Centre for Statistical & Survey Methodology Working Paper Series

Faculty of Engineering and Information Sciences

2013

Incorporating household type in mixed logistic models for people in households

Robert Graham Clark
University of Wollongong, rclark@uow.edu.au

National Institute for Applied Statistics Research Australia

The University of Wollongong

Working Paper

03-13

Incorporating Household Type in Mixed Logistic Models for People in Households

Robert Graham Clark

National Institute for Applied Statistics Research Australia, University of Wollongong, Wollongong NSW 2522. Phone +61 2 4221 5435, Fax +61 2 4221 4845. Email: anica@uow.edu.au

Incorporating Household Type in Mixed Logistic Models for People in Households

Robert Graham Clark

National Institute for Applied Statistics Research Australia, University of Wollongong.

Summary. Generalized linear mixed models (GLMMs), particularly the random intercept logistic regression model, are often used to model binary outcomes for people in households. A challenge in fitting these models is that the degree of dependency between co-householders often depends on the type of household, such as households of related people, households of unrelated people, and single person households. The use of a different variance component for each household type is investigated using two representative datasets, on voting behaviour and health risk factors and outcomes, and a simulation study. Variance components are found to be significantly different across household types in the examples. Models which ignore this understate covariate effects for household types with lower variance components, typically single person households.

Keywords: Clustered binary data, Generalized linear mixed models, Household surveys, Logistic regression, Marginal Models, Multilevel models

1. Introduction

Multi-level models, also known as mixed or random effect models, have been one of the major fields of progress in mathematical statistics in the last 25 years (e.g. Goldstein 2011). These models allow standard models for uncorrelated data, including the general linear model and the generalized linear model, to be naturally extended to handle correlated data, possibly with highly structured and complex dependency structures. This is done by assuming that there are some latent, unobservable variables (called random effects), conditional on which the outcome variable or variables of interest are independent. The random effects are typically assumed to be normally distributed with unknown variances and possibly covariances. The random effects can sometimes be considered to capture the context affecting the observations, for example data from different people in the same household or neighbourhood might be supposed to be related because they are located in the same household or area and are subject to many of the same conditions and influences, or influence each others' outcomes. This context might be captured to some extent by observed covariates, but some unobserved dependencies often remain, which are modelled by random effects. Conditional on these random effects, the observations may then be assumed to be independent.

Household surveys are a common and important source of data on health, social, political and economic issues. This paper is concerned with modelling binary data on people where multiple people are observed from each household. These will be referred to as household surveys, but the term is used in a broad sense to include non-survey investigations provided that data for people come grouped in households, dwellings or families. Households may be of very different types or compositions; (1) single person households, (2) households of related people, and (3) households of related people, is probably the simplest possible typology. These three types will be denoted "1", "r" and "u". More complex categorisations of households are sometimes used (e.g. Johnston *et al.* 2005). Let Y_{ij} be a binary outcome of interest with values 0 and 1, for person j in household i of type $k \in \{1, r, u\}$, and let $\boldsymbol{x_{ij}}$ be associated covariates. The following logistic random intercept model, a special case of a generalized linear mixed model (GLMM), will be assumed:

$$\left.\begin{array}{l} P\left[Y_{ij}=1|u_i\right]=g^{-1}\left(\boldsymbol{\beta^T x_{ij}}+u_i\right) \\ Y_{ij}, Y_{i'j'} \text{ independent for all } i, j, i', j' \text{ conditional on } \boldsymbol{u} \\ u_i \sim N\left(0, \sigma_k^2\right) \qquad \text{where household } i \text{ is of type } k \end{array}\right\} \tag{1}$$

where the link function g is the logistic function $g(t) = log(t/(1-t))$. The random intercepts u_i capture unobserved household context and the parameter $\boldsymbol{\beta}$ measures the effect of the covariates, conditional on this context. The variance component σ_k^2 reflects the degree of similarity between values of the outcome variable from people in the same household, or equivalently, the importance of household context in determining Y_{ij}. A value of $\sigma_k = 0$ implies independent observations with no context applying, while large σ_k implies highly positively associated values of Y_{ij} and $Y_{ij'}$ for people in the same household i. The notation σ_1^2, σ_u^2 and σ_r^2 will be used to distinguish variance components for single person, unrelated persons and related persons households, respectively.

The degree of similarity within households, can vary significantly by household type and size. For example Johnston *et al.* (2005, p.213) find that the level of agreement in voting intentions is higher for couples than for unrelated adults, and decreases with household size. See also page 115 of Krenzke and Rust (2010) where the between-person correlation of literacy scores varies substantially by within-household relationship. These results suggest that models where σ_k varies by k will better capture the reality of within-household relationships. In practice, however, it is commonly assumed that $\sigma_k = \sigma$ for all k. This paper explores the consequences of ignoring differences in the σ_k^2, and the feasibility and benefits of the more general model. A secondary question is whether main effect and interaction terms for household type should be included in the fixed effect part of the model, either instead of or as well as varying σ_k.

Neuhaus and McCulloch (2011) considered the related problem of fitting generalized linear mixed models when the cluster size is related to the cluster random effects, or to the dependent variable. It was assumed that random effects were uncorrelated with fixed effect covariates, since it was already known that correlations of this type sometimes result in highly biased coefficient estimates (Neuhaus and McCulloch, 2006). It was concluded, based on a theoretical argument and empirical results, that informative cluster sizes result in biased estimates of intercepts and mean response, but little bias in estimates of slope parameters. It is not clear whether these findings will apply to household surveys, where clusters are much smaller, frequently containing only one unit.

McCulloch *et al.* (2008, pp.246-247) cite Heagerty and Zeger (2000) and Heagerty and Kurland (2001) who show that if the variance structure is mistakenly assumed not to depend on covariates, then biased estimates of the coefficients for fixed effects can result. They suggest using a generalized estimating equations approach, where the focus is on fitting a marginal model, and associations within clusters are dealt with in an ad hoc way. This approach would not be ideal for modelling household survey data, where the extent of within-household dependency is of interest in its own right.

Section 2 develops model (1), particularly the identifiability of σ_1, since in a single person household there is only one observation per cluster. An intuitively appealing alternative is to assume that single person households are similar to unrelated person households, i.e. $\sigma_1 = \sigma_u$. Section 3 then tests alternative sub-cases of (1) empirically by analysing two datasets: a voting intentions variable from wave 1 (1991) of the British Household Panel Survey (similar to the data analysed by Johnston *et al.* 2005), and self-assessed health status variable collected in the 1995 Australian National Health Survey (ABS, 2001). In both cases, a set of fixed effect covariates is included in all models considered. All possible sets of interactions between these covariates and household type, and a range of restrictions to σ_k, are fitted to each dataset. Models are fitted by maximum likelihood with quadrature evaluation of integrals, although it is expected that the findings would also broadly apply to models fitted by Bayesian methods.

Section 4 is a simulation study, designed to evaluate whether σ_1 can be reliably estimated as a separate parameter, and to study the effect of assuming $\sigma_k = \sigma$ on fixed effect estimates and variance component estimates. Section 5 contains conclusions.

2. Identifiability of the Proposed Model

It is clear that the parameters in model (1) are identifiable with the possible exception of σ_1. The case when fixed effects are nested within household type will be considered first. The first line of (1) can then be modified to

$$P\left[Y_{ij} = 1 | u_i\right] = g^{-1}\left(\boldsymbol{\beta_k^T x_{ij}} + u_i\right). \tag{2}$$

The model and data are then fully separable by household type. Hence the estimation of $\boldsymbol{\beta_1}$ and σ_1 requires fitting of model (2) to data from single person households only, in other words a two-level logistic regression model where there is only one observation per cluster. If a probit model is used instead, with link function g defined by the standard normal distribution function, then σ_1^2 is non-identifiable for the probit model, and for the logistic model it is only very weakly identifiable (section 3 of Rabe-Hesketh and Skrondal 2001; subsection 3.3.1 of Skrondal and Rabe-Hesketh 2007).

When fixed effects are not nested within household type, so that the model given by (1) applies, the identifiability of σ_1 is less clear. A heuristic argument will now be presented suggesting that this parameter is identifiable:

(i) The marginal probability that Y_{ij} is equal to 1 is obtained by integrating out the random effect u_i:

$$P\left[Y_{ij} = 1\right] = \int g^{-1}\left(\boldsymbol{\beta^T x_{ij}} + u_i\right) f_k\left(u_i\right) du_i$$

where f_k is the density of u_i for household type k. For the logistic-normal model with small σ_k^2, this is approximately equal to

$$P\left[Y_{ij} = 1\right] \approx g^{-1}\left\{\left(1 + c^2\sigma_k^2\right)^{-1/2} \boldsymbol{\beta^T x_{ij}}\right\} \tag{3}$$

where $c = 16\sqrt{3}/(15\pi)$ (Breslow and Clayton, 1993, formula 18). (If a probit model is fitted, with g replaced by the normal distribution function, (3) holds exactly without the requirement of small σ_k^2, but with c replaced with 1 - see formula 8 of Rabe-Hesketh and Skrondal 2001, special case of two categories). Hence we can write

$$P\left[Y_{ij} = 1\right] \approx g^{-1}\left(\boldsymbol{\beta_M^T x_{ij}}\right) \tag{4}$$

where

$$\boldsymbol{\beta_{Mk}} = \left(1 + c^2\sigma_k^2\right)^{-1/2}\boldsymbol{\beta} \quad (5)$$

are approximate "marginalised" parameters. The marginalised parameters are a dilution of the conditional parameters $\boldsymbol{\beta}$ since $\left(1 + c^2\sigma_k^2\right)^{-1/2}$ is between 0 and 1. (For a discussion of marginal dilution, see pages 236-238 of McCulloch *et al.* 2008.)

(ii) In multi-person households, the random effects model is standard and the variance component reflects the degree of similarity of the dependent variable within households. So it is clear that σ_r^2 and σ_u^2 are identifiable. The coefficients in $\boldsymbol{\beta}$ are also identifiable, even if only the data from multi-person households is used.

(iii) In single-person households, each observation is independent. Assuming that (3) is an adequate approximation, $\boldsymbol{\beta_{M1}}$ are the coefficients of a single-level logistic regression, and therefore can be identified using just single person households.

