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Keywords
abductive case based reasoning, deductive CBR, systems diagnosis, clinical reasoning

Disciplines
Business | Social and Behavioral Sciences

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Abductive Case Based Reasoning

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Abstract: This article will introduce abductive case-based reasoning (CBR) and attempt to show that abductive CBR and deductive CBR can be integrated in clinical process and problem solving. Then it provides a unified formalization for integration of abduction, abductive CBR, deduction and deductive CBR. This article also investigates abductive case retrieval and deductive case retrieval using similarity relations, fuzzy similarity relations and similarity metrics. The proposed approach demonstrates that the integration of deductive CBR and abductive CBR is of practical significance in problem solving such as system diagnosis and analysis, and will facilitate research of abductive CBR and deductive CBR.

1 Introduction

As is well known, abduction and deduction play a fundamental role in problem solving [2][10]. In particular abduction seems to be a basic reasoning component in activities such as system explanation [28] and diagnosis [10][63] as well as system analysis[1]. Abduction is becoming an increasingly popular term in many fields of AI, such as system diagnosis, planning, natural language processing and motivation analysis, and logic programming [33][46][10][28][63]. Kindler et al. applied abduction and deduction to the laboratory medicine problem solving process [26]. Console et al. [10] introduced an interesting relation

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1. Simon HA observed that abduction is also the main subject of the theory of problem-solving [33] (p.94).

between abduction and deduction and showed that abduction can be reduced to deduction on a transformed (completed) domain theory that explicitly contains the assumption that all the direct explanations of an event have been represented. Recently, CBR has been shown to play an important role in explanatory or abductive reasoning tasks like diagnosis and explanation [44][55]. For example, Portinale and Torasso [44] developed a diagnostic system ADAPtER (abductive diagnosis through adaptation of past episodes for re-use), which combines case-based and abductive reasoning. However, they focused more on model-based diagnosis rather than abduction [44]. Another example is case-based explanation [28]. In most AI views, explanations are treated as deductive proofs [55]. Abductive reasoning systems build their proofs by non-deductive methods, and additional assumptions may be required for those proofs to apply [28]. However, their view is fundamentally the same in that if the abductive assumptions were shown to be true, the resulting explanation would be considered a deductive proof [55]. Abduction has been also drawn increasing attention in philosophy and cognitive science. For example, Magnani [33] integrates philosophical, cognitive and computational issues on abduction, examines some cases of reasoning in science and medicine, shows the connections between abduction, induction, and deduction, and argues that abduction is a logic of scientific discovery [33]. However, his investigation is basically from a viewpoint of philosophy and cognitive science. Case-based reasoning also has no influence on his study on abduction.

The case-based approach explicitly treats explanations as plausible reasoning chains that may be implicit. However, there is a lack of a theoretical treatment towards integration of deductive CBR and abductive CBR, although some researchers paid attention to an abductive basis for CBR [24]. There is also no unified treatment of the relationship between abduction, deduction, and CBR. This article attempts to show that abductive CBR and deductive CBR can be integrated in clinical process and problem solving. Then it provides a unified formalization for integration of abduction, abductive CBR, deduction and deductive CBR. This article also proposes the transformation from abduction to abductive CBR and from deduction to deductive CBR. This article finally investigates abductive case retrieval and deductive case retrieval using similarity relations, fuzzy similarity relations and similarity metrics. It should be noted that we divide case retrieval into abductive case retrieval and

deductive case retrieval. Case retrieval in abductive CBR is called abductive case retrieval, whereas case retrieval in deductive CBR is called deductive case retrieval.

The rest of this article is organized as follows: Section 2 examines abduction and deduction with two examples. Section 3 examines abductive CBR and deductive CBR as an extension of abduction and deduction respectively. Section 4 integrates abductive CBR and deductive CBR in clinical process and problem solving. Section 5 reviews similarity relations, fuzzy similarity relations and similarity metrics. Section 6 investigates abductive case retrieval and deductive case retrieval using similarity relations, fuzzy similarity and similarity metrics. Section 7 ends this article with a few concluding remarks.

2 Abduction and Deduction

This section will examine abductive reasoning and deductive reasoning with two examples and show that clinical reasoning and problem solving in general can be considered as an integration of abductive reasoning and deductive reasoning [55]. At first, it examines abduction and deduction in clinical processes. The goal here is not clinical data or knowledge modelling, but only a computational or logical understanding.

2.1 Abduction and Deduction in Clinical Processes

As is well known, the clinical process basically consists of the diagnosis and treatment of patients. Diagnosis is a process of finding possible diseases covering most symptoms and differentiation between the remaining explanations [33] (p.83). Diagnosis is also a judgement (or explanation) about what a particular illness is, made after making an examination of the symptoms of a patient. Its goal is to explain symptoms observed from the patient in the clinic [63]. The explanations for the observed symptoms are the basis for treatment. Treatment is a concrete solution to the illness of the patient based on the explanation descriptions of the diagnosis.

Example 1. Consider a concrete case of diagnosis and treatment happening in a normal day in the clinic. The doctor examines the patient and gets:

Symptom: dizziness.

He has the following medical knowledge (domain theory):

$$\{ \text{flu} \rightarrow \text{fever, infection} \rightarrow \text{fever, fever} \rightarrow \text{dizziness, fever} \rightarrow \text{no interest in eating} \}$$

and diagnoses that the patient has flu and tells the explanation to the patient. Then he completes the prescription which includes 10 tablets of "Aspirin" as he has the medical knowledge "$\text{flu} \rightarrow \text{Aspirin}$".

During the above process, the doctor has used two different reasoning paradigms: abductive reasoning and deductive reasoning. From a logical viewpoint, his diagnosis result is following the process of abductive reasoning:

He derives the explanation, "fever", from the symptom, "dizziness" and his knowledge "$\text{fever} \rightarrow \text{dizziness}$." Then he derives the explanation, "flu," from the just derived explanation, "fever," and his knowledge "$\text{flu} \rightarrow \text{fever}$." Therefore, his reasoning towards the satisfactory diagnosis is following the model of abduction or abductive reasoning [46][63]:

$$\begin{array}{r} P \rightarrow Q \\ \underline{\qquad Q} \\ \therefore P \end{array} \tag{1}$$

where P and Q represent compound propositions in a general setting. P is sometimes called a hypothesis in order to emphasize the difference between abduction and deduction, thus abduction provides a basis for hypothetical reasoning systems [20]. In medical diagnosis, $P \rightarrow Q$ is a form of general relation: disease $\rightarrow$ symptom.

Example 2. Another example of abduction is borrowed from [33] (p. 21) and has been heavily revised. It is a very simple example dealing with diagnostic reasoning. We will begin with the situation as it might be described in English: the knowledge (sentences) in the knowledge base includes:

1. If a patient is affected by a pneumonia, his/her level of white blood cells is increased.
2. John is affected by a pneumonia.
3. John's level of white blood cells is increased.

What we wish is to prove "John is affected by a pneumonia" using abduction (1). We first represent these facts in a first-order logic, and then show the proof as a sequence of applying abduction (1). To this end, let $P(x)$: x is affected by a pneumonia, $Q(x)$: x's level of white

blood cells is increased, $P(John)$: *John* is affected by a pneumonia*,* $Q(John)$: *John*'s level of white blood cells is increased. Then the above example can be formalized as:

1'. $\forall x(P(x) \to Q(x))$

2'. $P(John)$

3'. $Q(John)$

We use the substitution {*x/*John} (for detail see [48]) or an inference rule (elimination of quantifier) [45] and infer:

4'. $P(John) \to Q(John)$

From (4') and (3'), and abduction (1), we infer (2'); that is, John is affected by a pneumonia.

