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Abstract

A capacity model for shear strength of reinforced-concrete exterior beam-column joints subjected to quasi-static cyclic loading is established. In this model, only a fraction of this force is asserted to be transferred into the joint core, the remainder is assumed to be transferred into the adjacent column. A biaxial failure criterion of concrete is adopted to predict shear failure of the joint core caused by a combination of principal compression and tension stresses. The contribution of the shear reinforcement is accounted for by improving the tensile strength of the concrete. In addition, the special features of the proposed model include: first, the asynchronicity of concrete and shear reinforcement in tensile strength is taken into account and, second, the role of column axial stress is accounted for by its influence on both the magnitude and direction of principal tensile and compressive stresses at the joint core. The validity of the proposed model is evaluated by comparing the predicted shear strengths with 142 test results collected from the literature and with five other analytical models. This evaluation showed that the proposed model can predict shear strength with better reliability.

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Shear strength model of reinforced-concrete exterior joint under cyclic loading

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A capacity model for shear strength of reinforced-concrete exterior beam–column joints subjected to quasi-static cyclic loading is established. In this model, only a fraction of this force is asserted to be transferred into the joint core, the remainder is assumed to be transferred into the adjacent column. A biaxial failure criterion of concrete is adopted to predict shear failure of the joint core caused by a combination of principal compression and tension stresses. The contribution of the shear reinforcement is accounted for by improving the tensile strength of the concrete. In addition, the special features of the proposed model include: first, the asynchronicity of concrete and shear reinforcement in tensile strength is taken into account and, second, the role of column axial stress is accounted for by its influence on both the magnitude and direction of principal tensile and compressive stresses at the joint core. The validity of the proposed model is evaluated by comparing the predicted shear strengths with 142 test results collected from the literature and with five other analytical models. This evaluation showed that the proposed model can predict shear strength with better reliability.

Notation

A_g	cross-sectional area of column	h_c	width of column along the beam
A_{jh}, A_{jv}	total area of horizontal and vertical shear reinforcement, respectively	h_l	development length of beam longitudinal reinforcement
A_{sb}	area of beam tensile reinforcement	L	distance from beam end to beam–column interface
b_b, b_c	width of beam and column section, respectively	M_b	beam flexural moment at beam–column interface
b_j	effective joint width	r	strain ratio
d_b	average diameter of beam tensile reinforcement	T_{sb}	tension force of beam longitudinal reinforcement
f'_c	cylinder compressive strength of concrete	V_b, V_c	beam and column shear forces, respectively
f_{ct}	nominal tensile strength of concrete	V_{jh}	horizontal joint shear force
f_{ct}^*	nominal modified tensile strength of concrete considering the contribution of shear reinforcement	V_{jh}^c	shear force transferred to upper column
f_{jhy}, f_{jvy}	yield strength of horizontal and vertical shear reinforcement, respectively	$V_{jh,model}$	predicted joint shear strength
f_{jsy}	average nominal tensile strength of joint shear reinforcement	$V_{jh,test}$	test shear force
f_{sb}	beam longitudinal reinforcement tensile stress	V_{jh}^*	joint shear force resisted by the joint core
$f_{sb,JS}, f_{sb,BY-JS}$	beam longitudinal reinforcement tensile stresses corresponding to joint shear and beam yield joint shear failure mode, respectively (Equations 23 and 24)	X, Y	factors (Equations 13 and 20)
H	distance between upper and lower column inflection points	x	smaller of the distances from beam face to column face
h_b	height of beam cross-section	β	fraction factor
h_b'	internal moment arm of beam cross-section	ϵ_{cr}	cracking strain of concrete
		ϵ_{jsy}	yield strain of shear reinforcement
		θ	inclination of principal compressive stress from the column longitudinal direction
		κ	fraction of beam bar–concrete bond force transferred into upper column

μ_s, μ_E, μ_y	bond strength and bond strength in elastic and inelastic beam tensile reinforcement, respectively
$\mu(f_{sb})$	reinforcement–concrete bond strength
σ_y	column axial normal compressive stress
σ_1, σ_2	principal tensile and compressive stresses, respectively
τ_{jh}	joint shear stress

1. Introduction

Beam–column joints are considered as the critical regions of reinforced-concrete (RC) structures, as their failures lead to deterioration of the whole performance of frames under seismic loading. It has been recognised that beam–column joints resist both horizontal and vertical joint shear forces, which are developed by bending moments of opposite signs at member ends on either side of the joint core. The combination of column axial force and the joint shear forces result in principal tension and compression that leads to diagonal cracking and/or crushing of concrete in the joint core, leading to brittle shear failure of the joint. Therefore, to improve the safety of whole RC structures under seismic loading, the shear strength and ductility of the beam–column connections must be considered carefully so that brittle joint shear failure is avoided.

The shear capacity of RC joints is influenced by the concrete strength, joint panel geometry, reinforcement confinement, column axial load and reinforcement bond condition (Hitoshi, 2012; Kim and LaFave, 2007; Park and Mosalam, 2012a). Although much effort has been made in this area, there are still some inconsistencies in the standard codes for predicting shear strength of RC beam–column connections (ACI 318R (ACI, 2008); AIJ, 1999; Eurocode 8 (BS EN 1998-1; BSI, 2004); NZS 3101 (SNZ, 1995)). For example, NZS 3101 (SNZ, 1995) considers the influence of column axial load on the joint shear strength, but this is neglected in the other standards; each currently existing standard codes also has a different definition of effective joint width. Moreover, some researchers (Marques and Jirsa, 1975; Meinheit and Jirsa, 1977; Pantazopoulou and Bonacci, 1992) have concluded that column axial load has no coherent effect on the joints' shear strengths. Other researchers (Clyde *et al.*, 2000; Pantelides *et al.*, 2002) have confirmed that an increase in column axial load leads to improvement in the joint shear strength, while Park and Mosalam (2012b) showed that the joint shear strength is not clearly affected by the column axial.

The roles of shear reinforcement have also been debated by researchers. The horizontal shear reinforcement improves shear strength of the joint by confining the concrete core and by directly resisting a fraction of the shear force. Various models (Bakir and Boduroğlu, 2002; Hegger *et al.*, 2003; Kim *et al.*, 2009; Parker and Bullman, 1997; Paulay and Priestley, 1992; Sarsam and Phipps, 1985; Vollum and Newman, 1999) have

suggested an inconsistent contribution of shear reinforcement to the shear strength of the joint. This confirms that researchers have not stopped to debate whether or how the shear reinforcement can resist the shear force at the joint.

