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# Interaction forces in RC beams strengthened with NSM FRP round bars

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## **Abstract**

The near-surface mounted (NSM) strengthening method has attracted an increasing worldwide attention in the last decade. Although the bond efficiency between FRP and concrete in the NSM method is much improved compared with the externally bonded (EB) FRP strengthening method, debonding failures have also been often observed in reinforced concrete (RC) beams strengthened with NSM FRP bars. In such FRP-strengthened RC beams, debonding may initiate at either of the two ends of the NSM bar (i.e. end debonding), due to the existence of large localized interaction forces between the NSM bar and concrete near the bar ends. This paper presents an analytical solution to the interaction forces in RC beams strengthened with NSM FRP round bars, which are one of the most popular types of FRP bars used for NSM strengthening. The key elements of the proposed analytical solution are the two interfacial stiffness parameters (i.e. tangential interfacial stiffness and normal interfacial stiffness) and the eccentricity of the tangential interaction force to the centroid of the NSM bar. The accuracy of the analytical solution is verified with predictions from a sophisticated 3D finite-element (FE) model of a RC beam strengthened with a NSM round bar.

## **Disciplines**

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# INTERACTION FORCES IN RC BEAMS STRENGTHENED WITH NSM FRP ROUND BARS

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## ABSTRACT

The near-surface mounted (NSM) strengthening method has attracted an increasing worldwide attention in the last decade. Although the bond efficiency between FRP and concrete in the NSM method is much improved compared with the externally bonded (EB) FRP strengthening method, debonding failures have also been often observed in reinforced concrete (RC) beams strengthened with NSM FRP bars. In such FRP-strengthened RC beams, debonding may initiate at either of the two ends of the NSM bar (i.e. end debonding), due to the existence of large localized interaction forces between the NSM bar and concrete near the bar ends. This paper presents an analytical solution to the interaction forces in RC beams strengthened with NSM FRP round bars, which are one of the most popular types of FRP bars used for NSM strengthening. The key elements of the proposed analytical solution are the two interfacial stiffness parameters (i.e. tangential interfacial stiffness and normal interfacial stiffness) and the eccentricity of the tangential interaction force to the centroid of the NSM bar. The accuracy of the analytical solution is verified with predictions from a sophisticated 3D finite-element (FE) model of a RC beam strengthened with a NSM round bar.

## KEYWORDS

FRP, finite element (FE) analysis, interaction forces, interfacial stiffness, near-surface mounted (NSM), RC beams

## INTRODUCTION

The use of near-surface mounted (NSM) FRP composites for strengthening RC beams in flexure has become increasingly popular in the last decade (Hassan and Rizkalla 2004; Sena Cruz et al. 2006; De Lorenzis and Teng 2007; Oehlers et al. 2008). In the NSM FRP method, grooves are cut into the concrete cover of a concrete member for the embedding of FRP bars using an adhesive. A typical schematic of NSM FRP for flexural strengthening of RC beams is shown in Figure 1. Compared to the externally bonded FRP method, the NSM FRP method has a number of advantages including a reduced risk of debonding failure and better protection of the FRP reinforcement (De Lorenzis and Teng 2007). However, the improved bond effectiveness cannot eliminate the possibility of debonding failure in RC beams strengthened with NSM FRP bars (Zhang et al. 2013; 2014). The debonding failure in an NSM FRP-strengthened beam may initiate from a major intermediate crack (i.e. IC debonding), or from either of the two ends of an NSM bar (i.e. end debonding) (Teng et al. 2006). For the latter case which has been more often observed, the failure are closely related to the existence of large localized interaction forces between the NSM bar and concrete near the ends of the bar in such strengthened RC beams.

