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Brenton J. Prettejohn
University Of South Australia

Matthew J. Berryman
University of Wollongong, mberryma@uow.edu.au

Mark D. McDonnell
University Of South Australia

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Keywords

formation, adaptive, networks, model, impact, effects, network, topology, robustness, authority, consensus

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A model of the affects of authority on consensus formation in adaptive networks: impact on network topology and robustness

Brenton J. Prettejohn[a], Matthew J. Berryman[b], Mark D. McDonnell[a]

[a] *Institute for Telecommunications Research, University of South Australia, SA 5095, Australia*

[b] *Defence and Systems Institute, University of South Australia, SA 5095, Australia*

Abstract

Opinions of individuals in real social networks are arguably strongly influenced by external determinants, such as the opinions of those perceived to have the highest levels of authority. In order to model this, we have extended an existing model of consensus formation in an adaptive network, by the introduction of a parameter representing each agent's level of 'authority', based on their opinion relative to the overall opinion distribution. We found that introducing this model, along with a randomly varying opinion convergence factor, significantly impacts the final state of converged opinions and the number of interactions required to reach that state. We also determined the relationship between initial and final network topologies for this model, and whether the final topology is robust to node removals. Our results indicate firstly that the process of consensus formation with a model of authority consistently transforms the network from an arbitrary initial topology, to one with distinct measurements in mean shortest path, clustering coefficient, and degree distribution. Secondly, we found that subsequent to the consensus formation process, the mean shortest path and clustering coefficient are less affected by both random *and* targeted node disconnection. Speculation on the relevance of these results to real world applications is provided.

Keywords: social networks, opinion dynamics, consensus formation, complex network, network robustness, directed networks, sociopsychology, human dynamics, bounded confidence model

1. Introduction

Modelling interacting agents in social networks, and simulations of opinion formation processes in these models, is currently an active field of research in statistical physics. For example, the 'bounded confidence model' of opinion formation with continuous opinions has been studied extensively [1, 2, 3, 4, 5, 6] . However, the impact of an adaptively changing social network within that framework has only recently been introduced, and it was shown, for example, that convergence to a small number of opinion clusters is more likely when links within the network change adaptively [4, 5]. While complex network topologies have been studied for the bounded confidence model [7], their significance in the context of adaptive networks is unknown.

A considerable amount of evidence has been generated which supports the predominance of complex networks amongst naturally evolved and human designed systems [8, 9]. Such work includes research into network topology and biological evolution [10, 11], work on 'small-world' networks [12] showing the importance of 'random links' for connectivity, and research supporting the theory that natural networks tend to evolve toward having heavy tailed degree distributions, e.g. 'scale-free networks' [13].

Barabási and Albert have argued that the predominance of scale-free networks may be due in part to its natural robustness to network perturbations [13, 8]. Such networks tend to be robust in terms of metrics such as degree distribution, clustering coefficient, and mean shortest path, when randomly chosen nodes are removed. However, it is also known that disconnection of high degree nodes (often referred to as hubs) can have catastrophic outcomes on these same metrics [14, 15].

Many network simulation studies are based on the construction of constantly evolving networks, i.e. networks where nodes are added and removed on a continuing basis. The idea is that nodes are added to the network with a 'preferential attachment' bias, where nodes with a larger degree have a higher probability of attracting new nodes, or due to a 'fitness' quotient whereby some nodes are considered to be more *effective* at attracting connections from other nodes [16, 17]. Consequent research into construction and behavior of networks has tended to focus on such models, or networks with a static population predetermined at the point of creation [18, 19, 20, 21, 22].

However, this raises an interesting question, one that while being the subject of some conjecture [13, 23, 24] does not appear to have an easily found definitive answer—why do networks adopt and maintain the topologies that they do? Evolutionary principals of robustness may explain this to some extent—that is, networks that can best cope with perturbations are more likely to flourish. But, what about dynamic networks—i.e., where connections between nodes are made and broken over time—with relatively static population sizes, where definitions of fitness are complex, and depend on the network's purpose? Could it be that for naturally evolving networks, the function of the network contributes to its structural properties, so that robustness is a byproduct of function, and that in turn, the most robust structures emerge?

If so, perhaps some benefit to network robustness in designed systems can be provided if a method can be found for replicating such behaviour. This is especially the case where the network in question is deemed likely to be vulnerable to targeted node interference.

Recent sociophysics simulations, aimed at investigating opinion formation and consensus in terms of the emergence of global phenomena based solely on local interactions, have illustrated the importance of adaptability in the connections between agents in social networks [25, 4, 5]. In such work, an evolving random graph model is used to represent agents who currently communicate, and simulations of interactions between individuals are carried out, in order to map the difference between social groups that are able to dynamically re-form, and those which are not. As described in Section 2, the opinion dynamics closely follow the bounded confidence model [1]. Nodes in the model networks represent agents and edges in the model networks represent the potential for communication between pairs of nodes. Each agent has a continuously valued opinion that can be altered after an interaction between pairs of neighbours, provided the two neighbours' opinions are within some tolerance. Investigations within this model have included varying population sizes, as well as levels of adaptability—i.e., the likelihood of an agent cutting a link to a neighbor and creating a new one—as well as the number of interactions required to achieve the network's final state [4, 5]. Simulation results indicated that global consensus can only be formed in a static model when agents have relatively high tolerances for opinion difference, whereas when the ability for nodes to break old relationships and form new ones is introduced, consensus with relatively small tolerances becomes likely [4, 5].