In order to predict the shear strength of the joint, most of the theoretical models have been developed based on either the strut-and-tie (SAT) approach or the average plane stress approach with compatibility of strains and stress equilibrium. Despite being based on the SAT or average approach, in most of the existing models, the beam longitudinal reinforcement tensile force was assumed to be transferred totally into the joint core in terms of bond forces to cause joint shear failure. However, many experimental studies have shown that failure of the joint not only occurred at the joint core, but also developed into the upper and lower columns (Hadi and Tran, 2014, 2015; Wang *et al.*, 2012). In addition, a recent study (Tran *et al.*, 2014) showed that the contacting area between the beam longitudinal reinforcement and the surrounding concrete has a significant role in the joint shear strength. These facts lead to the idea that the beam longitudinal reinforcement tensile force transferred just a fraction of that force into the joint core. Therefore, in this paper, a new shear strength model based on the average plane stress approach is developed for exterior RC beam–column joints. In the proposed model, the beam bars' tensile force was assumed to be transferred partly into the joint core and partly into the adjacent columns. A biaxial ultimate strength envelope is employed to predict shear failure of the joint core caused by the combination of tension and compression stresses. Thus, both the compressive crushing of the strut and the tensile cracking of the concrete core were considered. The role of the joint shear reinforcement in the joint panel was considered by its contribution to the nominal tensile strength of concrete, in which the asynchronicity of concrete and shear reinforcement in tensile strength are considered. Additionally, the influence of column axial load is integrated into the model by contributing to the values of both the inclination angles and the magnitudes of the principal compression and tension stresses. A large database of the results obtained from 142 experimental tests was used to calibrate the proposed shear strength model.

2. Development of analytical model

2.1 Assumptions

In beam–column connections that are designed following the strong-beam–weak-column philosophy, the typical failure is the formation of column flexural cracks at the column–beam interfaces. These cracks reduce the possibility for the beam longitudinal reinforcement to transfer bond stress into the columns. Thus, most of the bond stress of the beam bars transfers predominantly into the joint core. However, in most connections, which are designed with the strong-column–weak-beam philosophy, the column's flexural cracks at the column–beam interfaces are insignificant, thus the fraction of the beam bars'

tensile force transferred into the column is significant, and therefore only a fraction of the beam bars' tensile force transfers into the joint core. This mechanism explains why diagonal cracks develop not only at the joint core but also at the columns close to the joint and the beam longitudinal reinforcement has a significant influence on the joint shear strength, as stated by Tran *et al.* (2014). For the above reasons, in this study, a fraction of the beam bars' tensile force was assumed to be resisted by the joint core and the remaining fraction was assumed to be resisted by the adjacent column. It is also worth mentioning that most of the existing models have assumed that the tension force of the beam reinforcement (T_{sb} in Figure 1) was totally clamped by the joint core, whereas Priestley (1993) recommended that only 50% of T_{sb} was clamped by the joint shear mechanism, because the other 50% was clamped by the diagonal compression strut at the adjacent column.

The proposed truss mechanism of the exterior beam–column connection is shown in Figure 1. The magnitude of beam tension force transferred into the column depends on the development length, h_l , of the beam reinforcement and the reinforcement–concrete bond strength, $(\mu(f_{sb}))$. This fraction depends on the level of beam reinforcement tensile stress (f_{sb}), because the reinforcement–concrete bond strength depends on the tensile strain of reinforcement and the bond can be deteriorated when the reinforcement is yielded. In other words, the bond stress in the upper face of the top of beam bars was assumed to transfer a fraction of the beam bars' tension force into the column. The considered anchorage detail of beam longitudinal bars was the 90° hooks on the top and bottom bars bent into the joint region, because this is conventional

and, with the presence of the hooks, the assumption that the beam tensile reinforcement does not slip is reasonable.

Figures 2(a), 2(b) and 2(c) show the forces at a RC exterior beam–column joint, the plane stress state at the centre of the joint and the assumed failure envelope of the concrete at the joint core, respectively. The assumption is that the joint shear strength is reached when the principal tensile and compressive stresses (σ_1 and σ_2) of concrete at the centre of the joint (Figure 2(a)) reach the failure envelope. From these figures, using the derivation methods presented by Tsonos (2007) and Wang *et al.* (2012), Equations 1–4 are established as

$$1. \quad \tau_{jh} = \sigma_1 / \tan \theta$$

$$2. \quad \sigma_2 = \sigma_y - \tau_{jh} \tan \theta$$

$$3. \quad \tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta} = \frac{2\tau_{jh}}{\sigma_y}$$

$$4. \quad \frac{\sigma_1}{f_{ct}^*} - \frac{\sigma_2}{f_c'} = 1$$

where $\tau_{jh} = V_{jh}^*/(b_j h_c)$ is the joint shear stress; $\sigma_y = N_c/(b_j h_c)$ is the normal vertical compressive stress; b_j is the effective joint

Figure 1. Assumed truss mechanism in exterior beam–column joint

Figure 2. Forces and failure mechanism of an exterior beam–column joint: (a) forces at an RC exterior beam–column joint; (b) plane stress state at the centre of a joint; (c) assumed failure envelope of the concrete at the joint core (Tasuji *et al.*, 1978)

width, taken as $b_j = \min(b_b + h_c, b_b + 2x)$, per ACI 318R (ACI, 2008); x is the smaller of the distances from the beam face to the column face; b_b and b_c are the width of the beam and the column sections, respectively; θ is the inclination angle of the principal compressive stress from the column longitudinal direction; and f'_c and f_{ct}^* are the compressive strength of the concrete and the nominal modified tensile strength of the concrete considering the contribution of the shear reinforcement, respectively.

It is noted that, in contrast to the models of Tsonos (2007) and Wang *et al.* (2012), in this model the Tasuji *et al.* (1978) failure criterion is applied, the joint shear force, V_{jh}^* , and the nominal modified tensile strength of the concrete, f_{ct}^* , are developed rationally. The derivations of V_{jh}^* and f_{ct}^* are presented in more detail in the following sections.

2.2 Global equilibrium

The global equilibrium of a RC exterior beam–column joint is shown in Figure 3. The beam flexural moment at the beam–column interface and the column shear force at the top face of the beam are presented as follows

$$5. \quad M_b = V_b L = A_{sb} f_{sb} h'_b$$

$$6. \quad V_c = \frac{L + h_c/2}{H} V_b$$

Figure 3. Global equilibrium in exterior RC beam–column joints: (a) when $f_{jsy} \geq f_{ct} + (\epsilon_{ct}/\epsilon_{jsy})f_{jsy}$; (b) when $f_{jsy} < f_{ct} + (\epsilon_{ct}/\epsilon_{jsy})f_{jsy}$

where V_b and V_c are the beam and the column shear forces, respectively; L is the length from the beam end to the beam–column interface; H is the height between the upper and lower column inflection points; A_{sb} , f_{sb} are the area and the stress of the beam tensile reinforcement at the beam–column interface, respectively; h'_b is the internal moment arm of the beam cross-section at the beam–column interface; h_c is the width of

column along the beam. The horizontal shear force at the joint is calculated as

$$7. \quad V_{jh} = A_{sb}f_{sb} - V_c = A_{sb}f_{sb} \left(1 - \frac{L + h_c/2}{H} \frac{h_b}{L} \right)$$