The above examples of diagnostic reasoning are an excellent way to introduce abduction [33] (p.18). In fact, abduction goes back to more than a hundred years ago. At that time, the American philosopher Charles Sanders Peirce defined "Abduction" as inference that involves the generation and evaluation of an explanatory hypothesis. Abduction is also fundamentally important for intelligent problem solving tasks such as diagnosis, natural language interpretation, hypothetical reasoning and (inductive) logic programming [20][34].

From an epistemological viewpoint [33] (p.25), there are two main meanings of the word abduction: (1) abduction is only to generate plausible hypotheses (selective or creative) and (2) abduction can be considered as an inference to the best explanation, which also evaluates hypotheses- This is the meaning of abduction accepted in this article, whereas the first meaning of abduction was accepted by Magnani [33]. More generally speaking, abduction is a method of scientific discovery, because scientific research consists of three stages: abduction, deduction and induction [22].

The study of abductive inference was slow to develop, as logicians concentrated on deductive inference and on inductive logic based on formal calculus [33] (ix, p.19). Abduction is the term currently used in the AI community for explanation-based generalization for a set of events from a given domain theory [9][22][23][55]. More specifically, abduction is the process of inferring certain facts and/or laws and hypothesises that render some sentences plausible, that explain or discover some (eventually new)

phenomenon or observation; it is the process of reasoning in which explanatory hypothesises are formed and evaluated [33] (p.18).

From a logical point of view, abduction is an unsound reasoning [20][46][63]. However, it has similar properties to those of other nonmonotonic logics proposed and studied in the AI literature [34]. For example, it shares declarative and computational properties with other forms of nonmonotonic reasoning [55]. Thus, abduction is a very useful kind of nonmonotonic reasoning, in particular for logic programming [22], and reasoning towards explanation in (system) diagnosis [63] and analysis in problem solving, which will be examined in more detail later.

It should be noted that most current rule-based diagnosis systems use knowledge of the form [43]:

$$\text{observation and knowledge of situation} \quad \rightarrow \quad \text{problem}$$

to express knowledge about the potential cause of an observation. For example, MYCIN expresses its knowledge in terms of rules of the form:

$$\text{symptom} \quad \rightarrow \quad \text{disease [CF]}$$

where CF is a certainty factor that represents a subjective evaluation of the rule's quality. The diagnosis task consists of matching rule symptoms and observed symptoms, accumulating the conclusions suggested by relevant rules, and ranking the conclusions by a simple arithmetic function on certainty factors. However, the rules above are the wrong way around: diseases result in symptoms, rather than symptoms in diseases. In other words, a diagnosis is not a logical consequence of our observations about a patient. In fact, exactly the opposite is the case, it is the observations that should be shown to be logical consequence of our knowledge and the diagnosis. Based on this idea, Poole and Goebel [43] uses an alternative formulation of the rules in their system, Theorist, which is a logic programming system that uses a uniform deductive reasoning mechanism to construct explanations of observations in terms of facts and hypotheses:

$$\text{problem} \quad \rightarrow \quad \text{observation}$$

where knowledge is expressed in terms of problems and the observations that consequently arise. For example, a medical diagnosis task would use rules of the form:

$$\text{disease} \quad \rightarrow \quad \text{symptom}$$

to encode the observable symptoms of diseases. This form of representation is more appropriate for expressing textbook knowledge of diseases, as it records what is known without any requirement for heuristic measures like certainty factors and experience record. Therefore, Poole and Goebel [43] have similar ideas to that in this research about diagnosis. However, they have not generalized their idea to a more general reasoning paradigm: abduction or abductive reasoning.

Let us turn to example 1. After having obtained the precise explanation for the symptoms of the patient, the doctor derives the treatment from the explanation "flu" and his knowledge of treatment "flu $\rightarrow$ Aspirin;" that is, "10 tablets of Aspirin." This is deductive reasoning, its general model is well-known *modus ponens (m.p.)*:

$$\begin{array}{l} P \rightarrow Q \\ \underline{P \qquad\quad} \\ \therefore\ Q \end{array} \tag{2}$$

where P and Q represent compound propositions in a general setting.

Deduction is a fundamental reasoning paradigm in traditional logic and mathematics [45]. It also has widespread applications in almost every academic field [55].

So far, this section has shown that the clinical process is an integration of abductive reasoning and deductive reasoning, as shown in Fig. 1. The cycle in Fig. 1 starts with the

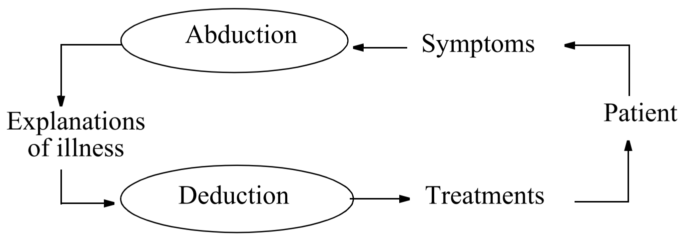

Fig. 1. Integration of abduction and deduction in clinical process [55]

patient showing symptoms and completes with treatments. Further, in a more general setting,

diagnosis is a process of obtaining a satisfactory explanation for a particular problem, made after making an examination of a system with the presence of some faults. Therefore, abductive reasoning can be applied to more general situations such as system diagnosis. The rest of this section will look into abduction and deduction in problem solving.

It should be noted that medical diagnostic reasoning can be described in terms of abduction, deduction and induction, which was given by Magnani in his epistemological model of diagnostic reasoning [33] (p.23), where induction is the final testing of an abduced hypothesis: by completing the whole cycle of the epistemological model it produces the best explanation. In order to keep "symmetry" of abduction and deduction in this article, we do not go into induction, its function will be realized by case adaptation or case revision (see later).

2.2 Abduction and Deduction in Problem Solving

The notation of explanation and analysis is basic in many human behaviors. In particular, in any intelligent system such as an expert system there is a subsystem to explain the reasoning process to the user. Reasoning towards explanation and analysis is a fundamental task in many of problem solving activities investigated by the AI community [63]. In what follows, problem solving is decomposed into analysis and reasoning. The analysis process in problem solving will be shown as mainly abductive reasoning, whereas the reasoning process is mainly deductive reasoning, with an example in propositional logic, borrowed from [53].

Example 3. Prove $P \to \neg\neg Q, \neg R \to P, \neg Q \Rightarrow R$

As is known, this is a non-trivial problem for an undergraduate student studying propositional logic [55]. It is better to decompose this problem solving into two phases: analysis and reasoning: Abductive reasoning is performed in the analysis phase, whereas deductive reasoning is usually performed in the reasoning phase, as discussed in Ref. [55]:

1. Analysis

(1) Because of R, $\neg R \to P$ is transformed into $\neg P \to R$. Then $\neg P$ is derived from $\neg P \to R$ and R based on abductive reasoning.

(2) Because of $\neg P$, $P \to \neg\neg Q$ is transformed into $\neg Q \to \neg P$. Then performing abductive reasoning, $\neg Q$ is derived from $\neg Q \to \neg P$ and $\neg P$.

(3) Because in the hypotheses there is also $\neg Q$. Thus it can conclude that this formula is provable and the analysis phase is finished.

It should be noted that the above transformations are logically equivalent. Further, it is interesting to note that during the analysis phase, the abductive reasoning chain (in reverse) is obtained without taking logical equivalence formulas into account:

$$\neg Q,\ \neg Q \to \neg P,\ \neg P,\ \neg P \to R,\ R.$$

which is just the main deductive reasoning chain in the below reasoning phase. Therefore, after having obtained the abductive reasoning chain from R to $\neg Q$, it is easy to prove the formula under consideration.

