Model development to capture the improvement of shear strength of soil using Australian native vegetation

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MODEL DEVELOPMENT TO CAPTURE THE IMPROVEMENT OF SHEAR STRENGTH OF SOIL USING AUSTRALIAN NATIVE VEGETATION

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BSc. Eng. (Hons)

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THESIS DECLARATION

I, Muditha Anuradhi Pallewattha, declare that this thesis, submitted in fulfilment of the requirements for the award of Doctor of Philosophy, in the School of Civil, Mining & Environmental Engineering, Faculty of Engineering and Information Sciences, University of Wollongong, is wholly my own work unless otherwise referenced or acknowledged. The document has not been submitted for qualification at any other academic institution.

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Muditha Anuradhi Pallewattha

2017
ABSTRACT

The use of vegetation for ground improvement is a sustainable, environmental friendly and cost effective approach. For railway corridors, this technique is now increasingly looked at for improving the shear strength and stiffness of subgrade soil apart from obvious environmental benefits attributed to wind barrier controls, and for reducing the effects of greenhouse gases. The increase in soil shear strength and stiffness is mainly due to the suction induced by root water uptake and the mechanical reinforcing effect provided by the tree roots.

This doctoral research mainly focuses on investigating the integrated behaviour of suction and root reinforcement in shear strength improvement of soil while past researches have considered these two aspects separately as independent components. The most rational way of capturing the true behaviour of vegetated ground is to treat the geo-hydraulic and mechanical properties as an integrated system, in view of the fact that root-permeated soil often remains in unsaturated condition due to the continual climatic process of evapo-transpiration. A series of laboratory and field investigations was carried out to examine the behaviour of a suction-reinforcement integrated system, and accordingly, a mathematical model was developed to support the experimental observations. A MATLAB simulation was also carried out based on the governing equations developed herein. The effect of coupled suction-reinforcement approach on the increased shear strength and the potential root failure modes were identified through direct shear testing. The theoretical predictions were found to be in good agreement with the laboratory results. Furthermore, the results obtained from a field investigation conducted on a site located at the University of
Wollongong campus verified the intricate relationships between the root water potential and measured matric suction variations.

A two-dimensional finite element analysis (plane strain) was carried out using PLAXIS-2D to simulate and demonstrate the native vegetation process in a practical application, in which the complex 3D root system was simplified to a 2D approximation. The field results reported by others (e.g. Potter, 2005 and Fatahi, 2007) were used in a finite element analysis and then the results of the initial settlement between non-vegetated and vegetated ground were compared to the increments of suction. The increase in soil shear strength along a ‘green’ rail corridor was captured in this PLAXIS simulation, further supported by MATLAB analysis based on the writer’s mathematical model. In particular, the hardening soil model available in PLAXIS was adopted for the root-permeated section, and the application and reliability of the model could be further validated by simulating the direct shearing process with and without root reinforcement. The FEM analysis indicates that the initial settlement of rail corridors with vegetation can be as much as 50% less than that of non-vegetated ground, and this benefit is further accrued with the increase of suction generated in the soil.
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LIST OF PUBLICATIONS

Pallewattha M., Indraratna B. and Heitor A., Increase in shear strength and alterations of root failure patterns with soil suction in root permeated soil. (In preparation for the submission to computers and geotechnics journal)


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LIST OF SYMBOLS

Tr = Tensile force develops in the roots

τr = Tangential component resisting shear displacement

σr = Normal component resisting shear displacement

tr = Average tensile strength of roots per unit area of soil.

ΔS = Increase of shear strength due to root permeation.

σN = Applied normal stress

ϕ = Friction angle of soil.

Ar = Root area ratio

τ’ = Maximum tangential stress

z = Thickness of the shear zone

As = Total cross-sectional area of the shear surface

j = Number of slipping root size classes

m = Number of non-slipping root size classes

ni = Number of roots in each size class

di = Diameter of root in each size class

Li = Root length in each size class

fi = Fractional of each leaf expressed in terms of the total leaf area of the canopy

Rni,i = Net radiation flux density absorbed by each leaf

Da = Vapour pressure deficit of air

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\( r_{a,t} = \text{Boundary layer resistance of each leaf} \)

\( r_{s,t} = \text{stomatal resistance of each leaf} \)

\( S = \text{Slope of saturation vapour pressure curve at the ambient air temperature} \)

\( \gamma = \text{Psychometric constant} \)

\( \rho_a = \text{Air density} \)

\( C_p = \text{Specific heat capacity of air at constant pressure} \)

\( \theta_{soil} = \text{Hydraulic head of soil} \)

\( \theta_{plant} = \text{Hydraulic heads of plant} \)

\( R_{soil} = \text{resistance to the water flow in soil.} \)

\( T = \text{Transpiration rate per unit of the soil surface area} \)

\( L(z) = \text{Length of root per unit soil volume} \)

\( z_{\text{max}} = \text{Maximum depth of the root zone} \)

\( f(\psi) = \text{Factor related to the soil suction} \)

\( F(T_p) = \text{Factor related to the potential transpiration} \)

\( S(x, y, z, t) = \text{Root water uptake} \)

\( k = \text{Hydraulic conductivity} \)

\( \psi = \text{soil matric suction} \)

\( G(\beta) = \text{Root distribution function} \)

\( k_1, k_2, k_3 = \text{Experimental coefficients} \)
\( \beta(t) = \) Root density at a time in a given point

\( \beta_{max}(t) = \) Maximum root density

\( \tau = \) shear strength of unsaturated soil,

\( c' = \) effective cohesion

\( \phi' = \) angle of frictional resistance

\( \sigma_n - u_a = \) net normal stress

\( u_a - u_w = \) matric suction

\( \chi = \) a parameter dependent on the degree of saturation

\( \phi^b = \) angle indicating the rate of increase in shear strength with respect to a change in the matric suction

\( \theta_w = \) volumetric water content

\( \theta_s = \) saturated volumetric water content

\( a = \) suction related to the inflection point on the curve,

\( n = \) soil parameter related to slope at the point of inflection,

\( m = \) soil parameter related to the residual water content, and

\( h_r = \) suction related to the volumetric residual water content

\( \theta_r = \) Residual volumetric water content

\( \pi = \) Osmotic suction

\( T_s = \) Surface tension

\( \Delta \tau_T = \) Total shear strength increment
\[ \Delta \tau_R = \text{Increase in shear strength due to the effect root reinforcement only} \]

\[ \Delta \tau_C = \text{Increase in shear strength due to the integrated root- suction system} \]

\[ \Delta \tau_S = \text{Increase in shear strength due to an increase in soil suction from tree transpiration} \]

\[ \sigma_{n1}, \sigma_{n2}, \sigma_{n3} = \text{Applied normal stresses in the tests} \]

Ts= Tensile strength of the root

\[ \Delta x = \text{Elongation of the root} \]

\[ \Delta f_{Resist1}, \Delta f_{Resist2}, \Delta f_{Resist3} = \text{Resisting forces due to the shear displacement as the roots stretch, slip, and are pulled out with a soil annulus} \]

\[ n1 = \text{Number of roots broken during shear displacement} \]

\[ T_i = \text{Tensile stress generated by the ‘i’th stretching root} \]

\[ n2 = \text{Number of roots which slip without breaking} \]

\[ R_{ri} = \text{Bond stress of the ‘i’th slipping root developed between the root and soil} \]

\[ A_{ri} = \text{Circumferential area of roots undergoing frictional displacement} \]

\[ n3 = \text{Number of roots that slip with a soil annulus} \]

\[ R_{si} = \text{Bond strength between the root and soil ‘i’th root which was pulled out with the soil annulus} \]

\[ l_{eff}, d_{eff} = \text{Effective length and circumferential diameter of the soil annulus which slips} \]

\[ \beta = \text{Angle the deformed root takes to the horizontal plane} \]
\[ A_T = \text{Total area of the shear plane} \]

\[ A_{\text{root}}(z) = \text{Cross sectional area of the root across the shear plane} \]

\[ B_r(\psi) = \text{The root-soil interface friction} \]

\[ B_s(\psi) = \text{Sol-soil interface friction} \]

\[ R_n = \text{Force component normal to the root} \]

\[ R_u = \text{Upward force component} \]

\[ V = \text{Vertical load applied onto a root element} \]

\[ H = \text{Horizontal force acting on a root element} \]

\[ \Delta d = \text{Shear displacement} \]

\[ \Delta t = \text{Thickness of the shear zone} \]

\[ L = \text{Length of the root,} \]

\[ L_1, L_2 = \text{Lengths of the root which lie above and below the shear zone} \]

\[ \Delta L_1 = \text{Elongated length of root} \]

\[ \delta E = \text{mobilised energy} \]

\[ E_T = \text{Total energy generated by the frictional work done} \]

\[ T_i = \text{Mobilised tensile stress} \]

\[ \beta^2, \alpha = \text{Angles related to the spatial distribution.} \]

\[ (\psi_m) = \text{Measured matric suction} \]

\[ \psi_{\text{exp}} = \text{Expected matric suction for the relevant moisture content close to the root zone} \]
$\pi_R = \text{root water potential}$
Chapter 1 Introduction

1.1 General

The vast increase in population and the high demand for infrastructure facilities in metropolitan areas has led to the construction of large number of massive earth structures, including major highways and railways. Since most Australian metropolitan areas are located along the coastal belt, these new infrastructure facilities are mainly built on a bed of soft soil. In fact, a recent statistical report published by the Department of Infrastructure, transport, regional development and local governments noted that the current Australian Rail Network is more than 44,000km long and needs modern development strategies to cater for the fast and increasingly sophisticated transport requirements of this country. This means that local civil engineers are facing a number of challenges in order to provide sustainable, reliable,
and cost effective infrastructure solutions while working with soft soil basins that require ground improvement.

The current ground improvement methods which are widely used have been proven in terms of performance but often are not very cost effective. This in turn has driven an increasing larger interest in alternative ground improvement methods which are cost effective and promote more sustainable practise in the industry. The Green Corridor concept whereby ground conditions are improved with native vegetation is one such methods.

Even though this is a relatively new concept in the construction industry, it has been used in slope stabilization for centuries to prevent erosion and provide stability, albeit carried out without proper engineering quantification or design.

1.2 Description of the problem

Tree roots are the main component, when the effect of vegetation on ground improvement is considered because tree roots, (a) reinforce the soil through their mechanical properties, (b) increase suction through the root water uptake induced by evapotranspiration, and (c) dissipate excess pore water along the shorter paths. However, when assessing the influence of vegetation, previous studies focused on the mechanical and the hydraulic effect of tree roots separately, which does not result in reliable answers because suction influences on mechanical properties of tree roots.

Furthermore, most of these experimental studies substituted wood anchors and artificial fibres for natural roots which led to unrealistic results due to an improper justification between the way natural roots and substitutions fail.
It is therefore necessary to observe true root failure in the root system while observing its changes due to the variations in suction due to tree transpiration.

1.3 Applications of green corridors in ground improvement

Ground improvement techniques which are commonly used in engineering projects are intrusion of geosynthetics, chemical agents such as cement and lime and vertical drains to accelerate the rate of consolidation (Kitsugi 1989, Chmeisse 1992, Indraratna 2000). The green corridor concept is now being widely used beside railway lines because it is a cost effective and environmental friendly method of ground improvement.

Figure 1.1. Schematic diagram of shear resistance to ground movement applied by the root zone.
A mature tree can provide 30kPa suction at the wilting point, and a well spread out woody root system can apply a good resistance to the shear displacement. Increase in the generated suction in the tree integrated soil system is higher than 30% - 100% compared to the bare soil. This variation in the percentage occurs with the tree species, maturity and climate conditions. This integrated system of suction and root reinforcement acts like an external anchor which can resist the shear displacement which takes place underneath the railway ballast due to the train load (Figure 1.1).

In this study the true effect of the vegetation on increasing the shear strength of soil by mechanical and hydraulic means is assessed via a series of direct shear test using a large shear box (300mm x 300mm x 200mm) followed by monitoring the soil suction, moisture content, and the root water potential of the vegetated ground in the field. A theoretical model is also developed to evaluate the experimental and field observations and then a MATLAB simulation was carried out to simplify the tedious process of calculation. A PLAXIS finite element simulation is done on a practical application and then the effect of vegetation on ground improvement was verified numerically.

1.4 The objective and scope of this study

The main objective of this study is to develop a model which could predict the improvement of the shear strength of soil due to tree root permeation. This study consists of laboratory experiments on naturally grown roots under varying suction values to capture the true behaviour of roots during shear displacement, developing a theoretical model, performing field experiments to capture the true ground behaviour...
of vegetated ground, and a numerical simulation for model validation followed by finite element modelling on a practical application.

The specific objectives related to the laboratory experiments and field measurements are as follows:

- Identifying the true behaviour of naturally grown roots during shear displacement.
- Verification of the effect of soil suction – moisture integrated system on the improvement of the shear strength of soil due to root permeation.
- Identifying the parameters required to develop a theoretical model, and developing methodologies to calculate and measure the parameters required in theoretical model computations.
- Monitoring the soil suction and moisture variation in vegetated ground using field and laboratory experiments.

The specific objectives related to developing a theoretical model are as follows:

- Developing theoretical expressions to evaluate the resistance to shear displacement generated through the different root failure modes.
- Developing a theoretical procedure with experimental coefficients to evaluate the main parameters such as the tensile strength generated in roots at given displacement.
• Developing a theoretical model to evaluate the total increase in the shear strength of soil due to tree root permeation while incorporating the experimental results. (i.e. considering suction-moisture integrated system, root failure modes.)
• Implementing a logical test to evaluate the occurrence of root failure at any given displacement without actually carrying out the experiments.

