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Year 3 Children's Understanding of Fractions: Are we Making Progress?

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The aim of the study reported here was to examine the quality of understandings developed by young children in the area of fractions and decimals. Analysis of data showed that the existence of great disparity in Year 3 children's knowledge base of fractions. We discuss these results in light of levels of competence that are expected in K-6 curriculum documents and with reference to past research on students' knowledge of fractions. The results of this small study raise doubts about the progress being made in the teaching of fractions.

Background

A principal aim of the primary mathematics curriculum is that students develop a sound understanding of the number system and become confident in using such understandings in describing real-world phenomena and solving problems (National Council of Teachers of Mathematics, 2000; Australian Education Council, 1990). Within the general topic of number, teachers and researchers have paid particular attention to fractions and decimals, for these numbers are used in making sense of problems in daily life of children and adults alike.

A major outcome expected in the areas of fraction and decimals is that primary school students should use these numbers and their relations flexibly to solve problems and reason mathematically (see *Mathematics K-6 Outcomes and Indicators*, Board of Studies - New South Wales, 1998). Despite the critical conceptual link provided by fractions between mathematics strands such as space and measurement, this area continues to present difficulties for some young children in primary schools. Indeed, analysis of state-wide performance of recent Year 5 students in New South Wales (NSW Department of Education, 1999) shows that students' understanding of and reasoning with fractions and decimals is less than satisfactory.

The situation in NSW is similar to what has been observed earlier in Victoria (Clements and Del Campo, 1987) and in other countries. Ellerton and Clements (1994) referred to the general area associated with fractions in Australia as a "weeping sore in mathematics education." In commenting on the poor performance in fractions among American students Brown et al (1988, p. 245) commented that the computational activity of these students seemed to be carried out "without developing the underlying conceptual knowledge about fractions."

Taken together, these findings suggest that the expectations held by mathematics curriculum developers for a sizeable group of students in the upper primary school are unrealistic. It is clear that a sizeable number of students will not achieve the following objective: 'Students develop an understanding of the parts of a whole, and the relationships between the different representations of fractions' (Board of Studies - New South Wales, 1998:61).

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The findings of the state-wide testing in NSW are frustrating. Many Year 5 students, despite having experienced the mathematics activities presented by their teachers, still have difficulty in using their knowledge of fractional and decimal numbers to solve problems that are regarded as being appropriate ones for that level of schooling. Yet, in their work with Australian preschool children, Hunting and his colleagues showed that while these young children did not usually understand fractions language, many of them had developed knowledge about sharing that would provide a sound basis for construction of future knowledge of rational number (e.g., Hunting & Sharpley, 1988). It seems clear that between the time of development of this rudimentary understanding and the later primary years many of these students do not develop knowledge structures that will support the problem solving with fractions and decimals that is demanded in Stage 3 of the K-6 curriculum and beyond.

This area of mathematical learning among young children has received considerable attention from researchers in recent years (e. g., Behr, Harel, Post & Lesh, 1992; Bezuk & Bieck, 1993; Baturo & Cooper, 1995; Owen & Super, 1993). More specifically, a significant body of research has focused attention on the development of rational number knowledge from children's understanding of whole numbers. Rational numbers are complex in character and provide important prerequisite conceptual tools for the growth and understanding of other number types and algebraic operations in secondary school. Young children are exposed to these number concepts at an early age in a variety of real-life situations, such as measuring and dividing continuous quantities, and quantitative comparison of two quantities. These experiences should enable young children to develop an understanding of rational numbers which matures through learning situations they encounter in the classroom. As the students move through the primary years we might anticipate that there will be a gradual integration of their understandings and the formal representations of mathematics.

Almost two decades ago Hiebert and others suggested that we were being too optimistic in anticipating such an integration of form and understanding (e.g., Hiebert, 1984, 1989). Hiebert argued that one of the factors contributing to the poor levels of performance of many students on fraction and decimal problems could be represented as a problem in establishing connections between form and understanding. One interest in this study was to see if this construction of the problem was still valid.

