H_M \) in a magnetic sheath and by \( H_{sat} \) in a surrounding free space is described by the Maxwell equations

\[
\text{curl} \ H = 0, \quad \text{div} \ H = 0,
\]

FIG. 1. Cross-sectional view of a superconductor filament covered by a coaxial cylindrical magnetic sheath.
the latter of which is valid in the whole space. Imposing an
insulating nonmagnetic layer of thickness much less than \(d\)
and \(R\) between the superconductor and the magnet sheath,\(^{10}\)
the boundary conditions read

\[
B_{n,SC} = \mu_0 \mu H_{n,M}, \quad B_{t,SC} = \mu_0 H_{t,M};
\]

\[
\mu H_{n,M} = H_{n,\text{out}}, \quad H_{t,M} = H_{t,\text{out}},
\]

for the normal (\(n\)) and tangential (\(t\)) components on the
superconductor/magnet interface [Eq. (3a)] and on the outer
magnet surface [Eq. (3b)], respectively. In addition, the field \(\mathbf{H}_{\text{out}}\) has to asymptotically approach the external field \(\mathbf{H}_0\). In
cylindrical coordinates, \((\rho, \varphi, z)\) coaxial with the filament
the solution of Eqs. (1) and (2) is

\[
B_{\rho,SC} = \mu_0 \mu_0 \rho \mathcal{A}_{SC}(I_0(\rho/\lambda) - I_2(\rho/\lambda))] \sin \varphi,
\]

\[
B_{\varphi,SC} = \mu_0 \mu_0 \rho \mathcal{A}_{SC}(I_0(\rho/\lambda) + I_2(\rho/\lambda)) \cos \varphi,
\]
in the superconductor;

\[
H_{\rho,M} = H_0 \left[ \mathcal{A}_{M1} - \mathcal{A}_{M2} R^2 / \rho^2 \right] \sin \varphi,
\]

\[
H_{\varphi,M} = H_0 \left[ \mathcal{A}_{M1} + \mathcal{A}_{M2} R^2 / \rho^2 \right] \cos \varphi,
\]
in the magnet sheath; and

\[
H_{\text{out}} = \left[ 1 + A_{\text{out}} \right] (R + d) / \rho^2 \sin \varphi,
\]

\[
H_{\varphi,\text{out}} = \left[ 1 - A_{\text{out}} \right] (R + d) / \rho^2 \cos \varphi,
\]
in the space around the filament. The coefficients \(\mathcal{A}_{SC}\), \(\mathcal{A}_{M1}\), \(\mathcal{A}_{M2}\), and \(A_{\text{out}}\) are given by

\[
\mathcal{A}_{SC} = 4 \mu_0 \Delta,
\]

\[
\mathcal{A}_{M1} = 2 \left[ (\mu - 1) I_0(R/\lambda) + (\mu - 1) I_2(R/\lambda) \right] / \Delta,
\]

\[
\mathcal{A}_{M2} = 2 \left[ (\mu - 1) I_0(R/\lambda) + (\mu + 1) I_2(R/\lambda) \right] / \Delta,
\]

\[
A_{\text{out}} = \left[ (\mu - 1)^2 - (\mu + 1)^2 R^2 I_2(R/\lambda) \right]

\[
+ \left( \mu^2 - 1 \right) \left[ 1 - R^2 I_2(R/\lambda) \right] I_0(R/\lambda) / \Delta,
\]

where

\[
\Delta = \left[ (\mu + 1)^2 - (\mu - 1)^2 R^2 I_2(R/\lambda) \right]

\[
+ \left( \mu^2 - 1 \right) \left[ 1 - R^2 I_2(R/\lambda) \right] I_0(R/\lambda).
\]

The Meissner current density in the superconductor only has
the \(z\) component which equals

\[
J_z(\rho, \varphi) = \mathcal{A}_{SC} \frac{2 H_0}{\lambda} I_1(\rho/\lambda) \cos \varphi.
\]

A limiting case of the hollow magnetic cylinder may be ob-
tained from Eqs. (4)–(8) by setting \(\lambda \to \infty\) which results in a
nonzero-homogeneous field inside the hole as expected from
Ref. 7.