The specific objectives related to model validation using numerical simulation and implementing a practically applicable method are as follows:

• Simulating a theoretical model using the simulation program available, such as MATLAB, to conduct a rigorous analysis and verify the model using the experimental results.
• Developing a graphical user interface to conduct a rigorous analysis of the theoretical model which could be used by the practising engineers.
• Conducting a finite element simulation on practical applications using commercially available software which could be used to predict the effects of vegetated ground.

1.5 Organisation of the Thesis

Chapter 1 is the introduction, and Chapter 2 is a comprehensive literature review of previous studies. The introduction to the general concept of using vegetation for ground improvement which have developed over the decades are discussed first, followed by a discussion of the effects of root reinforcement and soil suction. After that, the prevailing knowledge of root systems and tree transpiration are discussed in relation to root permeated soil. The chapter concludes with a discussion of previous studies on unsaturated soil mechanics related to vegetated ground.
Chapter 3 explains the laboratory experimental procedure used to determine the increase in shear strength due to root permeation. The variations in increase in shear strength with the variations of soil suction and applied normal stress are determined in the experiments and the analysis of the experimental results followed by the modified Mohr Coulomb model related to the results are shown this chapter.

Chapter 4 describes a new theoretical model developed to capture the increase in shear strength due to root permeation according to the experimental observations. The theoretical procedures developed to evaluate the important parameters in the main model are explained in the chapter with the explanations to the methods of obtaining the other parameters. The logical test performed to evaluate the root failure methods quantitively without tedious experimental procedures and the simulation of the MATLAB program used to carry out a rigorous analysis is also present in this chapter. Finally, the theoretical model verification using the experimental results are also presented.

Chapter 5 describes the laboratory and field experimental procedures set up to monitor the true behaviour of soil suction, moisture content and root water potential in vegetated ground. The findings and analysis of the results related to variations of the soil suction and moisture, as well as the root water potential are presented with the possible conceptual development to explain the observed results.

Chapter 6 describes the finite element simulation conducted on the field data observed in Miram, Victoria, Australia using the commercially available PLAXIS 2D 2015 finite element package. The comparison between the possible initial settlements obtained using the finite element simulations of vegetated and non-vegetated ground are presented in this chapter.
Chapter 7 presents the conclusions and recommendations for future works, followed by the bibliography and appendices.
Chapter 2 Literature Review

2.1 General

Bioengineering aspects of native vegetation in relation to geotechnical engineering has been tried to some extent over the previous decades to increase soil stiffness, stabilise slopes, and control erosion. While past research studies undertaken have been mainly focussed on the quantification of the effect that native vegetation has on the shear strength of soil, none could provide adequate description and quantification of parameters that could be used in design. Furthermore, the lack of proper details regarding the quantification and design methodologies has been the main factor that has hindered the more widely use of this method in practice.

The Green Corridor concept relies on (a) the mechanical strengthening provided by the tree roots due to the anchoring effect of main roots, (b) the improvement in cohesion due to hair roots and (c) an increase in the matric suction of soil induced by the root water uptake.
Most of the previous research studies that quantify the mechanical strengthening effect of tree roots is based mainly on empirical equations, and in many instances these equations only focus on particular tree species or conditions (Docker and Hubble 2008). It is also difficult to modify these equations to represent other conditions because they are interpreted experimentally, which limits the findings of early research in this area. Moreover, the transpiration effect of trees on soil has not been accounted properly. Indraratna et al (2006) addressed most of these missing aspects by calculating the matric suction induced by the transpiration of trees due to root water uptake in the vadose- zones. However, the effect of root reinforcement has not been covered so far.

2.2 Root reinforcement effect

Tree roots can increase the shear strength of soil by mechanical means. Over the past few decades the increase in the shear strength of soil with tree roots has been discussed and examined in numerous different ways by various research groups. Docker and Hubble (2001) suggested that tree roots can provide mechanical strength to soil in two main ways

1. Increase the shear strength due to the anchoring effect of larger, stiffer roots
2. An increase in shear strength due to the apparent cohesion provided by smaller roots

Wu et al. 1979, Waldron and Dakessian (1981) and Docker and Hubble (2009) studied the effect of mechanical strengthening generated through root reinforcement as an
increase to the shear strength ($\Delta S$) in saturated conditions. Warden (1979) and Wu et al. (1981) developed a simple root model to mathematically explain the behaviour of roots under a shearing action, but according to Docker and Hubble (2001), the results from using this model are only 50% of the actual experimental results because of oversimplification of the root system behaviour. Following that Operten and Friedman (2000), Natasha Pollen (2007) and Wang (1974) developed different root models by considering different root behaviours.

### 2.2.1 Development of simple root model - mathematical model

Waldron (1977) and Wu et al. (1979) independently developed a simple model to evaluate the contribution of the tree roots to the shear strength of soil (i.e. to determine $\Delta \tau$). This model simulates an idealised situation where the vertical roots extend across a potential sliding surface in a slope. It consists of a flexible, elastic root extending vertically across a horizontal shear zone of thickness $z$, as shown in Figure 2.1.
As Figure 2.1 shows, soil is sheared as the tensile force $T_r$ develops in the roots. This force can be resolved into a tangential component ($\tau_r$) which resists shear, and into a normal component ($\sigma_r$) which increases the confining stress on the shear plane. The average tensile strength of roots per unit area of soil is $t_r$ while $\theta$ is the angle of shear distortion of the root.

$$\tau_r = t_r \sin \theta \quad \text{and} \quad \tau_r = \sigma_r \cos \theta$$ \hspace{1cm} (2.1)

$t_r$ is the average tensile strength of roots per unit area of soil, and $\theta$ is the angle of shear distortion of the root. According to Waldron (1981), $\Delta S$ can be added directly to the coulomb equation, as shown in Equation 2.2, because there is no change in the friction angle.
\[ \tau = c + \Delta S + \sigma_N \tan \phi \]  

In Equation 2.2, \( \tau \) is the shear strength of soil, \( c \) is the cohesion of soil, \( \sigma_N \) is the applied normal stress, and \( \phi \) is the friction angle of soil. Figure 2.2 represents the behaviour of Mohr-coulomb envelopes in reinforced and unreinforced soils.

Figure 2.2. Mohr-Coulomb envelopes for reinforced and unreinforced soils with circles describing failure by (a) slippage and, (b) reinforcement rupture (after Hausmann, 1976).

The critical confining stress varies for different soil-fibre systems and is a function of properties such as the tensile strength and modulus of the fibres, the length/diameter ratio of fibres, and the frictional characteristics of the fibres and soil (Gray & Ohashi, 1983).
The contribution of the root to shear strength ($\Delta S$) is then given by Equation 2.3

$$\Delta S = \sigma_r \tan\phi + \tau_r = t_r (\cos\theta \tan\phi + \sin\theta)$$  \hspace{1cm} (2.3)$$

Where, $\theta$ is the shear distortion, $\phi$ is the friction angle and $\sigma_r$ is normal stress.

The average tensile strength of the roots per unit area of soil ($t_r$) is determined by multiplying the average tensile strength of the roots by the fraction of the shear surface cross section occupied by roots:

$$t_r = T_r \frac{A_R}{A}$$  \hspace{1cm} (2.4)$$

Assuming that the shear distortion ($\theta$) is known or can at least be estimated, this model can then estimate the maximum possible contribution that roots make to the soil strength by measuring the tensile strength ($T_r$) of the roots and the fraction of soil cross-sectional area occupied by the roots ($A_R/A$).

Even though it has limited applicability resulting from imposed simplified assumptions, this method has been used in numerous investigations over the years with some success (Coppin and Richards 1990, Wu et al 1979). This model only assumes that the tensile strength of roots is fully mobilised during failure, it does not consider that roots may slip or be pulled out of the soil before failure. This model has therefore been extended by Waldron and Dakessian (1981) to include a spectrum of root diameters to account for the possibility that roots not only stretch, but also slip through the soil and as well as break.

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With this extension, the model considers a ‘progressive’ failure where, as roots slip through the soil they continue to have a reinforcing increment. Therefore, total root reinforcement consists of contributions from the slipping (Equation 2.5) and non-slipping/stretching (Equation 2.6) of roots:

\[ \Delta S_s = \left[ \pi \tau' \delta / 2A_s \right] \sum_{i=1}^{j} n_i L_i \]  

(2.5)

\[ \Delta S_s = \left[ \pi (\tau' z)^{1/2} \gamma \delta / 2A_s \right] \sum_{i=1}^{j} E_i 1/2 n_i d_i 3/2 \]  

(2.6)

where \( \tau' \) is the maximum tangential stress; \( z \) is the thickness of the shear zone; \( \gamma \) is \((\sec \theta)^{1/2}\); \( \delta \) is \((\sin \theta + \cos \theta \tan \phi )\); \( A_s \) is the total cross-sectional area of the shear surface; \( j \) is the number of slipping root size classes; \( m \) is the number of non-slipping root size classes; \( n_i \) is the number of roots in each size class; \( d_i \) is the diameter of root in each size class; \( L_i \) is the root length in each size class; and \( E_i \) is the modulus of root in each size class. Its value rather than the root strength and limited root reinforcement in a saturated clay loam permeated with barley and pine roots, led to the failure of different roots at different displacements.

Waldron & Dakessian (1981) suggest that the most significant unmeasured component is the strength of the soil root bond. With these findings, they suggest that assuming
all roots fail in tension could simultaneously lead to large overestimates of increased shear strength of the soil root systems.

### 2.2.2 Field in-situ shear tests and empirical models

Docker and Hubble (2001) conducted many in-situ field shear tests for tree species such as *Casuarina galuca, Eucalyptus amplifolia, Eucalyptus elata and Acasia floribunda* and reported the following results:

\[
S_r = 60.61RAR - 1.78 \quad (2.7)
\]

\[
S_r = 38.12RAR + 0.85 \quad (2.8)
\]

\[
S_r = 47.44RAR + 0.07 \quad (2.9)
\]

\[
S_r = 116.43RAR + 8.25 \quad (2.10)
\]

In Equations 2.7 to 2.10, \( S_r \) is the increase in shear strength and \( RAR \) is the root area ratio which is the ratio between the area of the roots along the shear plane and the area of the shear plane. These equations are related to the *Casuarina galuca, Eucalyptus amplifolia, Eucalyptus elata and Acasia floribunda*, respectively. However, these empirical relationships do not contain the effect of suction generated through transpiration and the applicability of these empirical relationships to different root systems with a different orientation is limited because the only variable is the root area ratio.
Figure 2.3 shows three stages of root reinforcement for three idealised and identical roots which are estimated from the results of direct in-situ shear tests carried out for this particular investigation. The three stages of failure depicted are;

- Stage 1: Prior to the application of an applied shear force, the roots are at rest across the potential shear plane. In the simplest model, they are assumed to be extended perpendicular to this plane.
• Stage 2: An applied shear force causes the roots to deflect in a wide shear zone. At this stage, most of the roots provide resistance through a tensile force \( T \) that is mobilised as soil pressure \( P \) acts against the root.

![Image of shear stress versus displacement plots for four tree species and soil-only tests.](image)

Figure 2.4. Average shear stress versus displacement plots for the four tree species and the soil-only tests.

• Stage 3: Sufficient displacement of the block has mobilised the full tensile strength \( Tu \) in a sufficient quantity of roots to cause a reduction in the measured shear resistance.

Docke and Hubble (2000) carried out in-situ shear tests on blocks of soil containing the roots of four riparian tree species. Figure 2.4 shows the plots of average shear stress against the displacement of the tested species. Accordingly, the greatest shear resistance is provided by \textit{A. floribunda}, followed by the other species (\textit{E. elata}, \textit{E. amplifolia}, and then \textit{C. glauca}) with only a small discernible difference between them.
Therefore, Docker and Hubble (2009) suggest that A. floribunda roots provide a much higher tensile strength.

With the results from these experiments, Docker and Hubble (2009) describe two types of root failures;

Type 1: Failure occurs after reaching the maxium shear resistance before testing has been completed. This type of failure exhibits a definite decrease in resistance as the displacement increases.

Type 2: Failure occurs before reaching maximum resistance or with continuously increasing shear resistance throughout the test, or with no recorded reduction from the original state.

Figure 2.5. Shear resistance over block displacement for two types of roots (modified after Docker and Hubble 2009).
Figure 2.5 (a) is a diagrammatic representation of the two distinct tests explained above. Type 1 exhibits a reduction in shear resistance after reaching a peak in the same manner as a soil only test, but with higher peak resistance values and at greater displacements. Type 2 exhibits little or no reduction of shear resistance throughout the test, where the final shear resistance generally becomes peak resistance.

These facts indicate that the spatial distribution of roots contributes to soil reinforcement more than all the other factors, while Figure 2.5(b) shows the identification of the main root shapes which facilitates this procedure using the Docker and Hubble (2009) categorisation.

2.2.3 Other approaches for quantifying the mechanical effect of root permeated soil

The Docker and Hubble (2001), Waldren and Dakassian (1981), and Wu et al (1979) models focus mainly on an increase in the shear strength of soil with root action because the characterisation of soil in terms of its shear strength parameters is significant. This approach seems to be more reliable and easier to comprehend.

More research studies have also been carried out to try to understand the probable mechanisms of root reinforcement via laboratory and field experiments on root-reinforced soil (e.g. Broms, 1977; Tumay et al., 1979; Collios et al., 1980; Gray and Ohashi, 1983; Shewbridge and Sitar, 1989 tested soil with low modulus fabric and fibres. Kassif and Kopelowitz, 1968; Endo and Tsurata, 1969; O’Loughlin, 1974a,b; Waldron, 1977; Waldron and Dakessian, 1981; Ziemer, 1981; Terwilliger and Waldron, 1990; Abe and Ziemer, 1991; Zhou et al., 1997; Wu and Watson, 1998; Ekanayake and Phillips, 1999; Abernethy and Rutherford, 2001). Burroughs and
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Thomas (197); Riestenberg and Sovonick-Dunford (1983); Terwilliger and Waldron (1991); Riestenberg (1994) studied root interaction with landslide shear surfaces. Past studies findings confirm that the root reinforcement of soil is a significant consequence of soil-root interaction and therefore has implications for the stability of vegetated slope across a range of environments.