2. Reasoning

Based on the results of above abductions in the analysis phase, this phase performs deductive reasoning as that in propositional logic.

Proof	Explanations
1. $\neg Q$	(hypothesis)
2. $P \to \neg\neg Q$	(hypothesis)
3. $\neg Q \to \neg P$	(contrapositive (2))
4. $\neg P$	(m.p. (1), (3))
5. $\neg R \to P$	(hypothesis)
6. $\neg P \to R$	(contrapositive)
7. R	(m.p. (4), (6))

Deduction is an inference that refers to a logical implication. Differing from abduction, in deduction the truth of the conclusion of the inference is guaranteed by the truth of the premises on which it is based [33] (p.21). Generally speaking, many textbooks do not discuss the analysis phase in problem solving. They usually only provide standard solutions to problems. However, this is insufficient for class teaching. During teaching, lecturers

sometimes have to use such methods to instruct the students to improve their ability of analysing and solving problems. Therefore, problem solving can also be considered as an integration of abductive reasoning and deductive reasoning: Abductive reasoning is performed to get a satisfactory analysis in order to perform deductive reasoning to solve the problem. In other words, abductive reasoning is a necessary condition for performing deductive reasoning towards problem solving in some cases[1].

Forward chaining and backward chaining are well-known concepts in AI [48] (p.272). More specifically, one can start with the sentences in the knowledge base and generate new conclusions that in turn can allow more inferences to be made. This is called *forward chaining*. Forward chaining is usually used when a new fact is added to the knowledge base and its consequences should be generated. The theoretical foundation of forward chaining is previously mentioned *modus ponens*. Alternatively, one can start with something that is to be proved, find implication sentences that would allow one to conclude it, and then attempt to establish their premises in turn. This is called *backward chaining*, because it uses *modus ponens* backwards. Backward chaining is normally used when there is a goal to be proved, according to Russell and Norvig [48]. Backward chaining is commonly used in RBESs to enable a hypothesis to be tested and explained [66] (pp.8-9). This mimics human problem-solving strategies, for example, in medical diagnosis. When one is sick, his doctor often hypothesizes using knowledge of what the possible cause of his illness may be. Doctors then try to confirm their hypothesis by looking for characteristic symptoms or by performing certain tests. If these do not confirm their hypothesis, they will think of another illness and test that hypothesis. This problem solving strategy is often referred to as *generate and test* and has been used successfully by many expert systems, particularly in diagnosis. However, based on the above discussion, the theoretical foundation of backward chaining is the basic reasoning model of abduction (see (1)). Furthermore, the theoretical foundation of Prolog and most other logic programming languages is also abduction, because they are based on backward chaining [48] (p.313). Therefore, this subsection proposes a new insight into

1. It is also easy for the readers to give a figure here similar to Fig. 1

backward chaining and its relation to Prolog and most other logic programming languages: abduction and deduction exist in AI in a "symmetric" way.

Production system rule interpreters that start with facts that are matched to the left-hand side (LHS) term(s) of production rules are called forward chaining production systems. Production system rule interpreters that start with desired goal states that are matched to the right-hand side (RHS) term(s) of production rules are called backward chaining production systems [67] (pp.156-157). The forward chaining production systems are based on deductive reasoning, whereas the backward chaining production systems are based on abductive reasoning from a logical viewpoint (for more detail see [48] (pp.272-275)).

3 Abductive CBR and Deductive CBR

This subsection will demonstrate that abductive CBR and deductive CBR are an extension of abductive reasoning and deductive reasoning respectively. It then shows that abductive CBR and deductive CBR can be integrated in diagnosis, explanation, and problem solving. It begins with the evolution from abduction to abductive CBR.

3.1 From Abduction to Abductive CBR

As has been previously shown, abductive reasoning is a kind of explanation-oriented reasoning [63]. Diagnosis is a process of deriving an explanation of the symptoms based on the observations by the doctor of the patient and it can be considered as an abductive reasoning [10][55]. In fact, in clinical practice, a doctor usually first observes a particular patient's mouth, eyes, and body temperature, etc. and gets all possible symptoms of the patient. These symptoms can trigger a reminder of previous cases he has met. Prior experiences then play an important role in getting the exact explanation for the symptoms of the patient. Therefore, diagnosis or the process of explaining the symptoms is not only abduction, but also an experience-based reasoning. The experiences also play a pivotal role in the analysis phase of the problem solving. For example [55], why was $P \rightarrow \neg\neg Q$ not at first selected in the last section for analysis? Because CBR means reasoning based on previous cases or experiences, abductive CBR can be considered as the reasoning combining abduction and experience-based reasoning, briefly:

$$\text{Abductive CBR} = \text{Abduction} + \text{Experience-based reasoning} \tag{3}$$

An important experience principle in the diagnosis is "most similar symptoms result from most similar illness". Based on this principle, the doctor comes to the conclusion that the illness of the patient is most similar to the illness that he experienced last week. This is not only an experience-based reasoning but also a similarity-based reasoning. Thus, similarity-based reasoning is an operational form of experience-based reasoning [55]. In fact, similarity-based reasoning has played an important role in experience-based reasoning as shown in the CBR literature [18][27][30][55]. Therefore Eq.(3) can be specialized as a form of reasoning combining abduction and similarity-based reasoning:

$$\text{Abductive CBR} = \text{Abduction} + \text{Similarity-based reasoning} \tag{4}$$

Similarity-based reasoning is also very important in performing experience-based reasoning in the analysis phase of problem solving, because common sense is used in problem solving such as in mathematics "Two problems are similar, if they have similar explanations" [55]. For example, in case-based explanation, the first criterion for selecting likely explanations is experience in similar situations: Explanations of new situations are considered most plausible if they have applied in similar prior situations [28]. Therefore, the analysis of problem solving can be considered as a kind of abductive CBR [55].

Based on Eq.(4), (1) can be extended to the following reasoning model:

$$\begin{array}{l} P \to Q \\ Q \sim Q' \\ \underline{\qquad Q'} \\ \therefore P' \end{array} \tag{5}$$

where P, P', Q, and Q' represent compound propositions, Q and Q' are similar in the sense of a certain similarity (see Section 5). This is a theoretical foundation for abductive CBR [55], in particular for similarity-based abductive case retrieval, which will be examined in more detail in Section 6.

3.2 From Deduction to Deductive CBR

Now the subsection will turn to look at the evolution from deduction to deductive CBR. As was shown in the previous subsection, treatment in the clinical process can be considered as a deductive reasoning. Further, after having obtained a satisfactory diagnosis, the doctor not

only performs deduction but also uses his experience in the past for writing the prescription or performing treatment for the illness of the patient. Thus, experience-based reasoning plays an important role in deductive reasoning such as treatment. In fact, it is obvious that experience also plays a pivotal role in the reasoning phase of problem solving. For example, if $\neg R \to P$ is used as the first step of deductive reasoning in the example of the previous subsection, one might not know which will be the next step [55]. Therefore, deductive CBR can be considered as the form of reasoning combining deduction and experience-based reasoning, briefly [55]:

$$\text{Deductive CBR} = \text{Deduction} + \text{Experience-based reasoning} \tag{6}$$

This research prefers deductive CBR rather than CBR, because CBR[1] is an extension of deductive reasoning from a logical viewpoint [16]. If CBR is only used, one can't see the influence of deductive reasoning on CBR. Further, it seems that there is certain "symmetry" between abductive CBR and deductive CBR.