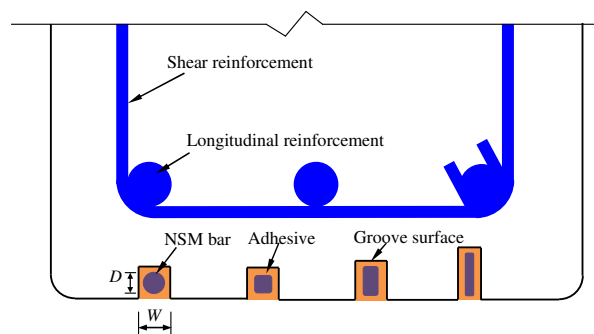


Figure 1. Schematic of NSM FRP strengthening systems

Recently, Zhang and Teng (2013) proposed a closed-form solution to the interfacial interaction forces in RC beams strengthened with NSM rectangular FRP bars. Zhang and Teng's (2013) solution includes the establishment of approximate equations for the interfacial stiffness parameters, and its accuracy has been demonstrated by a 3D linear elastic finite element (FE) model. Following Zhang and Teng's (2013) work, this paper develops a closed-form analytical solution to the interaction forces in RC beams strengthened with NSM round FRP bars, with the key issue being on the establishment of interfacial stiffness parameters (Figure 2) and the eccentricity of tangential interaction force to the centroid of the NSM bar. In this paper, the governing equations and solutions proposed by Zhang and Teng (2013) are first presented, followed by determination of the two interfacial stiffness parameters and the eccentricity of tangential interaction force particularly for round NSM FRP bars. The proposed analytical solution is then verified by a sophisticated 3D FE model of RC beam strengthened with an NSM round bar.

## GOVERNING EQUATIONS AND SOLUTION

The governing equations of interaction forces between NSM FRP and concrete ( $F_t$  and  $F_n$ , i.e., the interaction forces in the tangential direction and the normal direction respectively) for RC beams strengthened with NSM bars subjected to a uniformly distributed load (UDL)  $q$  can be expressed as (Zhang and Teng 2013):

$$\frac{d^2 F_t(x)}{dx^2} - k_t \left( \frac{d_o^2}{E_b I_b + E_f I_f} + \frac{1}{E_b A_b} + \frac{1}{E_f A_f} \right) F_t(x) - k_t \left( \frac{d_o}{E_b I_b + E_f I_f} \right) V_T(x) = 0 \quad (1)$$

$$\frac{d^4 F_n(x)}{dx^4} + k_n \left( \frac{1}{E_b I_b} + \frac{1}{E_f I_f} \right) F_n(x) + k_n \left( \frac{y_b}{E_b I_b} - \frac{y_f}{E_f I_f} \right) \frac{dF_t(x)}{dx} + k_n \frac{1}{E_b I_b} q = 0 \quad (2)$$

where  $x$  is the distance from the NSM bar end in the longitudinal direction;  $E_f$ ,  $A_f$  and  $I_f$  are the elastic modulus, cross-sectional area and second moment of area of the NSM FRP bar respectively;  $E_b$ ,  $A_b$  and  $I_b$  are the elastic modulus, cross-sectional area and second moment of area of the original beam respectively;  $E_a$ ,  $G_a$ , and  $t_a$  are the elastic modulus, shear modulus and thickness of the adhesive layer respectively;  $d_o$  is the distance between the centroids of the original beam and the NSM bar;  $y_b$  and  $y_f$  are the eccentricities of the tangential interaction forces to the centroid of the beam and that of the NSM bar respectively; and  $V_T$  is the shear force in the beam.  $k_t$  and  $k_n$  are the tangential interfacial stiffness and the normal interfacial stiffness respectively, which can be defined as the interfacial interaction forces between the NSM bar and concrete per unit length corresponding to a unit relative displacement between the NSM bar and concrete in the designated direction (as shown in Figure 2).