2.3 Suction Effect of Tree Roots on Soil

The root water uptake of trees increases the matric suction of adjacent soil due to a reduction in the moisture content which therefore makes the tree-soil matrix unsaturated for almost one whole year. Trees like Pinus radiata can absorb a water content equal to its own weigh per day from the soil underneath and most mature trees can generate suction in the soil-root system of up to 30MPa (Fatahi 2007). The main factor that affects the root water uptake is the rate of transpiration of the tree, and this depends mainly on the environmental parameters and the physiology of the tree(s).

An unsaturated soil-root matrix increases the shear strength of the soil because the matric suction is an important variable in the shear strength equation (Fredlund and Rahardjo 2012). It could therefore be concluded that the previous research only considered tree roots water uptake behaviour and its effect on the soil matrix when developing the soil-root interaction theories.

The humidity, temperature, wind speed, and the soil moisture condition (soil water potential) and tree physiology are the main environmental factors which affect the transpiration of trees. The amount of water vapour density present in the surrounding air (the humidity) is usually expressed as the vapour density and the pressure of relative humidity; it is also affected by the temperature and wind speed. According to Fick’s
law, the diffusion rate of transpiration is directly proportional to the difference in vapour pressure between a leaf and the surrounding atmosphere, and it is inversely proportional to the sum of the resistance of water flow encountered in the atmosphere (Fatahi 2007).

If plant species have no inhabitant acclimatisation the air temperature regulates transpiration by controlling the vapour pressure. A leaf can be 50°C to 100°C higher than the ambient air if it is fully exposed to sunlight, and at this rate the leaf stomata remains open and transpiration will occur even at 100% relative humidity where the vapour is condensed once it is released from the leaf. This is known as the ‘steaming jungle’ phenomenon that is very common in tropical jungles. (Hopkins 1999). Nobel (1991) suggested that wind speed controls transpiration by changing the resistance and effective length of the diffusion path of water vapour, whereas at higher wind speeds, the rate of transpiration increases as the diffusion paths are depleted, and vice-versa, and even though leaf transpiration is affected by wind speed, it is also subjected to acclimatisation in the plant species.

The other factors affecting transpiration is the leaf area, the number of stomata present in a leaf, and other biological features. Each individual factor adds up to the total transpiration rate of the plant, which means the higher the leaf area the higher the rate of transpiration. Based on Green (1993), the transpiration of a whole plant can be calculated using Equation 2.11
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\[ T_p = \sum_i f_i \left[ sR_{n,i} + \frac{0.93\rho_a C_p D_a}{r_{a,i}} \right] \frac{S + 0.93\gamma(2 + \frac{r_{s,i}}{r_{a,i}})}{S + 0.93\gamma(2 + \frac{r_{s,i}}{r_{a,i}})} \]  

(2.11)

where \( f_i \) is the fractional of each leaf expressed in terms of the total leaf area of the canopy, \( R_{n,i} \) is the net radiation flux density absorbed by each leaf, \( D_a \) is the vapour pressure deficit of air, \( r_{a,i} \) is the boundary layer resistance of each leaf, \( r_{s,i} \) is the stomatal resistance of each leaf, \( S \) is the slope of saturation vapour pressure curve at the ambient air temperature, \( \gamma \) is the psychometric constant, \( \rho_a \) is the air density and \( C_p \) is the specific heat capacity of air at constant pressure.

The condition of the surrounding soil also impacts on the rate of transpiration of a tree because the root water uptake is inversly proportional to the soil water potential, so the relationship between soil suction and the soil moisture content follows the soil water characteristic curve. Therefore, Fatahi, (2007) proposed a soil reduction factor that is a function of the moisture content of soil. The model suggested by Feddes et al. (1976) for root water uptake in relation to soil moisture content is as follows;
In Figure 2.6, $\theta_w$ is the moisture content at wilting point and $\theta_d$ is the minimum possible moisture content at maximum root water uptake, $\theta_{an}$ is the maximum value of the moisture content which can generate the maximum root water uptake; there is no root water uptake between $\theta_{an}$ to $\theta_{sat}$ which is known as the saturated moisture content range. Feddes et al. (1976) modelled an accurate and simplified model which can be used to evaluate the root uptake parameters, whereas Gardner (1960), Whisler (1968), Molz and Remsons (1970) and Hillel (1976) developed different models to calculate the amount of root water uptake; they are described briefly as follows:

The hydraulic conductivity of soil affects the root water uptake of soil. Gardner (1960) carried out a quantitative study on root water uptake and developed the following equation to quantify the root water uptake.

![Figure 2.6. Water uptake – Moisture content relationship; modified after Feddes et al. (1976).](image)
\[ S(x, y, z) = b. (\delta - \psi - z). k. \beta(x, y, z) \] (2.12)

where \( S \) is the rate of root water uptake, \( b \) is a constant, \( \delta \) is the water potential of plant roots, \( \psi \) is soil suction, \( z \) is the depth below soil surface, \( k \) is unsaturated hydraulic conductivity, \( \beta(x,y,z) \) is the root density as the length of root per unit of soil volume.

Whisler (1968), suggested a linear relationship for \( S(x, y, z) \) with Equation 2.13.

\[ S(z) = f(\beta). k. (h_p - h_s) \] (2.13)

where \( f(\beta) \) is root density function, \( h_p \) is water potential of roots, and \( h_s \) is water potential of soil.

Molz and Remsons (1970) developed the equation below for the root water uptake based on diffusivity and not on matric suction.

\[ S(x, y, z) \propto f(D(\theta)) \] (2.14)

where \( D(\theta) = k. \frac{d\psi_m}{d\theta} \), \( D(\theta) \) is diffusivity, \( \theta \) is volumetric water content, and \( k \) is the hydraulic conductivity which is considered in the diffusivity function.

In 1974, Feddes et al. introduced Equation 2.15, for root water uptake;
\[ S = -\frac{k \cdot [h_r(z) - h(z)]}{b(z)} \]  

(2.15)

where, \( h_r(z) \) is the pressure head at the soil water interface, \( h(z) \) is the pressure head in the soil, and \( b(z) \) is an empirical function representing the geometry of flow.

Hillel (1976) proposed a relationship to predict the root water uptake as shown in Equation 2.16.

\[ S = \frac{(\theta_{soil} - \theta_{plant})}{(R_{soil} + R_{root})} \]  

(2.16)

where \( \theta_{soil}, \theta_{plant} \) are the hydraulic heads of soil and plant, and \( R_{soil}, R_{root} \) are resistance to the water flow in soil related to hydraulic conductivity.

In all these equations, the root water uptake is shown as a function of hydraulic conductivity and the difference in water potential between soil and root. Selim and Iskandar (1978) introduced an equation for the root water uptake by considering the rate of transpiration shown in Equation 2.17.

\[ S = \frac{T \cdot L(z) \cdot k}{\int_{0}^{z_{max}} L(z) \cdot k \cdot dz} \]  

(2.17)

\( T \) is the transpiration rate per unit of the soil surface area, \( L(z) \) is the length of root per unit soil volume, \( z_{max} \) is the maximum depth of the root zone, and \( z \) is the depth below the soil.
Indraratna et al. (2006) developed a relationship for root water uptake based on the potential transpiration of a tree and the reduction factors due to soil suction, as shown in Equation 2.18.

\[ S(x, y, z, t) = f(\psi)G(\beta)F(T_p) \]  

(2.18)

where \( f(\psi) \) is computed using Feddes et al. (1974), \( F(T_p) \) is the factor related to the potential transpiration by referring to the relationship developed by Nimah and Hanks (1973), as represented in Equation 2.19.

\[
F(T_p) = \frac{T_p(1 + k_4z_{\text{max}} + k_4z)}{\int_{v(t)} G(\beta) (1 + k_4z_{\text{max}} + k_4z) dv}
\]  

(2.19)

where \( G(\beta) \) is root density effect and \( k_4 \) is an experimental coefficient.

Considering all the relationships shown above, it is understandable that the root water uptake is directly proportional to the shape of the root system, soil suction, and the potential occurrence of transpiration which is related to the leaf system of trees.
2.3.1 Variation of suction under the root-soil matrix

According to recent field investigations conducted in Australia, native vegetation close to railway lines helps to reduce the vulnerability of undrained failure in soils. Potter (2006) carried out extensive tests to investigate the feasibility of using vegetation for improving the ground adjacent to existing rail infrastructure. Potter (2006) concluded that planting vegetation along the sides of railway lines can increase the resilient modulus of the subgrade material which incorporates a typical rail track substructure. Figure 2.7 shows the soil suction contours under sections of tracks with and without vegetation. Suction in the vegetated ground is higher than in the ground without vegetation, which leads to an increase in shear strength of soil underneath the vegetated ground. Cameron (2001) also showed that ground near trees is more prone to desiccation than the ground further away.
Figure 2.7. Field measurements of total suction at the Miram site in July 2004, (a) non-vegetated, and (b) vegetated (after Potter 2006).

Before ground desiccation can be used for engineering purposes, it must be quantified, so Fatahi (2007) developed a model which incorporated the ground desiccation phenomenon with the root water uptake theory. Equation 2.20 shows the theory introduced to capture the variation in moisture in conjunction with the root water uptake of a tree.

\[
\frac{\partial \theta}{\partial t} = \nabla \cdot (k \nabla \psi) - \frac{\partial k}{\partial z} - S(x, y, z, t) \quad (2.20)
\]
In this equation, \( k \) is the hydraulic conductivity, \( \psi \) is soil suction, \( z \) is vertical coordinate and \( t \) is time. \( S(x, y, z, t) \) is the root water uptake, as described in the previous chapter, and it has been inserted as the sink factor into Equation 2.20.

The root distribution function which is represented as \( G(\beta) \) in Equation 2.18, is described as Equation 2.21

\[
G(\beta) = \frac{\tanh(k_3 \beta(t))}{\int \tanh(k_3 \beta(t)) \, dv}
\]  

(2.21)

Where \( k_3 \) is a experimental coefficent and \( \beta(t) \) is the root density at a time in a given point which is represented in Equation 2.22

\[
\beta(r, z, t) = \beta_{max}(t) e^{-k_1(t)|z-z_0(t)|-k_2(t)|r-r_0(t)|}
\]

(2.22)

In Equation 2.22, \( \beta_{max}(t) \) is the maximum root density, and \( k_1 \) and \( k_2 \) are the experimental parameters. The root zone is assumed to be an inverted conical shape, where \( r \) is the radius of the circle and \( z \) is the hight of the circle.
According to recent research by Docker and Hubble (2001), Dabson and Moffat (1995), Sudmeyer (2002) and Landsberg (1999), Fatahi (2007) as shown in Figure 2.8, the shape of a root with maximum density is a circle where $r = r_0(t)$ at depth of $z = z_0(t)$ and there is an exponential reduction in root density in the vertical and radial directions.

Figure 2.8. Schematic diagram for soil-plant atmosphere system (after Fatahi 2007).

Equation 2.20 can be used to calculate the instantaneous moisture content according to the root water uptake, and the subsequent suction values can be computed by relating them to the appropriate soil water characteristic curve. The instantaneous moisture content with the water uptake and the subsequent suction values can be calculated by relating them to the appropriate soil water characteristic curve using Equation 2.20.
Fatahi (2007) used a two dimensional finite element analysis to predict how the soil moisture content and matric suction are distributed around a tree. In this case study, the numerical analysis was based on the basic effective stress theory of unsaturated soils incorporated into the ABAQUS finite element code. Indraratna et al. (2006) carried out a finite element analysis using the aforementioned governing equations, and also used several more case studies to verify this model. Figures 2.9 and 2.10 show the suction calculated using the ABAQUS finite element model for the parameters mentioned in Fredlund and Huang’s (2001) analysis.
Figure 2.10. Variation of Matric suction after one month (after Indraratna et al. 2006).

Figure 2.9. Variation of Matric suction after six months (after Indraratna et al. 2006).
Figures 2.9 and 2.10 show the suction developed in a tree in the vicinity (initial condition) is depicted by the dashed lines while the continous lines show the increased suction after one month and one year. The suction generated due to water uptake in tree roots increases significantly over time. Fatahi (2007), developed a model which provides a matric suction value underneath a tree.

Ng et al (2012) carried out a comprehensive test program under well controlled atmospheric conditions to identify and compare the suction induced in silty sand with and without areas vegetated with Bermuda grass. The peak suction induced within the root zone in grassed soil was 86 kPa after being almost saturated for 20 days. This is 1.5 times higher than that measured in bare soil with no grass. These observations also mentioned that the influence zone of the vertical suction in grassy soil was up to four times the root depth, and the water flow below this depth was influenced less by the root-water uptake process. Moreover, the influence zone in the lateral suction region was outside the diameter (diameter of the grassy spot) of the grassy spot. When subjected to a ponding magnitude equivalent to a return period of 100 years, a similar suction was retained at a particular depth within the root zone in grassy and bare soils, although the initial suction induced by evapotranspiration in the grassy soil was twice as much as in the bare soil. However, at a depth directly below the root zone, the grassy soil retained a suction that was 9 kPa higher than the bare soil. When the grassed soil was subjected to the same amount of ponding, but at an order of magnitude higher than the initial suction, the suction retained was 40% higher than that measured in bare soil. In Ng et al (2013), an artificial rainfall event with a return period of 100 years was created and the effects of soil density on the distribution of soil suction induced by
introducing a grassy patch, and the water infiltration rates of silty sand was investigated. The suction of the vegetated silty sand and the water balance and soil water retention ability calculations were explored against the influence of the dry density of soil (in terms of RC), i.e., at 70%, 80%, and 95%. The variation of suction against time, as well as with and without vegetation with different soil densities is shown in Figure 2.11. According to the study the rate of water filtered to the ground is slightly higher in bare soil than vegetated soil, which means the suction retained by vegetated soil is high.