From a sociological point of view, we propose that this model can be extended in two ways that enable an improved match with real social interactions. The first extension is the inclusion of a more realistic randomly varying opinion-convergence factor. The second is the inclusion of an external determinant, namely the *perceived authority disparity* between the agents in the social network. In considering the first point, there is general acceptance [26, 27] that when a consensus is reached between individuals (or indeed groups), as often as not that consensus is not the result of an absolute meeting of the minds. It is rather a working agreement, which originates from, as well as facilitating, a more subtle convergence of actual opinion. The second point is discussed below in Section 2.2.2. We have previously shown that these adaptations provide notable, and arguably more sociologically representative outcomes for opinion convergence modelling [28], as has subsequent work that also underlines the impact of both a continuous opinion convergence factor, and external determinants [6].

After including these two factors, we investigated their impact on the final distribution of opinions, and also the significance of initial network topology for the consensus formation process [28]. We found that the addition of these two factors resulted in a more compact opinion spread, with less interactions being required for the network to converge to its final state, i.e. where no more topological changes could be made (see Fig. 1). We also found that initial network topology had no significant affect on the final opinion state, but also noticed that after consensus formation, some statistics of the final networks had similar values regardless of initial topology. It is this latter finding that we investigate in a detailed way in this paper.

Precisely how to model the topology of local and global social networks, as well as other networks occurring both naturally and by design, has been a topic of research for some time. From Milgram, in the '60s [29], through Strogatz and Watts' seminal work [12] and to this day, it has generally been agreed that both social networks, and most other naturally occurring networks, are not entirely unstructured by nature. Indeed, there is a growing body of research that suggests these networks have a complex structure that includes high clustering coefficient, low mean shortest path length, and in some instances a heavy tailed degree distributions [13, 8, 30, 31, 32, 33, 34, 35, 36, 37].

The initial network topology in [4, 5] was an Erdős-Rényi network model. This produces an artificially random network, in the sense that the number of neighbours for each node does not vary much, and there is no local clustering. In this paper we compare and contrast consensus formation in simulations of adaptive networks whose initial state is one of three canonical networks: the Erdős-Rényi *random* network, the Watts-Strogatz *small-world* network [12] or the Barabási-Albert *scale free* network [13]. Furthermore, we measure the affect on network structure of a consensus formation process, with particular interest in benefit to network efficiency and robustness. A detailed description of the methods used to produce these models, and the metrics used to monitor changes is provided in Section 2.

Section 3 describes the results of our simulations. We found that the consensus formation process has a profound affect on the *final* network topology. We identify in Sections 3 and 4 two significant findings from the simulations. First that the consensus formation process, regardless of initial topology, results in similar outcomes for all three metrics (clustering coefficient, mean shortest path, and degree distribution), which are also distinct in each case from those found in the three initial networks. Secondly, we found that this 'new' topology is demonstrably more robust in terms of clustering coefficient and mean shortest path, when subjected to both targeted, and non-targeted node disconnections.

2. Model formulation and extensions

2.1. Original model

The consensus formation framework we use is a well known model (see, e.g. [1, 4]) that is predicated on a simple and virtually axiomatic principle. If two agents (network nodes) each have opinions that differ only within a tolerance that would make communication productive, and they are offered the opportunity to communicate, then the disparity between the opinions will be reduced.

Several variables are integral to the model of [4], which is initially an Erdős-Rényi undirected random graph [38, 39]. We assume the network has size N, and initially has average degree $\bar{k} = 10 \ll N$, as studied in [4]. This mean-degree provides a network that is sufficiently sparse for us to highlight the affects of node disconnection—see Section 2.3.2.

In the adaptive network model, a sequence of interactions between a randomly chosen pair of network nodes are simulated. The choice of a first node, i, is made uniformly randomly across the entire network, and is independent for each interaction. The second node j is chosen uniformly randomly from the neighbours of node i. After the t-th interaction of the entire network, agent i has opinion, $o(i,t)$ $(i \in 1, .., N)$, which is an initially uniformly random value between 0 and 1, representing each node's opinion. Tolerance, d, is an arbitrarily set value used to determine if opinions are close enough for productive communication, or sufficiently divergent to require rewiring.

Note that d is identical for all pairs of nodes and is constant within a single simulation.

The network topology is altered during simulations according to the approach of Kozma and Barrat [4]. Firstly, a variable called w is set to an arbitrarily chosen constant between 0 and 1 for the entire simulation. Then, for each interaction between nodes i and j, with probability w the two nodes' opinion difference is considered (note that w is identical for all node pairs). Different values of w affect the time required for the network to reach a final state. However, this does not simply rescale results over time. Nodes have more than one neighbour, and as a consequence, if node i and node j have opinions outside the tolerance threshold, but they do not break their connection when selected because w is such that their opinions difference is not considered, then it is possible that one or both will change opinions independently before the next opportunity to communicate occurs. On a global level the result is that the more likely nodes i and j are to break their connection, the more likely the network is to fracture into several opinion clusters, rather than reach global convergence of opinions. This effect has been investigated in [4].