The multifaceted nature of fraction number has made the task of describing its growth difficult. Several attempts have been made to capture the complexity of fraction numbers and children' construction of these numbers. The most detailed analysis of fraction numbers has been undertaken by Kieren (1988). His analysis showed the fraction number knowledge consists of many interwoven strands. He identified eight, hierarchically related, levels in his description of fraction number thinking. An important outcome of this model is the specification of cognitive structures that provide the basis for the maturing of fraction numbers among young children. These structures or schemas which appear at levels three and four consists of what he referred to as *subconstructs*: partitioning, unit forming, quotients, measures, ratio and operations. These subconstructs among others play a key role in young children's understanding and interpretation of fractions. The model shown in Figure 1 builds on Kieren's representation of fraction subconstructs and also draws out potential links among the constructs (Chinnappan, 2000). These links are important for the analysis of the organisational quality of the knowledge of fractions.

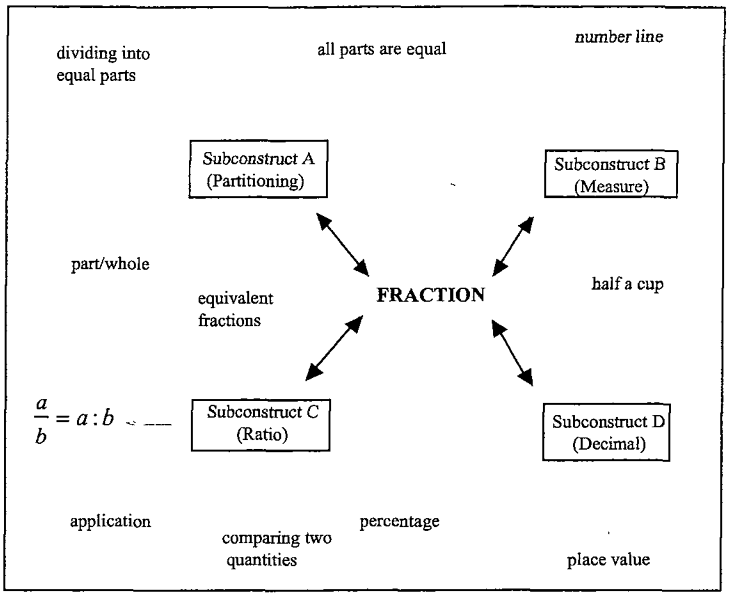


Figure 1. Knowledge components in a Fraction schema.

Although we do ultimately want to investigate the growth in complexity of students' fraction knowledge our concern here was with the gathering of necessary preliminary information about this knowledge. Our objective here was to gain information about how well-prepared students in early primary school might be for a discussion about fractions. Put more crudely, we wanted to find out how well prepared these students might be to get into the 'game' of fractions. We wanted to know if they had the common language, diagrammatic and symbolic vocabularies to participate in the discussions that would occur in their classrooms and whether this vocabulary was linked to well-developed schemas. Thus our initial focus was not on the complexity of the students' structure of fraction knowledge per se. It was focussed on the elements of the interaction that might typically occur between teacher (or textbook) and student.

Method

Participants

Twenty four children in Year 3 (aged between 8 and 9) from a regular suburban school participated in the study. The school was situated in a middle-class suburb of a metropolitan city and was not selective in its intake. The sample contained 13 boys and 11 girls. The children had studied whole numbers and fractions within the Number Strand of the K-6 mathematics curriculum in the previous two years of primary school and at the time of the study had completed the topic on fractions in Year 3.

Tasks and Procedure

A range of tasks were developed for the purposes of assessing children's knowledge about proper fractions. One set of tasks required children to respond to a series of questions that focussed on comprehension of simple sentences containing fraction words. The child's comprehension of the sentences was checked first and then they were asked to identify number words and those words that referred to fractions. Their knowledge of the labels used for fraction words was also assessed. 'Half' and 'quarter' were the fraction words used in this first set of tasks.