![Figure 2.11. Variations of suction induced at depths of 30mm with time for bare and vegetated silty sand (after Ng et al. 2013).](image)
In contrast to that, Cameron (2001), Fatahi (2007), Fredlund and Hung (2001), Indraratna (2006), Potter (2005) and Ng et al (2012) showed that vegetated ground can retain and increase the matric suction.

2.4 Spatial distribution of roots
The shape of root distribution has a major effect in the root water uptake and the suction profile of roots; the studies carried out concluded that the spatial distribution of roots plays a significant role in quantifying the vegetation of soil stabilisation, and moreover, the mechanical failure of roots in its shearing ability, is affected by the shape and size of the root system.

2.4.1 Root systems
The functions of roots include anchorage, the absorption of water, minerals, and nutrients, the synthesis of various essential compounds such as growth regulators, and the storage of food in root crops (Kramer 1995). It is known that different tree species have different root systems, and the properties of soil also affect the depth and spatial distribution of the root system. Figure 2.12 (a) and (b) show the two main root systems in trees.

The root systems of trees in well-drained soil are shaped as shown in Figure 1(b), with a combination of lateral and oblique roots and no real tap root. Only a small percentage of tree root systems are shaped as shown in Figure 2.12 (a) (Kramer 1995).
Figure 2.12. Illustration of different root systems (a) Tap root and (b) Fibrous root system.

According to Ghestem et al. (2011), Leung (2015) and Lynch (1995) the definition of root architecture can be found when the geotechnical aspects of the vegetated environment is considered. Figure 2.13 shows the different root architecture introduced by these studies.
2.4.2 Environmental factors affecting the spatial distribution of roots

The texture and structure of soil, as well as its aeration, moisture, temperature, pH, salinity; the presence of toxic elements such as lead, copper, aluminium, competition with other plants, and presence of bacteria, fungi, and soil inhabiting animals are the environmental factors which affect the growth of tree roots, as mentioned by Kramer (1995). The effects of these factors are discussed below.
Soil texture and structure

The properties of soil such as restrictions on root penetration have a direct effect on root growth, whereas aeration and the water content only have an indirect effect. Figure 2.14 shows the effect of different levels of compaction on the spiral distribution of soil.

![Figure 2.14: Root system of young barley plants grown in the field in soils with different bulk densities](Modified after Gilmen 1980)

Aeration

A good exchange of gas in the soil is needed for a proper spiral distribution of the root system, but this exchange of gas is influenced by poor structure. Moreover, according to Kramer (1995), low levels of oxygen would result in poor root type and low levels of nitrogen would limit the nitrogen fixation of roots by legumes.


Chapter 2

Temperature

Maintaining an optimum temperature is needed for the maximum spatial distribution of trees. The effect of temperature on the shape of the root system is clearly shown in Figure 2.15.

![Temperature Effect on Root System](image)

Figure 2.15. The influence of the root zone temperature on root morphology and the shoot growth of potato seedlings (modified after Sattelmacher et al. 1990).

Water content

The availability of water in the soil has a direct relationship to the growth of the root system because too much or too little has an adverse effect on their growth. Too much water reduces the oxygen level which then inhibits growth, whilst not enough water leads to a reduction in the soil and a cessation of root growth. Furthermore, if a plant begins to wilt, the root system can reach permeant death. This is why soil water is very important for healthy root growth; Figure 2.16 shows the variation of root depth due to different recorded rain fall precipitations.
Root competition

The presence of adjacent plants affects the size of the root system. Competition by root systems reduces the size of spatial redistribution, and according to Waisel et al. (2002), even though roots are always interwined with each other, some roots do not allow some varities of roots to grow close to them.

2.4.3 Quantification of the root system

To predict the root architecture for fellow researches, measuring the root system is important even though the root systems of similar species vary. Root systems can be measured by excavation, auger, monolith, profile wall, glass wall and radioactive
tracers, but these processes are tedious and labour intensive, unlike the use of radioactive traces. Some of these methods are explained by Böhm (1980) and Waisel et al. (2002) as follows;

Excavation

Excavation is better for trees and shrubs on stiff and dry sandy soils than for grass or annual crops. In this method, a deep trench is dug with vertical sides some distance away from the roots, and then compressed air is applied parallel to the roots because the maximum force in roots are parallel to the direction of root growth. To interpret the root system in a more reasonable manner, it is important to draw figures and take photographs.

Monolith

From a 1m long trench with a depth equal to the maximum root depth, monoliths can be extracted from the sides. Metal sheets are driven from the sides to extract the monoliths. Soil should be removed by washing the roots and photographs should be taken of the bare root system.

Auger

To obtain samples with least amount of disturbance to the root system, a hand auger or other mechanical method is used to extract samples, which are then broken horizontally and washed with water to remove the soil.
Profile wall

Here the roots are mapped through the wall of trench that is dug to the required length and depth. The dry weight of the roots is required, so another method must be followed.

Glass wall

Root mapping is done using a glass wall which is placed along the walls of the trench.

Radio active traces

Here a radio active tracer is injected into the stem of plants and a soil-root sample is taken to measure the radioactivity. The predictivity of the root distribution depends on the level of radioactivity.

Trees grown in areas where the different environmental factors differ, have very different root systems. Indraratna et al. (2006) concluded that the suction force applied onto the soil surrounding a mature tree by its roots is ten times higher than the suction capacity of a partial vacuum of prefabricated vertical drains. The main aim of this thesis is to develop a model to capture the root based suction in conjunction with evapotranspiration and the importance of the combined effect of root reinforcement and suction.

2.5 Soil shear strength and vegetation

The increase in shear strength resulting from the mechanical effects of a tree can be included in the Mohr Coulomb equation, as explained in section 2.2, because the friction angle is not affected. However, if the suction generated by the root water
uptake is added to the shear strength of the unsaturated shear strength equation, it should be re assessed.

2.5.1 Shear strength of unsaturated soil

The theories used in the mathematical equation for saturated soils are based on developing theories for unsaturated soil.

The normal and shear components of the stress tensors are related to a mathematical equation and the shear strength failure criteria of saturated soil can be extended to represent unsaturated soil conditions. Tergazi’s principle of effective stress for unsaturated soil has been extended by Bishop (1956) to propose an equation for determining the strength of unsaturated soil. Bishop’s original equation can be arranged as shown in Equation 2.23.

\[
\tau' = c' + (\sigma_n - u_a) \tan \phi' + (u_a - u_w) \chi \tan \phi'
\] (2.23)

Where

\( \tau' \) = shear strength of unsaturated soil,

\( c' \) = effective cohesion,

\( \phi' \) = angle of frictional resistance,

\( \sigma_n - u_a \) = net normal stress,

\( u_a - u_w \) = matric suction,

\( \chi \) = a parameter dependent on the degree of saturation.
The value of $\chi$ was assumed to vary from 1 to 0 to represent the transition from fully saturated condition to a totally dry condition; even though several investigators found limitations with respect to quantifying the parameter $\chi$ both theoretically and experimentally.

Fredlund et al. (1978) based their findings on the Mohr Coloumb failure criterion, so the extended shear strength equation for an unsaturated soil can be written like Equation 2.24. Accordingly, any two of the three shear state variables could be used to form an appropriate shear strength equation, however, the stress state variables $\sigma_n - u_a$ and $u_a - u_w$, are the combinations of stress state variables that are commonly used to address practical engineering problems. The linear form of the shear strength equation can be written as follows when using $\sigma_n - u_a$ and $u_a - u_w$ as the stress state variables:

$$\tau' = c' + (\sigma_n - u_a) \tan \phi' + (u_a - u_w) \tan \phi^b$$ (2.24)

where $c'$ = intercept of the "extended" Mohr-Coulomb failure envelope on the shear stress axis where the net normal stress and the matric suction at failure are equal to zero; this is also referred to as "effective cohesion",

$(\sigma_n - u_a) = \text{net normal stress state on the failure plane at failure}$,

$(u_a - u_w) = \text{matric suction on the failure plane at failure}$,

$u_a = \text{pore-air pressure on the failure plane at failure}$,
\( \phi' \) = angle of internal friction associated with the net normal stress state variable \((\sigma_n - u_a)\) and

\( \phi^b \) = angle indicating the rate of increase in shear strength with respect to a change in the matric suction.

As mentioned earlier, the shear strength equation for unsaturated soil is an extension from the shear strength equation used for a saturated soil, and while unsaturated soil uses two stress state variables, saturated soil only uses one variable [i.e., the effective normal stress \((\sigma_n - u_w)\)].

Figure 2.16. Extended Mohr-Coulomb failure envelope for unsaturated soils. (after Friedlund et al. 2012).
Similarly, a Mohr circle is drawn for saturated soils in terms of effective stress axis $\sigma_n - u_w$, the unsaturated soils use the net normal stress axis $\sigma_n - u_n$. The location of the Mohr circle plot in the third dimension is a function of the matric suction. The surface tangent drawn to the Mohr circle at the failure point is known as the failure envelope for Mohr-Coulomb failure in unsaturated soil; this also can be defined as the shear strength of unsaturated soil. For saturated conditions, the failure point is the line intersecting the extended Mohr-Coulomb failure envelope and the frontal plane.

Figure 2.16 shows a planar failure envelope which intersects the shear stress axis to give a cohesion intercept $c'$. The envelope has sloping angles of $\phi'$ and $\phi^b$ with respect to the $\sigma_n - u_a$ and $u_a - u_w$ axes, respectively. Both of these angles are assumed to be constants. To relate the shear strength parameters to the stress state variables, the cohesion intercept $c'$ and the slope angles $\phi'$ and $\phi^b$ are used to depict other factors which resulted from the shear strength test. The density, void ratio, degree of saturation, mineral composition, the stress history, and the strain rate are some of these resultants. All these resultant factors have been combined and expressed mathematically in relation to the shear strength parameters according to Fredlund et al (2012).

Jennings and Burland (1962) mention that normal stress has a greater effect on the mechanical behaviour of an unsaturated soil than changes in the matric suction, where the friction angle $\phi$ characterises an increase in the shear strength in relation to changes in the normal stress. Assuming linear failure conditions, the angle $\phi^b$ is used to characterise the increase in shear strength caused by an increase in the matric
suction. The results the value of $\phi^b$ and $\phi'$ appear to be consistently equal to or less than $\phi'$.

According to the available literature, the contribution made by the shear strength due to matric suction $\phi^b$ is assumed to be linear, whereas Gan et al. (1988) and Escario and Juca (1989) found that the variation of shear strength in relation to soil suction is non-linear. Equation 2.24 can be used for the linear and non-linear variation of shear strength with respect to suction.

### 2.5.2 Effect of the soil water characteristic curve on shear strength of unsaturated soil

The relationship between the soil suction and degree of saturation $S$ or gravimetric water content $w$, or the volumetric water content is shown in the soil-water curve in Figure 2.17. This is a conceptual and interpretative tool to help to understand the behaviour of unsaturated soils. The distribution of the soil, water, and air phases changes what happens when the stress state changes as soil moves from being saturated to unsaturated state. Figure 2.17 shows the typical soil-water characteristic with various zones of desaturation of soil.
When the soil suction increases, the wetted contact area between the particles of soil decreases; the rate at which the shear strength of unsaturated soils change is related to the wetted contact area between soil particles and/or aggregates, which explains the existing relationship between the soil-water characteristics and the shear strength of unsaturated soils. When the entire soil-water characteristic curve (i.e., 0 to 1,000,000 kPa) and the saturated shear strength parameters are used, a more general non-linear function predicted by Vanapalli et al. (1996) and Fredlund et al. (1996) is given in Equation 2.25.

\[
\tau' = [c' + (\sigma_n - u_a) \tan \phi'] + [(u_a - u_w)\theta^k \tan \phi']
\]  
(2.25)
Where

\( k = \) fitting parameter used for obtaining a best-fit between the measured and predicted values, and

\( \theta = \) normalised water content \( \frac{\theta_w}{\theta_s} \)

The contribution of shear strength due to suction constitutes the second part of Equation 2.25, which is:

\[
\tau_{us} = [(u_a - u_w)\theta^k \tan \phi]
\] (2.26)

To use Equation 2.25, the entire soil-water characteristic curve data (i.e., 0 to 1,000,000 kPa) and the saturated shear strength parameters are required. A best-fit soil-water characteristic curve can be obtained in terms of \( a, n, \) and \( m \) parameters using the equation proposed by Fredlund and Xing (1994), as shown below,

\[
\theta_w(\psi) = \theta_s \left[ 1 - \frac{\ln \left( \frac{1 + \frac{\psi}{h_r}}{1 + 10^6} \right)}{\ln \left( \frac{1 + \frac{10^6}{h_r}}{\exp(1) + \left( \frac{\psi}{a} \right)^m} \right)} \right] (2.27)
\]

Where

\( \psi = \) soil suction,

\( \theta_w = \) volumetric water content,

\( \theta_s = \) saturated volumetric water content,
a = suction related to the inflection point on the curve,

\( n = \) soil parameter related to slope at the point of inflection,

\( m = \) soil parameter related to the residual water content, and

\( h_r = \) suction related to the volumetric residual water content.

Vanapalli et al. (1996) proposed another equation for predicting the shear strength of unsaturated soils without using the fitting parameter \( K \). This equation is given below

\[
\tau' = [c' + (\sigma_n - u_a) \tan \phi'] + [(u_a - u_w) \frac{\theta_w - \theta_r}{\theta_s - \theta_r} \tan \phi']
\]  

(2.28)

Where

\( \theta_w = \) volumetric water content,

\( \theta_s = \) saturated volumetric water content and

\( \theta_r = \) residual volumetric water content.

