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Active seismic response control of tall buildings based on reduced order model

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Abstract

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Active Seismic Response Control of Tall Buildings Based on Reduced Order Model

Haiping Du, John Boffa, and Nong Zhang

Abstract— **This paper applies the dynamic model reduction method to obtain a reduced order model of an experimental tall building which has twenty floors and is 2.5m high. The experimental model is designed to imitate a real tall building with an active mass damper, and is used to study the available modelling and active control strategies for real tall buildings. Based on the dynamic model reduction method, a reduced order model is used within a H_∞ controller design to suppress excessive vibrations induced by seismic excitation. The reduced order model effectively describes only those frequency characteristics from the full order model that are of interest. The controller design based on the low order model can be directly applied to the full order model without introducing the control and observer spillover problems. Numerical simulations confirm that the low order model is acceptable at describing the relevant dynamic characteristics of the full order model and that it can be used effectively to mitigate the vibration caused by seismic disturbances.**

I. INTRODUCTION

ACTIVE control of civil engineering structures to reduce the excessive vibration caused by strong winds or earthquakes, has received considerable attention in recent years [1]. Various control strategies, such as H_2 (LQG) and H_∞ control, neural network control, fuzzy logic control, adaptive control, sliding mode control, independent modal space control etc., have been proposed and developed to attenuate the effects of structural vibration. Even though some robust controllers can tolerate minor structural and parametric uncertainties, it is always necessary to obtain the most accurate model of the plant as possible, in order for the maximization of the control effectiveness. Normally, model based control strategies such as the H_2 (LQG), need an accurate plant model, which can easily be approximated by using high order modeling techniques, but is obviously restricted by the unavailability of the infinite dimensional model. Based on these high order plant models, feasible controllers are very difficult to find to fulfill the necessary performance requirements. In many cases, even higher order controllers are designed to deal with the high order plant, and it becomes evident that the size of the model should be reduced, to decrease the cost in hardware realization and to increase the computational efficiency for real-time applications. Hence, the development of an accurate yet low order model for civil structures is necessary for both analysis and control purposes.

A low order controller can be obtained either by the reduction of a high order controller (which is designed based on high order plant model), or by directly designing a low order controller from the high order plant model. Alternatively, a low order controller can be indirectly designed by obtaining a reduced (low order) plant model in the first place. As mentioned above, designing a high order controller based on a high order plant model is not always feasible, especially for complex structures. Controller reduction itself is also a problem that should be studied to guarantee that the reduced order controller can avoid control and observer spillover problems [2] when it is applied to the original high order plant. The method of designing a low order controller directly from the high order plant has had some success, particularly when heuristic approaches are used [3], but it still faces many challenges. Therefore, the indirect method of designing a low order controller, by first reducing the order of the plant model, is still the most practical and effective method at present.

In control engineering, model reduction in terms of balanced truncation and Hankel norm approximation algorithms for state-space modeling is often used. However, in vibration control of civil structures, especially for tall buildings, these methods still require excessive computational efforts. Therefore, condensation techniques for plant model reduction of tall structures in second order form are often used. The static condensation method is presented in [4]. This method is only exact for static analysis, and often lacks accuracy for dynamic analysis, especially for the high frequency range. A mode-displacement method was presented in [5]. This method can produce a lower order plant model in terms of retained model coefficients, natural frequencies and a few modal coordinates. However, this method can only work well when the complete plant system remains in principal coordinates, and is inapplicable to a real physical plant. Zhang [6] presented a dynamic condensation

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method, which retains the dynamic characteristics of the original plant in an accurate manner, by selectively keeping only those few modes that are of interest. Three different model reduction methods are compared in [7] by computer simulation and it is concluded that the dynamic model reduction method performs in a superior manner.

In this paper, we will first use the dynamic model reduction method to obtain a reduced order plant model of the 20-storey building; we will then design a H_∞ controller based on the reduced plant model; and finally, we will apply the designed low order controller to the full order building model to reduce the excessive vibration excited by a seismic disturbance. Because the lower frequencies of the full order model can be described exactly by the reduced order model, the H_∞ controller designed from the low order model can be used to control the full order model without influencing the high residual modes that cause the control and observer spillover problems. The results of numerical simulations are presented at the end of this paper, to validate the acceptance of the reduced order model and the performance of the designed controller.