The solutions to Eqs. 1 and 2 are in the following form:

$$F_t(x) = B_1 \cosh(\lambda x) + B_2 \sinh(\lambda x) + m V_T(x) \quad (3)$$

$$F_n(x) = e^{-\beta x} [C_1 \cos(\beta x) + C_2 \sin(\beta x)] - n_1 \frac{dF_t(x)}{dx} - n_2 q \quad (4)$$

The constants in Eqs. (3) and (4) are as follows:

$$B_1 = \frac{k_t y_b q a}{E_b I_b 2\lambda} (L-a) - \frac{k_t}{\lambda^3} \left( \frac{d_o}{E_b I_b + E_f I_f} \right) q \quad (5) \quad B_2 = -B_1 = -\frac{k_t y_b q a}{E_b I_b 2\lambda} (L-a) + \frac{k_t}{\lambda^3} \left( \frac{d_o}{E_b I_b + E_f I_f} \right) q \quad (6)$$

$$C_1 = \frac{k_n}{2\beta^3 E_b I_b} [V_T(0) + \beta M_T(0)] - \frac{k_n}{2\beta^3} \left( \frac{y_b}{E_b I_b} - \frac{y_f}{E_f I_f} \right) F_t(0) + \frac{k_t q \lambda^2}{2\beta^3} \left( \frac{y_b E_f I_f - y_f E_b I_b}{E_b I_b + E_f I_f} \right) \left[ \frac{y_b a}{2E_b I_b} (L-a) - \frac{1}{\lambda^2} \left( \frac{d_o}{E_b I_b + E_f I_f} \right) \right] [\lambda \cosh(\lambda x) - \beta \sinh(\lambda x)] \quad (7)$$

$$C_2 = -\frac{k_n}{2\beta^2 E_b I_b} M_T(0) + \frac{k_t q \lambda^2}{2\beta^2} \left( \frac{y_b E_f I_f - y_f E_b I_b}{E_b I_b + E_f I_f} \right) \left[ \frac{y_b a}{2E_b I_b} (L-a) - \frac{1}{\lambda^2} \left( \frac{d_o}{E_b I_b + E_f I_f} \right) \right] \cosh(\lambda x) \quad (8)$$

$$\lambda^2 = k_t \left( \frac{d_o^2}{E_b I_b + E_f I_f} + \frac{1}{E_b A_b} + \frac{1}{E_f A_f} \right) \quad (9) \quad m = \frac{k_t}{\lambda^2} \left( \frac{d_o}{E_b I_b + E_f I_f} \right) \quad (10) \quad \beta = \sqrt[4]{\frac{k_n}{4} \left( \frac{1}{E_b I_b} + \frac{1}{E_f I_f} \right)} \quad (11)$$

$$n_1 = \left( \frac{y_b E_f I_f - y_f E_b I_b}{E_b I_b + E_f I_f} \right) \quad (12) \quad n_2 = \left( \frac{E_f I_f}{E_b I_b + E_f I_f} \right) \quad (13)$$

where  $M_T(0) = \frac{qa}{2}(L-a)$  and  $V_T(0) = q(\frac{L}{2}-a)$  are the total bending moment and the total shear force in the strengthened beam at  $x=0$  (i.e. at the NSM bar end) respectively,  $L$  and  $a$  are the span of the beam and the distance from the bar end to the nearest support respectively.

As can be seen from Eqs. 3 to 13, most parameters are directly associated with either the material properties of FRP/concrete or the geometric properties and can thus be easily obtained, except for (1) the two interfacial stiffness parameters (i.e.,  $k_t$  and  $k_n$ ), and (2) the eccentricity of tangential interaction force (i.e.,  $y_f$ ) to the centroid of the NSM bar. Zhang and Teng (2013) proposed equations of the above-mentioned parameters for RC beams strengthened with NSM rectangular bars and this paper provides the equations for RC beams strengthened with NSM round bars.

## INTERFACIAL STIFFNESS PARAMETERS

In the present study, the interfacial stiffness parameters were obtained directly based on their definitions by making use of FE models developed in ABAQUS (2012). According to the definitions of interfacial stiffness parameters, the interaction force between the NSM bar and concrete per unit length needs to be found out for a unit relative displacement between the two in the designated direction. Therefore, the FE models consisted of an NSM round bar, the surrounding adhesive layer and the groove surface which was set to be a fixed boundary of the adhesive layer. With such FE models, the interfacial stiffness was just equal to the total reaction force of the boundary (i.e. the groove surface) when a unit displacement (i.e. 1 mm) was applied to the NSM bar in the designated direction. The schematic of the FE models used to obtain the interfacial stiffness parameters is shown in Figure 2.