Equation 2.28 can also be written in terms of the degree of saturation \( S \), or the gravimetric water content \( w \), to predict the shear strength that will yield similar results. The residual volumetric water content \( (\theta_r) \) should be estimated using the soil-water characteristic curve for this equation.

Figure 2.17 is a graphical explanation of the residual state of saturation. The intersecting point of the tangent drawn through the point of inflection on the straight-line portion of the soil-water characteristic curve and the line extending the 1,000,000kPa along the curve is the residual saturation.
Vannapali and Fredlund (2000) studied Equations 2.25, 2.28 and the other two proposed models on three different soils and found that they can be used for all types soil with a suction range from 0 - 15000 kPa. Therefore, considering its simplicity and accuracy, Equation 2.28 is used in this study.

### 2.6 Soil suction

The idea of soil suction is used extensively in this study to explain the effect that vegetation has on the shear strength of soil. Therefore, a clear understanding of the different types of suction values is needed because total soil suction is defined in terms of the free energy or the relative vapour pressure (relative humidity) of soil moisture.

\[
\psi = -\frac{RT}{V_{v0} \sigma} \ln \left( \frac{H_{v-}}{U_{v0}} \right) \quad (2.29)
\]

Where

\( U_v \) = Partial water pressure of pore water pressure and

\( U_{v0} \) = saturation vapour pressure of water vapour over flat surface of pure water.

Total suction consists of two components, the matric suction \( u_a - u_w \) and the osmotic suction (\( \pi \)) (Equation 2.30)

\[
\psi = (u_a - u_w) + \pi \quad (2.30)
\]

Both components are due to differences in the relative humidity of soil vapour.
Matric Suction.

Surface tension causes a meniscus to form at the soil-air interface which then reduces the vapour pressure in the water. As the vapour pressure decreases and becomes more negative, the radius of the curvature of the meniscus and the matric suction is related indirectly. The soil pores decrease in size as the particles of soil decrease in size; this then becomes the size of the radius of curvature and as a consequence, the matric suction pressure. The vapour pressure also decreases as the degree of saturation decreases.

The soil matric suction is described in terms of capillary forces, i.e., the capillary rise acting on soil. The surface tension and the attractive forces between the soil ions and the water molecules in the absorbed water is the reason for the capillary rise, but for capillary action to occur, the total upward force due to surface tension is made equal to the downward force due to the weight of the water table in the tube.

The matric suction pressure can then be given as shown in Equation 2.31.

\[ u_a - u_w = \rho_w gh_c = \frac{2T_s}{R_s} \] (2.31)
The pore size of pore water pressure cannot be calculated directly because the surface tension has forces perpendicular and parallel to the surface which results in compressive forces acting on the soil particles. This variation of soil properties is affected by the variation in pore sizes and particle orientation. The angle $\alpha$ varies depending on the conditions, e.g., as soil becomes wet or begins to dry; this results in the hysteresis effects (the properties of soil depend on its history) in soils.

**Osmotic Suction**

Osmotic suction is a significant portion of the total soil suction, and is caused when the soil vapour pressure and the humidity has decreased due to the presence of dissolved ions in water.
Figure 2.19 can be used to illustrate osmotic suction; The pressure needed to equalise the flow of water from the solution to the pure water is equal to the osmotic pressure of the solution. (Tindall and Kunkel, 1999)

![Diagram of osmotic suction](image)

Figure 2.19. schematic diagram showing variations in the head in osmotic suction.

a) Water flows through the membrane into the solution due to osmotic suction in the solution. b) Water flows through the membrane into pure water due to pressure on the solution.

2.7 Summary
Tree roots can increase the shear strength of natural soil by imparting mechanical strength through root reinforcement and increasing suction through the root water uptake. Theoretical and empirical models have been developed to capture the root reinforcement effect and suction effect separately. However, the reinforcing and
hydrological effects should both be used to capture the true effect of tree roots on the shear strength of soil.

Most of attempts to quantify the effect of root reinforcement have used saturated conditions. It has also been reported that the evaluations of theoretical studies were higher than the experimental studies because unreasonable assumptions were made during the theoretical computations. (i.e., roots only fail in tension during shear displacement).

A large number of field studies have proven that the soil suction below vegetated ground can be increased, and theoretical models have been developed to successfully capture this effect. However, the studies developed to capture the effect of the vadose zone have only evaluated the increase of suction due to root water uptake, while the root reinforcement effect has been discarded.

The spatial distribution of the root system is an important parameter when evaluating the suction and root reinforcement effect because tree root systems vary from one species to another, as does the soil conditions. This is why previous studies have used reasonably different root systems when considering the geotechnical engineering aspects.

To better evaluate how a tree root system improves the shear strength of soil, the root reinforcement and suction must be considered simultaneously, and furthermore, the effect that soil suction has on reinforcement must also be considered.
Chapter 3 Experimental procedures to understand the behaviour of root-suction integrated system

3.1 Introduction

The soil structure of vegetated ground remains in a partially saturated state, as stated in Chapter 2, and it is certain that the depth and degree of saturation of the vadose zone increases as the soil structure interacts with trees. As a result, the parameters developed for mechanical strengthening in saturated conditions are unrealistic, therefore a combined effect must be considered and the parameters should be defined accordingly. Previous studies to quantify the increase in shear strength (Wu et al. 1979, Waldron and Dakessian 1981, Docker and Hubble 2009) were carried out in saturated condition, and the studies which assessed the increase of soil suction due to root water uptake
(Indraratna et al. 2006, Potter 2006) did not consider the effect of mechanical properties of roots. Therefore, an experimental setup was developed to capture the effect of root reinforcement and an increase in evapotranspiration based suction of a soil reinforced with roots.

The following factors are important parameters with which to evaluate the effect of root reinforcement in geotechnical engineering because they have been considered in most of the concepts and models developed earlier.

- The amount of roots present in the soil matrix (eg. Root Area Ratio)
- The mechanical properties of roots (eg. Tensile strength of a root)
- The failure patterns of a root.
- Orientation of the root in the soil matrix.

In a real situation, the increase in shear strength due to root permeation or \( \Delta \tau \) value should vary with the soil suction because of the change in applied stress over the surface of a root. Therefore, direct shear tests were carried out for different values of soil suction to capture its effect in root reinforcement. The initial hypothesis developed for this test is as follows.

By considering the simple root model developed by Waldron (1977) and Wu et al. (1979) the values of the shear strength increment (\( \Delta \tau \)) can be defined. According to Waldron (1981), \( \Delta \tau \) can be added directly to the coulomb equation, as shown in Equation 3.1 because there is no change in the friction angle. Three different \( \Delta \tau \) values have been defined, as follows:

\[
\Delta \tau_R = \text{Increase in shear strength due to the effect root reinforcement only. (in a saturated condition),}
\]
\[ \Delta \tau_C = \text{Increase in shear strength due to the integrated root-suction system} \]
\[ \Delta \tau_S = \text{Increase in shear strength due to an increase in soil suction from tree transpiration.} \]

The total increase in the shear strength of root permeated soil (\( \Delta \tau_T \)) in relation to the soil matrix alone can be represented according to Equation (3.1)

\[ \Delta \tau_T = \Delta \tau_C + \Delta \tau_R + \Delta \tau_S \]  

(3.1)

3.2 Experimental Procedure

In this study a direct shear test was carried out using the large shear box which is shown Figures 3.1 and 3.2a.

![Diagram of direct shear test for root permeated soil](image)

Figure 3.1. Schematic Diagram of direct shear test for root permeated soil.
A root permeated soil specimen was used and five different suction values were selected, e.g. 0kPa, 50kPa, 100kPa, 150kPa, and 200kPa. This suction range was selected based on laboratory observations of the visible wilting point, i.e. 200kPa. Three tests were carried out for each suction value and three different normal stresses were considered, i.e., 10kPa, 20kPa and 30kPa. Furthermore, 15 additional tests were carried out on unreinforced direct shear specimens for comparison.

Figure 3.2: (a) Soil specimen with tree root ready to be sheared. (b) Suction sensor has been installed. (c) Sand pile has been installed.

### 3.2.1 Preparation of specimens

A number of wooden boxes with internal dimensions 300mm(W) x 300mm(L) x 200mm(H) (Figure 3.3a) were used to plant the trees. Polyurethane sealer was applied
inside the boxes to prevent deterioration due to water absorption; if the wood deteriorates the texture of the soil becomes contaminated, making it difficult to obtain an equivalent root system.

The soil was initially compacted inside the wooden boxes using the steel plate and wooden collar shown in Figure 3.3c to obtain a dry density of 1350kgm^-3 using the steel plate and wooden collar as shown in Figure 3.3c. Every sample was prepared using earlier mentioned compacting method, under the 18% of gravimetric moisture content which was observed in the in-situ density tests in the field. The two layers of soil with the same height and the same weight were used for the compaction of each sample.

![Figure 3.3](image.png)

Figure 3.3. (a) dimensions of the boxes (b) potted plants to be grown (c) the steel plate and collar used to aid compaction.

All the boxes were scanned with CT scanning to check the uniformity of the compacted soil before do the planting (only the specimens with almost same lump
sizes and voids were used for planting); several scanned images are shown in Figure 3.4; and then recently germinated plants (15-20mm tall) with undistorted root systems were planted in the boxes. The plants were selected to obtain the same possible growth rate and ensure an equal and undistorted final root system, thus making it easier to make a direct comparison between the samples. These plants grew under the same environmental conditions for a period of 12 months.

![CT scanned images for several boxes](image)

Figure 3.4. CT scanned images for several boxes.

*Selection of the tree*

The tree species used in this study was an *Eucalyptus botryoides*, a species of eucalyptus with a good spreading root system, and which is commonly available in NSW. When soil is to be strengthened using trees, evergreen trees with a good root system must be selected, and the nativity of the tree to the region is an added advantage. All of these factors were considered and *Eucalyptus botryoides*,...
commonly known as ‘Bangalay’ or ‘Southern Mahogany’ was selected; it also has good tolerance to droughts and salinity, and is considered to be a small to tall tree.

The large 300mm x 300mm x 200mm shear box was used to carry out the test. The soil specimen with the plant was transferred from the wooden box to the brass shear box with minimal disturbance to the specimen; this was only possible by performing a large number of trails. A suction sensor was installed inside the shear box close to the shear zone of the soil, as shown in Figures 3.1 and 3.2b. The specimens were allowed to reach the required level of suction for several days. A 15mm diameter vertical sand pile was installed inside the shear box to check the width of the shear zone, as shown in Figure 3.2c.

Subsequently, the direct shear tests were carried out at three different normal stress values (10kPa, 20kPa, 30kPa) at each level of suction; the specimen was sheared at a rate of 2.5mm/min. The plant was kept alive and suction was monitored during the test. All the tests relevant to each level of suction and normal stress were repeated on the specimens without plant roots, to evaluate the increment of suction due to plant roots.

After the test, the different root failure patterns were carefully examined and quantitatively evaluated. The roots were also mapped to evaluate the typical failure mechanisms, root area ratio and the orientation of the roots, as shown in Figure 3.5.
Chapter 3

Experimental procedure

The tensile strength of the roots is determined using the universal testing machine (UTM) shown in Figure 3.6. The top and bottom of the root were prepared using epoxy resin and sand mix to fix to the jaws of the universal testing machine. A direct shear test with a larger vertical root with slipping condition was also carried to evaluate the bond strength between soil and root material. The details of the calculation procedure of bond strength using the laboratory results will be presented in Chapter 4. Furthermore, a direct shear test with a larger vertical root which was fixed to the upper and lower plates of the shear box was conducted to evaluate and verify the possible strain of the roots inside the soil. The detail verification procedure will be explained in next section of this chapter. To obtain the stiffness of the material under the different root area ratios and suction values, another set of tests were performed under 10kPa, 20kPa and 30 kPa normal stresses using the large shear box.

Figure 3.5 Root mapping of the samples.
Figure 3.6. (a) Testing tensile strength of roots, (b) Both ends of the root specimen are prepared using epoxy resin and sand, and (c) Measuring the diameter of the root at different points before the experiment.
3.3 Results and discussion

Three different possible methods of root failure were identified while analysing the post-tested specimen, as shown in Figure 3.7; some roots failed under tension, as shown in Figures 3.7a and 3.7b. During shearing, these roots are fixed into the body of the soil from the end and therefore they undergo stretching. Tensile stress is generated in the roots while they are stretching, and when this stress reaches the tensile strength of root, it breaks. This mobilised tensile strength is generated due to the applied earth pressure in shearing action. According to Docker and Hubble (2001), roots fail only in tension due to the tensile force mobilised along the root during shearing, but this argument is valid more in saturated soil and in unsaturated condition other failure patterns as shown Figure 3.7a can be observed.
The plant roots which were pulled out, as shown in Figures 3.7a and 3.7b, experienced pure slipping and the bond strength between the root and soil interface contributes to an increase the shear resistance inside the root-soil block due to this failure pattern. In addition to the above two failure patterns, some plant roots were pulled out with a soil annulus, as shown in Figures 3.7a and 3.7c. The soil annuluses pulled out during the shearing are almost equal in number and in size, this may be due to the equal growth in fibrous roots around the main roots. This is the main reason for considering the

Figure 3.7 (a) Root system extracted after the shear test, (b) Roots slipping out without breaking, (c) Roots breaking at shearing and (d) Roots slipping out with a soil annulus.
number and size of the soil annulus to be fixed, which can be defined as a parameter related to the growth of the plant roots.

### 3.3.1 Soil properties

![Particle size distribution and other soil properties used in the tests.](image)

The soil used in these experiments is classified as low plastic silty sand (SM) according to the USCS and particle size distribution curve, while the other soil properties are shown in Figure 3.8. The soil water characteristic curve determined for a soil specimen compacted at a dry density of 1350kg/m$^3$ is shown in Figure 3.9. The interpolation proposed by Van Genuchten (1980) is also shown, and the fitting parameters (i.e. $a=2.1$, $m=0.5$ and $n = 3$) were determined using the least square method.
Figure 3.9. Soil water retention curve of the soil.
3.3.2 Analysis of direct shear test results; Stress strain and vertical displacement

Figure 3.10 shows the results of the direct tests carried out under saturated conditions for root permeated soil and unreinforced specimen under normal applied stresses of 10kPa, 20kPa and 30kPa.