II. Reduced order model

A 20-storey experimental tall building model was set up in the laboratory to simulate a real tall building structure as shown in Fig. 1. An active mass damper was installed on the top storey to supply the active control force. The building model consists of 20 lumped mass floors which are each separated equally and are supported by two elastic steel columns. The two steel columns supply the stiffness and damping of the structure and are more representative of a real building when compared to a previous single column design. The Finite Element Method (FEM) was used to create the full (high) order mathematical model of the building (plant) with two degrees of freedom per floor. The influence coefficient method (for continuous structures) was also employed in this study, to obtain very similar results as the FEM, with the advantage that it uses only half of the degrees of freedom that the FEM requires. The linearized one-dimensional equation of motion for the 20 degree-of-freedom (DOF) structure equipped with active mass damper and subjected to earthquake excitation can be written as

$$\overline{M}\ddot{X}(t) + \overline{C}\dot{X}(t) + \overline{K}X(t) = Hu(t) + E\ddot{x}_g(t) \tag{1}$$

where $X(t)$ is the relative displacement of each floor with respect to ground; $u(t)$ is the control force; H defines the location of the control force; $\ddot{x}_g(t)$ is the earthquake ground acceleration; E denotes the influence of earthquake excitation; $\overline{M}$, $\overline{C}$, and $\overline{K}$ are the mass, damping, and stiffness matrices of the building model, respectively.

The equation of motion can then be converted to a state-space equation as

$$\dot{\bar{x}}(t) = \overline{A}\bar{x}(t) + \overline{B}_1 w(t) + \overline{B}_2 u(t) \tag{2}$$

where $\overline{A} = \begin{bmatrix} 0 & I \\ -\overline{M}^{-1}\overline{K} & -\overline{M}^{-1}\overline{C} \end{bmatrix}$, $\overline{B}_1 = \begin{bmatrix} 0 \\ \overline{M}^{-1}E \end{bmatrix}$, $\overline{B}_2 = \begin{bmatrix} 0 \\ \overline{M}^{-1}H \end{bmatrix}$, $\bar{x}(t) = \begin{bmatrix} X(t) \\ \dot{X}(t) \end{bmatrix}$, $w(t) = \ddot{x}_g(t)$.

For the obtained high order model in equation (2), which is a 40 DOF state-space equation without active mass damper, a feasible controller could not be found after several trials due to the large size of the model. In practice, it is very difficult to obtain a high order model for a real plant exactly. Even if one manages to obtain a high order model, it cannot be expected to control all modes of the plant especially the highest ones. Therefore, the design of a high order controller based on a high order plant model is not a practical solution in this case. In addition to this, the high order control model cannot be reduced based on its state-space equation (by using the state-space equation model reduction approach), such as balanced truncation method and Hankel norm approximation method, due to the computational challenge. Therefore, a model reduction method in terms of physical parameters such as mass, damping and stiffness parameters must be used first, before the state-space equation can be used. For doing so, Guyan [4] presented a static model reduction method to reduce the size of mass and stiffness matrices. However, the reduced mass matrix produced by this method does not preserve its accuracy while the reduced stiffness matrix does. In [5], a mode-displacement method was presented. It can produce a lower order plant model in terms of retained model coefficients, natural frequencies and a few of the modal coordinates. The mode-displacement works well when the complete plant system remains in principal coordinates, but it is inapplicable to a real physical plant model. Zhang [6] presented a dynamic model reduction method that can produce the reduced model formulated from condensed mass, damping and stiffness coefficient matrices and retain a small number of lowest modes of the original system. After comparing the three different model reduction methods with experimental validation in [8], the dynamic model reduction method is proven to be applicable in both the theoretical analysis and the experimental test. Therefore, this method will be used here to obtain the reduced order model for our control purposes.