(iv) From (ii) we are able to identify $\boldsymbol{\beta}$, and from (iii) we can identify $\boldsymbol{\beta_{M1}}$. We can therefore identify σ_1^2 using (5). The dilution of $\boldsymbol{\beta_{M1}}$ relative to $\boldsymbol{\beta}$ provides information about σ_1^2. Hence all of the parameters in the model should be identifiable, including the problematical σ_1^2.

The preceding heuristic argument suggests that σ_1^2 is identifiable. It is not proposed to fit the model using these steps - instead, full maximum likelihood using quadrature evaluation of integrals is used in Section 3 and 4.

Even if σ_1 is formally identifiable, it is not clear that it can be reliably estimated with real datasets. One option is the standard approach of assuming $\sigma_1 = \sigma_u = \sigma_r$. Another approach would be to assume that single person households behave similarly to households of multiple unrelated people, i.e. $\sigma_1 = \sigma_u$, while allowing σ_r to differ. This approach is appealing, because it allows the within-household homogeneity to differ for unrelated- and related-person households, which is the main motivation for the new models. Yet another alternative would be to assume that $\sigma_1 = 0$. This approach may seem natural, because the usual reason for using a mixed model is to model dependencies within clusters, and there are no such dependencies in clusters size 1. Moreover, in single-level data, random effects models are generally not applied, which also supports zeroing σ_1. However, when there are both single-member and multi-member clusters, assuming a variance component for the latter but not the former seems unnatural, because the random effects capture unobserved household level context, which does not cease to exist when people live alone. In the case of using logistic GLMMs for repeated measures, time varying components are not usually assumed to zero for subjects who happen to have only one observation recorded (Gelman and Hill, 2007, p.276). The difficulty in household data is that single-person households are socially and economically distinctive, so that it may not be appropriate to assume a common variance component. The next section will assess the various possible simplifying approaches empirically.

3. Empirical Comparison of Alternative Models

3.1. Datasets

Two datasets were used to assess various special cases of model (1) and (2).

The British Household Panel Survey is a longitudinal survey of British households which commenced in 1991, and which continues on an annual basis. All members of selected households older than 15 years are interviewed, on a range of social, political, economic and health topics. In order to test the models discussed in this paper, cross-sectional data from wave 1 of the Teaching/Sampler dataset is used (BHPS, 2011). Multilevel logistic regression models are used to assess how voting intentions relate to explanatory variables, in an analysis loosely based on Johnston *et al.* (2005). These authors also fit logistic multilevel models with households and people as levels. Some of the descriptive analyses of within-household agreement were broken down by household type, and the greater homogeneity of the voting intentions of couples and smaller households was noted. However, in the multilevel models, random effects were assumed to have equal variance for all households. The analysis in this section will generalize this by examining different submodels of (1).

Voting intentions are coded as Conservative ("avote" variable equal to 1) or Other. The explanatory variables used are: Financial Situation (AFISIT) (coded to 5 levels, from "doing alright" to "finding it very difficult"), change in financial situation (AFISITC, coded to three levels in the survey, but simplified here to "better off" and "about same or worse off"), social class of current job (AJBRGSC, coded to 7 levels in the survey, simplified here to "skilled white collar" or "other"), and whether or not social class of current job was missing or not. These variables represent the most powerful explanators in the regression analyses of Johnston *et al.* (2005). The levels for each variable are grouped to ensure reasonable sample sizes at each level, with an eye to which levels have similar effect estimates in Johnston *et al.* (2005).

Households with more than 4 adults are excluded to simplify calculations, representing 7.3% of adults aged 15 years or more. Households with missing data on the voting intentions or explanatory variables are also excluded. These represent only a small fraction of households. The final dataset consists of 8258 adults in 4691 households, for an average of 1.8 adults per household. There are 1849 single adults households, 2506 related adult households and 336 unrelated adult households. Forty percent of people intended to vote Conservative.

The second dataset used is the confidential unit record release of the 1995 Australian National Health Survey (ABS, 2001). This was a national probability survey of Australian households where all household members were surveyed on health behaviours, conditions, status, and service access. Adults aged 18 years and over are used, and households with more than 4 adults are excluded; only 1.8% of adults lived in these households. Self-assessed health status (measured in 5 levels, very good to very poor, recoded here to fair/poor or other, with 17% of respondents in the former category) is regressed against agegroup (18-29, 30-44, 45-59, 60+) by sex and socio-economic quintile (with 1 representing the most disadvantaged). Socio-economic status is based on 1991 Census data for the collectors district, an areal unit containing around 200 dwellings on average. This survey was also conducted in 2001, 2004-2005 and 2007-2008. Data from 1995 was used because subsequent surveys selected only one adult respondent per household. After excluding households with missing values of the explanatory or response variables, there are 37886 people in 20134 households. There are 5806 single-person, 12349 related person and 1979 unrelated person households.

3.2. Model Fitting

Let m be the number of households and n_i be the number of people in household i. Given the random effects u_i, the conditional likelihood of model (1) is $L_c = \prod_{i=1}^{m} L_{ci}$ where

$$\begin{aligned} L_{ci} &= \prod_{j=1}^{n_i} \left(g^{-1}\left(\boldsymbol{\beta^T x_{ij}} + u_i\right)\right)^{y_{ij}} \left(1 - g^{-1}\left(\boldsymbol{\beta^T x_{ij}} + u_i\right)\right)^{1-y_{ij}} \\ &= \prod_{j=1}^{n_i} \left(1 + e^{\boldsymbol{\beta^T x_{ij}} + u_i}\right)^{-1} e^{y_{ij}\left(\boldsymbol{\beta^T x_{ij}} + u_i\right)} \end{aligned}$$

This can be written $L_{ci}(u_i)$ to emphasise that the conditional likelihood is a function of the random effects.

Random effects are modelled as normally distributed; let $\phi\left(.;\mu,\sigma^2\right)$ denote the normal density function with mean μ and variance σ^2 . The likelihood is obtained by integrating out the random effects u_i multiplied by their density. This gives $L = \prod_{i=1}^{m} L_i$ where

$$L_i = \int L_{ci}(u_i)\, \phi\left(u_i; 0, \sigma^2_{k(i)}\right) du_i \tag{6}$$

and $k(i)$ is the household type of household i. A closed form expression for (6) does not exist. Maximum likelihood estimation therefore requires numerical integration for each household i, for each iteration of the estimates of $\boldsymbol{\beta}$ and $\left\{\sigma_k^2\right\}$.

A number of packages are available in the R statistical environment (R Development Core Team, 2012) for fitting random effects mixed models by numerical maximum likelihood, but none appear to be able to implement (1) in its full generality. The glmmML package (Broström and Holmberg, 2011) was able to fit this model to the two datasets used in this paper when $\sigma_k = \sigma$, but had no facility for separate variance components by household type. The glmer function in the lme4 package (Bates *et al.*, 2011) should in principle have been able to fit the model with unequal variance components, by defining random slopes, but in practice returned errors even when σ_k were assumed to be equal. This package's algorithm may have had difficulty because the proportion of clusters which contain only one observation is higher for people-within-households than for other applications.

As existing packages were unsuitable, the models are fitted by maximum likelihood, with the integral in (6) calculated by Gaussian quadrature with 25 points. The package fastGHQuad (Blocker, 2011) is used to calculate quadrature points and weights. A short Fortran program is called from R to speed up the calculation of L. The likelihood is then optimized numerically with respect to $\boldsymbol{\beta}$, σ_1, σ_r and σ_u. This is done by grid search on $\sigma_1 \in \{0, 0.1, 0.2, \ldots, 3\}$, with the remaining parameters optimised for each value of σ_1 using the optim function with the L-BFGS algorithm. This approach results in successful convergence in all cases in this and the next section. Grid search is used for σ_1 because the flatness of the likelihood with respect to this parameter for some models resulted in non-convergence of some models when the optim function is used to optimise all parameters. All computer code used is included in the supplementary online materials.

Given the complexity of this implementation, the code was validated by comparing the results to the glmmML package for models with equal σ_k. Parameter estimates agree to a large number of decimal places. The simulation study which is described in section 4 also confirms that the procedure is able to reproduce the parameters of the generating distribution.

3.3. Evaluation of Alternative Models

The base set of fixed effects described in §3.1 is fitted in all models. A main effect for household type (3 levels) and interactions between household type and the base set of fixed effects are also considered. Every possible subset of these household type effects is fitted, with the restriction that interactions were only included when the corresponding main effects were also in the model. This gives a total of 17 models for the voting dataset and 11 models for the health dataset. These models are all special cases of model (2) with some elements of $\boldsymbol{\beta_k}$ constrained to be equal across k.

Six alternative submodels for the variance components are considered: (1) $\sigma_k = 0$; (2) $\sigma_k = \sigma$ (usual approach in practice); (3) $\sigma_1 = \sigma_u$; (4) $\sigma_1 = \sigma_r$; (5) $\sigma_1 = 0$; and (6) σ_k unrestricted.

Each combination of the variance component model and the fixed effect model is fitted, giving a total of 102 models for voting and 66 models for health. The aim is then to identify one or more models which fit the data well according to some goodness of fit criteria. The problem of model selection in generalized linear mixed models remains very open. A recent review (Müller *et al.*, 2013) of this problem for the simpler special case of linear mixed models summarises a range of model selection methods including the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC). Müller *et al.* (2013) note that the best performing method depends very much on what criterion is defined as the objective. The few theoretical results which exist on model choice in mixed models generally apply under somewhat restrictive circumstances.

Given that there is no general agreement on the best model choice criterion, models are selected here using three different approaches: the AIC, the BIC and asymptotic hypothesis testing. Let the deviance D be defined as negative 2 times the log of the maximised likelihood for a model. Let p be the number of parameters in the model, including elements of $\boldsymbol{\beta}$, and σ_k depending on which constraints have been applied to the parameters. Let n be the total sample size of people in all households. The two information criteria are then defined as $AIC = D + 2p$ and $BIC = D + p\log(n)$. Both of these criteria reward better fit to the data (as measured by the deviance) but penalise large models to encourage parsimony. These forms of the AIC and BIC are sometimes called the *marginal* AIC and BIC, reflecting the fact that the random effects u_i have been integrated out in calculating the likelihood, which is therefore marginal with respect to these effects (see §3.1 of Müller *et al.* 2013 citing Vaida and Blanchard 2005). Conditional versions of the criteria may also be used, but these are more appropriate when the aim is to predict new observations from the same clusters with the same random effects. The aim here is instead to model the process as captured by the parameters $\boldsymbol{\beta}$ and σ_k. The conditional criteria are also more problematical to calculate as they generally use a so-called "effective sample size" allowing for non-independent observations instead of n, and it is unclear how this quantity should be defined for GLMMs except perhaps in a Bayesian framework (Spiegelhalter *et al.*, 2002).