Next, if the opinion difference between nodes i and j is larger than d, then their link in the network will be removed, and a new one created between node i and a node chosen uniformly randomly from all nodes $1, ..., N$ (excluding i and j).

If, on the other hand, the opinion difference is $|o(i,t) - o(j,t)| \leq d$, the opinions of interacting nodes i and j update according to

$$\begin{aligned} o(i,t+1) &= o(i,t) + \mu \left[o(j,t) - o(i,t)\right] \\ o(j,t+1) &= o(j,t) - \mu \left[o(j,t) - o(i,t)\right], \end{aligned} \tag{1}$$

where μ is an opinion-convergence factor. During simulations interactions continue until the final state of the network is reached, which is defined by [4] as the point at which further interactions will not result in any change in node opinions.

2.2. Model Extensions

2.2.1. Modeling variable opinion convergence

Throughout [4], the convergence factor is set at $\mu = 0.5$, and therefore a complete local agreement is reached every time two interacting agents alter their opinions. When discussing this approach, [4] make it clear that use of this complete agreement model, rather than one which makes room for variable convergence, was chosen primarily to simplify the process. However, as mentioned in the introduction, we aimed to adapt the model to be more analogous to 'real life', by replacing the constant μ of [4] with an opinion convergence model that is partially random and partially due to *authority* disparity.

Specifically, we set μ as a uniform random variable, $\mu \in [0, 0.5]$ (chosen independently for each interaction), and authority disparity is defined in the next section. Note that because of our inclusion of a random convergence factor, and also because we are only interested in final network topology here, rather than the actual opinions, we refined the definition of 'final network state' from [4]. Namely, we stopped our simulations at the point at which further interactions will not result in a change to network topology, i.e. when the opinions of all pairs of neighbours within the network are within tolerance.

2.2.2. Modeling authority

While the adoption of a random variable for the opinion-convergence factor provides a further measure of realism, we argue that this does not go far enough, as it only provides a representation of internal sociological factors. Aristotle proposed three modes of persuasion: pathos (appealing to the target's emotions), logos (applying a logical argument), and ethos (affect of the 'persuader's' perceived authority) [40, 41]. There is evidence for the importance of emotion and logic in the formation of opinion [42, 43]. Indeed, it has been argued that all three are present in any effective form of persuasive discourse [41]. We contend that emotion and logic can be thought of as encapsulated in the internal nature of the opinion-convergence factor in the model of [4], provided that a random opinion-convergence factor is used. However, whilst it is not university accepted, there is ample evidence to support the theory that authority has a significant impact, and is a determinant derived, at least in part, from external factors [44, 45, 46, 47].

In order to model this, we introduce a new variable to represent the 'authority level' of the i–th node, $a(i, t) \in [0, 1]$. Authority level is recalculated at an arbitrarily set frequency, i.e. after every f iterations, for each node i at time t according to the following algorithm. A single new constant parameter is required for our algorithm. This is a 'quorum' parameter, $c \in [0, 1]$; we require a set of 'leaders' to be chosen, such that the sum of their popularities is at least cN.

Algorithm 1:

Step 1: calculate the 'popularity', P_i, of each node's opinion, by counting how many other nodes in the network have opinions within tolerance d of node i; store the popularities in $\mathbf{P} = \{P_1, P_2, .., P_N\}$.

Step 2: Choose a set of 'leaders' by selecting nodes with the highest popularity, such that the opinion difference between each pair of leaders is larger than tolerance, d:

2a. Initialise: We denote the set of node indices corresponding to leaders as $\mathbf{L}$ and a set of nodes excluded for leadership as $\mathbf{F}$. Set the initially chosen leader: $\mathbf{L} = \{\text{argmax } \mathbf{P}\}$. Denote $\mathbf{F} = \varnothing$.

2b. Repeat until $\sum_{j=1}^{|\mathbf{L}|} P_j > cN$. Let $i = \text{argmax } (\mathbf{P} \setminus (\mathbf{L} \cup \mathbf{F}))$. If $|o(i,t) - o(j,t)| > d \ \forall \ j \ \in \mathbf{L}$, then add node i as a new leader: $\mathbf{L} := \mathbf{L} \cup \{i\}$; otherwise set $\mathbf{F} := \mathbf{F} \cup \{i\}$.

Note that where nodes have equal popularities, we choose between them uniformly randomly.

Step 3: Form 'peer groups'—disjoint sets of nodes—around each leader, by setting each node that is not a leader as a member of the 'peer group' of the leader with the opinion closest to its own.

Step 4: Rank each node within its peer group, according to the difference between its opinion and that of its leader's opinion;

Step 5: Calculate the authority level at the t-th interaction for each node i based on the size of its peer group, s_i, and its ranking within that group, r_i ($r_i = s_i$ for the leader and $r_i = 1$ for the node furthest in opinion from the leader), using the following equation.

$$a(i,t) = \frac{1}{2}\left(\frac{s_i}{N} + \frac{r_i}{s_i}\right). \tag{2}$$

Thus, here we weight equally the relative size of the peer group, and the ranking within the group. It can happen that many nodes have an opinion difference with all leaders that is outside tolerance. This is reflected by a low ranking, r_i, within a peer group, and the authority of such nodes is mainly due to its peer group size.