As proposed by Park and Mosalam (2012a), the horizontal shear force at the joint can be simplified as

$$8. \quad V_{jh} = A_{sb}f_{sb} \left(1 - \frac{L + h_c/2}{H} \frac{0.8h_b}{L} \right) \\ \approx A_{sb}f_{sb}(1 - 0.85h_b/H)$$

As a fraction of the shear force at the joint is transferred and resisted by the upper column, the shear force resisted by the joint core can be calculated as follows

$$9. \quad V_{jh}^* = V_{jh} - V_{jh}^c$$

where V_{jh}^c is the shear force transferred to the upper column in terms of bond stress. As can be seen in Figure 1, the magnitude of V_{jh}^c can be derived as follows

$$10. \quad V_{jh}^c = \kappa n \pi d_b \int_0^{h_1} \mu(f_{sb}) dx = 3.54 \kappa \sqrt{n A_{sb}} \int_0^{h_1} \mu(f_{sb}) dx$$

where the bond fraction κ illustrates the fraction of the bond force along the length h_1 of the beam longitudinal reinforcement that is transferred into the upper column. In ideal cases, when no flexural crack has occurred along the top beam bars, $\kappa = 0.5$ (half of the bond force in the development length h_1 transfers into the upper column); in most cases, $0 < \kappa < 0.5$; n is the number of beam longitudinal bars in tension with average diameter d_b . Note that $\mu(f_{sb})$ is the bond stress distribution along the beam bars in the distance h_1 (Figure 1) as a function of the tensile stress of the beam bar f_{sb} . The x -axis is depicted in Figure 1, where the origin of the axis starts at the outside edge of the column core as recommended by ACI 352R (ACI, 2002) for determining critical sections for development of longitudinal member reinforcement. In this study, the bond fraction κ and the distance h_1 are investigated in the following section by varying them from 0.3 to 0.5 and $0.5h_c$ to $0.8h_c$, respectively.

2.3 Fraction factor

The shear force resisted by the joint core can be expressed in terms of a fraction factor β as follows

$$11. \quad V_{jh}^* = V_{jh} - V_{jh}^c = \beta V_{jh} \approx \beta A_{sb}f_{sb}(1 - 0.85h_b/H)$$

The fraction factor β illustrates the ratio of shear force resisted by the joint core. The magnitude of the fraction β is related to the bond deterioration of the beam tension reinforcement. As explained above, when the shear force transferred to the upper column in terms of bond stress is insignificant, most of the beam bars' bond stress transfers into the joint core, thus the fraction factor β leads to a maximum value of 1 (100% of tension force of beam reinforcement claimed by the joint core). The fraction factor β reaches the lowest limit of 0.5 (50% of tension force of beam reinforcement claimed by the joint core and the other 50% claimed by the adjacent column) when the beam bars-concrete bond is perfect and the development length of the beam tension reinforcement is sufficient. In the proposed model, the bi-uniform bond strength model proposed by Lehman and Moehle (2000) was adopted to express the bond strength of the beam tension reinforcement. The bond strength in elastic beam tensile reinforcement, $\mu(f_{sb} < f_{sby}) = \mu_E = 1.0(f'_c)^{1/2}(\text{MPa}^{0.5})$ and that in the inelastic beam tensile reinforcement, $\mu(f_{sb} \geq f_{sby}) = \mu_y = 0.5\mu_E$. From Equations 10 and 11, β is derived as follows

$$12. \quad \beta = \frac{V_{jh}^*}{V_{jh}} = 1 - \frac{3.54 \kappa l_h \mu_s \sqrt{n}}{f_{sb}(1 - 0.85h_b/H) \sqrt{A_{sb}}} \\ = 1 - X \frac{\mu_s}{f_{sb}} \geq 0.5$$

where

$$13. \quad X = \frac{3.54 \kappa l_h \sqrt{n}}{(1 - 0.85h_b/H) \sqrt{A_{sb}}}$$

$\mu_s = \mu_E$ if $f_{sb} < f_{sby}$ (elastic beam tensile reinforcement);
 $\mu_s = 0.5\mu_E$ if $f_{sb} \geq f_{sby}$ (inelastic beam tensile reinforcement); f_{sby} is the average yield stress of beam longitudinal tension bars.

2.4 Determination of the modified concrete tensile strength, f_{ct}^* , and the inclination angle, θ

In the proposed model, the concrete and the joint shear reinforcement, which includes the intermediate longitudinal steel bars of the column passing through the joint (excluding the edge bars), A_{jv} and the horizontal steel shear reinforcement between the top and bottom main steel bars of the adjacent beam, A_{jh} were assumed to be idealised as an equivalent homogeneous material in a biaxial plane stress state. The contribution of the joint shear reinforcement is accounted for by increasing the nominal tensile strength of the idealised homogeneous material. A similar assumption has been used in strut-and-tie models (Tan *et al.*, 2001; Wang and Meng, 2008) for predicting shear strength of deep beams and in the average plane stress model (Wang *et al.*, 2012) for predicting shear strength of beam-column connections. However, one of the superior features of the proposed model is that the asynchronous nature of concrete and shear

Figure 4. Stress–strain relationship of concrete, steel reinforcement and concrete + steel reinforcement

reinforcement in tensile strength is considered. The way to determine the modified concrete tensile is presented below.

The average tensile stress–strain relationships of the concrete and the joint shear reinforcement are assumed as shown in Figure 4. The nominal tensile strength, f_{ct} , of concrete is reached at cracking strain, ϵ_{cr} , as given by ACI 318R (ACI, 2008)

$$14. \quad f_{ct} = 0.556\sqrt{f'_c}$$

The average nominal tensile strength, f_{jsy} , of the joint shear reinforcement is reached at yield strain ϵ_{jsy} and can be determined as follows

$$15. \quad f_{jsy} = \frac{A_{jh}f_{jhy} \cos \theta + A_{jv}f_{jvy} \sin \theta}{b_j h_c / \sin \theta}$$

where f_{jhy} and f_{jvy} are the yield strength of the horizontal and the intermediate vertical shear reinforcement of the joint, respectively.