A second set of tasks involved children having to exhibit their understanding of fractions via diagrams. In this case the child was asked to represent the fraction words in diagrams and also to illustrate fractional quantities in diagrams. The students were also asked express fraction words in mathematical symbols, and to display their knowledge of both the size of different fractions and of equivalent fractions. In the final set of tasks, we attempted to assess children's understanding of the concept 'fraction' and of the symbols that were used in representing fractions. Figure 2 shows a selection of problems from the seven sets of fraction tasks.

Each child was interviewed individually and was asked to respond to all the focus questions and problems. Children's responses were also probed to generate more complete data about their knowledge of fractions. All sessions were audio-taped and transcribed for subsequent analysis.

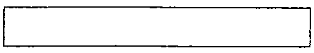
Set 1

'For your breakfast your mother filled half a glass with milk'

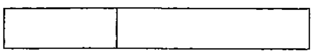
What did your mother do?
Draw a picture to show me what your glass of milk looked like when your mother had finished putting the milk in it.
Pick out any words in the sentence that refer to numbers, or parts of numbers.
Does that number have a special name?
What does 'a half' mean?
Can you write 'half' any other way?

Set 2

John cut a strip of paper into five parts. His teacher said that each part is 'one-fifth'.
Look at this diagram of the strip of paper. Show me how much *is one fifth.*



Look at this second diagram.
Is the shaded part equal to one fifth?
If it isn't could you change the diagram so that one fifth was shaded?



Set 3

In the number $\frac{2}{5}$, what does 2, - and 5 mean?

Figure 2. Fraction tasks.

Results and Discussion

To facilitate the description of students' task performance we formed the eight composite measures shown in Table 1. These measures summarise performance across all the interview questions.

Table 1
Definitions of Composite Measures

Identification of fraction words:	Identification by the student of words that refer to numbers, including fraction words.
Knowledge of labels:	Knowledge that the term 'fraction' referred to words such as 'half' and 'quarter'.
Representing fractions in mathematical symbols:	Ability to represent a fraction using a/b format.
Meaning of fraction words:	Knowledge of partitioning of a whole into equal parts
Representing fractions in diagram form:	Ability to represent fractions in simple diagrams or to modify diagrams to represent fractions
Meaning of a/b symbols:	Knowledge of the meaning attached to the a/b representation of a fraction
Relating and ordering fractions:	Ability to write an equivalent fraction and to order fractions
Explanation of fraction ordering:	Knowledge of how to establish the relative sizes of different fractions

The mean percentage scores for the total group of students on each of the composite variables are shown in Table 2. The figures in the table show that there was a high degree of variability in the scores on each set of tasks. The students performed best on the tasks requiring them to recognize and label the simple fraction words in the sentences provided as stimulus materials. Their knowledge of the meaning of fraction words and of how to represent fraction words in both diagrams and mathematical symbols was only moderately good. They were least successful on the tasks that required them to order and relate fractions written in a/b form.

Table 2
Group Scores (%) for Composite Measures (n=24)

Measures	Possible Score for each student	Percentage scores Mean	SD
Identification of fraction words	6	76.4	34.0
Knowledge of labels	6	68.8	33.4
Representing fractions in mathematical symbols	4	58.3	45.8
Meaning of fraction words	8	56.3	40.4
Representing fractions in diagram form	12	54.9	28.1
Meaning of a/b symbols	2	43.8	45.0
Relating and ordering fractions	6	16.7	27.8
Explanation of fraction ordering	2	8.3	24.1

Clearly, the large variation in performance indicates that the pattern of performance shown in Table 2 hides much interesting detail. Inspection of the performance of individual students shows a very substantial disparity in performance of students within this class. This disparity is illustrated more fully in Table 3 where the patterns of performance of four students is shown.