We choose this model for authority as there exists an accepted correlation between authority and popularity [48, 49, 50]. Clearly many factors could be considered in the calculation of popularity and by extension authority [51]. However, here we select popularity of opinion, since opinion is the central variable of interest here, and it therefore has an existing relevance, whilst also providing a valid external determinant.

The magnitude of the difference in authority between two communicating nodes, $|A_{i,j}| := |a(i,t) - a(j,t)| \in [0,1]$, is used to skew the altered opinions toward the more authoritative node's opinion, via the following extension of Eqn. (3). When two neighbours interact, they alter their opinions only if $|o(i,t) - o(j,t)| \leq d + |A_{i,j}/3|$, in which case

$$\begin{aligned} o(i,t+1) &= o(i,t) + \mu(1 - A_{i,j})\,[o(j,t) - o(i,t)] \\ o(j,t+1) &= o(j,t) - \mu(1 + A_{i,j})\,[o(j,t) - o(i,t)]\,. \end{aligned} \tag{3}$$

Recall that we set μ uniformly random, $\mu \in [0, 0.5]$. In our definition of authority difference, if node i has more authority, then $A_{i,j}$ is positive, while if node j has more authority, $A_{i,j}$ is negative. Hence, in the extreme case where $A_{i,j} = 1$, node i's opinion is unchanged, while node j's is changed to $o(j,t+1) = o(j,t)(1-2\mu) + 2\mu o(i,t))$. Thus, only when $\mu = 0.5$ does node j take on node i's opinion exactly. Otherwise, node j's opinion change is doubled relative to the model of [4] with the same μ. By the criteria for convergence, if both nodes have the same relative authority, then tolerance is simply d. Otherwise, note that authority disparity is also used to affect the level of tolerance, which is increased by one third of the authority difference.

2.3. A network topology perspective

Although our previous work was concerned with how a model of authority affected the process of consensus formation itself [28], our results suggested that the process of consensus formation may have a significant affect on how the network topology changes during convergence. In order to better understand the affect of network function on network topology, the following adaptations and metrics were applied to our original model.

2.3.1. Modeling different initial network topologies

Our model of authority is applied in three distinct initial undirected network topologies with the same size N and mean degree $\bar{k}$. We then analyze changes in network topology caused by opinion convergence, by considering *clustering coefficient* (CC), as defined by Watts and Strogatz [12], *mean shortest path* (MSP), and *degree distribution* (DD) [8].

A comparison is also made between each network's *proximity ratio* [52]. This metric uses the Erdős-Rényi topology as a baseline to provide an indication of 'small worldliness,' and is defined as

$$S = \frac{\frac{C}{L}}{\frac{C_{\mathrm{R}}}{L_{\mathrm{R}}}}, \tag{4}$$

where C_{R} and L_{R} are the CC and MSP of a random network with the same N, and $\bar{k}$ as a network for which the clustering coefficient is C and the MSP is L. According to [12], an arbitrary network is 'small-world' when $C \gg C_{\mathrm{r}}$ and $L \simeq L_{\mathrm{r}}$, whereas according to [52] a network is 'small-world' when $S \gg 1$. Given that $S = 1$ for a random network, these two definitions are in agreement, but proximity ratio additionally provides a means of quantitatively comparing networks—for further discussion, see [11].

Differing combinations of these metrics have been used previously [53, 54, 52, 10, 55] to define network characteristics. We have applied them here to provide a before and after comparison of each initial topology, and to assess the affect of the consensus formation process as well as node disconnection.

In addition, the 'largest connected component' or giant component is found, to check for fragmentation. Fragmentation is not expected to occur (depending on the topology and whether disruption is targeted or random) until at least 1% of nodes are removed [56]. Our results support this, as the largest connected component for each network—even after the maximum number of disconnections—was almost always equal to the original size of the network minus the number of disconnections. Of the 25000 simulations run for targeted node removal there were only three instances where the number of disconnected nodes exceeded the number disconnected by one. Of the same number of simulations run for randomly disconnected nodes it did not occur at all.

The first topology we consider is a *random* network created using the Erdős-Rényi (ER) method, where for a network with N nodes, there is a constant probability, p, that any given node is connected to any of the $N-1$ remaining nodes [39]. The second topology is the Watts-Strogatz *small world* (WS) model [12]. Creation of this topology begins with a ring-lattice network of N nodes, where each node i is connected to k nodes, with $k/2$

on either side in the ring-lattice. Random re-wirings are then applied to the network with a constant probability of b. Higher b produces a final network topology that is less clustered and more like the Erdős-Rényi model. Smaller b creates a so-called 'small world,' resulting in a network with a substantially higher clustering coefficient than an Erdős-Rényi random network model, but a mean shortest path only slightly longer than the Erdős-Rényi random network model.

The final model we employ is the Barabási-Albert *scale free* model. As described in [13], this model uses a preferential attachment method for the addition of new nodes to the network. An initial network is created with m_0 nodes. Each new node is connected to m existing nodes, which are chosen with a probability that increases with their current degree, thus conforming to the 'rich get richer' concept [23]. This provides a network topology which has a CC slightly higher than the ER model, but significantly lower than the WS model, an MSP which is comparable with both the ER and WS models, and a degree distribution which follows a power law such that $P(n) \sim n^{-3}$.