Figure 4 also shows the nominal total tensile strength f_{ct}^* of the concrete and the steel reinforcement. Because the concrete and the steel reinforcement cannot reach their strength simultaneously, in cases when $f_{jsy} \geq f_{ct} + (\epsilon_{cr}/\epsilon_{jsy})f_{jsy}$ (Figure 4(a)), the nominal modified tensile strength f_{ct}^* can be determined as $f_{ct}^* = f_{jsy}$, otherwise (Figure 4(b)), it can be determined as $f_{ct}^* = f_{ct} + (\epsilon_{cr}/\epsilon_{jsy})f_{jsy}$. Therefore, the nominal modified tensile strength can be calculated as follows

$$16. \quad f_{ct}^* = \text{Max} \left\{ f_{jsy}, f_{ct} + \frac{\epsilon_{cr}}{\epsilon_{jsy}} f_{jsy} \right\}$$

In general, the cracking strain ϵ_{cr} of pure concrete is much lower than the yield strain of joint shear reinforcement. In RC the cracking strain of the concrete may be increased but cannot reach the yield strain of the shear reinforcement. Therefore, the strain ratio $r = \epsilon_{cr}/\epsilon_{jsy}$ is assumed to be smaller than unity and, in this model, this ratio is investigated in the subsequent section by varying it from 1/1 to 1/10. The strain ratio represents the possibility of the concrete and the shear reinforcement to work together in resisting the principal tensile stress σ_1 . When this ratio is close to unity, it means the concrete and the shear reinforcement work well together in resisting the shear load and vice versa.

Replace $\tau_{jh} = \sigma_1/\tan \theta$ from Equation 1 into Equation 3, obtaining $\tan \theta/(1 - \tan^2 \theta) = \sigma_1/\sigma_y \tan \theta$. Solving this equation, the following is obtained

$$17. \quad \tan \theta = \sqrt{\frac{\sigma_1}{\sigma_y + \sigma_1}}$$

Assuming that when the shear strength of the joint is reached, the nominal principal tension stress at the centre of the joint reaches the modified tensile strength, f_{ct}^* , the inclination angle θ , therefore, can be calculated as $\tan \theta = (f_{ct}^*/\sigma_y + f_{ct}^*)^{1/2}$, in which the modified tensile strength, f_{ct}^* , can be calculated using Equation 16. However, for simplicity, in this model, Equation 18 is proposed to calculate the inclination angle θ

$$18. \quad \tan \theta = \sqrt{\frac{1.15f_{ct}}{\sigma_y + 1.15f_{ct}}}$$

2.5 The proposed joint shear strength

Combining Equations 1, 2, 4 and 11 yields the following expression for the stress of the tensile beam reinforcement at

the beam–column interface, f_{sb}

$$19. \quad f_{sb} = \frac{1}{\beta \tan \theta} \frac{1 + \sigma_y / f'_c}{1 / f'_{ct} + 1 / f'_c} \frac{b_j h_c}{A_{sb}(1 - 0.85 h_b / H)}$$

$$= Y \frac{b_j h_c}{\beta A_{sb}(1 - 0.85 h_b / H)}$$

where

$$20. \quad Y = \frac{1}{\tan \theta} \frac{1 + \sigma_y / f'_c}{1 / f'_{ct} + 1 / f'_c}$$

From Equation 8 the shear strength of the joint can be expressed as follows

$$21. \quad V_{jh} = A_{sb} f_{sb} (1 - 0.85 h_b / H) = \frac{b_j h_c}{\beta} Y$$

From Equations 12 and 19 the stress of the beam tensile reinforcement at the beam–column interface, f_{sb} , can be calculated as follows

$$22. \quad f_{sb} = b_j h_c Y / [A_{sb}(1 - 0.85 h_b / H)] + \mu_s X$$

As mentioned above, when the beam tensile reinforcement has not yielded, the bond strength, $\mu_s = \mu_E 1.0 (f'_c)^{1/2} (\text{MPa}^{0.5})$, Equation 22 can be rewritten as

$$23. \quad f_{sb} = f_{sb,JS} = b_j h_c Y / [A_{sb}(1 - 0.85 h_b / H)] + \sqrt{f'_c} X$$

When the beam tensile reinforcement has yielded, the bond strength, $\mu_s = 0.5 \mu_E = 0.5 (f'_c)^{1/2} (\text{MPa}^{0.5})$, Equation 22 can be expressed as

$$24. \quad f_{sb} = f_{sb,BY-JS}$$

$$= b_j h_c Y / [A_{sb}(1 - 0.85 h_b / H)] + 0.5 \sqrt{f'_c} X$$

The shear strength of the joint, therefore, can be determined as follows.

In cases where $f_{sb,JS} < f_{sby}$ (f_{sby} is the average yield strength of beam tensile reinforcement), the beam–column connections have failed in joint shear failure mode without beam bar yielding (JS mode), the fraction factor, β , and the shear strength, V_{jh} , of the joint are calculated by using Equations 8 and 12, respectively, with the bond strength, $\mu_s = 1.0 (f'_c)^{1/2}$ and $f_{sb} = f_{sb,JS}$.

Figure 5. Procedure of the proposed analytical model

In cases where $f_{sb,BY-JS} \geq 1.25 f_{sby}$, the beam–column connections are assumed to be failed by formation of beam hinges (BH) close to the beam–column interfaces (Hwang and Lee, 2002; Tsonos, 2007), the fraction factor, β , is calculated by using Equation 12 with the bond strength, $\mu_s = 0.5 (f'_c)^{1/2}$ and the shear strength of the joint is calculated using Equation 8 with $f_{sb} = 1.25 f_{sby}$.

In other cases, the beam–column connections have failed in joint shear failure mode with yielding of beam bars (BY-JS mode), the fraction factor, β , is calculated by using Equation 12 with the bond strength, $\mu_s = 0.5 (f'_c)^{1/2}$ and the shear strength of the joint is calculated using Equation 8 with $f_{sb} = f_{sb,BY-JS}$. Consequently, the joint shear strength of the exterior beam–column connection can be calculated using the procedure illustrated in Figure 5.

3. Verification of the proposed model

From the published literature (Table 1), 142 experimental RC exterior beam–column joints have been collected to verify the proposed model. The failure modes of the collected specimens were JS, BY-JS or BH, detail of the selection criteria can be seen in Tran *et al.* (2014). The collected database covers a broad range of various parameters including joint reinforcement ratio, concrete strength, column axial average stresses and h_b/h_c ratio, as summarised in Table 1. In the database, 46 specimens failed in JS mode, 52 specimens failed in BY-JS mode and the remaining 44 specimens failed in BH mode. The concrete strengths of the specimens range from 15.4 to 93.8 MPa; the column axial stresses range from -4% to 40% of its axial strength; and h_b/h_c ratio ranges from 0.9 to 2. The test shear force, $V_{jh,test}$, and the predicted shear strength, $V_{jh,model}$, calculated following the proposed model are also presented in Table 1. The summarised test shear forces, $V_{jh,test}$, were either collected from the reported values or derived using