The third model fitting approach is based on asymptotic significance tests. All models are compared to any models that they are nested within (i.e when one model is a special case of another). If the simpler model is significantly worse than the more complex model (p-value less than 0.05), it is ruled out of contention; otherwise the more complex model is ruled out. The remaining set of models are defined to be acceptable. The p-values are calculated using the usual chi-square approximation to the difference between the deviances of nested models, which gives asymptotically correct p-values when the variance components under the null hypothesis are not on the boundary 0 (Cox and Hinkley, 1974, pp.322-323). For testing $\sigma_1 = 0$, the approximation $-2\Delta \log(L) \sim \frac{1}{2}\chi_0^2 + \frac{1}{2}\chi_1^2$ is used (Stram and Lee , 1994). This approach is the invention of the author, but does seem to reflect many practitioners' method of choosing a GLMM, albeit usually from a smaller set of candidate models.

Table 1 shows the subset of models for voting intention which are viable according to at least one of the three criteria. The AIC and BIC are relative to the lowest value of each over all models. Models where the AIC or the BIC are within 1.5 of the minimum value are shown, as these would usually be considered nearly as plausible as the best models. The table also shows whether each model passed the hypothesis test criteria or not. The models where $\sigma_k = \sigma$ and there are no interactions involving household type are also included, as these reflect the usual practice. Models are in increasing order of AIC, which is perhaps the most commonly used criterion in practice. The table clearly shows that the existing practice, shown in the first two rows, is unacceptable according to all three criteria. Four models had AIC within 1.5 of the minimum:

- Models 1 and 3 have household type as a main effect only with no interactions. In both models, the σ_k are all unequal, with $\hat{\sigma}_1 < \hat{\sigma}_u < \hat{\sigma}_r$. In model 1, $\hat{\sigma}_1$ is set to 0, while in model 2, $\hat{\sigma}_1 = 0.8$. The other two variance components are much higher with $\hat{\sigma}_u = 2.6$ and $\hat{\sigma}_r = 3.0$.

- Model 2 has $\sigma_k = \sigma$, i.e. the usual common variance component setup. However, it has considerably more parameters in the fixed effect part of the model, with a household type main effect and an interaction between household type and AFISIT (current financial situation).

- Model 4 is the most complex of the first 4 models, with $0 = \sigma_1 < \sigma_u < \sigma_r$ and an interaction between household type and AFISIT.

Models 1 and 2 are the only ones which pass the significance test criterion. Models 1 and 3 have reasonably low BIC, but model 2 has very large BIC, reflecting the fact that the BIC heavily penalises models with more parameters when the sample size is reasonably large. The model with the lowest BIC, model 5, is similar to model 1, but without even a household type main effect; however, this model has quite high AIC. On balance, any of models 1, 2 or 3 would be acceptable choices, but model 2 is considerably less parsimonious than the other two.

Table 2 shows the same set of results for the health dataset. Any of the first four models could be considered acceptable according to their AIC. Model 4 has a much lower BIC than models 1-3, reflecting that it requires 5 fewer parameters, while still fitting the data well. All four models have $\sigma_r \neq \sigma_u$. Model 4 is the only of the four where σ_1 is a free parameter. This freedom seems to result in a smaller fixed effects model being acceptable, whereas models 1-3 all include an interaction between household type and age. Model 4 has the lowest BIC due to its smaller number of parameters.

A surprising result in Table 2 is that $\hat{\sigma}_1$ is much higher (at 2) than $\hat{\sigma}_u$ and $\hat{\sigma}_r$ (at about 1) in model 4. This seems counter-intuitive, given that one might expect household context to be stronger when related adults live together. It might indicate that adults living alone are a more heterogenous group, and this between-person heterogeneity is being captured by σ_1.

Fixed Effects involving Type	Variance Component Model	p	Dev.	AIC	BIC	hypo-thesis tests	$\hat{\sigma}_1$	$\hat{\sigma}_r$	σ_u
1. hhtype	$\sigma_1 = 0$	12	17.6	0.0	0.3	yes	0.0	3.0	2.6
2. hhtype*AFISIT	$\sigma_k = \sigma$	19	4.2	0.7	46.1	yes	3.0	3.0	3.0
3. hhtype	unequal σ_k	13	16.5	0.9	7.7	no	0.8	3.0	2.6
4. hhtype*AFISITC	$\sigma_1 = 0$	14	14.7	1.2	14.4	no	0.0	3.0	2.6
5. none	$\sigma_1 = \sigma_u$	10	34.2	12.6	0.0	no	2.2	3.1	2.2
6. hhtype	$\sigma_k = \sigma$	11	38.6	19.0	12.8	no	2.9	2.9	2.9
7. none	$\sigma_k = \sigma$	9	45.4	21.8	2.8	no	2.9	2.9	2.9

Table 1: Selected Models for British Household Panel Survey Voting Intentions Variable (Conservative vs Other). Akaike Information Criteria (AIC) and Bayesian Information Criteria (BIC) are presented relative to the lowest values over all models. The number of parameters, p, includes variance components and coefficients of fixed effects.

Fixed Effects involving Type	Variance Component Model	p	Dev.	AIC	BIC	hypo-thesis tests	$\hat{\sigma}_1$	$\hat{\sigma}_r$	σ_u
1. hhtype*age	$\sigma_1 = 0$	22	11.6	0.0	38.3	yes	0.0	1.1	0.9
2. hhtype*age	$\sigma_1 = \sigma_u$	22	11.7	0.1	38.4	yes	0.9	1.1	0.9
3. hhtype*age	$\sigma_1 = \sigma_r$	22	12.3	0.7	39.0	yes	1.1	1.1	0.9
4. hhtype	unequal σ_k	17	22.8	1.3	0.0	yes	2.0	1.1	0.9
5. hhtype	$\sigma_k = \sigma$	15	45.8	20.3	3.2	no	1.1	1.1	1.1
6. none	$\sigma_k = \sigma$	13	110.6	81.1	48.2	no	1.1	1.1	1.1

Table 2: Selected Models for Australian National Health Survey Self-Assessed Health (Fair or Poor vs Good or Better). Akaike Information Criteria (AIC) and Bayesian Information Criteria (BIC) are presented relative to the lowest values over all models. The number of parameters, p, includes variance components and coefficients of fixed effects.

3.4. *Identifiability of* σ_1

Figure 1 shows the profile deviance of σ_1, for both datasets, when the fixed effects model includes (a) main effect for household type plus other covariates, and (b) main effects for household type and other covariates and interactions between household type and all other variables. In case (a), in the voting dataset, the profile is fairly flat between 0 and the maximum likelihood estimate (shown with a star), and then rises steeply in the positive direction. In the health dataset, the deviance has a clear and sharp minima at the maximum likelihood point. In case (b), σ_1 is practically non-identifiable for both datasets. It is not strictly non-identifiable, since the profile is not perfectly flat if shown on a finer scale. Thus, it is clear that there is a choice between modelling differences between single person households and other households using the variance components, or using interaction terms in the fixed effects model. Tables 1 and 2 show that the former is more parsimonious.

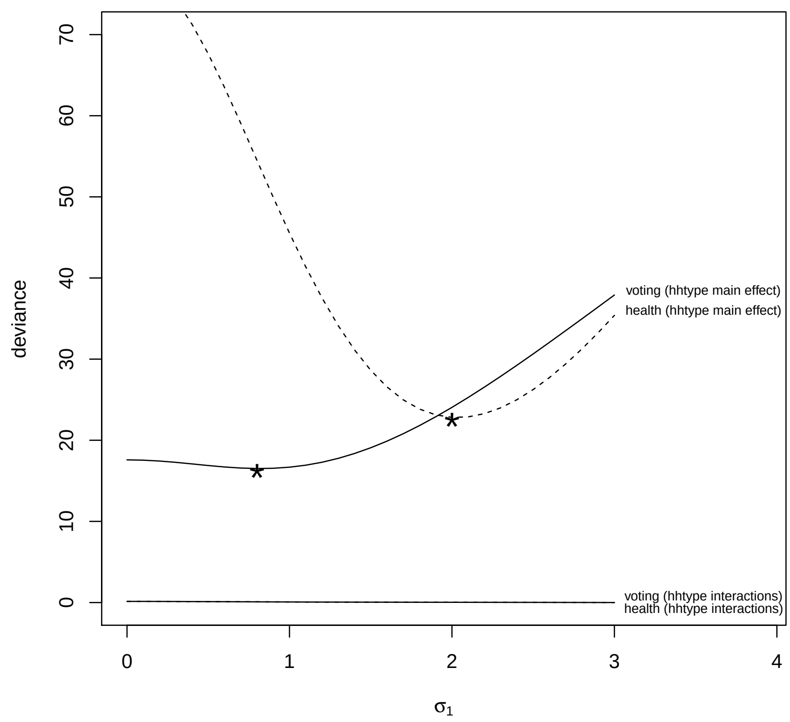

Fig. 1. Profile Likelihood Plot showing Minimized Deviance with σ_1 held fixed at values between 0 and 3. Profiles shown with household type as a main effect fixed effect, and with interactions between household type and other variables.

3.5. Fixed Effect Coefficient Estimates

Table 3 shows the maximum likelihood estimates for the fixed effects, and asymptotic estimates of the standard errors (bracketed), for the voting dataset. Fixed effect models include various covariates plus household type, but no interactions between household type and other factors. The first column of estimates are calculated assuming $\sigma_k = \sigma$ while the second column allows unequal σ_k. Table 3 shows that the first model exaggerates each effect, for example the coefficients AFISIT (financial status) range from the baseline of 0 to -2.2 for "finding it very difficult". In contrast, the second column, has coefficients ranging from 0 to -1.9. Similar effects apply to the other variables. Assuming $\sigma_k = \sigma$, when in fact σ_1 is probably much lower than σ_u and σ_r, seems to result in an expansion in the fixed effects.