2.3.2. Targeted and random node removal

Further refinements to the model were necessary to enable quantitative measurement of the affect of consensus formation on network robustness. Specifically, similar to processes reviewed in [8], we simulated removal of all edges from either specifically targeted (those with the highest degree within the remaining network), or randomly chosen nodes. The removed edges were not replaced. Removal of edges enabled us to quantify the affect of both targeted and random network decay, and the ability of the consensus formation process to vitiate those effects.

3. Results

All of the following results are averaged over 1000 runs, with $N = 1000$, and $\bar{k} = 10$ for all initial topologies. For Watts-Strogatz the probability of rewire is $b = 0.01$, which results in a highly clustered 'small-world.' For Barabási-Albert the size of the seed network is $m_0 = 5$ with each node in the seed network initially connected to $m = 2$ nodes. For the consensus formation process the number of iterations between authority calculations is set at $f = 10000$, the threshold for a quorum is set at $c = 0.5$, and the probability of a rewire when opinion difference exceeds d is $w = 0.25$.

3.1. Opinion distribution after the consensus formation process

Fig 1 shows the affect of introducing random μ and our 'authority' model to the consensus formation process of [4]. Fig. 1(a) shows that over a range of tolerance levels, d, there is a substantially larger number of interactions required to reach the final state with the addition of random μ alone, and that the addition of an authority factor results in substantially less interactions being required to reach the final state than in the original model. Fig. 1(b) shows the difference in numbers of opinion clusters between each model also over a range of tolerance levels. Clearly both have a significant affect; 'authority' has an impact for all d, whereas random μ has a more pronounced impact for a relatively high d.

3.2. Topology before and after the opinion formation process

Figs 2 and 3 show that the clustering coefficient (CC), degree distribution (DD) and mean shortest path (MSP) are markedly changed by the consensus formation process, with final values highly dependent on the tolerance, d.

As d decreases, the final clustering coefficient (Fig. 2(a)) and mean shortest path (Fig. 2(b)) begin to converge to a common value, although this value changes with d. For small d, the convergence for each initial network becomes more pronounced, until at $d \simeq 0.035$ they are indistinguishable. Convergence of these metrics is at CC $\simeq 0.05$ (slightly higher than Barabási-Albert model), rises to CC $\simeq 0.11$ at $d \simeq 0.01$ before dropping away sharply. Mean shortest path converges at MSP$\simeq 3.6$ (significantly higher than both the Barabási-Albert and Erdős-Rényi models) with $d \simeq 0.1$. MSP continues to reduce until reaching MSP$\simeq 3.2$ (equivalent to the Erdős-Rényi model) at $d \simeq 0.005$ (Fig. 2(b)).

Fig. 3(a) shows the degree distribution for the three original network topologies—Watts-Strogatz, Erdős-Rényi, and Barabási-Albert—both before and after the consensus formation process is run with $d = 0.001$. Fig. 3(b) shows mean degree distributions after the consensus formation process has been run with $d = 0.01$ and $d = 0.1$. In each case, a single distinct distribution is formed regardless of the initial topology. This 'new' degree distribution appears almost identical to the Erdős-Rényi degree distribution, particularly in the cases of $d = 0.001$ and $d = 0.1$, where the difference is almost negligible. The largest difference is for $d = 0.01$, where there is a degree distribution that has a tail which is slightly heavier than than Erdős-Rényi and considerably more so than Watts-Strogatz, and significantly less so than Barabási-Albert. However, regardless of the degree distribution, for both $d = 0.01$ and $d = 0.1$, clearly the final network is not the same as the Erdős-Rényi network, because CC and MSP differs from that of Erdős-Rényi. On the other hand, at $d = 0.001$, all three metrics are indistinguishable from the Erdős-Rényi network.

3.3. Network Metrics and Robustness: A Comparison

Fig. 4 shows the comparative difference to proximity ratio (PR) or 'small worldliness', caused to each of the four topologies as a result of both random and targeted node disconnection. The affect of disconnection is recorded increasing from 1 node through to 25 nodes. The consensus formation process is run with $d = 0.01$, and all figures are averaged over 1000 runs. 'Begin' refers in the case of Barabási-Albert, Erdős-Rényi, and Watts-Strogatz to the same version of each topology considered in Section 3.2 before node disruption. In the case of 'PCF' it refers to an Erdős-Rényi network after one run of the consensus formation process has been run—i.e. Post Consensus Formation (PCF). Regarding the data in Fig. 4(d), we did not show data for other initial topologies that give rise to a PCF topology, because proximity ratio (as shown in Figure 4(d)) is a function of clustering coefficient and average path length. Fig. 2 shows that both of these metrics converge in the PCF network at $d = 0.01$ for each initial topology.

It should also be noted that the giant connected component (GC) was measured for all of these simulations and fragmentation did not occur for any of the network topologies.