Table 1. Verification of the proposed analytical model

Reference	Exterior joint	b_c : mm	h_c : mm	b_b : mm	h_b : mm	f'_c : MPa	A_{sb} : mm ²	f_{sby} : MPa	A_{jh} : mm ²	f_{jhy} : MPa	A_{jv} : mm ²	f_{jvy} : MPa	$V_{j,test}$: kN	$V_{j,model}$: kN	$V_{jh,test}/V_{jh,model}$	Test/predicted failure modes
Ehsani and Wight (1985)	1B	300	300	259	480	33.6	2013	331	881	438	568	490	575	526	1.09	BY-JS/JS
	2B	300	300	259	439	35.0	2013	331	881	438	568	490	587	546	1.07	BY-JS/BY-JS
	3B	300	300	259	480	40.9	2013	331	1321	438	568	490	572	623	0.92	BH/BY-JS
	4B	300	300	259	439	44.6	2013	331	1321	438	568	490	586	648	0.90	BH/BY-JS
	5B	340	340	300	480	24.3	2322	331	881	438	1020	414	679	575	1.18	BY-JS/JS
	6B	340	340	300	480	39.8	1729	331	881	438	568	490	504	563	0.90	BH/BH
Ehsani <i>et al.</i> (1987)	1	340	340	300	480	64.7	1168	455	1321	455	568	455	549	579	0.95	BH/BH
	2	340	340	300	480	67.3	1420	455	1321	455	568	455	669	704	0.95	BH/BH
	3	300	300	259	439	64.7	1252	455	1321	455	568	455	596	630	0.95	BH/BH
	4	300	300	259	439	67.3	1561	455	1321	455	774	455	736	785	0.94	BY-JS/BH
Ehsani and Alameddine (1991)	LL8	356	356	318	508	55.8	2040	421	1548	421	774	421	860	894	0.96	BY-JS/BY-JS
	LH8	356	356	318	508	55.8	2040	421	2322	421	774	421	837	932	0.90	BY-JS/BH
	HL8	356	356	318	508	55.8	2564	421	1548	421	1020	421	986	975	1.01	JS/BY-JS
	HH8	356	356	318	508	55.8	2564	421	2322	421	1020	421	985	1053	0.94	BY-JS/BY-JS
	LL11	356	356	318	508	73.8	2040	421	1548	421	774	421	768	932	0.82	JS/BH
	LH11	356	356	318	508	73.8	2040	421	2322	421	774	421	933	932	1.00	BH/BH
	HL11	356	356	318	508	73.8	2564	421	1548	421	1020	421	967	1108	0.87	JS/BY-JS
	HH11	356	356	318	508	73.8	2564	421	2322	421	1020	421	1020	1172	0.87	BY-JS/BH
	LL14	356	356	318	508	93.8	2040	421	1548	421	774	421	877	932	0.94	BY-JS/BH
	LH14	356	356	318	508	93.8	2040	421	2322	421	774	421	890	932	0.95	BY-JS/BH
Hwang <i>et al.</i> (2005)	HH14	356	356	318	508	93.8	2564	421	2322	421	1020	421	1032	1172	0.88	BY-JS/BH
	OT0	420	420	320	450	67.3	2040	430	0	—	1584	421	997	933	1.07	BY-JS/BH
	3T44	420	420	320	450	76.8	2040	430	2322	498	1584	421	1065	933	1.14	BH/BH
	3T3	420	420	320	450	69.0	2040	430	639	471	1584	421	1132	933	1.21	BY-JS/BH
	2T4	420	420	320	450	71.0	2040	430	774	498	1584	421	1080	933	1.16	BY-JS/BH
	1T44	420	420	320	450	72.8	2040	430	774	498	1584	421	1039	933	1.11	BY-JS/BH
	3T4	420	420	320	450	75.2	2040	491	1161	436	1584	458	1110	1065	1.04	BH/BH
	2T5	420	420	320	450	76.6	2040	491	1200	469	1584	458	1162	1065	1.09	BH/BH
Wong and Kuang (2008)	1T55	420	420	320	450	69.7	2040	491	1200	469	1584	458	1126	1065	1.06	BH/BH
	BS-L-300	300	300	260	300	34.1	942	520	0	—	0	—	505	435	1.16	JS/JS
	BS-L-450	300	300	260	450	30.9	942	520	0	—	0	—	316	404	0.78	JS/JS
	BS-L-600	300	300	260	600	36.4	942	520	0	—	0	—	284	397	0.72	JS/BY-JS
	BS-L-V2	300	300	260	450	32.6	942	520	0	—	314	500	399	400	1.00	JS/JS
	BS-L-V4	300	300	260	450	28.3	942	520	0	—	628	500	403	391	1.03	JS/JS
Tsonos (2007)	BS-L-H1	300	300	260	450	33.3	942	520	157	500	0	—	389	400	0.97	JS/JS
	BS-L-H2	300	300	260	450	42.1	942	520	314	500	0	—	479	485	0.99	BY-JS/BY-JS
	A1	200	200	200	300	35.0	314	500	283	540	157	500	172	157	1.10	BH/BH
	E1	200	200	200	300	22.0	462	495	283	540	308	495	232	184	1.26	JS/BY-JS
	E2	200	200	200	300	35.0	308	495	283	540	308	495	164	152	1.08	BH/BH
	G2	200	200	200	300	22.0	462	495	201	540	308	495	222	175	1.27	JS/JS
Clyde <i>et al.</i> (2000)	Test 2	305	457	305	406	46.2	2564	454	0	—	774	470	947	977	0.97	JS/JS
	Test 4	305	457	305	406	41.0	2564	454	0	—	774	470	982	955	1.03	JS/JS
	Test 5	305	457	305	406	37.0	2564	454	0	—	774	470	941	896	1.05	JS/JS
	Test 6	305	457	305	406	40.1	2564	454	0	—	774	470	927	901	1.03	JS/JS
Kuang and Wong (2006)	BS-L	300	300	260	450	30.9	942	520	0	—	0	—	316	400	0.79	JS/JS
	BS-U	300	300	260	450	31.0	942	520	0	—	0	—	341	401	0.85	JS/JS