The appendix contains the corresponding table for the health dataset, where the two sets of fixed effect estimates are much closer together, and the pattern is less consistent, although it appears that the first column of estimates are slightly attenuated. As discussed in §3.3, σ_1 is apparently the largest of the variance components for this dataset.

A feature of Table 3 is that the estimated standard errors of the fixed effect estimates are reduced when unequal σ_k are allowed. It might be expected that introducing these extra parameters would result in a penalty to the precision of other parameter estimates. It seems that effect of incorrectly assuming equal σ_k is to somewhat inflate variance component estimates, so that removing this assumption results in better estimated precision. This phenomenon is not seen in the the corresponding table for the health dataset, possibly reflecting that σ_1 is the largest variance component rather than the smallest.

Variable	Parameter Estimate (standard error)	
	$\sigma_k = \sigma$	unequal σ_k
intercept	0.43 (0.108)	0.64 (0.077)
AFISIT5 doing alright	-0.72 (0.085)	-0.63 (0.071)
AFISIT just about getting by	-1.36 (0.090)	-1.18 (0.073)
AFISIT5 just about getting by	0.00 (n/a)	0.00 (n/a)
AFISIT finding quite difficult	-1.84 (0.135)	-1.54 (0.109)
AFISIT finding very difficult	-2.22 (0.181)	-1.85 (0.140)
AFISITC better off than last year	-0.21 (0.078)	-0.16 (0.067)
AJBRGSC not skilled white collar	-0.46 (0.078)	-0.36 (0.064)
social class not reported	-0.78 (0.081)	-0.67 (0.072)
HHTYPE 1	0.00 (n/a)	0.00 (n/a)
HHTYPE r	0.06 (0.092)	-0.37 (0.072)
HHTYPE u	-0.52 (0.165)	-0.84 (0.165)

Table 3: Fixed Effect Parameter Estimates for British Household Panel Survey Voting Intentions Variable (Conservative vs Other)

3.6. Observed vs Predicted Counts for Single Person Households

The results in Table 1 and 2 give support for either allowing a separate variance component for single person households, or including fixed effect interactions with household type. However, those results are based on information criteria calculated from large samples with thousands of households. It is not clear whether the predictions from the new models are really all that different from the standard approach of assuming $\sigma_k = \sigma$ and including household type as a main effect, if at all. Table 5 shows that the standard approach fits the observed data visibly worse than the more general model, in one respect. The observed and predicted proportions of people voting Conservative by AFISIT (current financial status) are tabulated for people living alone. Predicted proportions were calculated by summing the fitted probability from the model for the relevant subset of the sample, and dividing by the subsample size. Results are shown for (i) the base model plus a household type main effect with equal σ_k, (ii) model i plus an interaction between household type and AFISIT, and (iii) model i but with σ_k unconstrained. It can be seen that models (ii) and (iii) reproduce the observed proportions closely. However, model (i) is at odds with the data, and its predicted proportions differ from the observed by up to 8 percentage points. Model (i) attenuates the effect of AFISIT on voting, with the predicted proportion voting conservative by financial situation varying from 26% to 50%, whereas it should range from 19% to 58%. This dilution is what one might expect from equation (5), as σ_1 is overstated in model (i).

The appendix contains similar tables by AFISIT with all household types included, and by household type only. In both cases, observations and predictions were fairly close for all three models. The same tables have been produced for the health dataset, with agegroup in place of AFISIT. The results were very similar to the voting dataset, except that model (i) exaggerated rather than attenuated the agegroup effect. This is as expected from (5) since σ_1 is understated by model (i) for this dataset.

AFISIT (current financial situation)	Number of People	Observed Proportion	Predicted Proportion $\sigma_k = \sigma$	$\sigma_k = \sigma$ + interaction	unequal σ_k
all	1849	38.8	38.7	38.8	38.8
living comfortably	429	58.3	50.3	58.3	57.1
doing alright	448	43.1	41.9	43.1	43.2
just about getting by	645	30.9	34.2	30.8	31.4
finding quite difficult	190	25.8	29.5	25.7	25.3
finding very difficult	137	19.0	25.6	18.9	20.2

Table 4: Observed and predicted proportions voting conservative for single person households only, by current financial situation (AFISIT)

4. Simulation Study

A simulation study with 500 replications was conducted, to test the modelling procedure and computer code, and to further explore the benefit or otherwise of the general variance component setup in model (1). Each simulated dataset contained 625, 1250 or 2500 households, where 40% of households were single person, 40% had two related people, and 20% had two unrelated people. A single covariate x was simulated for each person, from a uniform discrete distribution on $\{-1, -0.8, \ldots, 0.8, 1\}$, independently of all other people. A term of $-0.25 * (\text{household type} = 1) + 0.25 * (\text{household type} = u)$ was sometimes added to x, to test the effect of the covariate being associated with household type.

Binary variables Y_{ij} were simulated using model (1), where the linear predictor was $\alpha + \beta x + u_i$ where $\alpha = 0$ and $\beta = 1$. This model implies that approximately half of the values of Y_{ij} will be equal to 1. The standard deviations of the random effects, $(\sigma_1, \sigma_r, \sigma_u)$ were set to either: (a) $(0.75, 3, 1.5)$ (roughly based on the voting dataset from section 3); (b) $(2, 1, 1)$ (roughly based on the health dataset from section 3); (c) $(0.25, 1, 0.5)$ (similar to case a, but with smaller variance components); or (d) (2,2,2) (to match the usual assumption made in practice).

Various sub-models of model (1) were fit to the data, using maximum likelihood with integrals evaluated by 25-point quadrature, as in Section 3. To save space, selected results are shown for case (a) with 2500 households where x was not associated with household type. The full set of tables are contained in the appendices.

Table 5 shows the means of the parameter estimates over the 500 simulations, along with the true values, under three alternative assumptions for the variance components σ_1, σ_r and σ_u. Standard errors are shown in brackets. It can be seen that the first two simplifications result in biased estimates of β. These biases, while not negligible, are not particularly large compared to the standard errors of $\hat{\beta}$. The real failure of the simplified models is in the estimated variance components, as would be expected when the true variance components differ greatly between household types. The equal variance component model results in a substantial under-estimation of the variance component in related person households, and corresponding under-estimates of the other two variance components. The homogeneity of related-person households, which is often one of the key focuses of multilevel modelling of household surveys, is noticeably understated, which would be a significant failing of this approach in practice. The second simplification of $\sigma_1 = \sigma_u$ lessens the problem but does not remove it, as σ_1 is still over-estimated, with households of unrelated people apparently the main driver of the grouped variance parameter.

Fitting the most general model, with $\sigma_1 \neq \sigma_r \neq \sigma_u$, results in close to unbiased estimates of all parameters. This is not surprising given that the data was generated from this model, but it was not obvious in advance that σ_1 could be estimated well enough to avoid biases in the estimates for this and the other parameters. What is surprising is that the standard error of $\hat{\beta}$ is very similar for all three variance models. The additional parameters due to allowing unequal variances do not detract from the precision of the fixed effect. This may be because data from the three household types are weighted more efficiently when the household-level heterogeneity is modelled correctly.

Table 6 examines the proportions of people where $Y_{ij} = 1$, broken down by household type and $x = -0.5$ vs $x = 0.5$. The mean of the fitted predictions of these proportions over the 500 simulations are shown, under three alternative variance models, with the mean predictions for $x = -0.5$ and $x = 0.5$ separated by a "-" in each case. The final column shows the true expected values under the generated model. Expected/predicted proportions where $Y_{ij} = 1$ were obtained by integrating out u_i over its true/estimated distribution. It can be seen that the first two models compress the probabilities where $x = \pm 0.5$ for single person households, bringing them much closer together than the correct values shown in the last column: about 30% closer in the case of the first model. The reverse happens for households of related people. Thus models with grouped variance components substantially understate the effect of the covariate on Y for people living alone, and substantially overstate it for related-people households. The most general model avoids these problems.

Parameter	Expected Value of Estimator in model where:			True Value
	$\sigma_k = \sigma$	$\sigma_1 = \sigma_u$	unequal σ_k	
σ_1^2	5.73 (0.85)	2.18 (0.59)	0.81 (0.96)	0.562
σ_r^2	5.73 (0.85)	9.44 (1.56)	9.20 (1.51)	9.000
σ_u^2	5.73 (0.85)	2.18 (0.59)	2.34 (0.63)	2.250
β	1.127 (0.108)	1.068 (0.101)	1.005 (0.106)	1.000

Table 5: Expected values of estimated model parameters (standard errors in brackets) for dataset with 2500 households with x independent of household type

Household Type	Mean prediction			True Expected Value
	$\sigma_k = \sigma$	$\sigma_1 = \sigma_u$	unequal σ_k	
1	0.425 - 0.576	0.406 - 0.595	0.380 - 0.622	0.391 - 0.609
r	0.423 - 0.574	0.439 - 0.559	0.435 - 0.563	0.443 - 0.557
u	0.425 - 0.576	0.406 - 0.595	0.403 - 0.598	0.412 - 0.588

Table 6: True Expected Proportion and Mean Predictions of Proportion where $Y_{ij} = 1$, for $x_{ij} = -0.5$ and $x_{ij} = 0.5$ (separated by -), by Household Type (1=single person, u=unrelated people, r=related people)

5. Conclusions

It is well known that the degree of similarity of a dependent variable within households depends on the type of household, but multilevel modelling of household survey has ignored this complication. This article confirms that the current standard approach of including household type only as a fixed effect, or not at all, is inadequate, at least for the two datasets analysed here. The best solution is to include household type as a fixed main effect and to allow separate variance components for each type. The parameters of this model, including the single person household variance component (σ_1), are well identified, although σ_1 becomes practically unidentifiable if all interactions between household type and other covariates are added. The advantages of the new model are:

i. It gives substantial improvements to global goodness of fit, as measured by the deviance, AIC and BIC.

ii. If σ_1^2 is smaller than the other variance components, as would often be the case, then the standard approach exaggerates the effect of covariates for single person households.

iii. The variance component for related person household would often be larger than the other variance components. The standard approach can then noticeably understate the homogeneity of related person households.

iv. The new model results in lower standard errors for the estimated coefficients of the fixed effects, in spite of requiring two additional variance component parameters.