It is common sense in the clinical process that "similar illnesses usually result in similar treatments". This is not only an experience-based reasoning but also a similarity-based reasoning. Thus, similarity-based reasoning is a special form of experience-based reasoning in the treatment phase of clinical processes. Further, similarity-based reasoning is very important in performing experience-based reasoning in the reasoning phase of problem solving, because there is an experience principle in this phase such as in mathematics "Similar problems have similar solutions" [15][55]. Therefore, Eq.(6) is specialized as a reasoning combining deduction and similarity-based reasoning; that is:

$$\text{Deductive CBR} = \text{Deduction} + \text{Similarity-based reasoning} \tag{7}$$

As is known, CBR solves new problems reapplying the lessons from specific prior reasoning episodes. A functional motivation for CBR is the principle that in a regularity in the world, similar problems have similar solutions [29]. When this principle holds, starting from similar previous solutions can be more effective than reasoning from scratch. Similarly, the functional motivation for abductive CBR is the principle that there is also a regularity in tasks

1.To our knowledge, nobody has examined CBR based on different reasoning paradigms from a logical viewpoint. We divide CBR into abductive CBR and deductive CBR in order to examine the influence of abduction and deduction on CBR.

such as diagnosis or analysis, similar symptoms result from similar illnesses [55]. This principle leads to the conclusion that it is *effective* for generating new explanations by retrieving prior explanations (analysis) for similar symptoms (problems) and adapting those retrieved explanations (analysis) to fit the new symptoms (problems) if possible. However, different understanding of similarity usually leads to different case retrieval and then to different abductive CBR and deductive CBR, which will be discussed in more detail in Section 6.

Eq.(7) can be expressed as the following reasoning model:

$$\begin{array}{l} P' \\ P' \sim P \\ \underline{P \to Q} \\ \therefore\ Q' \end{array} \tag{8}$$

where P, P', Q, and Q' represent compound propositions, Q and Q' are similar in the sense of similarity (see Section 5). This is the basic reasoning model of similarity-based reasoning. (8) is a theoretical foundation for deductive CBR [16], in particular for similarity-based deductive case retrieval (see Section 6).

It should be noted that CBR also has a close relation with memory-based reasoning (MBR) [49] and analogical reasoning (AR), because MBR is often considered a subtype of CBR which can be viewed as fundamentally analogical [30][61]. MBR systems solve problems by retrieving stored cases (precedents) as a starting point for new problem-solving [30] (p 13). However, its primary focus is on the retrieval process, and in particular on the use of parallel retrieval schemes to enable retrieval without conventional index selection. Parallel models can lead to very fast retrieval, but also raise new questions to address about the criteria for knowledge access.

CBR might be viewed as a particular form of analogical reasoning (AR) [13]. The latter has been investigated for a long time in AI and the interest in this research has been considerably renewed by the development of CBR [14]. Further, whereas CBR solves new problems and interprets new situations by applying analogous prior episodes [30] (p 13), research on analogy was originally more concerned with abstract knowledge and structural similarity, whereas research on CBR is more concerned with forming correspondences

between specific episodes based on pragmatic considerations about the usefulness of the result.

However, from the theoretical viewpoint, MBR and AR share the same reasoning paradigm: that is, *generalized modus ponens, or similarity-based reasoning,* although they stem from different real world scenarios [61]. Therefore, the relationship of CBR, MBR, AR, and Fuzzy reasoning can be summarized and shown in Fig. 2, in which deduction provides

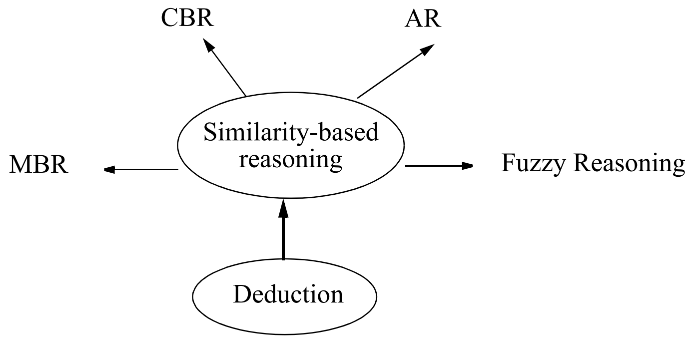

Fig. 2. Basic relations of CBR, MBR, AR, and RBES

the foundation for similarity-based reasoning, which is a basis for MBR, CBR, and AR as well as fuzzy reasoning. This is also the answer to why similarity and similarity assessment are pivotal in the mentioned fields [61].

Further, analogical reasoning has been used in explanation-based generalization (EBG), which is a deductive reasoning[1] from a program which satisfies the input conditions [23], therefore, EBG has currently become a hot topic in CBR.

So far, the relationship between abduction, abductive CBR and deduction, deductive CBR has been discussed respectively from a logical viewpoint. It has also shown that clinical process and problem solving are a form of reasoning combining abductive CBR and deductive CBR, as shown[2] in Fig. 3. Similar to Fig. 1, in Fig. 3 the cycle starts with the patient showing symptoms and completes with treatments. Because deductive CBR is a kind

1. In fact, EBG can be also considered as an abductive reasoning, because EBG is an integration of explanation-based reasoning and generalization. The former can be considered as a kind of abductive reasoning, whereas the latter can be considered as a kind of inductive reasoning. Therefore EBG can be considered as an integration of abductive reasoning and inductive reasoning.
2. It is easy to give a similar diagram for integration of abductive CBR and deductive CBR in problem solving.

of deductive (monotonic) reasoning, whereas abductive CBR is a kind of nonmonotonic reasoning, clinical process and problem solving is then an integration of traditional reasoning and nonmonotonic reasoning.

This also implies that any human professional activities usually involve not one reasoning paradigm but many reasoning paradigms such as abduction and deduction, abductive CBR and deductive CBR. Therefore it is interesting to examine for any concrete professional activity, how many different reasoning paradigms it must use. We believe that this is a necessary step for building an expert (intelligent) system to simulate such a professional activity.

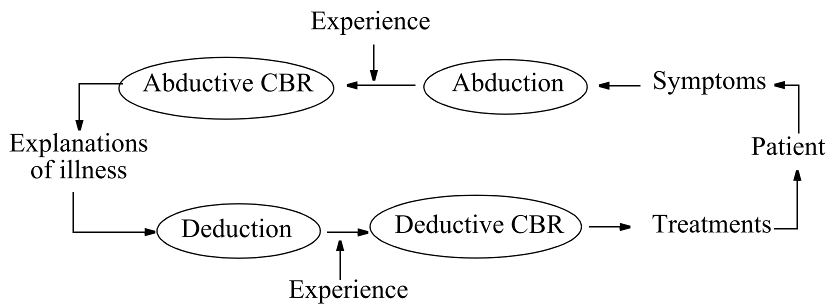

Fig. 3. Integration of abductive CBR and deductive CBR

4 Integration of Abductive CBR and Deductive CBR

This subsection will examine abductive CBR and deductive CBR from a viewpoint of knowledge-based systems (KBSs) and integrate the abductive CBR system and deductive CBR system with a knowledge-based model.

As has been shown, clinical process and problem solving are a form of reasoning combining abductive CBR and deductive CBR from a logical viewpoint. There has been an important influence of KBSs on CBR systems in most (deductive) CBR literature [27][65]. For example, the case base in the CBR system can be considered as a variant of the knowledge base in KBSs [16]. Therefore, a deductive CBR system can be considered as an integration of deductive reasoning and a KBS [55].