Table 3
Individual Scores of Four Students for Task Sets

Measure (possible score)	S3	S23	S11	S17
Recognition of fraction words (6)	6	6	0	0
Knowledge of labels (6)	6	6	0	2
Representing fractions in mathematical symbols (4)	4	4	0	0
Meaning of fraction words (8)	8	8	0	0
Representing fractions in diagram form (12)	11	12	2	2
Meaning of a/b symbols (2)	2	2	0	0
Relating and ordering of fractions (6)	6	4	0	0
Explanation of fraction ordering (2)	2	0	0	0

Here we have two pairs of Year 3 students with widely differing knowledge bases and very different capacities to effectively engage in discussions about fractions. Students 3 and 23 performed at a very high level and could cope with much of the fractions work being done in Year 5 or 6 classes. They have the vocabulary necessary to work on fractions problems and have quite complex knowledge related to the partitioning. Student 23, for example, not only responded correctly to all questions about language and symbols that were related to fractions but was also able to generate a correct solution to a problem which required him to work out the number of extra squares that had to be shaded so that $\frac{4}{5}$th of a given rectangle was shaded. Student 3 was able to give an acceptable interpretation of the a/b format and correctly ordered a set of fractions that included one improper fraction.

Students 11 and 17 showed very little evidence of even a rudimentary knowledge of fractions. Indeed their knowledge of even the language of fractions was limited, though they could both show half a glass of milk in diagram form. S17, for instance, could not recognise anything that referred to numbers in the sentence about breakfast and milk (Figure 2: Set 1, Question 4).

It seems obvious that if these pairs of students are to develop more powerful knowledge structures in the short term they need different and differently paced curricula. The first pair, along with several other students in this class, need to be engaged in activities that move them toward development of knowledge of the fraction subconstructs in the bottom half of Figure 1. For this group the expectations in curriculum documents are far too low. For example, according to the Outcomes and Indicators (Board of Studies- New South Wales, 1998, p. 61) students in Stage 2 of development (Years 3-4) are expected to 'represent numbers in tenths and hundredths using concrete material and grids, and record these numbers in words'. While these are necessary skills in the development of fraction schema, students S3 and S23 have exceeded this requirement, and are in need of, and ready for, more complex representations and operations involving fractions and decimals. We suggest that such students need exposure to subconstructs C and D of Figure 1.

The other pair of students is representative of a group that constituted about a quarter of this class, all of whom had a poorly developed understanding of the fraction concept. Although some of these students do have some of the common language and diagrammatic vocabulary, the inconsistency of their performance indicated that even this vocabulary was not strongly established. Students 11 and 17 are poorly placed to get into the fractions game. Once the teaching in class involved more than discussion of 'half', the performance we observed suggests that these two students would struggle to make meaning. Neither student would be likely to have the vocabulary to create links between what they already knew and what might be involved in discussion of 'one fifth' or '$\frac{1}{2}$'. There would be very little of Figure 1 that we could confidently judge to have been established by these students.

Conclusion

The patterns of performance here provide a somewhat more detailed account that shows what might lie behind the patterns of performance in state-wide testing. The results of this study shows that although the participants came from one Year 3 class there existed a wide disparity in their understanding and representation of fractions. The performance of many, if not most, of the students in this class was only weakly related to the specifications established for their year level in curriculum documents.

The results also suggest that the argument made by Hiebert (1984) about students in the US still holds for many Australian students. In his analysis Hiebert focussed on the problems of establishing links among a student's informal understanding and the formal symbols and procedures introduced in the course of mathematics classes. In this study we see strong evidence of problems in the first of Hiebert's sites of connection, the connection of understanding and the meaning of the symbols used in mathematics lessons. Hiebert's emphasis on the importance of connections should make more sense to us than it might have in 1984, or when Skemp (1971) made similar arguments at an earlier time. With our current desire to place all student learning with a constructivist framework (e.g, the South Australian Curriculum Standards and Accountability framework) the view that we should focus on the building of links between form and understanding seems all too obvious. Yet it seems that these links are not being established by many students.

In a recent paper Cobb and Bowers (1999) also use the crude language of the classroom 'game'. They remind us about the consequences if students lack the necessary vocabulary to take part in the game.

> 'All students must have a way to participate in the mathematical practices of the classroom community. In a very real sense, students who cannot participate in these practices are no longer members of the classroom community, from a mathematical point of view' (p. 9).

We must admit that students like Students 11 and 17 in our group might well have been excluded from the mathematical community in their classroom. And there is also the possibility that Students 3 and 23 might 'leave' that mathematical community unless they were provided with opportunities to develop their quite powerful understandings about fractions.

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