In most engineering applications, it is recommended that the lowest few natural frequencies and the corresponding modes of the original structural system be kept in the reduced model. For the system described in (1), if we choose n_c degrees of freedom of the original structural system to be retained in the condensed model, the mass matrix of the condensed model can be determined as [6]

$$M_c = X_c^{-1}\left(-\omega^2 I + B_c\right)^{-1}, \tag{3}$$

where X_c is the displacement vector of order n_c at the chosen master coordinates; ω is the excitation frequency; $B_c = \Phi\Lambda\Phi^{-1}$, where $\Lambda = \begin{bmatrix} \lambda_1^2 & 0 & \cdots & 0 \\ 0 & \lambda_2^2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \lambda_{n_c}^2 \end{bmatrix}$, $\Phi = \begin{bmatrix} \phi_{11} & \phi_{12} & \cdots & \phi_{1n_c} \\ \phi_{21} & \phi_{22} & \cdots & \phi_{2n_c} \\ \vdots & \vdots & \vdots & \vdots \\ \phi_{n_c1} & \phi_{n_c2} & \cdots & \phi_{n_cn_c} \end{bmatrix}$,

and λ_i represents the *i*th natural frequency, ϕ_{ji} represents the modal coefficient at *j*th master coordinate. Consequently, the condensed stiffness matrix is determined as $K_c = M_c B_c$. As damping always exists in actual structural systems and is difficult to be modelled accurately, the level of modal damping is determined by experience or by experimental modal testing on the system. Then, the original system in equation (1) is reduced to

$$M_c \ddot{X}_c(t) + C_c \dot{X}_c(t) + K_c X_c(t) = H_c u(t) + E_c \ddot{x}_g(t) \tag{4}$$

where M_c, C_c, K_c, H_c, and E_c are corresponding condensed matrices; $X_c(t)$ is the condensed displacement variable. Hence, the equation of motion for the reduced order model (4) is converted to a state-space equation as

$$\dot{x}(t) = Ax(t) + B_1 w(t) + B_2 u(t) \tag{5}$$

where $A = \begin{bmatrix} 0 & I \\ -M_c^{-1}K_c & -M_c^{-1}C_c \end{bmatrix}$, $B_1 = \begin{bmatrix} 0 \\ M_c^{-1}E_c \end{bmatrix}$, $B_2 = \begin{bmatrix} 0 \\ M_c^{-1}H_c \end{bmatrix}$, $x(t) = \begin{bmatrix} X_c(t) \\ \dot{X}_c(t) \end{bmatrix}$.

In this paper, the three lowest natural frequencies and the corresponding modes of the 20-storey building model were chosen to be retained in the reduced order model. The 7^{th}, 13^{th}, and 20^{th} original coordinates were chosen as the master coordinates.

For comparison, we list the calculated natural frequencies of the lowest three modes by different methods in Table I. In which, 'Full Order Model' represented the full order model obtained by FEM; 'DMRM' indicates the reduced order model obtained by the dynamic model reduction method [6]; 'Guyan' indicates the reduced order model obtained by the method presented in [4]; and 'Mode-Displacement' indicates the reduced order model obtained by the method presented in [5]. From Table I we can see that the dynamic model reduction method can obtain the accurate low frequencies compared with the full order model and has no problems with the coordinate transformation.

The open-loop dynamic response to seismic excitation for the low order model and the full order model are also compared (the results are discussed under Section IV). It is made clear by the results that the low order (reduced) plant model produces very similar response output in terms of displacement, velocity and acceleration, with those produced by the full order plant model. This confirms that the reduced order plant model is representative of the full order plant model in the low frequency range and can be successfully used for the controller design task.

III. Controller Design

In this paper, the H_∞ controller will be designed based on the reduced order model and will then be applied to the full order model to evaluate its performance. Due to its robustness, H_∞ control has been applied to many areas and disciplines. For seismic excited civil engineering structures, the H_∞ control using static full state feedback, dynamic output feedback, and static output feedback have all been studied. Due to space limitations, a comparison of control performances of these different control strategies will not be presented here, only that of the H_∞ dynamic output feedback controller. The H_∞ control theory is well known and the detailed derivations can be referred to in literature, therefore only the essential contents will be presented here.

The H_∞ control aims to reduce the effect of disturbance, e.g. seismic excitation $w(t)$, on the interested control output $z(t)$ such that the ratio of the L_2 norm of the control output $z(t)$ to the L_2 norm of the disturbance $w(t)$, with zero initial conditions, is smaller than $\gamma > 0$, that is, $\|z(t)\|_2 < \gamma \|w(t)\|_2$, where γ is the disturbance attenuation performance, and the L_2 norm is defined by

$$\|z(t)\|_2^2 < \int_0^\infty z'(t) z(t) dt, \quad \|w(t)\|_2^2 < \int_0^\infty w'(t) w(t) dt.$$