From the viewpoint of AI, the systems based on abductive reasoning has also been affected by the research of KBSs [29][46]. Thus, abductive CBR systems can also be considered as an integration of abductive reasoning and KBSs. In fact, CBR-based abduction

has been studied for many years [10][28][29][44][63]. For example, Portinale and Torasso [44] developed a diagnostic system ADAPtER, which tried to combine case-based and abductive reasoning.

Another example is case-based explanation [28][29], which generates new explanations by retrieving explanations of relevant prior episodes and adapting them to fit the new situation in light of the explainer's need for information [29]. The prior experiences of the explainer are fundamental to focusing on search for candidate explanations, and the motivation for explaining is reflected in both the explanation generation and selection processes [55]. Therefore, abductive CBR can be considered as an extension of case-based explanation.

In case-based explanation, the most important criterion for judging plausibility is similarity-based [29]: Explanations of new anomalies are favoured if they are similar to explanations that applied to similar prior anomalies. This similarity judgment is done implicitly through the case retrieval process; retrieval of stored explanations is aimed at retrieving explanations from similar prior situations [28].

Based on the above consideration, Leake proposed a process model for case-based explanation in [29]; that is:

- Problem characterization: Generate a description of what must be explained, i.e., the information that a good explanation must provide
- Explanation retrieval: Use the results of the problem characterization step as an index for retrieving relevant explanations of prior episodes from memory
- Explanation evaluation: Evaluate the retrieved explanations' plausibility and usefulness. Generate problem characterizations for any problems that are found
- Explanation adaptation: If problems were found, use the evaluator's problem characterization to select adaptation strategies for modifying the explanation to repair the problems. Apply the strategies and return to the explanation evaluation phase to evaluate the new explanation.

In what follows, an integrated knowledge-based model for both the abductive CBR system and deductive CBR system is proposed, shown in Fig. 4. This model can also be used

in clinical processes, because diagnosis and treatment in the clinical process is a special case of problem solving [55].

In this model, problem solving is decomposed into analysis and reasoning. Abductive CBR is performed in the analysis process, whereas deductive CBR is performed in the reasoning process [55]. For clarity, an *abductive case base* is used in the abductive CBR system and a deductive case base in the deductive CBR system instead of case base respectively. Similar to the inference engine in expert systems [36], an abductive CBR engine is used in the abductive CBR system and a deductive CBR engine in the deductive CBR system for the reasoning mechanism in each case. However, working memories are ignored in

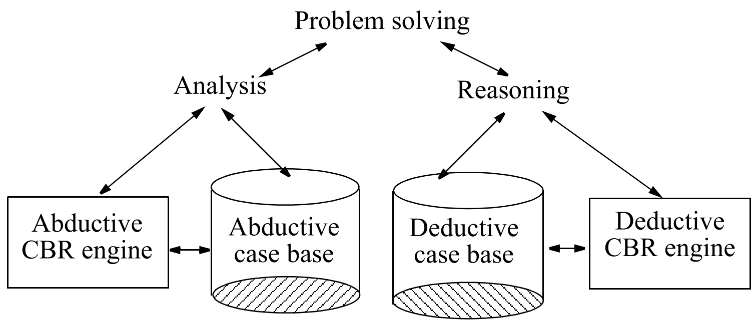

Fig. 4. A knowledge-based model of integrating abductive and deductive CBR

each case in the figure. In fact, it is important that the user interface can differ in the analysis and reasoning phases in any problem solving. In particular, in the user interface, the user should know what the problems are, what the premise set and conclusions are, etc. The user interface might consist of some kind of natural language processing system that allows the user to interact with the system in a limited form of natural language. Therefore the problem solving system consists of two subsystems: One is an abductive CBR system; another is a deductive CBR system. The major part of both systems is the (either abductive or deductive) case base and the abductive CBR engine or deductive CBR engine. In terms of the CBR systems, the abductive case base consists of explanation-oriented facts and rules about the subject or problems at hand; the deductive case base consists of reasoning-oriented facts and rules about the subject or problem solving available. The abductive CBR engine and the deductive CBR engine consist of all the processes that manipulate the (either abductive or

deductive) case base to derive information requested by the user based on either abductive CBR or deductive CBR.

Based on the above discussion, the following process cycle is of significance for abductive CBR systems:

- Repartition for building an abductive case base
- Retrieve the most similar abductive cases
- Reuse the abductive cases to attempt to give the explanation to the problem(s)
- Revise the retrieved abductive cases
- Retain the new abductive case as a part of a new abductive case base.

This is called the R^5 model of an abductive CBR system [55], which corresponds to the R^5 model for deductive CBR systems introduced by the authors [18]. In what follows, we investigate abductive case retrieval and deductive case retrieval based on similarity relations, fuzzy similarity relations and similarity metrics respectively. Both of them are a pivotal part of abductive CBR and deductive CBR.

5 Fundamentals of Abductive and Deductive Case Retrieval

Similarity is the core concept in both abductive and deductive CBR, because it is involved not only in case retrieval but also in case adaptation as well as in case base building [17]. In this section, we review similarity relations, fuzzy similarity relations and similarity metrics, which are all fundamental for investigating not only abductive CBR but also deductive CBR, in particular for abductive case retrieval and deductive case retrieval.

5.1 Similarity Relations

The concept of a similarity is a natural generalization of similarity between two triangles in the plane and between two matrices in mathematics [17]. More precisely:

Definition 1. A binary relation S on a non-empty set X is called a *similarity relation* provided it satisfies

(R) $\forall x, xSx$

(S) if xSy then ySx

(T) if xSy, ySz then xSz

The conditions (R), (S), and (T) are the reflexive, symmetric and transitive laws. If xSy, x and y is called similar, denoted as $x \sim y$ for convenience [47].

It should be noted that the similarity relation proposed here is identical to the equivalence relation in discrete mathematics [47][17]. However, the former is more important than the latter in the context of CBR, because similarity relations rather than equivalence relations play an important role in CBR. Thus, this treatment is different from the idea of Zadeh [68] in that Zadeh considered a similarity relation, which is frequently cited in fuzzy literature without further consideration, as a fuzzy one and as a generalization of the concept of an equivalence relation, whereas this research views Zadeh's similarity relations as fuzzy similarity relations (see the next subsection). Fuzzy similarity relations are a fuzzification of a similarity relation rather than an equivalence relation [17].

5.2 Fuzzy Similarity Relations

As an extension of similarity relations, fuzzy similarity relations were introduced by Zadeh [68] and have attracted research attention since then [5][13][38][69]. Fuzzy similarity relations have been also used in CBR in particular in case retrieval [12][13][41] and case base building [58]. This subsection will examine fuzzy similarity relations from a new viewpoint. For the sake of brevity and simplicity, it uses standard fuzzy set theory notation for operations min $\wedge$, max $\vee$, although there are many alternative choices for these operations available in fuzzy set theory [69]. S is still used to denote a fuzzy similarity relation if there is not any confusion arising.

Definition 2. A fuzzy binary relation, S, on a nonempty set X is a fuzzy similarity relation[1], if it is reflexive, symmetric and transitive [38][68], i.e.,

$$S(x, x) = 1 \tag{9}$$

$$S(x, y) = S(y, x) \tag{10}$$

$$S \geq S \circ S \tag{11}$$

1. The notation $S(x, y)$ is used for the membership $\mu_S(x, y)$, although the latter is commonly used in the fuzzy set literature.

where $\circ$ is the composition operation of fuzzy binary relations based on $\wedge$ and $\vee$ operation.