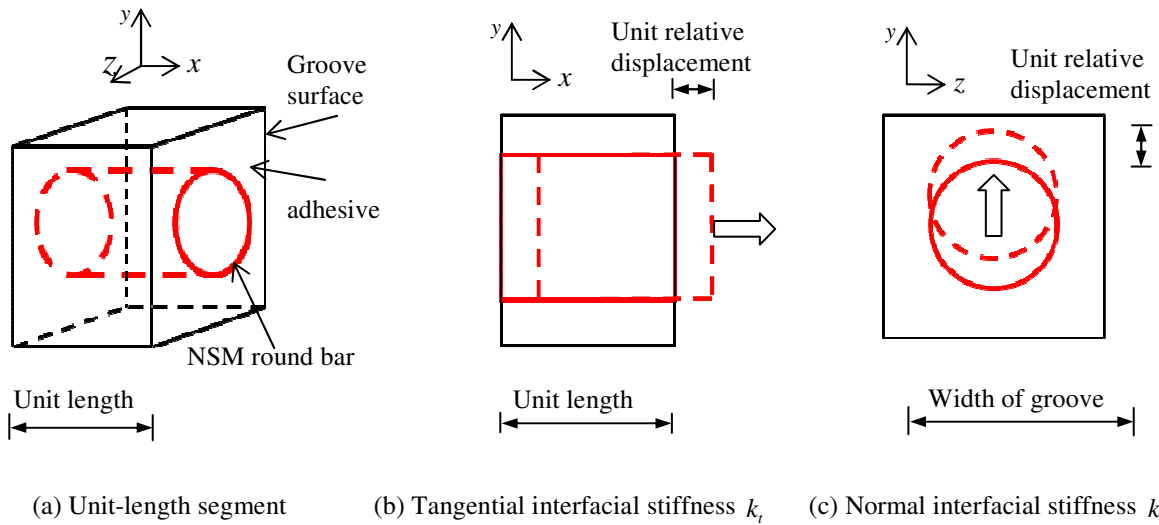


Figure 2. Schematic of the FE models used for the determination of interfacial stiffness parameters

In determining the interfacial tangential stiffness (Figure 2b), 8-node solid elements with a full Gauss integration were used and the adhesive layer is restrained against displacements in the two transverse directions (i.e. only displacements in the longitudinal direction were allowed). To save the computation time, the length of

the models was set to be 0.1 mm instead of the unit length of 1 mm; the reaction force obtained from these FE models was then multiplied by 10 to obtain the reaction force corresponding to a unit length. As the adhesive layer was under pure shear, such treatment does not affect the accuracy of the results. In evaluating the normal interfacial stiffness (Figure 2c), the problem was simplified as a plain strain problem in the transverse direction and 4-node quadrangle plane strain elements with a full Gauss integration were used, considering that the length of the FRP bar is usually much larger than the cross-sectional dimensions of the groove. Regression analyses of the FE results obtained from the numerical parametric studies using the above-mentioned FE models were conducted and the best-fit equations for the interfacial stiffness and eccentricity are as follows:

$$k_t = AG_a = 7.1G_a(R-1)^{-0.61} = 7.1G_a\left(\frac{1}{R-1}\right)^{0.61} \quad (14)$$

$$k_n = BE_a = 4E_a(R-1)^{-0.66} = 4E_a\left(\frac{1}{R-1}\right)^{0.66} \quad (15)$$

$$y_f = \frac{0.165}{W^{1.12}D^{0.12}} \quad (16)$$

Where  $D$  is the diameter of the FRP bar,  $W$  is the side length of the square groove, and  $R$  is the ratio of  $W/D$ .