All topologies show some decrease in proximity ratio after node disconnection. The most obvious decrease occurs in the Barabási-Albert (Fig. 4(c)) model. As is expected, the affect of randomly chosen nodes is relatively small for this topology. The Barabási-Albert topology before node disruption has a PR of slightly less than 4.5, which according to [52] means it cannot be considered 'small world'. The Watts-Strogatz model (Fig. 4(a)) is of course the model with the highest PR. While the affect of node disruption is significant, it still does not decrease to any where near the level of the other networks. The Erdős-Rényi network, as the baseline for this metric begins with a PR of 1. There is no significant change in PR caused by random node disconnection. Moreover, while the change caused by targeted disconnections is apparent, it is quite small when compared to the other two 'classical' topologies. The PCF topology is affected least by disconnections, although there does appear to be a slight decrease in PR as the number of targeted disconnections increases.

4. Discussion and conclusions

We have shown that the addition of a variable representing authority, and a random variable convergence factor, results in a narrower opinion spread in the final convergence state, requiring less iterations to reach the final unchanging state. Our results indicate that a system level variable—in this instance due to the sociological focus, modelled on 'authority'—provides a more directed local node interaction process. This in turn results in consensus formation being achieved with less iterations. It is interesting to note that while 'authority' is determined globally the nodes themselves use it only locally, that is each node need only to know its own authority level and that of the nodes with which it shares direct connections. By utilizing a randomly chosen convergence factor between nodes that fell within tolerance, a more inclusive global convergence occurs. This in turn provides less diverse final state opinion clusters.

In our previous work [28], results indicated that regardless of the initial topology, the PCF topology showed notably similar qualities. Factors such as clustering coefficient (CC) (Fig. 2(a)), and mean shortest path (MSP) (see Fig. 2(b)), maintained quite similar values when the consensus formation process was run at low tolerances. This raised the questions of exactly how much affect the consensus formation process might have on the topology of the host network, and whether it may be able to provide a process for a range of network types to autonomously reconfigure to a topology providing similar CCs, MSPs, and degree distributions (DDs), thereby increasing both predictability and robustness.

As is evidenced by the simulations here, the consensus formation process has considerable affect on the three factors most often used to quantify network topologies. CC is higher for a large range of d than both the Erdős-Rényi, and Barabási-Albert models (Fig. 2(a)). The DD (Fig. 3) has more sparsely connected nodes, at the expense of mid-range degree nodes, than normally found in a random network. MSP (Fig. 2(b)), converges for all three initial topologies at $d \simeq 0.1$ and continues to decrease to a point similar to Erdős-Rényi and significantly less than Watts-Strogatz at $d \simeq 0.01$. It is also notable that communication tolerance has a profound negative correlation with change in these metrics, between $d = 0.01$ and $d = 0.1$. This supports the conclusion that there is a 'range' of tolerance that results in a more communal (high CC) communication-friendly (low MSP) network structure, while too high or too low a level of tolerance will have an adverse affect on these attributes.

Perhaps one of the most interesting conclusions is that the consensus formation process at a very low tolerance generates measurements quite distinct from the three original topologies—see Figs 2 and 3.

Fig. 3 shows that (on a log scale) the network's PCF DD is trending very slightly more toward a power-law than the small world or random distribution. Clearly there are more very low degree nodes in this topology than in the Watts-Strogatz, or Erdős-Rényi topologies. Also evident is that this increase in the number of sparsely connected nodes comes at the expense of nodes with a degree close to the mean rather than from very high degree nodes.

Fig. 4 indicates that the consensus formation process has a beneficial affect on robustness after node disconnection in terms of 'small worldliness'. Further, the process itself provides a method of attaining a proximity ratio (PR) high enough to qualify the network as 'small-world', both before and after significant numbers of targeted and random node disconnections. Fig. 4 shows that PR for the PCF network is significantly higher than both Barabási-Albert, and Erdős-Rényi, though considerably less than Watts-Strogatz prior to any disruption. After node disconnection, the PCF network appears to have affected robustness notably, with consequence of disruption being insignificant.

It is argued that a significant contributing factor in the increased robustness, is the relatively large number of nodes with very low connectivity. We suggest that the presence of substantial numbers of nodes with significantly lower degree may also be associated with short MSP and high CC. Network topologies created by the consensus formation process with a sufficiently but not preclusively low tolerance, must—in order to more effectively 'communicate' the predominant opinion—rationalize pathways within the network.

It is argued that this behaviour can be termed emergent, that is, the behaviour at a global level is generated by local interactions governed by rules which require no specific universal knowledge or intent.

Our findings suggest potentially interesting new research avenues with real world applications. For example they are consistent with the idea of organizational structures which take a more horizontal rather than hierarchical approach (a greater number of more lowly connected nodes). Our results are also consistent with the hypothesis that function has a considerable influence on the final topology of evolving networks. This is evidenced by the fact that regardless of initial topology, the process of consensus formation in the dynamic network model we have studied leads to a final topology which displays measurements for a range of metrics that are both consistent and notably distinct from each initial topology.

It is also interesting that once this final state is reached, these measurements seem to be more robust to both targeted and random node removal. This suggests that once a network topology has been 'adapted' to facilitate its purpose, it is less globally vulnerable to local failures and thus more stable.