A more explicit form of Eq.(11) is [68]:

$$S(x, z) \geq \underset{q}{\vee} (S(x, y) \wedge S(y, z)) \tag{12}$$

The revised form of this definition was given by Ovchinnikov in 1991 [38]. Dubois and Prade [13] used the revised form for fuzzy similarity relations directly in 1994. The main difference between the definitions of Zadeh and of Ovchinnikov lies in that instead of Eq.(12), Ovchinnikov viewed the following model as max-min transitivity.

$$S(x, z) \geq S(x, y) \wedge S(y, z) \tag{13}$$

This is simpler than that used by Zadeh, because if the cardinality of the set is less than or equal to 3, then (12) coincides with (13) [17]. This extension has some advantages, if one examines in depth the relation between similarity and metric in the Euclidean space. For detail see [17].

5.3 Similarity Metrics

In CBR, similarity between any two problems or solutions is usually assessed. Thus it is a metric rather than only a relation that should be used to assess the similarity between any two problems, although we usually use similarity measure and similarity metric in CBR in the same way [17]. For this reason we prefer to use the term metric rather than measure in this context to investigate the similarity involved in CBR.

Definition 3. A relation, denoted by S_m, on a non-empty set X, is a similarity metric if it satisfies [17]:

1. S_m is a similarity relation in X
2. 1-S_m is a metric on X; that is, it is a function from $X \times X$ to $[0, 1]$, provided that

- for any $x, y \in X$, $S_m(x, y) = 1$ if and only if $x = y$
- for all $x, y \in X$, $S_m(x, y) = S_m(y, x)$
- for all $x, y, z \in X$,

$$S_m(x, z) \geq S_m(x, y) \wedge S_m(y, z) \tag{14}$$

where $\wedge$ is min operator. (14) in this definition is called the *similarity inequality*.

It should be noted that the similarity metric here, S_m, can not directly satisfy the *triangle inequality* [17]. Eq.(14) is motivated by the concept of fuzzy similarity relations given in [38], which is as Eq.(13) in this section. However, It is easy to show that such 1-S_m is a metric [17]. Further, we emphasize that the similarity metric is at first a traditional similarity relation and then just a metric (maybe to some extent), because we believe that the similarity between two objects is the necessary condition to discuss how similar they are. In practice, our first concern is usually if x and y are similar, then we would further ask how similar they are. In some cases we are only concerned with similarity relations rather than similarity metrics. Therefore, the separation between a similarity relation and a similarity metric is of practical significance.

Furthermore, there are many different types of similarity metrics that have been introduced in CBR. For further information please see [17].

6 Abductive Case Retrieval vs Deductive Case Retrieval

The previous sections have focused on abductive CBR and deductive CBR. Because case retrieval plays a pivotal role in CBR [18], it is necessary to divide case retrieval into *abductive case retrieval* and *deductive case retrieval*. Case retrieval in abductive CBR is called *abductive case retrieval*, whereas case retrieval in deductive CBR is called *deductive case retrieval*. In what follows, the section describes some more about abductive case retrieval and deductive case retrieval in a parallel way.

6.1 Abductive Case Retrieval

For brevity, assume that the abductive case base is denoted as $B = (E, P)$, where E is a subset of explanation descriptions in W_e, the possible world of explanations that is associated with similarity S_e. P is a subset of problem descriptions W_p associated with similarity S_p [17]. An abductive case is denoted as an ordered pair (e, p), where $e \in E$ and $p \in P$.

In the context of abductive CBR, if the current problem p' is similar to p of the case (e, p) in abductive case base B, then one can conclude that the explanation of p', e', is also similar to e, according to (5). However, it is obvious that different understanding of similarity leads to different abductive case retrieval. Abductive case retrieval can thus be examined taking into account similarity relations, fuzzy similarity relations, and similarity metrics respectively [59]. It should be noted that the models under consideration can be considered as the specialization of (5), but denoted by a production rule. So it is easy to use these production rules and (5) to perform abductive CBR in different settings.

6.2 Deductive Case Retrieval

For deductive case retrieval, assume that the deductive case base is denoted as $C = (P, Q)$, where P is a subset of the possible world of problem descriptions W_p associated with a similarity S_p and Q is a subset of the possible world of solution descriptions W_s associated with a similarity S_s [18]. A deductive case, c, is denoted as an ordered pair (p, s), where $p \in P$ and $s \in Q$. In the context of CBR, if the current problem p' is similar to p in the deductive case (p, s) in the deductive case base C, then one can conclude that the solution of p', s', is also similar to s, according to (8).

Based on the above terminology, we can discuss abductive case retrieval and deductive case retrieval either separately or in a unified way. In practice, abductive case retrieval and deductive case retrieval can be examined in a parallel way, based on the basic model of integration of abductive CBR and deductive CBR in Section 4.

6.3 Similarity-based Abductive Case Retrieval

According to the methods proposed in [56][59],

1. IF p_1 is similar to p_2 in the sense of S_p, THEN e_1 is similar to e_2 in the sense of S_e

where S_p and S_e are similarity relations, p_1 and p_2 are two problem descriptions in W_p; e_1 and e_2 are their corresponding explanation descriptions in W_e. In domain-dependent CBR systems, we can refer to p_1 as a current problem description, and e_1 to a potential explanation of p_1, which we attempt to obtain using abductive CBR.

This model reflects the basic hypothesis of abductive CBR: *similar symptoms result from similar illnesses,* and it also reflects the principle of case-based explanation given by Leake in [28]: "Similar events are explained in similar ways." This is the basic motivation for research and development of abductive CBR. This still reflects "Two problems are similar if the solutions are similar" [64], However, the solutions are sometimes unknown, thus it is a paradox. This paradox is resolved by the observation that the similarity of solutions can be stated a priori; e.g., two solutions can be regarded as similar if they are equal or if the solution transformation is simple.

It is worth noting that in some cases the implication from IF to THEN is fuzzy, Thus the above model can be weakened into the three following categories from a viewpoint of fuzzy logic:

2. IF p_1 is similar to p_2 in the sense of S_p, THEN it is possible that e_1 is similar to e_2 in the sense of S_e

where S_p and S_e are fuzzy similarity relations. In fact, there is a common sense: "The more similar are the problems, the more similar are the corresponding explanations". In order to model this abductive CBR principle, the concept of a fuzzy similarity relation is not enough to model "more similar to", because it requires a similarity metric.

3. IF p_1 is more similar to p_2 in the sense of S_p, THEN e_1 is more similar to e_2 in the sense of S_e

where S_p and S_e are similarity metrics. This model reflects "Two problems are more similar if the solutions are more similar" and therefore belongs to a special case of *abductive CBR*, which was mentioned in the previous section. More specially, this model can be formalized as the following model:

$$\text{IF } S_p(p_1, p_2) \geq S_p(p_1, p_3), p_3 \in W_p \text{, THEN } S_e(e_1, e_2) \geq S_e(e_1, e_3), e_3 \in W_e \quad (15)$$

Further, the following model is more useful for a domain-dependent abductive CBR system, because any abductive CBR algorithm should aim at the "most similar" problem and its corresponding explanations.

4. IF p_1 is most similar to p_2 in the sense of S_p, THEN e_1 is most similar to e_2 in the sense of S_e

In usual abductive CBR systems, if p_1 is the current problem, then the problems in the set of problems of abductive case base, P, are evaluated if they are similar to p_1 and how similar they are to p_1, therefore category 4 can be modelled as:

$$\text{IF } S_p(p_1, p_2) = \max_{\forall p \in P} S_p(p_1, p), \text{ THEN } S_e(e_1, e_2) = \max_{\forall e \in E} S_e(e_1, e) \tag{16}$$

This model can be considered as a theoretical foundation for abductive case retrieval in abductive CBR.

6.4 Similarity-based Deductive Case Retrieval

In the rest of this section we turn to investigate deductive case retrieval based on similarity relations, fuzzy similarity relations, and similarity metrics respectively.