Unequal variance components should therefore be evaluated whenever binary multilevel models are fitted to household survey data. A less parsimonious alternative which fits the data nearly as well is to include interactions between household type and other covariates.

Acknowledgements

Support for this work from a bilateral Economic and Social Research Council (RES-000-22-2595) and Australian Research Council (LX0883143) grant is gratefully acknowledged. I am grateful to Alan Welsh for his insightful advice on the paper. My colleagues on the grant, particularly David Steel and Mark Tranmer, also made useful suggestions.

References

Australian Bureau of Statistics (ABS) (2001). *National Health Survey Australia Confidentialised Unit Record File Technical Report.* http://abs.gov.au/AUSSTATS/abs@.nsf/DetailsPage/4324.0.55.0011995.

Bates, D., Maechler, M., and Bolker, B. (2011). *lme4: Linear mixed-effects models using S4 classes.* R package version 0.999375-42. http://lme4.r-forge.r-project.org/

BHPS (2011). British Household Panel Survey: Waves 1-11, 1991-2002: Teaching Dataset (Social and Political Attitudes) [computer file]. 2nd Edition. University of Essex. Institute for Social and Economic Research. UK Data Archive. ESDS Longitudinal. http://dx.doi.org/10.5255/UKDA-SN-5038-2

Blocker, A.W. (2011). *fastGHQuad: Fast Rcpp implementation of Gauss-Hermite quadrature.* R package version 0.1-1. https://github.com/awblocker/fastGHQuad

Breslow, N. E. and Clayton,D. G. (1993). Approximate inference in generalized linear mixed models. *Journal of the American Statistical Association 88*(421), pp. 9–25.

Broström, G. and Holmberg, H. (2011). *glmmML: Generalized linear models with clustering.* R package version 0.82-1. http://cran.r-project.org/web/packages

Cox, D. R. and Hinkley, D. V. (1974). *Theoretical Statistics.* Chapman and Hall: London.

Gelman, A. and Hill, J. (2007). *Data Analysis using Regression and Multilevel/Hierarchical Models.* Cambridge University Press: New York.

Goldstein, H. (2011). *Multilevel Statistical Models.* John Wiley & Sons: Hoboken, New Jersey.

Heagerty, P.J. and Kurland, B.F. (2001). Misspecified maximum likelihood estimates and generalised linear mixed models. *Biometrika 88*(4), 973–985.

Heagerty, P. and Zeger, S. (2000). Marginalized multilevel models and likelihood inference (with comments and a rejoinder by the authors). *Statistical Science 15*(1), 1–26.

Johnston, R., Jones, K., Propper, C., Sarker, R., Burgess, S. and Bolster, A. (2005). A missing level in the analyses of british voting behaviour: the household as context as shown by analyses of a 1992–1997 longitudinal survey. *Electoral Studies 24*(2), 201–225.

Krenzke, T., Li, L. and Rust, K. (2010). Evaluating within household selection rules under a multi-stage design. *Survey Methodology 36*(1), 111–119.

McCulloch, C.E., Searle, S.R. and Neuhaus, J.M. (2008). *Generalized, Linear and Mixed Models.* John Wiley & Sons: Hoboken, New Jersey.

Müller, S., Scealy, J.L. and Welsh, A.H. (2013). Model selection in linear mixed models. *Statistical Science*, to appear.

Neuhaus, J. M. and McCulloch, C. E. (2006). Separating between- and within-cluster covariate effects by using conditional and partitioning methods. *Journal of the Royal Statistical Society, Series B: Statistical Methodology 68*(5), 859–872.

Neuhaus, J.M. and McCulloch, C. M. (2011). Estimation of covariate effects in generalized linear mixed models with informative cluster sizes. *Biometrika 98*(1), 147–162.

R Development Core Team (2012). *R: A Language and Environment for Statistical Computing*. Vienna, Austria: R Foundation for Statistical Computing. ISBN 3-900051-07-0.

Rabe-Hesketh, S. and Skrondal, A. (2001). Parameterization of multivariate random effects models for categorical data. *Biometrics 57*(4), 1256–1263.

Skrondal, A. and Rabe-Hesketh, S. (2007). Redundant overdispersion parameters in multilevel models for categorical responses. *Journal of Educational and Behavioral Statistics 32*(4), pp. 419–430.

Spiegelhalter, D.J., Best, N.G, Carlin, B.P. and Van Der Linde, A. Bayesian measures of model complexity and fit *Journal of the Royal Statistical Society: Series B (Statistical Methodology) 64*(4), 583–639.

Stram, D.O. and Lee, J.W. (1994). Variance components testing in the longitudinal mixed effects model. *Biometrics 50*(4), 1171–1177.

Vaida, F. and S. Blanchard (2005). Conditional Akaike information for mixed-effects models. *Biometrika 92*(2), 351–370.

Appendices: Detailed Tables for Paper on Mixed Logistic Models for Household Surveys

Robert Graham Clark

National Institute for Applied Statistics Research Australia, University of Wollongong, Australia.

Outline

Appendix 1: Additional Tables for Empirical Study

Variable	Parameter Estimate (standard error)	
	$\sigma_k = \sigma$	unequal σ_k
intercept	-2.53 (0.061)	-3.02 (0.062)
AGE 18-29 years	0.00 (n/a)	0.00 (n/a)
AGE 30-44yrs	0.24 (0.053)	0.23 (0.055)
AGE 45-59yrs	1.00 (0.053)	1.01 (0.054)
AGE 60+yrs	1.94 (0.054)	2.00 (0.055)
SEX female	0.07 (0.056)	0.06 (0.056)
SOCIO-ECO Q1	0.37 (0.040)	0.40 (0.042)
SOCIO-ECO Q2	0.15 (0.040)	0.17 (0.042)
SOCIO-ECO Q3	0.00 (n/a)	0.00 (n/a)
SOCIO-ECO Q4	-0.20 (0.040)	-0.20 (0.042)
SOCIO-ECO Q5	-0.46 (0.041)	-0.46 (0.042)
female 30-44	-0.08 (0.072)	-0.09 (0.073)
female 45-59	-0.12 (0.071)	-0.11 (0.073)
female 60+	-0.26 (0.069)	-0.21 (0.071)
HHTYPE 1	0.00 (n/a)	0.00 (n/a)
HHTYPE r	-0.28 (0.033)	0.17 (0.043)
HHTYPE u	0.09 (0.047)	0.64 (0.060)

Table A1.1: Fixed Effect Parameter Estimates for Australian National Health Survey Self-Assessed Health (Fair or Poor vs Good or Better)

AFISIT (current financial situation)	Number of People	Observed Proportion	Predicted Proportion		
			$\sigma_k = \sigma$	$\sigma_k = \sigma$ + interaction	unequal σ_k
all	8258	39.8	39.9	39.9	39.9
living comfortably	2288	53.4	50.5	50.3	50.5
doing alright	2285	42.0	41.8	41.8	41.7
just about getting by	2578	32.1	34.0	34.2	34.0
finding quite difficult	712	26.5	29.2	29.3	29.5
finding very difficult	395	22.5	25.1	25.2	25.0

Table A1.2: Observed and predicted proportions voting conservative for all households, by current financial situation (AFISIT)

Age	Number of People	Observed Proportion	Predicted Proportion $\sigma_k = \sigma$	$\sigma_k = \sigma$ + interaction	unequal σ_k
all	37886	16.5	16.5	16.5	16.5
18-29yrs	9033	9.7	9.8	9.7	9.7
30-44yrs	12720	10.6	10.6	10.5	10.6
45-59yrs	8677	18.0	18.1	18.1	18.1
60+yrs	7456	33.3	32.9	33.0	33.1

Table A1.3: Observed and predicted proportions with fair or poor health in all households, by age

Age	Number of People	Observed Proportion	Predicted Proportion $\sigma_k = \sigma$	$\sigma_k = \sigma$ + interaction	unequal σ_k
all	5806	23.0	23.0	23.0	23.0
18-29yrs	868	14.2	12.0	14.2	13.8
30-44yrs	1790	15.0	13.3	15.0	14.8
45-59yrs	1073	23.6	22.2	23.6	22.3
60+yrs	2075	33.2	36.6	33.2	34.1

Table A1.4: Observed and predicted proportions with fair or poor health in single person households only, by age

Appendix 2: Additional Tables of Mean Parameter Estimates in Simulation Study when x is Not Associated with Household Type

Number of Households	Parameter	Expected Value of Estimator in model where: $\sigma_k = \sigma$	$\sigma_1 = \sigma_u$	unequal σ_k	True Value
625	σ_1^2	5.79 (1.58)	2.34 (1.26)	1.47 (2.17)	0.562
	σ_r^2	5.79 (1.58)	9.71 (3.15)	9.55 (3.11)	9.000
	σ_u^2	5.79 (1.58)	2.34 (1.26)	2.55 (1.38)	2.250
	intercept	-0.012 (0.387)	0.000 (0.311)	0.000 (0.301)	0.000
	hh type r	0.015 (0.391)	-0.001 (0.277)	-0.001 (0.270)	0.000
	hh type u	0.000 (0.258)	-0.001 (0.186)	0.000 (0.169)	0.000
	β	1.129 (0.221)	1.078 (0.189)	1.036 (0.187)	1.000
1250	σ_1^2	5.70 (1.07)	2.20 (0.80)	1.13 (1.50)	0.562
	σ_r^2	5.70 (1.07)	9.37 (2.04)	9.18 (1.98)	9.000
	σ_u^2	5.70 (1.07)	2.20 (0.80)	2.37 (0.87)	2.250
	intercept	0.002 (0.204)	0.002 (0.196)	0.001 (0.188)	0.000
	hh type r	-0.007 (0.238)	-0.006 (0.185)	-0.007 (0.181)	0.000
	hh type u	0.002 (0.165)	0.001 (0.126)	0.002 (0.114)	0.000
	β	1.130 (0.139)	1.071 (0.132)	1.021 (0.134)	1.000
2500	σ_1^2	5.73 (0.85)	2.18 (0.59)	0.81 (0.96)	0.562
	σ_r^2	5.73 (0.85)	9.44 (1.56)	9.20 (1.51)	9.000
	σ_u^2	5.73 (0.85)	2.18 (0.59)	2.34 (0.63)	2.250
	intercept	-0.015 (0.133)	-0.014 (0.132)	-0.013 (0.127)	0.000
	hh type r	-0.003 (0.139)	-0.002 (0.108)	-0.001 (0.103)	0.000
	hh type u	0.006 (0.117)	0.004 (0.090)	0.003 (0.075)	0.000
	β	1.127 (0.108)	1.068 (0.101)	1.005 (0.106)	1.000