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Tsonos <i>et al.</i> (1992)	S1	200	200	200	300	37.0	308	485	302	495	616	485	139	150	0.93	BH/BH
	S2	200	200	200	300	26.0	305	513	302	495	616	485	146	156	0.93	BH/BH
	S6'	200	200	200	300	29.0	616	485	302	495	0	—	202	212	0.96	JS/JS
Paulay and Scarpas (1981)	Unit 1	457	457	356	610	22.6	1885	296	1810	326	1257	296	754	585	1.28	BH/BH
	Unit 2	457	457	356	610	22.5	2790	297	1810	326	1257	296	990	868	1.14	BH/BH
	Unit 3	457	457	356	610	26.9	1885	296	942	316	1257	296	753	585	1.29	BH/BH
Pantelides <i>et al.</i> (2002)	Unit 1	406	406	406	406	33.1	2564	459	0	—	0	—	872	815	1.07	JS/JS
	Unit 2	406	406	406	406	30.2	2564	459	0	—	0	—	833	817	1.02	JS/JS
	Unit 3	406	406	406	406	34.0	2564	459	0	—	0	—	826	828	1.00	JS/JS
	Unit 4	406	406	406	406	31.6	2564	459	0	—	0	—	927	842	1.10	JS/JS
	Unit 5	406	406	406	406	31.7	2564	459	0	—	0	—	770	793	0.97	JS/JS
	Unit 6	406	406	406	406	31.0	2564	459	0	—	0	—	851	831	1.02	JS/JS
Chalioris <i>et al.</i> (2008)	JA-0	200	300	200	300	34.0	452	580	0	330	157	580	218	252	0.86	JS/BY-JS
	JA-s5	200	300	200	300	34.0	452	580	503	330	157	580	218	267	0.81	BH/BH
	JB-0	200	300	200	300	31.6	471	580	0	—	0	—	201	233	0.86	JS/BY-JS
	JB-s1	200	300	200	300	31.6	471	580	101	330	0	—	219	240	0.91	JS/BY-JS
	JCa-0	100	200	100	200	20.6	157	470	0	—	0	—	66	62	1.06	JS/JS
	JCa-s1	100	200	100	200	20.6	157	470	101	330	0	—	71	69	1.02	BY-JS/BY-JS
	JCa-s2	100	200	100	200	20.6	157	470	201	330	0	—	71	76	0.93	BY-JS/BY-JS
	JCb-0	100	200	100	200	23.0	236	470	0	—	0	—	84	81	1.05	JS/JS
	JCb-s1	100	200	100	200	23.0	236	470	101	330	0	—	97	88	1.10	JS/JS
	JCb-s2	100	200	100	200	23.0	236	470	201	330	0	—	88	95	0.93	JS/JS
Chun <i>et al.</i> (2009)	JC-1	500	500	350	500	61.7	1548	403	426	384	2322	403	793	664	1.19	BH/BH
	JC-2	500	500	350	500	60.1	3096	403	426	384	2322	403	1320	1327	0.99	BY-JS/BH
	JC-No. 11	650	520	450	505	31.0	3054	468	852	384	2036	458	1179	1324	0.89	BY-JS/BY-JS
Karayannis and Sirkelis (2008)	A1	200	200	200	300	36.4	157	574	0	—	0	—	76	92	0.82	BY-JS/BH
	A2	200	200	200	300	36.4	157	574	0	—	0	—	74	92	0.80	BY-JS/BH
	B1	200	200	200	300	36.4	157	574	402	574	0	—	82	92	0.90	BH/BH
Karayannis <i>et al.</i> (2008)	B2	200	200	200	300	36.4	157	574	402	574	0	—	84	92	0.92	BH/BH
	A0	200	200	200	300	31.6	157	580	0	—	0	—	83	93	0.89	BY-JS/BH
	A1	200	200	200	300	31.6	157	580	101	580	0	—	86	93	0.93	BH/BH
	A2	200	200	200	300	31.6	157	580	201	580	0	—	91	93	0.98	BH/BH
	A3	200	200	200	300	31.6	157	580	302	580	0	—	90	93	0.97	BH/BH
	B0	200	200	200	300	31.6	471	580	0	—	0	—	199	183	1.09	JS/JS
	B1	200	200	200	300	31.6	471	580	101	580	0	—	215	196	1.10	JS/JS
	C0	200	200	200	300	31.6	452	580	0	—	157	580	209	189	1.11	JS/JS
	C2	200	200	200	300	31.6	452	580	201	580	157	580	209	192	1.09	BY-JS/JS
	C3	200	200	200	300	31.6	452	580	302	580	157	580	222	204	1.09	BH/JS
Hwang <i>et al.</i> (2004)	C5	200	200	200	300	31.6	452	580	503	580	157	580	221	228	0.97	BH/BY-JS
	28-3T4	550	550	380	500	35.0	2040	491	774	436	3168	458	1290	1042	1.24	BH/BH
	28-0T0	550	550	380	500	33.0	2040	491	0	—	3168	458	1138	1042	1.09	BY-JS/BH
Alva <i>et al.</i> (2007)	LVP2	200	300	200	400	44.2	804	594	201	602	804	594	514	433	1.19	BY-JS/BY-JS
	LVP3	200	300	200	400	23.9	804	594	402	602	804	594	364	380	0.96	BY-JS/JS
	LVP4	200	300	200	400	24.6	804	594	201	602	804	594	327	361	0.91	JS/JS
	LVP5	200	300	200	400	25.9	804	594	402	602	804	594	380	395	0.96	BY-JS/JS
Durrani and Zerbe (1987)	J1	305	305	254	381	39.4	1136	414	774	531	1020	483	375	506	0.74	BH/BH
Liu (2006)	Unit RC-1	230	230	200	330	19.4	471	324	0	—	0	—	140	155	0.90	BY-JS/BY-JS
	Unit RC-6	250	250	250	330	25.9	452	307	56.5	384	226	307	177	148	1.20	BH/BH
	Unit NZ-7	250	250	250	330	30.0	452	307	283	384	226	307	190	148	1.29	BH/BH
Karayannis and Sirkelis (2005)	AJ1s	200	200	200	300	32.8	157	580	101	580	0	—	87	93	0.93	BY-JS/BH

(continued on next page)