Similar to the above discussion on abductive case retrieval, we can also obtain parallel results of deductive case retrieval. In what follows, we only give a summary about similarity-based deductive case retrieval instead of a thorough investigation. For detail see [59].

Case retrieval based both on similarity relations and on fuzzy similarity relations has been examined in [56] and [59]. In what follows, this section only attempts to investigate deductive case retrieval based on similarity metrics [59]:

1. IF p_1 is more similar to p_2 in the sense of S_p, THEN s_1 is more similar to s_2 in the sense of S_s
2. IF p_1 is more similar to p_2 in the sense of S_p, THEN s_1 is not more similar to s_2 in the sense of S_s
3. IF p_1 is not more similar to p_2 in the sense of S_p, THEN s_1 is more similar to s_2 in the sense of S_s
4. IF p_1 is not more similar to p_2 in the sense of S_p, THEN s_1 is not more similar to s_2 in the sense of S_s.

where S_p on the possible world of problems W_p and S_s on the possible world of solutions W_s are similarity metrics. Now each of them will be examined in some detail.

Category 1 reflects the CBR principle mentioned in this subsection. This category can be formalized as:

$$\text{IF } S_p(p_1, p_2) \geq S_p(p_1, p_3), p_3 \in W_p, \text{ THEN } S_s(s_1, s_2) \geq S_s(s_1, s_3), s_3 \in W_s \tag{17}$$

In fact, the following model is more useful for a domain-dependent CBR system, because any case retrieval algorithm is aimed at the most similar problem and its corresponding solutions.

1'. IF p_1 is most similar to p_2 in the sense of S_p, THEN s_1 is most similar to s_2 in the sense of S_s

This category can be modelled as:

$$\text{IF } S_p(p_1, p_2) \geq S_p(p_1, p), \forall p \in W_p, \text{ THEN } S_s(s_1, s_2) \geq S_s(s_1, s), \forall s \in W_s. \tag{18}$$

In usual CBR systems, if p_1 is the current problem or normalized inquiry, then it is problems in the set of problems of case base, P, rather than all problems in W_p which are evaluated if they are similar to p_1 and how similar they are to p_1, therefore (18) can be simplified as:

$$\text{IF } S_p(p_1, p_2) \geq S_p(p_1, p), \forall p \in P, \text{ THEN } S_s(s_1, s_2) \geq S_s(s_1, s), \forall s \in Q \tag{19}$$

where Q is the set of solutions in case base, which corresponds to P.

In fact, (18) and (19) can be improved with the following two models respectively [59]:

$$\text{IF } S_p(p_1, p_2) = \max_{\forall p \in W_p} S_p(p_1, p), \text{ THEN } S_s(s_1, s_2) = \max_{\forall s \in W_s} S_s(s_1, s) \tag{20}$$

$$\text{IF } S_p(p_1, p_2) = \max_{\forall p \in P} S_p(p_1, p) \text{ THEN } S_s(s_1, s_2) = \max_{\forall s \in Q} S_s(s_1, s) \tag{21}$$

The last two models can be considered as a theoretical foundation for the current mainstream in case retrieval studies [59]. It is also useful for any areas in which information search or retrieval play an important role.

Category 2 reflects some practical cases in real life [55]. It is worth noting from a logical viewpoint that this category might be invalid, although "IF p_1 is similar to p_2 in the sense of S_p, THEN s_1 is similar to s_2 in the sense of S_s," which is the first category in the previous section. In this case, "not more similar" lies between "more similar" and "similar" from a viewpoint of fuzzy logic. Therefore, this category can be considered as a weaker form of category 1 mentioned in the previous section [59].

Category 3 reflects a dual problem to category 2, which has not attracted much attention in CBR, because case retrieval is the initial point in the study on CBR [55].

As to category 4, it is also logically equivalent to the following category:

5. IF s_1 is more similar to s_2 in the sense of S_s, THEN p_1 is more similar to p_2 in the sense of S_p.

This category reflects "Two problems are more similar if the solutions are more similar" and therefore belongs to a special case of *abductive CBR*, which was mentioned in the previous section.

More specifically, this category can be formalized as:

$$\text{IF } S_s(s_1, s_2) \geq S_s(s_1, s_3), s_3 \in W_s \text{ THEN } S_p(p_1, p_2) \geq S_p(p_1, p_3), p_3 \in W_p \tag{22}$$

In fact, the following model might be more useful for an *abductive CBR*,

5'. IF s_1 is most similar to s_2 in the sense of S_s, THEN p_1 is most similar to p_2 in the sense of S_p

This category can be modelled as:

$$\text{IF } S_s(s_1, s_2) \geq S_s(s_1, s), \forall s \in W_s, \text{ THEN } S_p(p_1, p_2) \geq S_p(p_1, p), \forall p \in W_p \tag{23}$$

In the usual case retrieval, the model (23) can also be simplified as:

$$\text{IF} S_s(s_1, s_2) \geq S_s(s_1, s), \forall s \in Q, \text{ THEN } S_p(p_1, p_2) \geq S_p(p_1, p), \forall p \in P \tag{24}$$

and considered as a theoretical foundation for studies of abductive CBR. In fact, (23) and (24) can be replaced with the following two models respectively:

$$\text{IF } S_s(s_1, s_2) = \max_{\forall s \in W_s} S_s(s_1, s), \text{ THEN } S_p(p_1, p_2) = \max_{\forall p \in W_p} S_p(p_1, p) \tag{25}$$

$$\text{IF } S_s(s_1, s_2) = \max_{\forall s \in Q} S_s(s_1, s) \text{ THEN } S_p(p_1, p_2) = \max_{\forall p \in P} S_p(p_1, p) \tag{26}$$

These last two models can be considered as a theoretical foundation for *abductive CBR*.

From a logical viewpoint, "If P Then Q" means that if P is true then Q is at least true; that is, from a computational viewpoint, $t(P) \le t(Q)$, where $t(\ )$ is truth value of a proposition. If t is a similarity metric, then it is essentially the same as that in [13], in the latter, t is intentionally replaced by two different similarity metrics S and T. Based on this idea, (21) and (26) can be specified as:

$$S_p(p_1, p_2) = \max_{\forall p \in P} S_p(p_1, p) \le S_s(s_1, s_2) = \max_{\forall s \in Q} S_s(s_1, s) \tag{27}$$

$$S_s(s_1, s_2) = \max_{\forall s \in Q} S_s(s_1, s) \le S_p(p_1, p_2) = \max_{\forall p \in P} S_p(p_1, p) \tag{28}$$

The last two models are useful for implementing a concrete CBR system.

7 Similarity-based Models for Fuzzy Rule-based Case Retrieval

This section will provide similarity-based models of fuzzy rule-based case retrieval and its implementation from a viewpoint of fuzzy logic. This is a further development of similarity-based models for rule-based case retrieval discussed in the previous section, because of the uncertain knowledge and inexact matching that are involved in case retrieval. It can therefore be argued that CBR is a unifying mechanism for integrating rule-based reasoning and similarity-based reasoning. Therefore, this is also a further insight into integration of abductive CBR and deductive CBR from a logical viewpoint[1].

1. Sun [52] outlined a unifying mechanism for carrying out the basic processes of rule-based reasoning and similarity-based reasoning from a connectionist viewpoint.

7.1 Similarity Neighbourhood and Similarity Uniform Mapping

First of all, two concepts are introduced with respect of similarity metrics. Assume that (W_p, S_p) and (W_s, S_s) are two similarity systems in which S_p on the possible world of problems W_p and a relation S_s on the possible world of solutions W_s are similarity metrics (see Section 5.3).