Table A2.1: Expected values of estimated model parameters (standard errors in brackets) where x independent of household type, $(\sigma_1, \sigma_r, \sigma_u) = (0.75, 3, 1.5)$

Number of Households	Parameter	Expected Value of Estimator in model where: $\sigma_k = \sigma$	$\sigma_1 = \sigma_u$	unequal σ_k	True Value
625	σ_1^2	1.04 (0.45)	1.26 (0.78)	4.61 (3.17)	4.000
	σ_r^2	1.04 (0.45)	1.00 (0.52)	1.06 (0.54)	1.000
	σ_u^2	1.04 (0.45)	1.26 (0.78)	1.10 (0.73)	1.000
	intercept	0.001 (0.201)	0.001 (0.204)	0.000 (0.255)	0.000
	hh type r	0.006 (0.231)	0.004 (0.235)	0.004 (0.279)	0.000
	hh type u	-0.003 (0.161)	-0.003 (0.165)	-0.002 (0.224)	0.000
	β	0.936 (0.142)	0.942 (0.144)	0.999 (0.150)	1.000
1250	σ_1^2	1.00 (0.35)	1.17 (0.59)	4.47 (2.90)	4.000
	σ_r^2	1.00 (0.35)	0.94 (0.38)	1.00 (0.40)	1.000
	σ_u^2	1.00 (0.35)	1.17 (0.59)	1.03 (0.55)	1.000
	intercept	0.002 (0.122)	0.002 (0.124)	0.003 (0.152)	0.000
	hh type r	-0.005 (0.159)	-0.005 (0.162)	-0.005 (0.187)	0.000
	hh type u	0.003 (0.109)	0.002 (0.111)	0.002 (0.147)	0.000
	β	0.936 (0.101)	0.942 (0.104)	0.999 (0.112)	1.000
2500	σ_1^2	1.02 (0.27)	1.17 (0.40)	4.48 (2.43)	4.000
	σ_r^2	1.02 (0.27)	0.96 (0.29)	1.02 (0.30)	1.000
	σ_u^2	1.02 (0.27)	1.17 (0.40)	1.03 (0.37)	1.000
	intercept	-0.002 (0.081)	-0.001 (0.082)	-0.002 (0.103)	0.000
	hh type r	0.003 (0.092)	0.003 (0.094)	0.002 (0.112)	0.000
	hh type u	0.002 (0.081)	0.002 (0.083)	0.002 (0.113)	0.000
	β	0.932 (0.078)	0.937 (0.080)	0.997 (0.087)	1.000

Table A2.2: Expected values of estimated model parameters (standard errors in brackets) where x independent of household type, $(\sigma_1, \sigma_r, \sigma_u) = (2, 1, 1)$

Number of Households	Parameter	Expected Value of Estimator in model where:			True Value
		$\sigma_k = \sigma$	$\sigma_1 = \sigma_u$	unequal σ_k	
625	σ_1^2	0.75 (0.39)	0.36 (0.40)	0.82 (1.46)	0.062
	σ_r^2	0.75 (0.39)	1.09 (0.54)	1.11 (0.54)	1.000
	σ_u^2	0.75 (0.39)	0.36 (0.40)	0.40 (0.44)	0.250
	intercept	0.002 (0.184)	0.001 (0.180)	0.001 (0.188)	0.000
	hh type r	0.010 (0.213)	0.009 (0.200)	0.009 (0.208)	0.000
	hh type u	-0.004 (0.154)	-0.003 (0.144)	-0.003 (0.153)	0.000
	β	1.042 (0.138)	1.030 (0.133)	1.046 (0.136)	1.000
1250	σ_1^2	0.71 (0.31)	0.30 (0.30)	0.55 (0.92)	0.062
	σ_r^2	0.71 (0.31)	1.02 (0.40)	1.03 (0.40)	1.000
	σ_u^2	0.71 (0.31)	0.30 (0.30)	0.32 (0.32)	0.250
	intercept	0.001 (0.112)	0.002 (0.108)	0.001 (0.114)	0.000
	hh type r	-0.001 (0.152)	-0.001 (0.142)	-0.001 (0.149)	0.000
	hh type u	0.003 (0.110)	0.003 (0.102)	0.003 (0.110)	0.000
	β	1.034 (0.104)	1.018 (0.100)	1.028 (0.101)	1.000
2500	σ_1^2	0.73 (0.24)	0.25 (0.22)	0.31 (0.50)	0.062
	σ_r^2	0.73 (0.24)	1.04 (0.31)	1.04 (0.30)	1.000
	σ_u^2	0.73 (0.24)	0.25 (0.22)	0.28 (0.22)	0.250
	intercept	-0.004 (0.062)	-0.003 (0.060)	-0.003 (0.061)	0.000
	hh type r	0.001 (0.063)	0.001 (0.058)	0.001 (0.059)	0.000
	hh type u	0.004 (0.075)	0.003 (0.069)	0.004 (0.070)	0.000
	β	1.034 (0.080)	1.013 (0.077)	1.016 (0.080)	1.000

Table A2.3: Expected values of estimated model parameters (standard errors in brackets) where x independent of household type, $(\sigma_1, \sigma_r, \sigma_u) = (0.25, 1, 0.5)$

Number of Households	Parameter	Expected Value of Estimator in model where: $\sigma_k = \sigma$	$\sigma_1 = \sigma_u$	unequal σ_k	True Value
625	σ_1^2	4.13 (1.14)	4.43 (2.05)	4.64 (3.25)	4.000
	σ_r^2	4.13 (1.14)	4.16 (1.38)	4.15 (1.38)	4.000
	σ_u^2	4.13 (1.14)	4.43 (2.05)	4.44 (2.18)	4.000
	intercept	0.005 (0.279)	0.005 (0.282)	0.003 (0.288)	0.000
	hh type r	-0.001 (0.330)	-0.006 (0.337)	-0.008 (0.342)	0.000
	hh type u	-0.004 (0.216)	-0.004 (0.220)	-0.002 (0.225)	0.000
	β	1.008 (0.184)	1.009 (0.186)	1.002 (0.188)	1.000
1250	σ_1^2	4.04 (0.83)	4.17 (1.32)	4.43 (2.97)	4.000
	σ_r^2	4.04 (0.83)	4.05 (0.99)	4.03 (0.98)	4.000
	σ_u^2	4.04 (0.83)	4.17 (1.32)	4.18 (1.34)	4.000
	intercept	0.003 (0.176)	0.003 (0.177)	0.004 (0.176)	0.000
	hh type r	-0.006 (0.222)	-0.005 (0.224)	-0.005 (0.226)	0.000
	hh type u	0.003 (0.146)	0.003 (0.146)	0.002 (0.147)	0.000
	β	1.002 (0.127)	1.003 (0.129)	0.996 (0.136)	1.000
2500	σ_1^2	4.08 (0.67)	4.17 (0.99)	4.51 (2.48)	4.000
	σ_r^2	4.08 (0.67)	4.08 (0.76)	4.08 (0.76)	4.000
	σ_u^2	4.08 (0.67)	4.17 (0.99)	4.17 (1.02)	4.000
	intercept	-0.006 (0.123)	-0.006 (0.124)	-0.006 (0.126)	0.000
	hh type r	0.000 (0.142)	0.000 (0.143)	0.000 (0.144)	0.000
	hh type u	0.003 (0.109)	0.003 (0.110)	0.003 (0.113)	0.000
	β	1.002 (0.100)	1.003 (0.100)	1.001 (0.106)	1.000

Table A2.4: Expected values of estimated model parameters (standard errors in brackets) where x independent of household type, $(\sigma_1, \sigma_r, \sigma_u) = (2, 2, 2)$

Appendix 3: Additional Tables of Mean Parameter Estimates in Simulation Study and x IS Associated with Household Type

Number of Households	Parameter	Expected Value of Estimator in model where: $\sigma_k = \sigma$	$\sigma_1 = \sigma_u$	unequal σ_k	True Value
625	σ_1^2	5.78 (1.51)	2.29 (1.16)	1.49 (2.21)	0.562
	σ_r^2	5.78 (1.51)	9.71 (3.14)	9.56 (3.09)	9.000
	σ_u^2	5.78 (1.51)	2.29 (1.16)	2.50 (1.26)	2.250
	intercept	0.126 (0.332)	0.042 (0.313)	0.018 (0.306)	0.000
	hh type r	0.148 (0.394)	0.022 (0.287)	0.013 (0.278)	0.000
	hh type u	-0.128 (0.274)	-0.042 (0.190)	-0.019 (0.176)	0.000
	β	1.133 (0.203)	1.080 (0.188)	1.040 (0.187)	1.000
1250	σ_1^2	5.70 (1.09)	2.19 (0.85)	1.10 (1.48)	0.562
	σ_r^2	5.70 (1.09)	9.37 (2.05)	9.18 (1.99)	9.000
	σ_u^2	5.70 (1.09)	2.19 (0.85)	2.37 (0.94)	2.250
	intercept	0.116 (0.202)	0.040 (0.197)	0.012 (0.190)	0.000
	hh type r	0.141 (0.240)	0.012 (0.191)	0.000 (0.185)	0.000
	hh type u	-0.113 (0.161)	-0.036 (0.125)	-0.008 (0.115)	0.000
	β	1.129 (0.143)	1.071 (0.135)	1.020 (0.136)	1.000
2500	σ_1^2	5.73 (0.82)	2.16 (0.54)	0.78 (0.91)	0.562
	σ_r^2	5.73 (0.82)	9.43 (1.56)	9.20 (1.51)	9.000
	σ_u^2	5.73 (0.82)	2.16 (0.54)	2.32 (0.59)	2.250
	intercept	0.103 (0.134)	0.025 (0.134)	-0.007 (0.128)	0.000
	hh type r	0.155 (0.144)	0.023 (0.115)	0.008 (0.109)	0.000
	hh type u	-0.112 (0.117)	-0.036 (0.091)	-0.003 (0.079)	0.000
	β	1.126 (0.107)	1.068 (0.099)	1.005 (0.102)	1.000