### VERIFICATION OF THE ANALYTICAL SOLUTIONS

To verify the proposed analytical solutions (Eqs. 3 to 13), 3D FE analysis of a RC beam strengthened with an NSM round bar under a UDL were conducted using ABAQUS (2012) and the results are compared with analytical predictions. The modelled RC beam had a span of 2400mm, a height of 300mm and a width of 150mm. The diameter of the NSM round bar was 8 mm and the groove side length was 12 mm, leading to a length-to-diameter ratio of 1.5. The NSM round bar was symmetrically placed with respect to the mid-span of the beam, with a length of 1800mm; the distance from the NSM bar end to the nearest support was thus 300mm. A UDL of 30 N/mm was applied on the top surface of the beam. By taking advantage of symmetry, only a quarter of the beam was modelled, with symmetric boundary conditions applied (Figure 3). All of the materials were modelled using 8-node solid elements with a full Gauss integration.

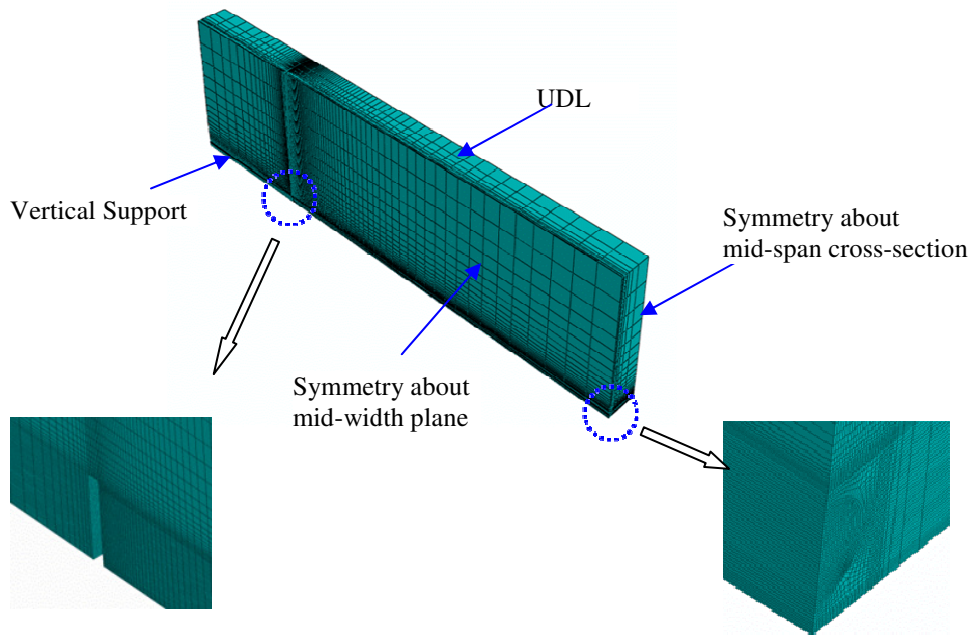


Figure 3. Mesh of the modelled beam

The interaction forces were obtained by integrating the relevant stresses along a prescribed path in the adhesive layer. The interaction forces obtained from the FE model are compared with the predictions from Eqs. 3 to 13 in Figure 4, from which it can be seen that the analytical predictions of both tangential interaction forces and normal interaction forces agree very well with the FE predictions, with gaps only existing in a very small region close to the NSM bar end. The errors of the analytical solution in that small region is due to the fact that the analytical solution does not take into account the boundary condition of zero shear stress at the bar end.