Throughout this section, Let r be a real number. The *open similarity disc* of radius $r > 0$ centred at p_0 means the set of problems p in W_p such that $S_p(p, p_0) > r$. Generally speaking, r is approximate to 1 in the case retrieval. The open similarity disc of radius r at p_0 is denoted as $S_p(p_0, r)$. Suppose $w > 0$ and (p_0, s_0) is a case in the case base C, then $S_s(s_0, w)$ is an open similarity disc of radius w at s_0 [59].

Definition 4. A function f: $W_p \to W_s$ is called *uniformly similar* on the domain W_p if for any $r > 0$ there exists a $w > 0$ such that

$$\text{if } p_1, p_2 \in W_p \text{ and } S_p(p_1, p_2) > r \text{ then } S_s(f(p_1), f(p_2)) > w \tag{29}$$

Further for any $p \in P$, $(p, f(p)) \in C$, where C is the case base. f is called a *uniform solution function*.

The motivation for introducing this concept is from the concept of conformal mappings in complex analysis and uniformly continuous functions in real analysis [17]. The goal of introducing this concept is to build a formal connection between W_p and W_s. This is an important condition for examining fuzzy rule-based case retrieval.

7.2 Fuzzy Rule-based Case Retrieval

This subsection examines similarity-based models for fuzzy rule-based case retrieval based on Zadeh's composite rule [69] by beginning with the following real world scenario in e-commerce.

A customer uses the interface of the e-commerce system, for example, CMB [57], to submit a requirement with the problem description p_0, which may be fuzzy and uncertain owing to its description in natural language. The search agent of CMB will search the case

base C in CMB to try to find if there is a case $c_1 = (p_1, s_1)$ in C, such that p_0 is completely matched over p_1; that is, $p_0 \equiv p_1$. If so, the goods with the solution description s_1 are the most satisfactory solution to the requirements of the customer according to the experience of CMB. Otherwise, the search agent has to activate the similarity-based mechanism, which is based on similarity metric S_p, to search the case base C in CMB to obtain the most similar p_1, which is in a case $c_1 = (p_1, s_1)$ in C, such that $S_p(p_0, p_1) = \max_{\forall p \in P} S_p(p_0, p)$ with similarity degree r. Then the most satisfactory solution to the requirements of the customer is s_0 such that $S_s(s_0, s_1) = \max_{\forall s \in Q} S_s(s_1, s)$ with similarity degree w (see Section 7.1), according to the experience of CMB.

It should be noted that s_0 is only the most satisfactory good to meet the requirements of the customer. However, it may not be matched completely to the requirements of the customer. In this case, case adaptation is necessary, if the customer asks to tune the requirements or the product with an adjustment of the problem descriptions.

As is known, a case (p, s) can be represented as a rule; that is, $p \to s$. Therefore, above discussion can be expressed in the following concise form :

$$\frac{p_0, p_0 \sim p_1, p_1 \to s_1, s_1 \sim s_0}{s_0} \tag{30}$$

where, p_0 is the problem description of the customer, $p_0 \sim p_1$ means that p_0 and p_1 are most similar, with the similarity degree r, in the sense of S_p, $p_1 \to s_1$ is the case retrieved from the case base C based on the similarity-based retrieval algorithm. $s_1 \sim s_0$ means that s_0 and s_1 are most similar, with the similarity degree w, in the sense of S_s, and s_0 is the most satisfactory solution to the requirements of the customer with the similarity degree $r \times k \times w$, where k is the certainty factor of $p_1 \to s_1$. Usually, $k = 1$ because the case in the case base is the result of experience, i.e. a successful solution to a previous problem [59].

Model (30) is an implementation-oriented realization of the CBR world. A special case is $s_0 \equiv s_1$; that is, s_0 is identical to s_1. This degenerates from model (30) to the cases that many studies have implicitly or explicitly done such as [52]. In the later case (30) is simplified as [16]:

$$\frac{p_0, p_0 \sim p_1, p_1 \rightarrow s_1}{s_1} \tag{31}$$

The rest of this subsection turns to fuzzy rule-based case retrieval. Assume that p_0 corresponds to $\tilde{P}_0$, $p_0 \sim p_1$ corresponds to $\tilde{F}_{01}$, $p_1 \rightarrow s_1$ corresponds to $\tilde{F}_{11}$, $s_1 \sim s_0$ corresponds to $\tilde{F}_{10}$, and s_0 corresponds to $\tilde{S}_0$. Then, according to the compositional rule of inference of Zadeh [69] and the above model (30), the following is obtained [16]:

$$\tilde{S}_0 = \tilde{P}_0 \circ \tilde{F}_{01} \circ \tilde{F}_{11} \circ \tilde{F}_{10} \tag{32}$$

where $\tilde{P}_0$ is a fuzzy set in W_p. $\tilde{F}_{01}$, $\tilde{F}_{11}$, and $\tilde{F}_{10}$ are a (fuzzy) similarity metric, a fuzzy rule and a fuzzy similarity metric in $W_p \times W_s$ respectively, and $\tilde{S}_0$ is a fuzzy set on W_s. This is a theoretical foundation for fuzzy rule-based case retrieval. In the case $s_0 \equiv s_1$, $\tilde{F}_{10}$ is an unit matrix, (32) is then simplified into [16]:

$$\tilde{S}_0 = \tilde{P}_0 \circ \tilde{F}_{01} \circ \tilde{F}_{11} \tag{33}$$

When, $\tilde{P}_0$, $\tilde{F}_{01}$, $\tilde{F}_{11}$ and $\tilde{S}_0$ are only a numerical similarity metric respectively, (32) is, essentially, degenerated into the form discussed in [52].

8 Concluding Remarks

This article showed that abductive reasoning and deductive reasoning can be integrated in the clinical process and problem solving. It argued that the theoretical foundation of backward chaining and Prolog as well as most other logic programming languages is abduction. After discussing the evolution from abduction to abductive CBR and that from deduction to deductive CBR, the article integrated abductive CBR and deductive CBR, and proposed a unified formalization for integration of abduction, abductive CBR, deduction and deductive

CBR. It also demonstrated that the integration of the abductive CBR system and deductive CBR system is of practical significance in problem solving such as system diagnosis and analysis. Finally we investigated abductive case retrieval and deductive case retrieval using similarity relations, fuzzy similarity relations and similarity metrics. The proposed approach will facilitate research and development of abductive CBR and deductive CBR as well as systems analysis and diagnosis as well as explanation-based reasoning.

From the viewpoint of symmetry, abduction and deduction have the same importance in problem solving. However, whereasarticle deduction is very popular, abduction seems to be strange, because few academics know the approach. This results from the academic tradition and mathematical logic, because the formal theory of mathematical logic is essentially based on modus ponens. Our subjects in every level of education, in particular in mathematics and logic, are based on such a formal theory of mathematical logic. In the past hundred years, some philosophers and mathematicians and in particular scientists in computer science have been aware of the importance of abduction in problem solving, and have attempted to use it in diagnostic reasoning, explanation-based reasoning, and other fields of problems solving. However, they have not tried to establish a formal theory of abduction that is similar to the formal theory of deduction. Further, in mathematical logic there are a significant number of reasoning paradigms which are essentially from the formal axioms of mathematical logic. A formal theory of abduction appears to only have two reasoning paradigms: one is the dual form of modus ponens and another is the dual form of syllogism, which has so far been ignored in research of abduction.

In future work we will try to establish a formal logical theory of abduction and abductive CBR. We will also try to use the proposed approach to automate the bargaining process and experience-based reasoning, which is an important aspect of electronic commerce.

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