Perhaps most interestingly, is the possibility that were 'opinion' and 'authority' replaced with other domain specific variables, this process may help in the configuration (dynamic or otherwise) of optimal network topologies for use in many different domains. It is hypothesised that different external determinants may provide new topologies which are 'tailored' to facilitate specific functions. One such implementation (the neuronal network of the *C. elegans* worm, which is the only fully mapped nervous system) seems to add support to this hypothesis, perhaps further supporting the possibility that topology evolves to support function [57].

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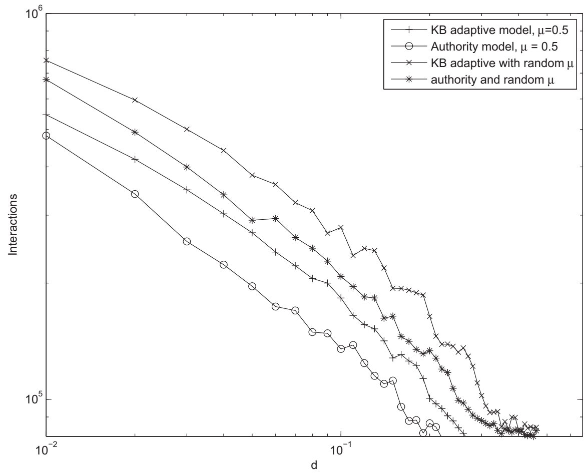
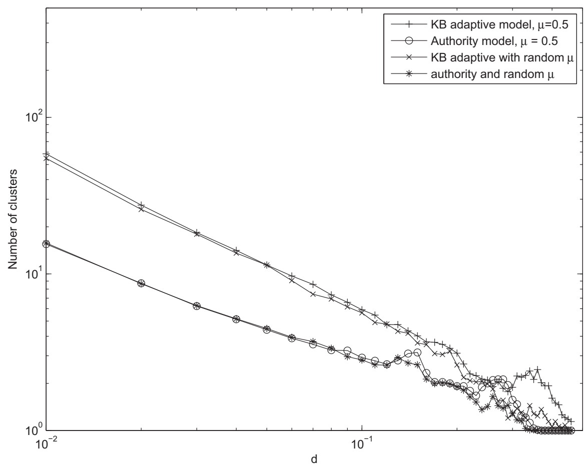
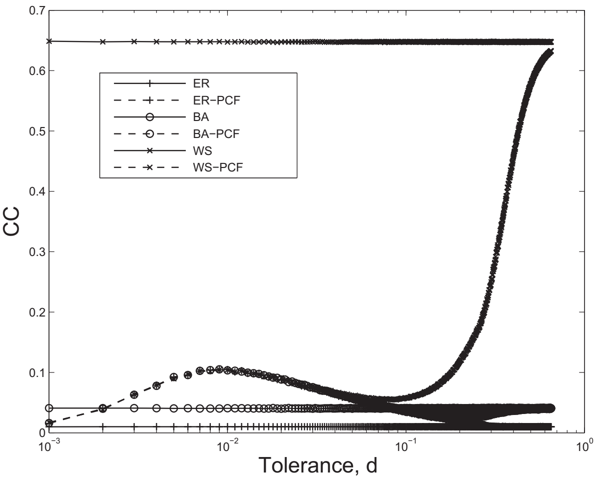


(a) Interactions

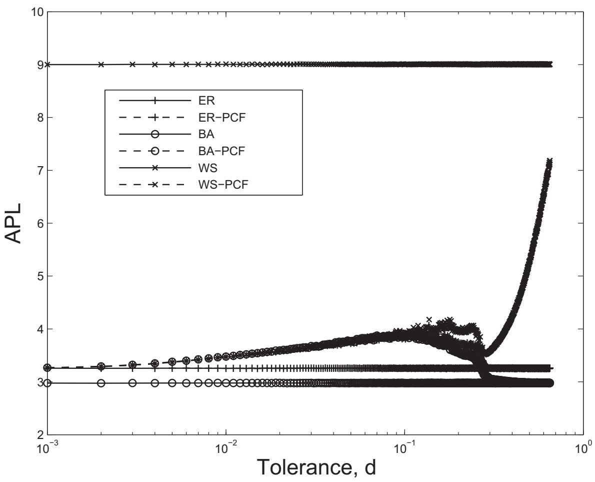


(b) Opinion Clusters

Figure 1: (a) A comparison of the number of interactions required for the dynamic network to reach its final state, for both the original model of [4] (KB adaptive), and our model (Authority), as a function of tolerance, d. (b) A comparison of the number of opinion clusters (groups of nodes whose opinions are within tolerance) for each model, as a function of d. In both (a) and (b) the networks are of size $N = 1000$ and mean degree $\bar{k} = 10$. The plots show results for both fixed opinion convergence factor, μ and uniformly random $\mu \in [0, 0.5]$.