For this study, we define the top floor acceleration as the control output, $z(t)$, i.e.,

$$z(t) = C_1 x(t) + D_{11} w(t) + D_{12} u(t) \tag{6}$$

where constant matrices C_1, D_{11}, D_{12} are drawn from matrices A, B_1, and B_2 to make $z(t) = \ddot{x}_{20}(t)$. The measured outputs are the relative displacements and velocities of the 7^{th}, 13^{th} and 20^{th} floors with respect to ground, and the relative displacement and velocity of the active mass with respect to top floor. The output then becomes

$$y(t) = C_2 x(t) + D_{21} w(t) + D_{22} u(t) \tag{7}$$

where $C_2 = I_{8\times8}$, $D_{21} = D_{22} = 0_{8\times1}$. Therefore, together with (5), we use the system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_1 w(t) + B_2 u(t) \\ z(t) &= C_1 x(t) + D_{11} w(t) + D_{12} u(t), \\ y(t) &= C_2 x(t) \end{aligned} \tag{8}$$

to design a controller with the form of

$$\dot{\zeta}(t) = A_K \zeta(t) + B_K y(t)$$
$$u(t) = C_K \zeta(t) + D_K y(t) \quad (9)$$

where ξ is controller state variable; A_K , B_K , C_K , D_K are controller matrices to be designed, such that the closed-loop system

$$\dot{x}_{cl}(t) = \widetilde{A} x_{cl}(t) + \widetilde{B} w(t)$$
$$z(t) = \widetilde{C} x_{cl}(t) + \widetilde{D} w(t) \quad (10)$$

where $x_{cl}(t) = [x(t) \quad \zeta(t)]^T$, $\widetilde{A} = \begin{bmatrix} A + B_2 D_K C_2 & B_2 C_K \\ B_K C_2 & A_K \end{bmatrix}$, $\widetilde{B} = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}$, $\widetilde{C} = [C_1 + D_{12} D_K C_2 \quad D_{12} C_K]$, $\widetilde{D} = D_{11}$,

is stable, and the H_∞ norm of the closed-loop transfer function from $w(t)$ to $z(t)$ is $\|T_{zw}\|_\infty = \frac{\|z(t)\|_2}{\|w(t)\|_2} < \gamma$ for $\gamma > 0$. If γ is minimized, then controller (9) is the optimal H_∞ controller.

By virtue of the Bounded Real Lemma, $\widetilde{A}$ is stable and $\|T_{zw}\|_\infty < \gamma$ if and only if there exists a symmetric matrix $P > 0$ with

$$\begin{bmatrix} \widetilde{A}P + P\widetilde{A}^T & \widetilde{B} & P\widetilde{C}^T \\ \widetilde{B}^T & -\gamma I & \widetilde{D}^T \\ \widetilde{C}P & \widetilde{D} & -\gamma I \end{bmatrix} < 0 . \quad (11)$$

This is a linear matrix inequality (LMI) for $P > 0$ and can be easily resolved by the Matlab LMI toolbox. Then, the controller (9) can be obtained from the solution of LMI [9].

IV. NUMERICAL RESULTS

In this section, the 20-storey building model is used to illustrate the application of the reduced order control for the seismic excited structures.

The parameters for the 20-storey building model are designed as follows: total lumped mass of each floor is 29 kg; the length, width, and height of the total lumped mass per floor is 354mm, 228mm, and 50mm respectively; the two columns are made from 100mm × 5mm bright flat steel, and they are placed 100mm apart; the unclamped length of the columns, i.e. the effective height of the building model, is 2.5m; the distance between each floor is 76mm; the active mass is approximately 22kg. The active mass is connected to the top floor by a linear motor. The linear motor forms part of the twentieth floor of the building and it provides the control force between itself and the active mass (21st floor).

After obtaining the abovementioned building full order mathematical model (40 DOF) by the finite element method, or the 20 DOF model, by the influence coefficient method, the reduced order model (3 DOF) can be formed. As discussed previously, it only includes the lowest three modes of the original model, and is obtained by using the dynamic model reduction method presented in Section II. The H_∞ controller, which aims to minimize the top floor acceleration when the building is subjected to seismic excitation, is then designed based on this reduced order model. Finally, the low order controller is applied to the full order plant model to mitigate the effects of earthquake disturbances.