Table A3.1: Expected values of estimated model parameters (standard errors in brackets) where x associated with household type, $(\sigma_1, \sigma_r, \sigma_u) = (0.75, 3, 1.5)$

Number of Households	Parameter	Expected Value of Estimator in model where: $\sigma_k = \sigma$	$\sigma_1 = \sigma_u$	unequal σ_k	True Value
625	σ_1^2	1.04 (0.45)	1.25 (0.82)	4.62 (3.17)	4.000
	σ_r^2	1.04 (0.45)	1.00 (0.52)	1.06 (0.54)	1.000
	σ_u^2	1.04 (0.45)	1.25 (0.82)	1.10 (0.77)	1.000
	intercept	-0.040 (0.197)	-0.037 (0.199)	0.013 (0.255)	0.000
	hh type r	-0.018 (0.240)	-0.012 (0.243)	0.021 (0.288)	0.000
	hh type u	0.038 (0.162)	0.035 (0.166)	-0.015 (0.229)	0.000
	β	0.936 (0.142)	0.941 (0.145)	0.999 (0.152)	1.000
1250	σ_1^2	1.00 (0.36)	1.18 (0.60)	4.50 (2.91)	4.000
	σ_r^2	1.00 (0.36)	0.94 (0.39)	1.00 (0.40)	1.000
	σ_u^2	1.00 (0.36)	1.18 (0.60)	1.03 (0.56)	1.000
	intercept	-0.045 (0.124)	-0.042 (0.126)	0.007 (0.161)	0.000
	hh type r	-0.039 (0.160)	-0.033 (0.164)	-0.001 (0.192)	0.000
	hh type u	0.050 (0.112)	0.047 (0.115)	-0.002 (0.157)	0.000
	β	0.936 (0.102)	0.942 (0.105)	0.999 (0.112)	1.000
2500	σ_1^2	1.02 (0.27)	1.18 (0.39)	4.42 (2.44)	4.000
	σ_r^2	1.02 (0.27)	0.96 (0.29)	1.03 (0.30)	1.000
	σ_u^2	1.02 (0.27)	1.18 (0.39)	1.03 (0.36)	1.000
	intercept	-0.045 (0.080)	-0.043 (0.081)	0.006 (0.110)	0.000
	hh type r	-0.024 (0.095)	-0.018 (0.097)	0.013 (0.121)	0.000
	hh type u	0.045 (0.082)	0.043 (0.084)	-0.006 (0.121)	0.000
	β	0.933 (0.079)	0.939 (0.080)	0.998 (0.087)	1.000

Table A3.2: Expected values of estimated model parameters (standard errors in brackets) where x associated with household type, $(\sigma_1, \sigma_r, \sigma_u) = (2, 1, 1)$

Number of Households	Parameter	Expected Value of Estimator in model where:			True Value
		$\sigma_k = \sigma$	$\sigma_1 = \sigma_u$	unequal σ_k	
625	σ_1^2	0.76 (0.38)	0.36 (0.39)	0.88 (1.49)	0.062
	σ_r^2	0.76 (0.38)	1.09 (0.55)	1.11 (0.55)	1.000
	σ_u^2	0.76 (0.38)	0.36 (0.39)	0.39 (0.42)	0.250
	intercept	0.018 (0.189)	0.003 (0.183)	0.015 (0.193)	0.000
	hh type r	0.036 (0.225)	0.008 (0.209)	0.018 (0.219)	0.000
	hh type u	-0.019 (0.158)	-0.004 (0.147)	-0.016 (0.160)	0.000
	β	1.044 (0.137)	1.031 (0.132)	1.049 (0.135)	1.000
1250	σ_1^2	0.71 (0.31)	0.28 (0.30)	0.55 (0.94)	0.062
	σ_r^2	0.71 (0.31)	1.02 (0.40)	1.03 (0.40)	1.000
	σ_u^2	0.71 (0.31)	0.28 (0.30)	0.31 (0.33)	0.250
	intercept	0.023 (0.111)	0.009 (0.108)	0.015 (0.115)	0.000
	hh type r	0.031 (0.154)	0.003 (0.144)	0.008 (0.151)	0.000
	hh type u	-0.019 (0.110)	-0.004 (0.103)	-0.010 (0.112)	0.000
	β	1.033 (0.102)	1.017 (0.098)	1.027 (0.100)	1.000
2500	σ_1^2	0.73 (0.24)	0.26 (0.22)	0.31 (0.49)	0.062
	σ_r^2	0.73 (0.24)	1.04 (0.31)	1.04 (0.30)	1.000
	σ_u^2	0.73 (0.24)	0.26 (0.22)	0.29 (0.23)	0.250
	intercept	0.017 (0.063)	0.002 (0.061)	0.003 (0.063)	0.000
	hh type r	0.035 (0.066)	0.006 (0.061)	0.007 (0.062)	0.000
	hh type u	-0.017 (0.077)	-0.002 (0.071)	-0.003 (0.073)	0.000
	β	1.034 (0.081)	1.013 (0.077)	1.015 (0.079)	1.000

Table A3.3: Expected values of estimated model parameters (standard errors in brackets) where x associated with household type, $(\sigma_1, \sigma_r, \sigma_u) = (0.25, 1, 0.5)$

Number of Households	Parameter	Expected Value of Estimator in model where: $\sigma_k = \sigma$	$\sigma_1 = \sigma_u$	unequal σ_k	True Value
625	σ_1^2	4.13 (1.14)	4.42 (1.99)	4.64 (3.25)	4.000
	σ_r^2	4.13 (1.14)	4.16 (1.38)	4.15 (1.38)	4.000
	σ_u^2	4.13 (1.14)	4.42 (1.99)	4.42 (2.08)	4.000
	intercept	0.011 (0.273)	0.014 (0.277)	0.015 (0.286)	0.000
	hh type r	0.004 (0.339)	0.008 (0.342)	0.009 (0.350)	0.000
	hh type u	-0.011 (0.216)	-0.014 (0.220)	-0.014 (0.230)	0.000
	β	1.007 (0.182)	1.007 (0.184)	1.001 (0.187)	1.000
1250	σ_1^2	4.04 (0.84)	4.19 (1.39)	4.47 (3.00)	4.000
	σ_r^2	4.04 (0.84)	4.05 (1.00)	4.04 (0.98)	4.000
	σ_u^2	4.04 (0.84)	4.19 (1.39)	4.19 (1.40)	4.000
	intercept	0.003 (0.178)	0.006 (0.180)	0.008 (0.183)	0.000
	hh type r	-0.007 (0.226)	-0.003 (0.232)	0.001 (0.234)	0.000
	hh type u	0.003 (0.148)	0.001 (0.151)	-0.002 (0.157)	0.000
	β	1.003 (0.127)	1.004 (0.129)	0.997 (0.134)	1.000
2500	σ_1^2	4.07 (0.66)	4.14 (0.95)	4.44 (2.51)	4.000
	σ_r^2	4.07 (0.66)	4.08 (0.75)	4.07 (0.76)	4.000
	σ_u^2	4.07 (0.66)	4.14 (0.95)	4.14 (0.96)	4.000
	intercept	-0.002 (0.121)	-0.002 (0.122)	0.002 (0.129)	0.000
	hh type r	0.005 (0.145)	0.007 (0.148)	0.011 (0.152)	0.000
	hh type u	-0.001 (0.111)	-0.002 (0.112)	-0.005 (0.121)	0.000
	β	1.002 (0.099)	1.002 (0.099)	1.000 (0.106)	1.000

Table A3.4: Expected values of estimated model parameters (standard errors in brackets) where x associated with household type, $(\sigma_1, \sigma_r, \sigma_u) = (2, 2, 2)$

Appendix 4: Additional Tables of Expected and Mean Predicted Proportions where $Y_{ij} = 1$ in Simulation Study and x independent of Household Type

Number of Households	Household Type	Mean prediction $\sigma_k = \sigma$	$\sigma_1 = \sigma_u$	unequal σ_k	True Expected Values
625	1	0.425 - 0.576	0.405 - 0.594	0.385 - 0.615	0.391 - 0.609
	r	0.423 - 0.574	0.439 - 0.561	0.435 - 0.563	0.443 - 0.557
	u	0.427 - 0.578	0.406 - 0.595	0.406 - 0.600	0.412 - 0.588
1250	1	0.424 - 0.576	0.406 - 0.595	0.383 - 0.618	0.391 - 0.609
	r	0.424 - 0.576	0.440 - 0.561	0.436 - 0.565	0.443 - 0.557
	u	0.423 - 0.575	0.405 - 0.594	0.401 - 0.597	0.412 - 0.588
2500	1	0.425 - 0.576	0.406 - 0.595	0.380 - 0.622	0.391 - 0.609
	r	0.423 - 0.574	0.439 - 0.559	0.435 - 0.563	0.443 - 0.557
	u	0.425 - 0.576	0.406 - 0.595	0.403 - 0.598	0.412 - 0.588

Table A4.1: True Expected Proportion and Mean Predictions of Proportion where $Y_{ij} = 1$, for $x_{ij} = -0.5$ and $x_{ij} = 0.5$ (separated by -), by Household Type (1=single person, u=unrelated people, r=related people) and x is independent of Household Type, $(\sigma_1, \sigma_r, \sigma_u) = (0.75, 3, 1.5)$