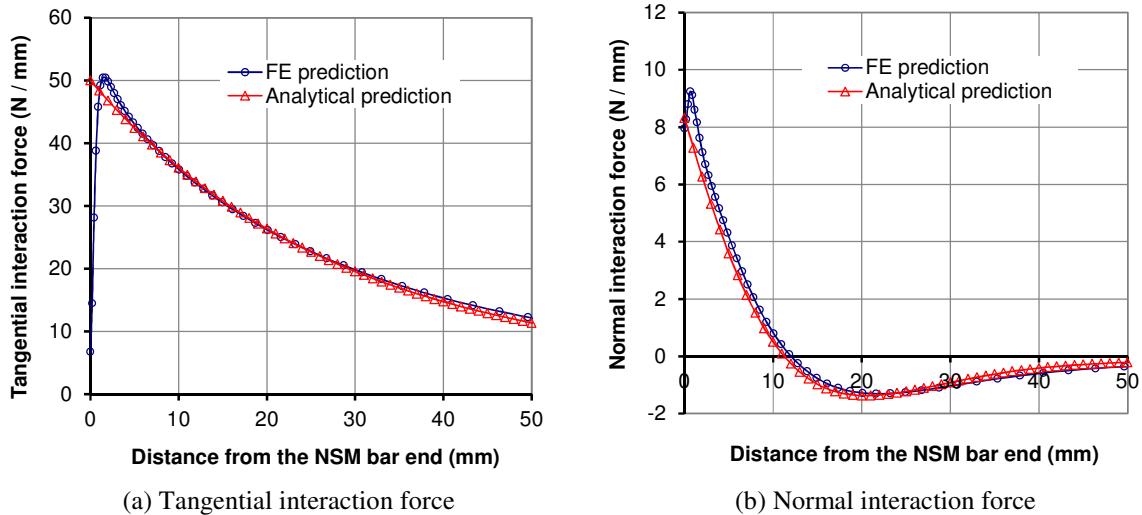


Figure 4. Comparison between FE predictions and analytical predictions

## CONCLUSIONS

An analytical solution to interaction forces between the NSM bar and concrete in RC beams strengthened with NSM round bars are presented in this paper, following the work conducted by Zhang and Teng (2013) for RC beams strengthened with NSM rectangular bars. The key elements of the proposed analytical solution are the two interfacial stiffness parameters (i.e. tangential interfacial stiffness and normal interfacial stiffness) and the eccentricity of the tangential interaction force. FE models developed using ABAQUS (2012) were adopted to conduct numerical parametric studies, the results from which were used to determine the interfacial stiffness parameters and eccentricity of the tangential interaction force. The accuracy of the proposed analytical solutions has been verified with a sophisticated 3D FE model of a RC beam strengthened with a NSM round bar. Both analytical predictions and FE predictions identified high interaction forces near the NSM bar end, which to some extent explain why end debonding failures commonly happen in RC beams strengthened with NSM round bars.

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## REFERENCES

- ABAQUS. (2012). *ABAQUS 6.12 User's Manual*, ABAQUS, Inc., Providence, RI.
- De Lorenzis, L., and Teng, J. G. (2007). "Near-surface mounted FRP reinforcement: an emerging technique for strengthening structures." *Composites: Part B*, 38(2), 119-143.
- Hassan, T. K., and Rizkalla, S. H. (2004). "Bond Mechanism of near-surface-mounted fiber-reinforced polymer bars for flexural strengthening of concrete structures." *ACI Structural Journal*, 101(6), 830-839.
- Oehlers, D. J., Haskett, M., Wu, C. Q., and Seracino, R. (2008). "Embedding NSM FRP plates for improved IC debonding resistance." *Journal of Composites for Construction*, 12(6), 635-642.
- Sena Cruz, J. M., Barros, J. A. O., Gettu, R., and Azevedo, Á. F. M. (2006). "Bond behavior of near-surface mounted CFRP laminate strips under monotonic and cyclic loading." *Journal of Composites for Construction*, ASCE, 10(4), 295-303.
- Teng, J. G., De Lorenzis, L., Wang, B., Rong, L., Wong, T. N., and Lam, L. (2006). "Debonding failures of RC beams strengthened with near-surface mounted CFRP strips." *Journal of Composites for Construction*, ASCE, 10(2), 92-105.
- Zhang, S. S. and Teng, J. G. (2013) "Interaction forces in RC beams strengthened with near-surface mounted rectangular bars and strips." *Composites Part B: Engineering*, 45(1), 697-709.
- Zhang, S. S. Teng, J. G. and Yu, T. (2013). "Bond-slip model for CFRP strips near-surface mounted to concrete." *Engineering Structures*, 56, 945-953.
- Zhang, S. S., Teng, J. G. and Yu, T. (2014). "Bond strength model for CFRP strips near-surface mounted to concrete." *Journal of Composites for Construction*, ASCE, 18, SPECIAL ISSUE: 10th Anniversary of IIFC, A4014003.