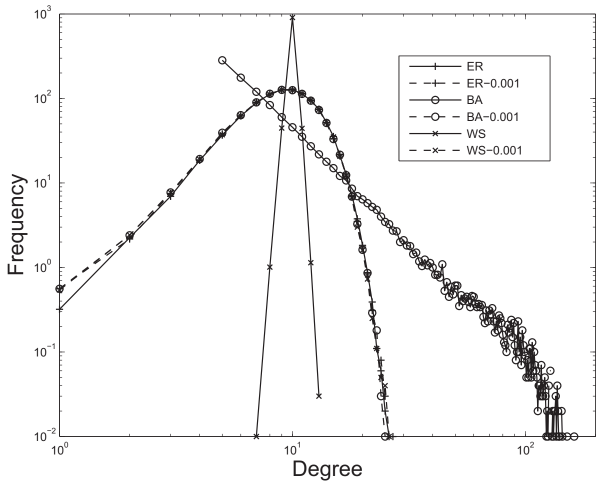


(a) Clustering Coefficient

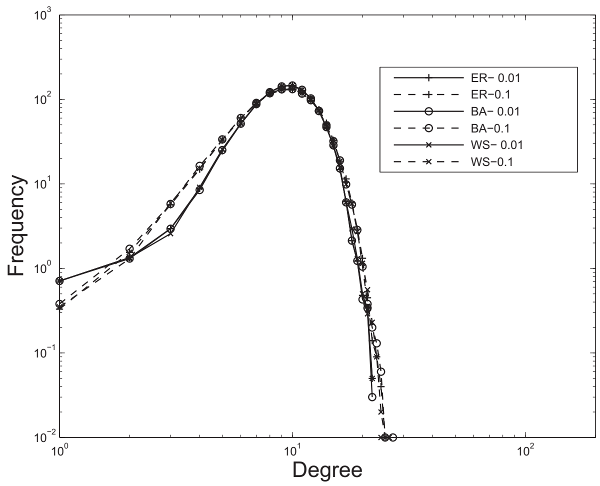


(b) Mean shortest path

Figure 2: A comparison of the impact of the consensus formation process on network topology. The figures show clustering coefficient (CC) and mean shortest path (MSP) as a function of tolerance, d, both for the initial network topologies (ER, BA, WS) and for the post consensus formation (PCF) networks (ER-PCF, ER-BA, ER-WS). Each network has size $N = 1000$ and mean degree $\bar{k} = 10$.

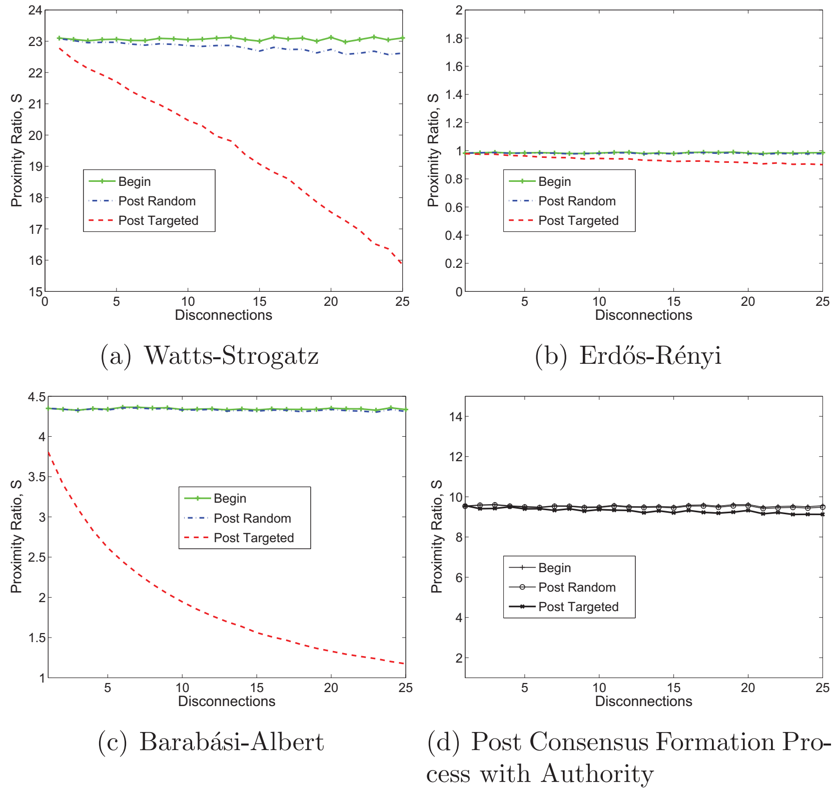


(a) Initial networks, and post consensus formation with $d = 0.001$



(b) Post consensus formation with $d = 0.01$ and $d = 0.1$

Figure 3: A comparison of the impact of the consensus formation process on the adaptive network's degree distribution (DD). The DD is shown for three cases of tolerance, d, for the three initial network topologies (ER, BA, WS) and for the post consensus formation (PCF) networks (ER-PCF, ER-BA, ER-WS). Each network has size $N = 1000$ and mean degree $\bar{k} = 10$.



(a) Watts-Strogatz

(b) Erdős-Rényi

(c) Barabási-Albert

(d) Post Consensus Formation Process with Authority

Figure 4: Comparison of the proximity ratio, S, (a measure of 'small-worldliness') for four different initial networks, before and after node disconnection for (i) targeted nodes and (ii) randomly chosen nodes, and before and after a second consensus formation process is run in the case of a network initialised to a post consensus formation (PCF) network, with tolerance $d = 0.01$. The network's size is $N = 1000$ and its mean degree is $\bar{k} = 10$.

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