Number of Households	Household Type	Mean prediction $\sigma_k = \sigma$	$\sigma_1 = \sigma_u$	unequal σ_k	True Expected Values
625	1	0.404 - 0.594	0.406 - 0.593	0.426 - 0.573	0.425 - 0.575
	r	0.405 - 0.595	0.403 - 0.596	0.404 - 0.595	0.398 - 0.602
	u	0.406 - 0.596	0.407 - 0.594	0.406 - 0.596	0.398 - 0.602
1250	1	0.405 - 0.596	0.406 - 0.595	0.427 - 0.574	0.425 - 0.575
	r	0.405 - 0.597	0.404 - 0.598	0.405 - 0.597	0.398 - 0.602
	u	0.404 - 0.595	0.405 - 0.594	0.404 - 0.595	0.398 - 0.602
2500	1	0.405 - 0.595	0.407 - 0.594	0.429 - 0.571	0.425 - 0.575
	r	0.405 - 0.595	0.404 - 0.596	0.405 - 0.595	0.398 - 0.602
	u	0.406 - 0.596	0.407 - 0.594	0.406 - 0.596	0.398 - 0.602

Table A4.2: True Expected Proportion and Mean Predictions of Proportion where $Y_{ij} = 1$, for $x_{ij} = -0.5$ and $x_{ij} = 0.5$ (separated by -), by Household Type (1=single person, u=unrelated people, r=related people) and x is independent of Household Type, $(\sigma_1, \sigma_r, \sigma_u) = (2, 1, 1)$

Number of Households	Household Type	Mean prediction $\sigma_k = \sigma$	$\sigma_1 = \sigma_u$	unequal σ_k	True Expected Values
625	1	0.389 - 0.609	0.382 - 0.616	0.385 - 0.613	0.379 - 0.621
	r	0.389 - 0.610	0.396 - 0.604	0.395 - 0.605	0.398 - 0.602
	u	0.391 - 0.612	0.385 - 0.619	0.384 - 0.619	0.384 - 0.616
1250	1	0.390 - 0.611	0.384 - 0.618	0.385 - 0.617	0.379 - 0.621
	r	0.391 - 0.611	0.397 - 0.605	0.396 - 0.606	0.398 - 0.602
	u	0.390 - 0.611	0.383 - 0.617	0.382 - 0.619	0.384 - 0.616
2500	1	0.391 - 0.611	0.383 - 0.618	0.381 - 0.620	0.379 - 0.621
	r	0.390 - 0.610	0.397 - 0.603	0.395 - 0.605	0.398 - 0.602
	u	0.391 - 0.611	0.384 - 0.618	0.382 - 0.620	0.384 - 0.616

Table A4.3: True Expected Proportion and Mean Predictions of Proportion where $Y_{ij} = 1$, for $x_{ij} = -0.5$ and $x_{ij} = 0.5$ (separated by -), by Household Type (1=single person, u=unrelated people, r=related people) and x is independent of Household Type, $(\sigma_1, \sigma_r, \sigma_u) = (2, 2, 2)$

Number of Households	Household Type	Mean prediction $\sigma_k = \sigma$	$\sigma_1 = \sigma_u$	unequal σ_k	True Expected Values
625	1	0.424 - 0.575	0.425 - 0.574	0.420 - 0.578	0.425 - 0.575
	r	0.425 - 0.575	0.425 - 0.576	0.425 - 0.576	0.425 - 0.575
	u	0.424 - 0.575	0.425 - 0.574	0.425 - 0.575	0.425 - 0.575
1250	1	0.425 - 0.576	0.425 - 0.575	0.422 - 0.579	0.425 - 0.575
	r	0.426 - 0.576	0.425 - 0.576	0.426 - 0.576	0.425 - 0.575
	u	0.425 - 0.575	0.425 - 0.575	0.425 - 0.575	0.425 - 0.575
2500	1	0.425 - 0.575	0.426 - 0.575	0.424 - 0.576	0.425 - 0.575
	r	0.425 - 0.574	0.424 - 0.575	0.424 - 0.575	0.425 - 0.575
	u	0.425 - 0.575	0.426 - 0.575	0.426 - 0.575	0.425 - 0.575

Table A4.4: True Expected Proportion and Mean Predictions of Proportion where $Y_{ij} = 1$, for $x_{ij} = -0.5$ and $x_{ij} = 0.5$ (separated by -), by Household Type (1=single person, u=unrelated people, r=related people) and x is independent of Household Type, $(\sigma_1, \sigma_r, \sigma_u) = (2, 2, 2)$

Appendix 5: Additional Tables of Expected and Mean Predicted Proportions where $Y_{ij} = 1$ in Simulation Study and x Associated with Household Type

Number of Households	Household Type	Mean prediction $\sigma_k = \sigma$	$\sigma_1 = \sigma_u$	unequal σ_k	True Expected Values
625	1	0.407 - 0.558	0.398 - 0.588	0.360 - 0.590	0.391 - 0.609
	r	0.424 - 0.575	0.439 - 0.561	0.436 - 0.564	0.443 - 0.557
	u	0.427 - 0.579	0.402 - 0.592	0.408 - 0.602	0.412 - 0.588
1250	1	0.409 - 0.561	0.399 - 0.589	0.359 - 0.594	0.391 - 0.609
	r	0.425 - 0.576	0.440 - 0.561	0.436 - 0.565	0.443 - 0.557
	u	0.428 - 0.580	0.401 - 0.591	0.407 - 0.603	0.412 - 0.588
2500	1	0.410 - 0.561	0.399 - 0.588	0.356 - 0.597	0.391 - 0.609
	r	0.423 - 0.574	0.439 - 0.559	0.435 - 0.563	0.443 - 0.557
	u	0.430 - 0.581	0.403 - 0.592	0.409 - 0.605	0.412 - 0.588

Table A5.1: True Expected Proportion and Mean Predictions of Proportion where $Y_{ij} = 1$, for $x_{ij} = -0.5$ and $x_{ij} = 0.5$ (separated by -), by Household Type (1=single person, u=unrelated people, r=related people) and x is associated with Household Type, $(\sigma_1, \sigma_r, \sigma_u) = (0.75, 3, 1.5)$

Number of Households	Household Type	Mean prediction $\sigma_k = \sigma$	$\sigma_1 = \sigma_u$	unequal σ_k	True Expected Values
625	1	0.413 - 0.603	0.413 - 0.601	0.432 - 0.580	0.425 - 0.575
	r	0.405 - 0.595	0.403 - 0.596	0.404 - 0.595	0.398 - 0.602
	u	0.409 - 0.599	0.411 - 0.598	0.409 - 0.600	0.398 - 0.602
1250	1	0.414 - 0.605	0.415 - 0.603	0.435 - 0.581	0.425 - 0.575
	r	0.405 - 0.597	0.404 - 0.598	0.405 - 0.597	0.398 - 0.602
	u	0.407 - 0.598	0.409 - 0.597	0.407 - 0.598	0.398 - 0.602
2500	1	0.414 - 0.604	0.415 - 0.602	0.436 - 0.578	0.425 - 0.575
	r	0.405 - 0.595	0.403 - 0.596	0.405 - 0.595	0.398 - 0.602
	u	0.409 - 0.599	0.411 - 0.599	0.409 - 0.599	0.398 - 0.602

Table A5.2: True Expected Proportion and Mean Predictions of Proportion where $Y_{ij} = 1$, for $x_{ij} = -0.5$ and $x_{ij} = 0.5$ (separated by -), by Household Type (1=single person, u=unrelated people, r=related people) and x is associated with Household Type, $(\sigma_1, \sigma_r, \sigma_u) = (2, 1, 1)$

Number of Households	Household Type	Mean prediction $\sigma_k = \sigma$	$\sigma_1 = \sigma_u$	unequal σ_k	True Expected Values
625	1	0.386 - 0.606	0.382 - 0.616	0.382 - 0.609	0.379 - 0.621
	r	0.389 - 0.610	0.395 - 0.604	0.395 - 0.605	0.398 - 0.602
	u	0.394 - 0.614	0.384 - 0.618	0.386 - 0.622	0.384 - 0.616
1250	1	0.386 - 0.606	0.382 - 0.616	0.380 - 0.612	0.379 - 0.621
	r	0.391 - 0.611	0.397 - 0.605	0.396 - 0.606	0.398 - 0.602
	u	0.392 - 0.613	0.383 - 0.617	0.384 - 0.621	0.384 - 0.616
2500	1	0.387 - 0.606	0.382 - 0.617	0.377 - 0.616	0.379 - 0.621
	r	0.390 - 0.610	0.397 - 0.603	0.395 - 0.605	0.398 - 0.602
	u	0.394 - 0.614	0.384 - 0.618	0.385 - 0.623	0.384 - 0.616

Table A5.3: True Expected Proportion and Mean Predictions of Proportion where $Y_{ij} = 1$, for $x_{ij} = -0.5$ and $x_{ij} = 0.5$ (separated by -), by Household Type (1=single person, u=unrelated people, r=related people) and x is associated with Household Type, $(\sigma_1, \sigma_r, \sigma_u) = (2, 2, 2)$

Number of Households	Household Type	Mean prediction $\sigma_k = \sigma$	$\sigma_1 = \sigma_u$	unequal σ_k	True Expected Values
625	1	0.423 - 0.573	0.424 - 0.573	0.420 - 0.577	0.425 - 0.575
	r	0.425 - 0.575	0.425 - 0.576	0.425 - 0.575	0.425 - 0.575
	u	0.424 - 0.574	0.425 - 0.574	0.425 - 0.574	0.425 - 0.575
1250	1	0.425 - 0.576	0.425 - 0.575	0.423 - 0.579	0.425 - 0.575
	r	0.426 - 0.576	0.425 - 0.576	0.426 - 0.576	0.425 - 0.575
	u	0.424 - 0.575	0.424 - 0.575	0.424 - 0.574	0.425 - 0.575
2500	1	0.425 - 0.575	0.425 - 0.575	0.424 - 0.576	0.425 - 0.575
	r	0.424 - 0.575	0.424 - 0.575	0.424 - 0.575	0.425 - 0.575
	u	0.426 - 0.576	0.426 - 0.576	0.426 - 0.576	0.425 - 0.575

Table A5.4: True Expected Proportion and Mean Predictions of Proportion where $Y_{ij} = 1$, for $x_{ij} = -0.5$ and $x_{ij} = 0.5$ (separated by -), by Household Type (1=single person, u=unrelated people, r=related people) and x is associated with Household Type, $(\sigma_1, \sigma_r, \sigma_u) = (2, 2, 2)$