Table 1. Continued

Reference	Exterior joint	b_c : mm	h_c : mm	b_b : mm	h_b : mm	f'_c : MPa	A_{sb} : mm ²	f_{sby} : MPa	A_{jh} : mm ²	f_{jhy} : MPa	A_{jv} : mm ²	f_{jvy} : MPa	$V_{j,test}$: kN	$V_{j,model}$: kN	$V_{jh,test}/V_{jh,model}$	Test/predicted failure modes
Le-Trung <i>et al.</i> (2010)	NS	167	167	134	200	36.5	314	324	0	—	0	—	120	102	1.17	BY-JS/BY-JS
	SD	167	167	134	200	36.5	314	324	50.3	459	0	—	114	107	1.07	BY-JS/BY-JS
Antonopoulos and Triantafillou (2003)	C1	200	200	200	300	15.6	462	585	0	—	0	—	116	115	1.00	JS/JS
	C2	200	200	200	300	19.0	462	585	0	—	0	—	115	129	0.89	JS/JS
	S-C	200	200	200	300	15.4	462	585	101	260	0	—	123	120	1.03	JS/JS
Ghobarah and Said (2002)	T1	250	400	250	400	30.8	1195	425	0	—	402	408	527	464	1.14	BY-JS/BY-JS
El-Amoury and Ghobarah (2002)	T0	250	400	250	400	30.6	1195	425	0	—	402	408	420	462	0.91	JS/BY-JS
Ghobarah and Said (2001)	T2	250	400	250	400	30.8	1195	425	0	—	402	408	502	442	1.14	JS/JS
Tsonos (2002)	A1	200	200	200	300	35.0	314	500	483	500	157	500	164	157	1.04	BH/BH
	E1	200	200	200	300	35.0	308	500	483	500	157	500	161	154	1.05	BH/BH
Ghobarah and El-Amoury (2005)	T-B10	250	400	250	400	30.0	598	425	628	454	402	408	280	278	1.01	BH/BH
	T-B53	250	400	250	400	30.0	1195	425	0	—	402	408	421	456	0.92	JS/BY-JS
Shrestha <i>et al.</i> (2009)	UC1	300	300	300	450	25.8	1759	532	0	—	0	—	293	382	0.77	JS/JS
Vatani-Oskouei (2010)	Specimen 1	350	350	350	400	24.3	1018	417	628	282	1018	417	400	466	0.86	BY-JS/BY-JS
	Specimen 2	350	350	350	400	19.6	1018	417	628	282	1018	417	425	424	1.00	BY-JS/BY-JS
Fisher and Sezen (2011)	C-2-RC	152	152	152	203	30.0	213	493	129	376	258	493	102	101	1.00	BY-JS/BH
	E-1-RC	152	152	152	203	30.0	329	448	129	376	258	493	113	107	1.05	BY-JS/JS
	B-1-RC	152	152	152	203	30.0	387	435	129	376	258	493	130	121	1.07	JS/JS
Megget (1974)	Unit A	330	380	255	460	22.1	1792	375	1548	317	774	365	547	542	1.01	BY-JS/JS
Murty <i>et al.</i> (2003)	Q1	200	250	200	400	25.6	628	382	0	—	402	382	156	173	0.90	JS/JS
	Q3	200	250	200	400	26.9	628	382	1032	280	402	382	211	225	0.94	BY-JS/BH
	R1	200	250	200	400	30.2	628	382	0	—	402	382	173	187	0.92	JS/BY-JS
	R3	200	250	200	400	27.1	628	382	1032	280	402	382	217	225	0.96	BH/BH
	S1	200	250	200	400	27.8	628	382	0	—	402	382	163	180	0.90	JS/JS
	S3	200	250	200	400	30.1	628	382	1032	280	402	382	198	225	0.88	BY-JS/BH
	Specimen I	406	406	356	457	27.6	2040	483	2064	365	1548	483	1040	807	1.29	BY-JS/JS
Chutarat and Aboutaha (2003)	Specimen A	406	406	356	457	33.1	800	483	2064	365	774	483	419	409	1.02	BH/BH
Kaku and Asakusa (1991)	No 1	220	220	160	220	31.1	531	391	226	250	0	—	248	217	1.14	BH/BY-JS
	No 2	220	220	160	220	41.7	531	391	226	250	0	—	250	227	1.10	BH/BH
	No 3	220	220	160	220	41.7	531	391	226	250	0	—	217	202	1.08	BY-JS/BY-JS
	No 4	220	220	160	220	44.7	531	391	56.5	281	0	—	239	227	1.06	BY-JS/BH
	No 5	220	220	160	220	36.7	531	391	56.5	281	0	—	221	216	1.02	BY-JS/BY-JS
	No 6	220	220	160	220	40.4	531	391	56.5	281	0	—	209	190	1.10	BY-JS/BY-JS
	No 7	220	220	160	220	32.2	531	391	226	250	314	395	249	227	1.10	BH/BH
	No 8	220	220	160	220	41.2	531	391	226	250	314	395	244	227	1.08	BH/BH
	No 9	220	220	160	220	40.6	531	391	226	250	314	395	236	224	1.05	BY-JS/BY-JS
	No 10	220	220	160	220	44.4	531	391	56.5	281	314	395	243	227	1.07	BH/BH
	No 11	220	220	160	220	41.9	531	391	56.5	281	314	395	231	227	1.02	BH/BH
	No 12	220	220	160	220	35.1	531	391	56.5	281	314	395	207	202	1.03	BY-JS/BY-JS
	No 13	220	220	160	220	46.4	531	391	226	281	314	395	209	201	1.04	BY-JS/BY-JS
	No 14	220	220	160	220	41.0	531	391	56.4	281	113	282	226	227	1.00	BY-JS/BH
	No 15	220	220	160	220	39.7	531	391	56.4	281	157	395	230	227	1.02	BY-JS/BH
	No 16	220	220	160	220	37.4	531	391	226	250	508	381	251	227	1.11	BH/BH
Megget and Park (1971)	Unit 1	330	381	254	457	28.4	1282	285	774	317	0	—	314	379	0.83	BH/BH
	Unit 3	330	381	254	457	35.8	1282	285	1032	250	0	—	313	379	0.83	BH/BH
Park and Paulay (1974)	S4	330	381	254	457	20.5	1282	296	1290	312	0	—	317	393	0.81	BY-JS/BH
Total	142												Average: COV:		1.00 0.118	

Equations 5 and 7 based on the maximum applied load measured from the test. In this study, the moment arm of the beam cross-section is assumed to be 80% of the total height of the beam cross-section, h_b .

It can be seen from Table 1 that the predicted shear strength of the exterior beam-column joints is closely in agreement with the test values. The proposed model predicts the joint shear strength of the database with a mean value of 1.00 for the ratio between the test results and the model prediction and the corresponding coefficient of variation of 11.8%. As the model predicts the shear strength of the joints by way of the beam tensile reinforcement stress, it can predict beam reinforcement yield and failure mode of the joints, as shown in Table 1. It is noted that, in Table 1, the predictions of joint shear strength are made with $h_l = 0.65h_c$, $\kappa = 0.40$ and $r = 0.5$, which are determined from Figure 6 corresponding to the best correlation with the experimental results, indicating that the analytical predictions are acceptable for the suggested ranges of h_l , κ and r .

In Figure 7, the evaluation results using the proposed model are compared with those from five existing joint shear strength models proposed by Hwang and Lee (2002), Tsonos (2007), Wang *et al.* (2012), Scott *et al.* (1994) and Kim *et al.* (2009). The

four former models are theoretical while the last one is an empirical model. These models were chosen for comparison because they are the latest available models and represent models developed basing on the existing approaches. The test-to-predicted average strength ratio (Avg) and its coefficient of variation (COV) are also included in Figure 7. It can be seen from the figure that a better accuracy of the proposed model is clearly demonstrated.