The open-loop and closed-loop frequency responses from the ground acceleration to the top floor acceleration for the full order model (20 DOF) and the reduced order model (3 DOF) are plotted in Fig. 2(a) and Fig. 2(b), respectively. It can be seen clearly in Fig. 2, that the designed controller effectively provides active damping to the lowest three resonance frequencies for both the full order model and the reduced order model. However, the controller does not influence the high order frequencies of the full order model too much and the control spillover problem is consequently avoided.

For the simulations of the time responses for both of open-loop and closed-loop systems, the recorded El Centro earthquake data was used. The original data had many dominate low frequency components and was sampled at 50 Hz. In order to shift the dominate frequency components to a broad range, the original earthquake data was re-sampled with sampling rate 400 Hz. The re-sampled earthquake signal is shown in Fig. 3.

Due to space limitations, only the time responses of displacement, velocity, and acceleration due to earthquake excitation, for the top floor, are plotted in Figs. 4-6, respectively. To confirm that the reduced order model retains accuracy in response output with those of the full order model, the time responses for the reduced order model are plotted as well. It can be seen in Figs. 4(a)-6(a) that the open-loop responses of the reduced order model and of the full order model are nearly the same, especially the displacement and velocity responses. The bigger difference existed in the top floor acceleration is mainly induced by the high order mode frequencies. Comparing the closed-loop responses shown in Figs. 4(a)-6(a) with the open-loop responses shown in Figs. 4(b)-6(b), we can see that the closed-loop responses are significantly reduced due to the application of active damping. Hence, the seismic excited vibration is successfully suppressed.

The peak response quantities of the 20-storey building model with and without the active mass damper are presented in Table II, denoted by ‘open-loop’ and ‘closed-loop’, respectively. In Table II, ‘Fl’ denotes the floor number, ‘Disp’ denotes the peak displacement, ‘Vel’ denotes the peak velocity, and ‘Acc’ denotes the peak acceleration, respectively, of the building model subject to simulated earthquake ground acceleration. As can be seen in Table II, a significant reduction in displacement, velocity, and acceleration is achieved when the building model is equipped with an active mass damper. This confirms again that the low order controller is effective in controlling the vibration of the full order plant model.

V. CONCLUSION

This paper studies the vibration suppression problem for a 20-storey building model subjected to seismic excitation. Due to the high order of the building model, a feasible controller cannot be found by using LMIs, and model reduction methods such as balanced truncation and Hankel norm approximation. The dynamic model reduction method was applied successfully to obtain a reduced order model for the 20-storey building model. A H_∞ controller was then designed based on this low order plant model, and was applied to attenuate the vibration of the original plant model while it was subjected to seismic excitation. Numerical simulations prove that this low order controller can control the higher order plant model very well. The reduced order plant model describes the lowest three modes of the original plant model very accurately. So much so, that the controller design based on the low order plant model can work well with the original model and no control and observer spillover problems are induced. Experimental validation will be done in the next step work.

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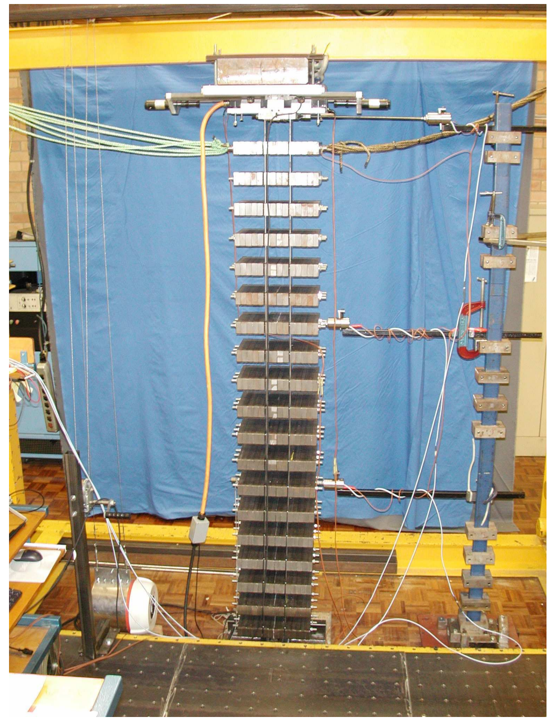
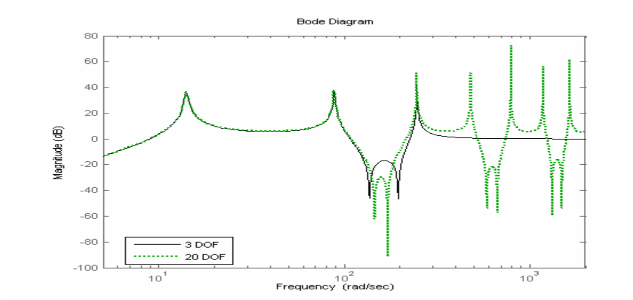
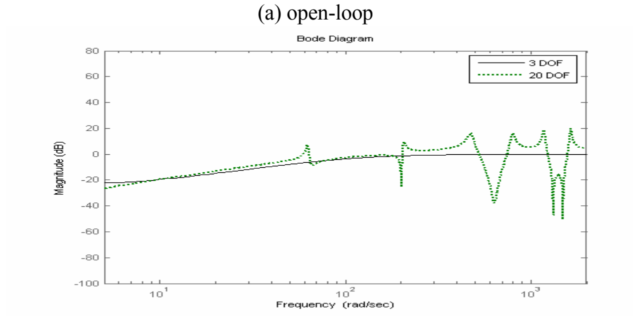