As expected, the shear strength of the root permeated soil specimens were higher than the unreinforced specimens, which agrees with previous studies carried out by Docker and Hubble (2001). The peak shear strength of root permeated soil at an applied normal stress of 10kPa is slightly higher than the unreinforced peak value at an applied normal stress of 30kPa. Therefore, the ability of root reinforcement to increase the shear strength of soil is clearly shown and this increasing value is related to the $\Delta \tau_R$. Increase
in shear strength due to the root reinforcement effect only, and under saturated conditions) as explained in an earlier section of this chapter. The vertical displacement against the horizontal displacement related to the above results are shown in Figure 3.11.

![Figure 3.11](image)  

**Figure 3.11.** Results of direct shear test of unreinforced specimens and root permeated specimens under saturated conditions; Horizontal displacement Vs Vertical displacement.

The results of vertical displacement (compression) of root permeated soil shows the slightly low value (0.01mm) compared to the equivalent unreinforced value; this implies a slight increase in vertical stiffness. However, the point of peak shear stress related to the root permeated soil specimens tends to move to the right hand side in the stress strain graph, unlike the equivalent unreinforced specimen. This phenomenon can
be explained as an increase of ductility in the root permeated specimen due to the tensile strength of the roots.

Figure 3.12. Results of direct shear tests of soil only specimens and root permeated specimens at different initial suction values; Plots of horizontal displacement Vs shear stress. $\sigma_N =$ applied normal stress.
Figure 3.12 shows the results of direct shear tests carried out with four different suction values (50kPa, 100kPa, 150kPa and 200kPa) for root permeated soil and equivalent unreinforced specimen. Figure 3.12 (a), again shows the increase in the peak shear value of root permeated soil compared to the equivalent unreinforced sample; this result was expected due to the intrusion of roots into the soil. Furthermore, the peak shear values of root permeated soil increase as the suction increases due to the effect of suction on the unsaturated shear strength. However, this increased value should be equal to the root permeated soil and equivalent unreinforced specimen if no coupling effect has been introduced as $\Delta \tau_C$ (Increase in shear strength due to the coupling effect). Table 3.1 shows the variation of $\Delta \tau_C$ and $\Delta \tau_R$ values, both of which confirm that root reinforcement and suction do not act separately, they actually integrate and improve the shear strength of vegetated soil.

Table 3.1: variation of $\Delta \tau_R$ and $\Delta \tau_C$ values with soil suction and applied normal stress.

<table>
<thead>
<tr>
<th>Suction (kPa)</th>
<th>10kPa $\Delta \tau_R$</th>
<th>10kPa $\Delta \tau_C$</th>
<th>20kPa $\Delta \tau_R$</th>
<th>20kPa $\Delta \tau_C$</th>
<th>30kPa $\Delta \tau_R$</th>
<th>30kPa $\Delta \tau_C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.8</td>
<td>0</td>
<td>2.8</td>
<td>0</td>
<td>2.8</td>
<td>0</td>
</tr>
<tr>
<td>50</td>
<td>2.8</td>
<td>0.7</td>
<td>2.8</td>
<td>3.2</td>
<td>2.8</td>
<td>4.4</td>
</tr>
<tr>
<td>100</td>
<td>2.8</td>
<td>2</td>
<td>2.7</td>
<td>4.9</td>
<td>2.8</td>
<td>7.8</td>
</tr>
<tr>
<td>150</td>
<td>2.7</td>
<td>4.9</td>
<td>2.7</td>
<td>5.3</td>
<td>2.8</td>
<td>9</td>
</tr>
<tr>
<td>200</td>
<td>2.8</td>
<td>5.1</td>
<td>2.8</td>
<td>11.3</td>
<td>2.8</td>
<td>12.8</td>
</tr>
</tbody>
</table>
In Table 3.1, the $\Delta \tau_R$ value does not change with the suction or applied normal stress, which agrees with the Docker and Hubble (2008) results, which showed that an increase in shear strength is only a function of the root area ratio. $\Delta \tau_C$ is the increase in shear strength related to the combined effect of suction and reinforcement, which can change with the suction and applied normal stress.

Figure 3.13. Plots of vertical displacement vs horizontal displacement.
Figure 3.13 shows the variation of vertical displacement with respect to horizontal displacement due to the direct shear test. The ultimate vertical displacement decreased in the test with high suction values (i.e. the ultimate vertical displacement of the 200kPa suction test is lower than the 150kPa test). This variation in vertical displacement of equivalent unreinforced sample and root permeated sample also show a reduction in the test of higher suction values. Therefore the vertical displacement is

![Figure 3.13](image)

Figure 3.14. Results of direct shear test of soil only specimens and root permeated specimens at 20 kPa applied normal stress, (a) Plots of horizontal displacement Vs Shear stress, (b) Plots of horizontal displacement Vs degree of saturation and (c) Plots of horizontal displacement Vs vertical displacement.

![Figure 3.14](image)
not a significant parameter when comparing the root permeated results to the unreinforced results in high suction values.

The variation of vertical displacement with horizontal displacement for different suction values at 20kPa applied normal stress is shown in Figure 3.14 (c). This confirms that the compressive displacement decreases as the suction increases. Figure 3.14(b) shows that variations in the degree of saturation due to vertical displacement during the tests is less than 1%, and therefore the degree of saturation has virtually no effect on the end results. Furthermore, the observed variation of suction variation during test is also only less than +/-2kPa, therefore the effect of suction variation during the test is also considered as negligible.

The results shown in Figure 3.15 are similar to Figure 3.14 for the test at 100kPa of initial suction. Variations in the degree of saturation during these tests are only 1% and therefore they had no effect on the test results. Therefore the whole effect of variation of degree of saturation on the results of tests can be considered negligible.
Figure 3.15. Results of direct shear test of soil only specimens and root permeated specimens at 100 kPa of initial suction. (a) Plots of horizontal displacement Vs Shear stress, (b) Plots of horizontal displacement Vs degree of saturation and (c) Plots of horizontal displacement Vs vertical displacement.
3.3.3 Analysis of peak shear stress results

The peak shear stress of the stress-strain graph is an important parameter to consider when computing the shear strength of soil. Figure 3.16 shows the variation of peak shear stress of the direct shear tests carried out for five different suction values under three different applied normal stresses.

Figure 3.16. Variation of peak shear stress with, (a) suction, (b) degree of saturation and (c) moisture content.
As expected, the peak shear stress of the unreinforced soil specimens and the root permeated specimens increased with the increase of the soil suction and applied normal stress. However, the unreinforced specimens show a more linear increase of peak shear stress with an increase in suction under each applied normal stress, whereas the root permeated soil specimens show a scattered variation under the same conditions (Figure 3.16a). This scattered behaviour may be led by the different root failure mechanisms acting at different failure stages and it will be theoretically explained in Chapter 4.

Table 3.2 represents the root mapping data and number of root failures in different modes. It shows that the number of broken roots suddenly increases at 100kPa and this leads to the sudden increase in peak shear stress at 100 kPa suction. The root area ratio of all the samples was almost equal (0.51%-0.56%) because the plants experienced equivalent ambient growth factors and initial selection of the plants with same growth. In fact these samples whose root area ratios deviated were discarded to avoid disputes when the results are compared.

Figures 3.16b and 3.16c represent the degree of saturation and volumetric moisture content at peak shear values. There is no noticeable change in the degree of saturation at peak stress in these test specimens with an equal initial suction. Therefore, the effect of the degree of saturation for peak shear stress is negligible.
Table 3.2: Number of root failures in different modes, root area ratios, and peak shear stress of root permeated soil and peak shear stress of the equivalent unreinforced specimen.

<table>
<thead>
<tr>
<th>Sample No</th>
<th>Vertical stress (kPa)</th>
<th>Matric Suction/Moisture content</th>
<th>τPeak (kPa)</th>
<th>Unreinforced τPeak (kPa)</th>
<th>RAR %</th>
<th>No. of broken roots</th>
<th>No. of roots with soil lump</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>0/0.43</td>
<td>11</td>
<td>8.2</td>
<td>0.56</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>50/0.36</td>
<td>13</td>
<td>11.5</td>
<td>0.56</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>100/0.35</td>
<td>16.2</td>
<td>14.8</td>
<td>0.54</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>50/0.36</td>
<td>17</td>
<td>15.2</td>
<td>0.55</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>20</td>
<td>50/0.36</td>
<td>23</td>
<td>18.3</td>
<td>0.55</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>30</td>
<td>100/0.35</td>
<td>28</td>
<td>22.2</td>
<td>0.52</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>10</td>
<td>100/0.35</td>
<td>28.6</td>
<td>25.8</td>
<td>0.54</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>8</td>
<td>20</td>
<td>150/0.34</td>
<td>36.9</td>
<td>30.5</td>
<td>0.52</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>9</td>
<td>30</td>
<td>150/0.34</td>
<td>49.8</td>
<td>35.6</td>
<td>0.52</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>150/0.34</td>
<td>41.5</td>
<td>33.5</td>
<td>0.52</td>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>11</td>
<td>20</td>
<td>150/0.34</td>
<td>45</td>
<td>38.2</td>
<td>0.52</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>12</td>
<td>30</td>
<td>200/0.32</td>
<td>53.8</td>
<td>43.4</td>
<td>0.52</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>13</td>
<td>10</td>
<td>200/0.32</td>
<td>46.5</td>
<td>41.1</td>
<td>0.53</td>
<td>10</td>
<td>4</td>
</tr>
<tr>
<td>14</td>
<td>20</td>
<td>200/0.32</td>
<td>58.3</td>
<td>45.5</td>
<td>0.51</td>
<td>12</td>
<td>3</td>
</tr>
<tr>
<td>15</td>
<td>30</td>
<td>200/0.32</td>
<td>64.8</td>
<td>50.6</td>
<td>0.51</td>
<td>12</td>
<td>3</td>
</tr>
</tbody>
</table>
Figure 3.17. Extended Mohr- Coulomb failure envelope to show the increase in shear strength by root permeated soil (Modified Fredlund et al. 2012).
Figure 3.17 shows the proposed positions of the $\Delta \tau_R$ and $\Delta \tau_C$ in an extended Mohr Coulomb envelope (Fredlund et al. 2012), while considering there is no change in the friction angle as per Wu et al (1978). The Mohr circles with thin continuous lines represent the results related to the unreinforced specimens, while the Mohr circles with dark continuous lines represent the results related to the root permeated specimens. The number of Mohr circles represented under each failure line are limited to two to retain some clarity in the graph (i.e. usually three circles are presented). The $\sigma_n1$ and $\sigma_n2$ in Figure 3.17 are the applied normal stresses in the tests and $\sigma_n3$ is also obtained in the actual direct shear test. The $(u_a - u_w)1$ is the suction value related to the test, and tests are carried out with five different suction values (i.e. 0 kPa, 50kPa, 100kPa, 150kPa and 200kPa). $\tau_1$, $\tau_2$ are the peak shear stress values in the failure plane related to the tests with $\sigma_n1$ and $\sigma_n2$ applied normal stresses; $\tau_{T1}$ and $\tau_{T2}$ are the peak shear stresses in the failure plane of the root permeated samples. Suffix ‘a’ and ‘b’ have been used to state the values related to the saturated condition and unsaturated condition accordingly. The extended Mohr coulomb envelope was developed to include the obtained results and tested values of direct shear test (i.e. $\tau_{T1a}$, $\tau_{T1a}$ should be on the intersection of the straight line goes through $\sigma_n1$ and Mohr circle, because both unreinforced and root permeated specimens were tested under the same applied normal stress). The lines go through the $\tau_{1a}$-$\tau_{2a}$ and $\tau_{T1a}$-$\tau_{T2a}$ should be parallel since there is no change in the friction angle due to root permeation. The value at the intersection of the line goes through the $\tau_{T1b}$-$\tau_{T2b}$ and vertical axis (shearstress) is $C' + \tau_R + \tau_C$ and at saturated condition it becomes $C' + \tau_R$, since there is no suction to generates coupling effect. Equation 3.2 explains the relationship between $\tau_{T1}$, $\tau_1$ and $\Delta \tau_T$. 

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\[ \tau_{T1} = \Delta \tau_{T} + \tau_{1} \] (3.2)

As stated in Equation 3.1, \( \Delta \tau_{C} + \Delta \tau_{R} + \Delta \tau_{S} \) is equal to \( \Delta \tau_{T} \), but \( \Delta \tau_{S} \) is an increase in the shear strength because the increment of suction by evapotranspiration is not shown in Figure 3.17. This is because the suction increment value \( (u_{a} - u_{w}) \) caused by evapotranspiration can be added directly to the unsaturated part of the shear strength equation, and is considered to be represented in \( (u_{a} - u_{w}) \) value in Figure 3.17. Table 3.3 shows the represented values of the variables in Figure 3.17 which were obtained during the direct shear tests.
Table 3.3: Values of the parameters presented in Figure 3.17.