The good prediction of the proposed model indicates that the adopted linear failure envelope and the proposed assumptions are acceptable. For specimens with BH failure mode, the Hwang and Lee (2002) model and Tsonos (2007) model give the same prediction as the proposed model because they are all based on the assumption that the beam fails when the tensile stress of beam reinforcement reaches the value of $1.25f_{sby}$. However, for specimens with JS or BY-JS, both these models give less accuracy than the proposed model. The reasons for these two models being less accurate can be explained as follows. The accuracy of the Hwang and Lee (2002) model depends significantly on the determination of the width of the diagonal strut, but it is difficult to estimate accurately. In the Tsonos (2007) model the contribution of shear reinforcement to the tensile strength of the concrete is ignored, while its contribution to improving the compressive strength of the concrete is exaggerated.

Figure 6. Statistics of evaluation results for investigated values of h_l , k and r : (a) $h_l = 0.5h_c$; (b) $h_l = 0.6h_c$; (c) $h_l = 0.65h_c$; (d) $h_l = 0.7h_c$; (e) $h_l = 0.8h_c$ (Note: h_l is the development length of beam longitudinal reinforcement (Figure 1); h_c is the width of column along the beam; $r = \epsilon_{cr}/\epsilon_{sjy}$ is the ratio between the cracking strain of concrete and the yield strain of joint shear reinforcement; COV, coefficient of variation of test-to-predicted shear strength ratio; κ is the fraction of beam longitudinal bond force transferred into upper column)

Figure 7. Comparison of evaluation results with five existing models: (a) proposed model; (b) Hwang and Lee (2002) model; (c) Tsonos (2007) model; (d) Wang *et al.* (2012) model; (e) Scott *et al.* (1994) model; (f) Kim *et al.* (2009) model

The lower accuracy of the remaining three models as compared with the proposed model can be explained as follows. The Scott *et al.* (1994) model neglects the contribution of joint shear reinforcement, while the Kim *et al.* (2009) model is developed based on empirical analysis with a limitation of the database and without consideration of the column axial load. The Wang *et al.* (2012) model is relatively less accurate because it neglects the influence of the column axial force on the direction of the compression and tension stresses on the joint panel. In addition, it assumes that the tensile strength of the concrete and shear reinforcement reach the same strain, whereas, in reality, the concrete and the reinforcement cannot reach their strength simultaneously. Figure 6 clearly shows that when $r=1$ (the concrete and the reinforcement reach their strength simultaneously) the proposed model gives less accurate predictions, while the best prediction corresponds to $r=0.5$.

4. Parametric study

It is known that the concrete compressive strength, f'_c , column axial stress level, $N_c/(b_c h_c f'_c)$, and the amount of shear reinforcement ($A_{jh} f_{jh} / (b_c h_c)$ and $A_{jv} f_{jv} / (b_c h_c)$) are among the key parameters that affect the shear strength of the joint. Moreover, in the proposed model the role of the column axial stress and the contribution of the shear reinforcement are

assumed in the specified ways. These parameters, therefore, were considered in this part of the study to evaluate the fitness of the proposed assumptions as well as the accuracy of the proposed model. Figure 8 presents the variation of test-to-predicted shear strength ratio with each of these parameters. It can be seen from the figure that the key parameters of the RC exterior joints have been very well captured by the proposed model. Figures 8(a) and 8(b) show that the test-to-predicted shear strength is independent of the horizontal and vertical joint shear reinforcement. This means that the proposed assumption about the role of the shear reinforcement and the proposed ratio $r = \varepsilon_{cr} / \varepsilon_{sjv} = 0.5$ were justified. Recently, an analytical model based on the plane stress concept was developed by Wang *et al.* (2012). In their model the influence of the column axial load on the direction of the principal stresses was neglected. The parametric study based on data from 47 test results showed that their model underestimated the joint shear strength slightly when the concrete strength or the column axial load was low. The proposed model, on the other hand, can capture very well the influence of both the column axial stress level and the concrete compressive strength (Figures 8(c) and 8(d)). This fact shows that the contribution of the column axial stress level has been considered reasonably and the proposed Equation 18 for the determination of the inclination angle θ is justifiable.

Figure 8. Influence of each of the key parameters on the test-to-predicted joint shear strength ratio: (a) influence of horizontal shear reinforcement; (b) influence of vertical shear reinforcement; (c) influence of column axial stress level; (d) influence of concrete compressive strength

5. Variation of the fraction factor

As discussed above, the fraction factor is the important parameter that controls the percent of shear force resisted by the joint and thus, it affects the accuracy of the predicted joint shear strength. From the above analysis, it can be seen that the magnitude of the fraction factor is proportional to h_c and $(f'_c)^{1/2}$, while it is inversely proportional to f_{sby} and d_b . The variation of the fraction factor with the $h_c(f'_c)^{1/2}/(f_{sby}d_b)$ ratio is shown in Figure 9. For the collected database, β ranges from 0.69 to 0.91 with an average value of 0.84 and it reduces with the increase of $h_c(f'_c)^{1/2}/(f_{sby}d_b)$ ratio. It can also be seen from Figure 9 that, for the joints failed with yielding of beam tensile bars (BY-JS and BH failure modes), the fraction factor is significantly high. The average values of the fraction factor for the joint failure in JS, BY-JS and BH modes are 0.8, 0.86 and 0.86, respectively. It is also noted that, in the collected database, the $h_c(f'_c)^{1/2}/(f_{sby}d_b)$ ratio ranges from 0.09 to 0.44. In some cases, when the T-connections are designed with $h_c(f'_c)^{1/2}/(f_{sby}d_b)$ ratio larger than 0.44, from Figure 9 it can be seen that the fraction factor tends to reduce to the minimum value of 0.5, which was proposed by Priestley (1993) for calculating the shear strength of a very strong column and beam but a weak joint.

Figure 9. Variation of fraction factor β with $h_c(f'_c)^{0.5}/(f_{sby}d_b)$ ratio

6. Conclusions

An analytical shear strength model for exterior beam–column connections was developed based on the average plane stress concept. In the proposed model, a fraction of the beam bars' tension force was assumed to be resisted by the joint core, while the remainder was assumed to be resisted by the adjacent

column. The ratio of joint shear force resisted by the joint core was expressed in terms of the fraction factor, which was formulated using the bond resistance of the concrete surrounding the beam longitudinal tensile reinforcement within the joint region. The biaxial failure envelope of concrete was adopted to predict the shear failure of the joint core caused by a combination of compressive and tensile stresses. The contribution of shear reinforcement and column axial load on shear strength of the joint was considered. The asynchronicity of concrete and shear reinforcement in tensile strength was accounted for in the model. Finally, a procedure for the proposed analytical model was outlined for design applications.

The proposed model was validated by comparison with results of a large database of exterior beam–column joints subjected to cyclic loading collected from the literature. These comparisons have shown the superiority of the proposed model in predicting exterior joint shear strength. A parametric study was also conducted using the collected database. This study illustrates that the proposed assumptions including the role of column axial stress and joint shear reinforcement are justified. Finally, the study proved that the key parameters of an exterior joint were successfully integrated in the proposed model.

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