Now, we can easily calculate the mean magnetization of both the
superconducting core and iron sheath which, due to the
geometry of the problem, only has a \(y\) component. The
magnetization of the superconductor is

\[
M_{SC} = \frac{1}{V_{SC}} \int_{V_{SC}} dV \mathbf{j}_z = -2 \mathcal{A}_{SC} H_0 I_2(R/\lambda),
\]

where the factor 2 is to account for the far ends of the
sample.\(^{11}\) The magnetization of the iron sheath is

\[
M_M = \frac{\mu - 1}{V_M} \int_{V_M} dV \mathbf{j}_z = (\mu - 1) H_0 A_{M1}.
\]

In the practically interesting case, \(R \gg \lambda\), they become

\[
M_{SC} = -\frac{4 H_0}{\mu + 1 - (\mu - 1) / (1 + d/R)^2},
\]

\[
M_M = \frac{2(\mu - 1) H_0}{(\mu + 1)^2 - (\mu - 1)^2 / (1 + d/R)^2}.
\]

Although the magnetization of the sheath (13) does not con-
tain \(\lambda\), it does not coincide with the magnetization of a
low magnetic cylinder which can be obtained from Eq. (11) by
setting \(\lambda \to \infty\):

\[
M_{HC} = \frac{2(\mu - 1) H_0}{(\mu + 1)^2 - (\mu - 1)^2 / (1 + d/R)^2} > 1.
\]

For the ratio of these two quantities an inequality,

\[
\frac{M_M}{M_{HC}} = \frac{(\mu + 1)^2 - (\mu - 1)^2 / (1 + d/R)^2}{(\mu + 1)^2 - (\mu - 1)^2 / (1 + d/R)^2} > 1,
\]

holds which means the magnetic flux density increase due to
the flux expelled from the superconductor.

Although in the critical state of superconductors only
part of the magnetic flux is expelled, the same inequality
\(M_M \geq M_{HC}\) should still be valid. Therefore, we conclude that
in previous considerations,\(^{4–6}\) the magnetization of the
sheath below \(T_c\) could be underestimated and, hence, the
magnetization of a superconductor together with the critical
current value could be underestimated too.

In the critical state of the superconductors the slope of the
hysteresis loop is given by the magnetization of the
sheath \(M_M\). The magnetization of the sheath under a
field \(H_0\) is

\[
M_M = H_0 A_{M1} / (\mu - 1).
\]

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\[
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\]
can be found. Combining Eqs. (13) and (14), it follows that the magnetization of the iron sheath at $T<T_c$ is about 3% larger than that at $T>T_c$. Let us note that this difference may reach 100% at smaller values of the parameters $d/R$ and $\mu$.

The response of the magnetic sheath is intrinsically non-linear such that is clearly visible in the normal state magnetization curve at $B_p>0.15$ T. The deviation of the total magnetic moment below $T_c$ at $B_p=0.3$ T from that in the normal state may be attributed to the nonlinearity due to the first magnetic flux entry into the superconductor. From the $B_p$ value, a critical current of the first vortex penetration may be estimated as follows.

First, from the magnetization of the superconducting core, the maximum value of screening current $j_s=j_c$ (R,0) can be found. Combining Eqs. (9) and (10), we obtain in the case $R \gg \lambda$

$$j_c = 2|M_{SC}|/\lambda,$$

with $M_{SC}$ from Eq. (12). Taking $\mu=46$, $d/R=0.5$, and $\lambda=1400$ Å for $T=30$ K from Ref. 12, we obtain for $H_0 = B_p/\mu_0$ the value of $j_c=5 \times 10^3$ A/cm$^2$. A practically important quantity is the average density of the screening current that may be defined as $j_c = J_c / \pi R^2$, where $J_c$ is determined by the integration of expression (9) over one-half of the superconductor cross section. In the limit of $R \gg \lambda$, we obtain

$$j_c = 8|M_{SC}|/\pi R,$$

(17)

and for the parameters used we found $j_c=1.8 \times 10^4$ A/cm$^2$ which is in a good agreement with the results of Refs. 4, 5, and 10.

In conclusion, we have developed a procedure of extracting the superconduction response from the low-field magnetic measurements on the iron sheathed superconductor filaments taking into account the difference between the magnetization of the magnet sheath below and above $T_c$.

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