<table>
<thead>
<tr>
<th>Sample no</th>
<th>(u_a−u_w)</th>
<th>σ kPa</th>
<th>τ kPa</th>
<th>τ_T kPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0kPa</td>
<td>σ1 = 10</td>
<td>τ1 = 8.2</td>
<td>τ_T1 = 11</td>
</tr>
<tr>
<td>2</td>
<td>0kPa</td>
<td>σ2 = 20</td>
<td>τ2 = 11.5</td>
<td>τ_T2 = 13</td>
</tr>
<tr>
<td>3</td>
<td>0kPa</td>
<td>σ3 = 30</td>
<td>τ3 = 14.8</td>
<td>τ_T3 = 16.2</td>
</tr>
<tr>
<td>4</td>
<td>50kPa</td>
<td>σ1 = 10</td>
<td>τ1 = 14.5</td>
<td>τ_T1 = 18</td>
</tr>
<tr>
<td>5</td>
<td>50kPa</td>
<td>σ2 = 20</td>
<td>τ2 = 18.3</td>
<td>τ_T2 = 23</td>
</tr>
<tr>
<td>6</td>
<td>50kPa</td>
<td>σ3 = 30</td>
<td>τ3 = 22.2</td>
<td>τ_T3 = 28</td>
</tr>
<tr>
<td>7</td>
<td>100kPa</td>
<td>σ1 = 10</td>
<td>τ1 = 24.8</td>
<td>τ_T1 = 29.6</td>
</tr>
<tr>
<td>8</td>
<td>100kPa</td>
<td>σ2 = 20</td>
<td>τ2 = 30.5</td>
<td>τ_T2 = 36.9</td>
</tr>
<tr>
<td>9</td>
<td>100kPa</td>
<td>σ3 = 30</td>
<td>τ3 = 35.6</td>
<td>τ_T3 = 49.8</td>
</tr>
<tr>
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<td>150kPa</td>
<td>σ1 = 10</td>
<td>τ1 = 33.5</td>
<td>τ_T1 = 41.5</td>
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<tr>
<td>11</td>
<td>150kPa</td>
<td>σ2 = 20</td>
<td>τ2 = 38.2</td>
<td>τ_T2 = 45</td>
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<td>150kPa</td>
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<td>τ3 = 43.4</td>
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</tr>
<tr>
<td>13</td>
<td>200kPa</td>
<td>σ1 = 10</td>
<td>τ1 = 33.5</td>
<td>τ_T1 = 46.5</td>
</tr>
<tr>
<td>14</td>
<td>200kPa</td>
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<td>τ2 = 38.2</td>
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</tr>
<tr>
<td>15</td>
<td>200kPa</td>
<td>σ3 = 30</td>
<td>τ3 = 43.4</td>
<td>τ_T3 = 64.8</td>
</tr>
</tbody>
</table>

Root mapping data is important in analysing the behaviour of roots inside soil during shearing. Therefore Table 3.4 shows more representative results of the root systems obtained while root mapping the post tested direct shear samples; this data will be used in MATLAB simulation to be explained in Chapter 4.
Table 3.4 Input data file for MATLAB simulation

<table>
<thead>
<tr>
<th>Diameter (m)</th>
<th>Youngs Modulus (kN/m²/mm)</th>
<th>Length (m)</th>
<th>Thickness of the shear zone (m)</th>
<th>L1 (m)</th>
<th>L2 (m)</th>
<th>Δd (m)</th>
<th>Moved length (m)</th>
<th>Tensile Strength (kN/m²)</th>
<th>Orientation α1</th>
<th>Orientation α2</th>
<th>Mean soil height (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.003</td>
<td>10000</td>
<td>0.15</td>
<td>0.02</td>
<td>0.0985</td>
<td>0.0985</td>
<td>0.033</td>
<td>0.018588</td>
<td>10000</td>
<td>0</td>
<td>30</td>
<td>0.098</td>
</tr>
<tr>
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<td>10000</td>
<td>0.15</td>
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<td>0.097</td>
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<td>45</td>
<td>0.142</td>
</tr>
<tr>
<td>0.002</td>
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<td>0.02</td>
<td>0.096</td>
<td>0.096</td>
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<td>60</td>
<td>30</td>
<td>0.098</td>
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<tr>
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<td>0.02</td>
<td>0.0985</td>
<td>0.096</td>
<td>0.033</td>
<td>0.0164</td>
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<td>100</td>
<td>60</td>
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</tr>
<tr>
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<td>0.0164</td>
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<td>30</td>
<td>0.098</td>
</tr>
<tr>
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<td>0.02</td>
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<td>10000</td>
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<tr>
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<td>0.02</td>
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<td>160</td>
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<td>0.02</td>
<td>0.096</td>
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<td>0.033</td>
<td>0.0164</td>
<td>10000</td>
<td>180</td>
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<td>0.098</td>
</tr>
<tr>
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<td>0.02</td>
<td>0.096</td>
<td>0.096</td>
<td>0.033</td>
<td>0.0164</td>
<td>10000</td>
<td>200</td>
<td>45</td>
<td>0.142</td>
</tr>
<tr>
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</tr>
<tr>
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<td>0.02</td>
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<td>0.033</td>
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<td>0.017224</td>
<td>10000</td>
<td>260</td>
<td>45</td>
<td>0.142</td>
</tr>
</tbody>
</table>
3.3.4 Shear modulus

The shear modulus can be defined as how the material responds to shear stress, it is usually symbolised as `G`. The relationship between the shear modulus (G), Young’s modulus (E) and Poisson’s ratio (V) is shown in Equation 3.3.

\[ G = \frac{E}{1 + V} \]  

Equation 3.3

Figure 3.18 shows a schematic diagram which represents the stress strain curve for an elasto-plastic material where the shear modulus is obtained by the first derivative of shear stress over the shear strain. Therefore, the increment of shear stress over the strain with respect to displacement is an important parameter. Figure 3.19 shows the increment of shear stress with respect to displacement.

![Stress strain curve for an elasto-plastic material.](image-url)
Figure 3.19. The increment of shear stress over strain (dτ/dε) with respect to displacement (0 kPa suction).

The increment of shear stress over strain (dτ/dε) of the unreinforced specimens tested at 0kPa suction, lagged behind from root permeated specimens which began from a displacement of 2.2mm to 6mm, and can be identified as the displacement zone where the peak shear values occur. The initial dτ/dε of the root permeated sample is also higher than the unreinforced specimen, while the unreinforced specimens show a steadier variation of dτ/dε and the root permeated specimens have more scattered variations. It can be observed that root permeated samples show more ductile behaviour than soil only specimens reinforcing effect of the roots in root permeated soil. Figure 3.20 represent the variation of dτ/dε over the displacement of specimens tested under four different suction levels (i.e. 50kPa, 100kPa, 150kPa and 200kPa).
The variation of $\frac{d\tau}{d\varepsilon}$ values for the root permeated samples tend to increase with the high suction values, while all the root permeated specimens were more ductile than the unreinforced specimens; this leads to an increase in the peak shear value of the root permeated specimens.

Figure 3.20. The increment of shear modulus with respect to displacement $d\tau/dc$. 

The variation of $d\tau/dc$ values for the root permeated samples tend to increase with the high suction values, while all the root permeated specimens were more ductile than the unreinforced specimens; this leads to an increase in the peak shear value of the root permeated specimens.
3.3.5 Root-soil bond coefficient

The bond coefficient is an important parameter for computing the adhesive force resisting the shear displacement generated by the roots. Literature indicates that there is no good method for testing to compute the adhesive force, so a vertical root was tested using the large shear box and the adhesive force was calculated based on work done in a system is equal to the stored energy. Figure 3.21 shows a schematic diagram of the direct shear test with a vertical root where a 18mm diameter root is fixed to the shear box from the top and sheared via a motorised application. The unreinforced specimen was also tested for the same conditions. The results and the procedure for calculating the bond stress are illustrated in the Chapter 4.

![Figure 3.21. Schematic diagram of a direct shear test with a vertical root.](image)
3.4 Summary
A laboratory direct shear test was carried out with five different suction levels and three different applied normal stresses for the root permeated specimens and equivalent unreinforced specimens, to capture the effect that suction has on the reinforcement effect of soil. This integrated soil suction-reinforcement effect was introduced as $\Delta \tau_C$ as an additional component to an increase in the shear strength of soil due to root permeated soil; it is presented in Equation 3.1.

Three root failure mechanisms were identified during root mapping, as shown in Figure 3.7, and the quantitative results are shown in Table 3.2. Figures 3.12 and 3.13 show the stress vs horizontal displacement and vertical displacement vs horizontal displacement graphs; the results confirmed the positive effect which soil suction has on the root reinforcement of soil.

Tables 3.1 and 3.3 show the values of $\Delta \tau_C, \Delta \tau_R, \Delta \tau_S$ and the $\tau_{T1}$ values, while the proposed positions of these values in extended Mohr-coulomb envelope are shown in Figure 3.17. Figure 3.16 shows the results of the variations of peak shear of the unreinforced specimens and root permeated specimens with respect to suction. The peak shear results of unreinforced specimens show a gradual increase while the root permeated soil has a scattered increase due to the sudden change in the failure patterns of the roots. This phenomenon will be explained mathematically in Chapter 4.

The shear modulus of the soil is an important parameter in the shear strength of soil, and Figures 3.18 and 3.19 show the variations in the shear modulus over the horizontal displacement after direct shear tests; they confirmed that the behaviour was more ductile while the modulus of root permeated soil increased. The overall results showed that root permeation increased the strength of shear soil, while the reinforcement-
suction integrated portion has played a major role in the total increase in shear strength due to root permeation.
Chapter 4 Development of a mathematical-analytical model

4.1 General

The shear strength of an integrated tree root – soil system is influenced by root reinforcement and soil suction (Cameron 2001, Docker and Hubble 2001, Fatahi 2007). However, to quantify the real effect that tree roots have in an integrated system, the root reinforcement and suction must be considered simultaneously, as described and verified in an earlier chapter with experimental evidence. An integrated root - soil system is always in an unsaturated condition due to evapotranspiration by the tree. The variation of suction in root permeated ground has been comprehensively studied and presented by Indraratna et al. (2006).
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Docker and Hubble (2001) and Docker and Hubble (2008) laid a platform to carry out a direct shear test in root permeated soil to capture the true effect of roots grown naturally. Most tests in previous studies that capture the effect of root reinforcement (Pollen and Simon 2005, Thomas and Pollen-Bankhead 2010) used substitute roots such as fibres and wood anchors rather than natural roots, and therefore the model predictions obtained using substitute roots are unrealistic compared to tests conducted with real roots (Docker and Hubble 2008). The mathematical model developed to capture the true improvement of the shear strength by considering the root failure mechanisms observed during these tests is described in this chapter subsequently.

4.2 Increase in shear strength of soil due to root permeation

Vanappali et al (1996) proposed a relationship for estimating the shear strength of unsaturated soil (Equation 4.1):

\[
\tau' = [c' + (\sigma_n - u_a) \tan \phi'] + [(u_a - u_w) \left(\frac{\theta_w - \theta_r}{\theta_s - \theta_r}\right) \tan \phi']
\]  

(4.1)

where \(\tau'\) is the shear strength of soil, \(\phi'\) is the interface friction angle with respect to net normal stress, \(\sigma_n\) is the normal stress, \(u_a - u_w\) is the matric suction, \(c'\) is cohesion, \(\theta_w\) = volumetric water content, \(\theta_s\) = saturated volumetric water content and \(\theta_r\) = residual volumetric water content. Vannapali and Fredlund (2000) reported that accurate predictions can be obtained using Equation 4.1 for all types of soil with a suction range of 0 - 15000 kPa; therefore, since Equation 4.1 is both simple and accurate, it has been used in this study.

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Waldron (1981) suggested that the increase in shear strength due to root permeation \( (\Delta \tau) \) can be added directly to the Coulomb equation, as shown in Equation 4.2, because there is no change in the friction angle.

\[
\tau = c + \Delta \tau + \sigma_N \tan \phi
\]  
(4.2)

Where \( \tau \) is the shear strength of soil, \( c \) is the cohesion of soil, \( \sigma_N \) is the applied normal stress, and \( \phi \) is the friction angle of the soil. Combining Equations 4.1. and 4.2. then the shear strength of a root permeated media can be expressed as follows.

\[
\tau' = [c' + (\sigma_n - u_a)\tan \phi'] + [(u_a - u_w)\left(\frac{\theta_w - \theta_r}{\theta_s - \theta_r}\right)\tan \phi'] + \Delta \tau
\]  
(4.3)

Furthermore, the total increase in shear strength due to root permeation \( (\Delta \tau_T) \) can be express as in Equation 4.4 by considering the different modes of shear gain explained in Chapter 3.

\[
\Delta \tau_T = \Delta \tau_C + \Delta \tau_R + \Delta \tau_S
\]  
(4.4)

Where \( \Delta \tau_R = \text{Increase in shear strength due to the effect of root reinforcement only.} \)

(In a saturated condition), \( \Delta \tau_C = \text{Increase in shear strength due to integrated root-suction system,} \) and \( \Delta \tau_S = \text{Increase in shear strength due to the increase in soil suction due to evapotranspiration.} \)
As stated in Chapter 3, the fraction generated from an increase in shear strength due to an increase in soil suction due to evapotranspiration ($\Delta \tau_S$) can be added directly to the unsaturated portion of Equation 4.3 because the effective results of $\Delta \tau_S$ originate as an increase in the soil suction, and therefore it can be added as an increment of suction to the equation. Then Equation 4.3 yields to Equation 4.5, as follows:

$$\tau' = \left[ c' + (\sigma_n - u_a) \tan \phi' \right] + \left[ (u_a - u_w) + \psi \left[ \frac{\theta_w - \theta_i}{\theta_e - \theta_r} \right] \tan \phi' \right] + (\Delta \tau_C + \Delta \tau_R)$$

(4.5)

In this equation, $(u_a - u_w)$ is the variation of soil suction induced by the evaporation processes and $\psi$ is an increase in soil suction by transpiration due to the availability of a tree (this occurs only at the presence of tree). These two suctions are usually measured as one, as stated separately in Equation 4.5 for clarity.