Fig. 1. 20-storey building model with active mass damper



(a) open-loop



(b) closed-loop

Fig. 2. Frequency response from ground acceleration to top floor acceleration

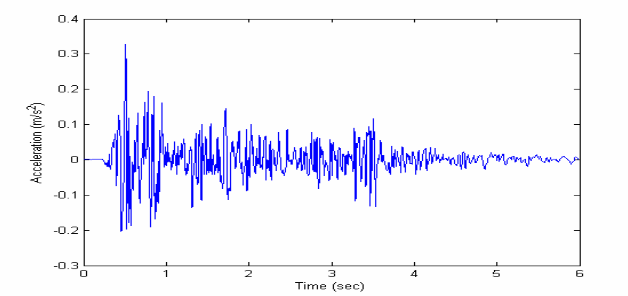
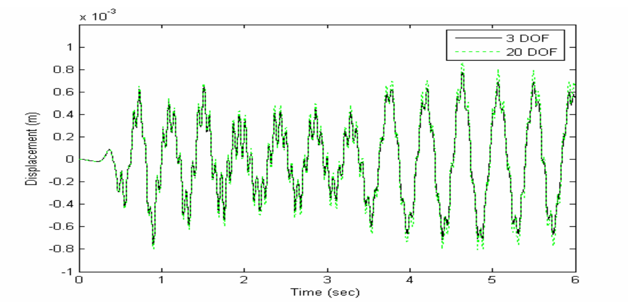
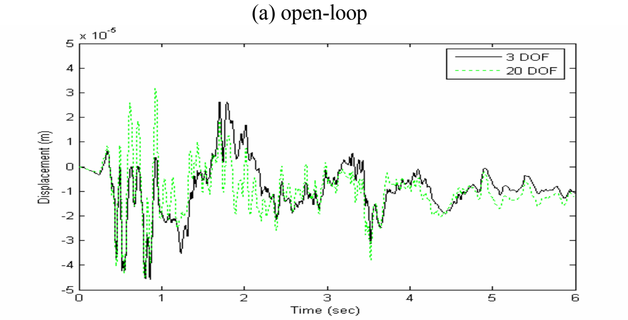


Fig. 3. El Centro earthquake ground acceleration

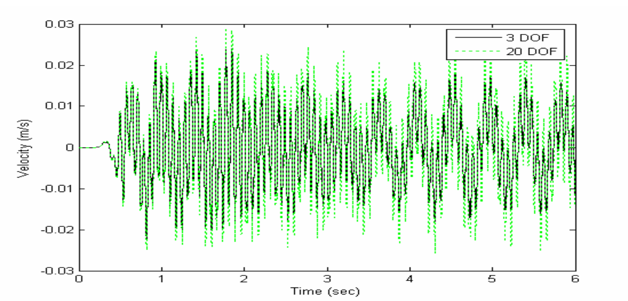


(a) open-loop

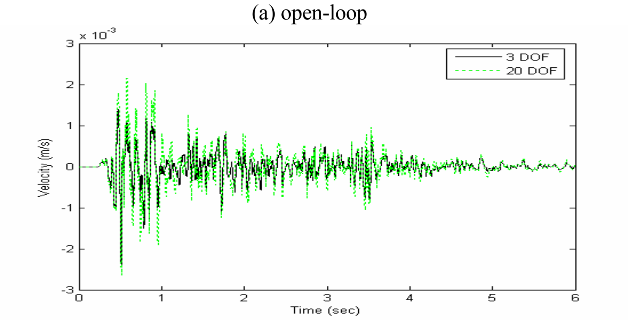


(b) closed-loop

Fig. 4. Displacement of top floor

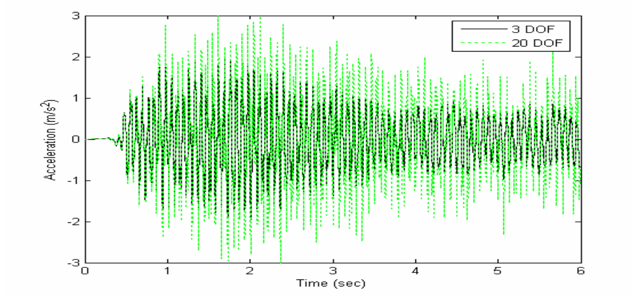


(a) open-loop

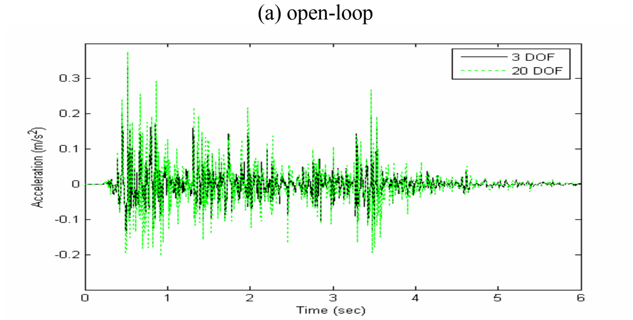


(b) closed-loop

Fig. 5. Velocity of top floor



(a) open-loop



(b) closed-loop

Fig. 6. Acceleration of top floor

TABLE I
COMPARISON OF THREE LOWEST FREQUENCIES

Model	Mode Frequency (rad/sec)		
	1st	2nd	3rd
Full Order Model	14.1548	88.4742	247.1603
DMRM	14.1548	88.4742	247.1603
Guyan	14.1594	88.9778	252.9634
Mode-Displacement	14.1548	88.4742	247.1603

TABLE II
PEAK QUANTITIES FOR EVERY FLOOR

Fl	Open-loop			Closed-loop		
	Disp (mm)	Vel (m/s)	Acc (m/s^2)	Disp (mm)	Vel (m/s)	Acc (m/s^2)
1	0.0082	0.0009	0.2925	0.0057	0.0004	0.2241
2	0.0302	0.0032	0.7005	0.0212	0.0014	0.2839
3	0.0619	0.0061	1.2265	0.0440	0.0029	0.3803
4	0.0993	0.0089	1.6580	0.0719	0.0046	0.5472
5	0.1414	0.0116	1.8684	0.1027	0.0063	0.6975
6	0.1871	0.0138	2.0109	0.1340	0.0080	0.8868
7	0.2331	0.0151	2.0490	0.1639	0.0097	0.9987
8	0.2778	0.0160	2.0627	0.1906	0.0113	1.0344
9	0.3201	0.0161	1.8558	0.2125	0.0127	1.0089
10	0.3597	0.0162	1.5186	0.2282	0.0137	0.9239
11	0.3964	0.0167	1.6804	0.2369	0.0144	0.9135
12	0.4309	0.0162	1.7680	0.2379	0.0146	0.8998
13	0.4631	0.0149	1.7989	0.2311	0.0142	0.8868
14	0.4923	0.0127	1.6675	0.2164	0.0134	0.9503
15	0.5183	0.0106	1.3211	0.1944	0.0121	0.9526
16	0.5419	0.0094	1.1340	0.1658	0.0103	0.8482
17	0.6125	0.0120	1.0937	0.1317	0.0081	0.7082
18	0.6971	0.0155	1.2309	0.0951	0.0065	0.4757
19	0.7835	0.0219	1.9762	0.0657	0.0046	0.4050
20	0.8708	0.0291	3.1788	0.0457	0.0026	0.4191