### 4.2.1 Shear generation due to three failure patterns

The three failure patterns were observed during the direct shear tests, as follows:

- Roots undergone to pure slipping during the shearing. (Figure 4.1a)
- Roots broken during shearing; Tensile failure (Figure 4.1b)
- Roots come out with a soil annulus (Figure 4.1c)

It was assumed that these failure patterns did not overlap, e.g. roots experience one of the three types of failure. For instance, the plant roots pulled out shown in Figure 4.1a experienced pure slipping so the bond between the root and soil interface influences the increase of shear resistance of the root-soil block, and moreover, some of the roots failed under tension, as shown in Figure 4.1b. During shearing these roots are fixed into the body of the soil from one end, as shearing progresses these roots stretch and
once the tensile strength of the root is exceeded, they break. This mobilised tensile strength is generated by the applied earth pressure during shearing. In addition, some plant roots pulled out with a soil annulus as shown in Figure 4.1c.

Figure 4.1. Three root failure patterns.

Waldron (1977) showed that the shear resistance increased by the stretching roots which has same diameter ($\Delta \tau_1$) is equal to, as follows:

$$\Delta \tau_1 = \alpha_r T_N \delta$$  \hspace{1cm} (4.6)

Where $\alpha_r = A_r / A_S$, and $A_S$ is the total shearing cross section whereas $A_r$ is the total area of the root across the shearing plane, $T_N$ is the maximum tensile stress developed in the root at any displacement and $\delta$ can be expressed, as follows:

$$\delta = (\cos \beta i + \sin \beta \tan \phi)$$  \hspace{1cm} (4.7)

Where $\beta$ is the angle of the deformed root to the horizontal plane as shown in Figure 4.2. This model can be applied to the slipping, stretching or breaking roots having the same diameter.
In this study, Waldron (1977) model was extended to estimate the tree roots contribution to the shear strength increment ($\Delta \tau_T$) at a given displacement, by considering the three failure patterns seen in the experiments (Chapter 3), and as shown in Equations 4.8 - 4.10. Equations 4.8 - 4.10 can be used for vertical roots (Figure 4.2) with a different diameter, because the root area and the tensile properties, or the bonding properties of the roots are taken into account for each root separately.

Figure 4.2. Three failure modes for vertical roots. (1) Root fails under tension $t$ = mobilised tensile stress before failure, $T_s$ = Tensile strength of the root. $\Delta x$ = Elongation of the root. (2) Root pulled out (Pure slipping along soil) (3) Root pulled out with a soil annulus.

In this study, Waldron (1977) model was extended to estimate the tree roots contribution to the shear strength increment ($\Delta \tau_T$) at a given displacement, by considering the three failure patterns seen in the experiments (Chapter 3), and as shown in Equations 4.8 - 4.10. Equations 4.8 - 4.10 can be used for vertical roots (Figure 4.2) with a different diameter, because the root area and the tensile properties, or the bonding properties of the roots are taken into account for each root separately.

\[
\Delta f_{Resist 1} = \sum_{i=1}^{n_1} Ari(T_i(cos\beta_i + sin\beta_itan\phi)) \tag{4.8}
\]

\[
\Delta f_{Resist 2} = \sum_{i=1}^{n_2} Ari(R_{ri}(cos\beta_i + sin\beta_itan\phi)) \tag{4.9}
\]
\[ \Delta f_{Resist3} = \sum_{i=1}^{n_3} \text{leff. } \pi \text{def}. \left( Rsi (\cos \beta_i + \sin \beta_i \tan \phi) \right) \] (4.10)

In the above equations, \( \Delta f_{Resist1} \), \( \Delta f_{Resist2} \) and \( \Delta f_{Resist3} \) are the resisting forces due to the shear displacement as the roots stretch, slip, and are pulled out with a soil annulus accordingly; \( n_1 \) is the number of roots broken during shear displacement, \( T_i \) is tensile stress generated by the \( 'i' \)th stretching root, \( n_2 \) is the number of roots which slip without breaking, \( R_{ri} \) is the bond stress of the \( 'i' \)th slipping root developed between the root and soil, \( A_{ri} \) is the circumferential area of roots undergoing frictional displacement against shear displacement, \( n_3 \) is the number of roots that slip with a soil annulus, \( Rsi \) is the bond strength between the root and soil \( 'i' \)th root which was pulled out with the soil annulus, \( \text{leff} \) and \( \text{deff} \) are the effective length and circumferential diameter of the soil annulus which slips. \( \beta \) is the angle the deformed root takes to the horizontal plane, as shown in Figure 4.2.

Equations 4.8 - 4.10 expresses the resisting force to the shear displacement instead of the resisting stress, by considering that the resisting force of the \( 'i' \)th root ( \( \Delta f_{Resist\ i} \) ) is equal to \( (\Delta \tau_i)x(\ A_{ri}) \) where \( (\Delta \tau_i) \) and \( (\ A_{ri}) \) are the resisting stress and the area of the \( 'i' \)th root. The summation of all the resisting forces was taken to the limit of \( 'n' \)th number which is the total number of roots which experienced different types of failures.

These equations were developed in terms of resisting forces rather than shear stresses, by considering the convenient derivation of forces along each root separately. However, the shear stress can be computed for the direct shear test results by dividing
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the total resisting forces on the shear plane by the total effective area, as per Gray and Leiser’s (1982) modifications for a simple root model (Equation 4.11)

\[ t_r = T_R \frac{A_R}{A} \]  \hspace{1cm} (4.11)

where \((t_r)\) is the mobilised tensile strength of the soil per unit area of soil and \(T_R\) is the tensile strength developed in a root and \(\frac{A_R}{A}\) is the root area ratio.

Equation 4.12 represents the total resisting force to the shear displacement, which is \(F_{Resist}\), along the shear plane under the direct shear condition.

\[ F_{Resist} = \left(\tau_{unsat}(A_T - A_{root}(z))\right) + \sum_{i=0}^{n1} Ari(T_i(cos\beta_i + sin\beta i tan\phi) \]
\[ + \sum_{i=0}^{n2} Ari(R_{ri}(cos\beta_i + sin\beta i tan\phi) \]
\[ + \sum_{i=0}^{n3} leff.\pi eff. (Rsi (cos\beta_i + sin\beta i tan\phi) \]  \hspace{1cm} (4.12)

In Equation 4.12, \(\tau_{unsat}\) represents the shear strength of the soil alone, \(A_T\) is the total area of the shear plane, and \(A_{root}(z)\) is cross sectional area of the root across the shear plane. The first part of the Equation 4.10 \(\left(\tau_{unsat}(A_T - A_{root}(z))\right)\) represents the shear generated from soil to soil interaction, while the other part represents the resisting force generated by the three failure patterns of the roots.

In this model each root is assumed to have undergone only one failure mode during the whole shearing process. Furthermore, it is assumed that no elongation occurred in the roots as pure slipping and slipping with a soil annulus. The upper bound value of the generated tensile stress is the tensile capacity of the root, a material property which
depends on the cell structure of the root. According to basic botany, the root cell structure changes from being plasmolysed (desiccated cells) to a turgid (moisturised stiff cells) condition when water is available (Willmer and Beattie 1978). Turgid cells are stiffer than plasmolysed, but there should be a good engineering quantification to evaluate the variation in strength with variations in the cell structure (Stokes and Matthee 1996, Saifuddin and Osman 2014); thus far this has been considered to be negligible compared to other parameters in this study.

### 4.2.2 Calculation of $B_r(\psi)$ and $B_s(\psi)$

The root-soil interface friction ($B_r(\psi)$) and the sol-soil interface friction ($B_s(\psi)$) is governed by the level of suction in the soil. According to Hamid and Miller (2009) the interface friction for an unsaturated vertical shear interface can be illustrated as Equation 4.13.

$$\tau_f = c' + (\sigma_n - u_a)\tan\delta' + (u_a - u_w) [\tan\delta' \left(\frac{\theta - \theta_r}{\theta_s - \theta_r}\right)]$$

(4.13)

where $\tau_f$ is the interface frictional stress with respect to net normal stress and $\delta'$ is the interface friction angle with respect to net normal stress. In a vegetated soil system, $\tau_f$ and $\delta'$ should be defined relevant to the soil-root interface. Figure 4.3 shows the basic forces acting on an inclined root in root permeated soil. Equation 4.14 and 4.15 represent the normal and upward forces acting on the small element of root.
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\[ R_n = V \cos \alpha + H \sin \alpha \]  \hspace{1cm} (4.14)

\[ R_u = V \sin \alpha - H \cos \alpha \]  \hspace{1cm} (4.15)

Figure 4.3. Basic forces acting on an inclined root.

where \( R_n \) is the force component normal to the root and \( R_u \) is the upward force component, \( V \) is the vertical load applied onto a root element and \( H \) is the horizontal force acting on a root element. The force resisting the upward movement of root \( R_r \) is represented in Equation 4.16

\[
R_r = c' + (V \cos \alpha + H \sin \alpha) \tan \delta' + (u_a - u_w) \left[ \tan \delta' \left( \frac{\theta - \theta_r}{\theta_s - \theta_r} \right) \right] \\
- (V \sin \alpha - H \cos \alpha)
\]  \hspace{1cm} (4.16)

\( H' \) in equation 4.16 is directly proportional to the lateral earth pressure and ‘V’ is directly proportional to the applied normal stress. Therefore, the bond strength is influenced by the soil suction and normal stress. The \( \tan \delta' \) in Equation 4.16 can be defined as the root-soil bond coefficient which is a material property that is represented as ‘\( \tan \lambda \)’ in further references. Then Equation 4.16 yields to Equation 4.17, as follows;

\[
R_r = c' + \left[ (\sigma_N - u_a) \cos \alpha + \left( k_0 \left( (\sigma_N - u_a) + \gamma h \right) \sin \alpha \right) \tan \lambda \\
+ (u_a - u_w) \left[ \tan \delta' \left( \frac{\theta - \theta_r}{\theta_s - \theta_r} \right) \right] - ((\sigma_N - u_a) \sin \alpha \right) \\
- \left( k_0 \left( (\sigma_N - u_a) + \gamma h \right) \sin \alpha \right) \cos \alpha) 
\]  \hspace{1cm} (4.17)
4.2.3 Calculation of bond coefficient (\(\tan \lambda\))

The bond coefficient was calculated using a 18mm vertical root and it was sheared under pure slipping, as explained in Chapter 3 (Figure 3.21). Figure 4.4 is a schematic diagram of a vertical root which has undergone pure slipping under a direct shear application.

![Diagram of a vertical root](image)

Figure 4.4. (a) Schematic diagram of a vertical root which underwent shearing, (b) Enlarged view of the segment of a root.

In Figure 4.4, \(\Delta d\) is the shear displacement, \(\Delta t\) is the thickness of the shear zone, \(L\) is the length of the root, \(L_1, L_2\) are the lengths of the root which lie above and below the shear zone, and \(\Delta L_1\) is the length of root \(\Delta L_1\); they can be calculated according to Equation 4.18, because \(\Delta L_1\) cannot be measured directly. The difference in area between the graphs in the unreinforced soil test and the root permeated test were used to calculate the bond coefficient. The total energy mobilised by the root element from pure slipping can be considered as equal to the difference in area between the unreinforced soil test and root permeated soil because there was only pure slipping.
\[
\Delta L_1 = L_2 - (L - L_1 - \sqrt{(\Delta d^2 + \Delta t^2)}) \quad (4.18)
\]

The mobilised energy (\(\delta E\)) by a small root element (Figure 4.4b) at a \(z\) vertical movement can be expressed as equation 4.19.

\[
\delta E = R_r 2\pi r \delta z z \quad (4.19)
\]

Where \(R_r\) is the resisting force to the upward movement of a root per unit area, \(r\) is the radius of the root, and \(\delta z\) is its thickness. The energy mobilised by the frictional work done can be calculated by integrating the small root element from 0 to \(\Delta L_1\).

\[
\text{Total frictional energy generated by the frictional work done} = \int_{z=0}^{z=\Delta L_1} R_r \times 2\pi rz \, dz \quad (4.20)
\]

Then total energy generated by the frictional work done (\(E_T\)) yields to the Equation 4.21, while Equation 4.22 can be obtained by substituting the value of \(\Delta L_1\) to Equation 4.21.

\[
E_T = [f r \pi z^2], \ z = 0, z = \Delta L_1 \quad (4.21)
\]

\[
E = R_r \pi r \left( L_2 - (L - L_1 - \sqrt{(\Delta d^2 + \Delta t^2)})^2 \right) \quad (4.22)
\]
Figure 4.5 shows the results of the direct shear test obtained for an 18mm vertical root which was sheared under pure slipping, and the equivalent soil sample. The difference in area between these two graphs is equal to the energy generated by the frictional work done by the root.

The bond coefficient was calculated using Equation 4.23, which was yielded by Equations 4.17, 4.19 and 4.22.

\[
\tan \lambda = \frac{\Delta A}{\pi \tau \left( L^2 - (L - L_1 - \sqrt{(\Delta d^2 + \Delta t^2)})^2 \right)} - (u_a - u_w) \left[ \tan \delta' \left( \frac{\theta - \theta_r}{\theta_s - \theta_r} \right) \right] + \left( (\sigma_N - u_a) \sin \alpha - k_0 ((\sigma_N - u_a) + \gamma h) \sin \alpha \cos \alpha \right) \right) - c'
\]

\[
\Delta A \text{ in Equation 4.23 is the difference in area of the direct shear tests carried out with a vertical root (Figure 4.5), and unreinforced soil test under the same test conditions.}
\]

The bond coefficient was 0.21 to 0.22 for the three tests carried out using a vertical root.
4.2.4 Calculation of generated tensile strength \( (T_i) \)

When there is no slipping (the root is fixed at the bottom), \( \Delta L1 \) value in Equation 4.18 turns into the elongation of the root. The strain \( (\varepsilon) \) of the root at each horizontal displacement can be calculated using Equation 4.24.

\[
\varepsilon = \frac{L2 - (L - L1 - \sqrt{\Delta d(t)^2 + \Delta t^2})}{L}
\]  

(4.24)

The tensile energy mobilised in the root \( (E(t)) \) can be expressed as Equation 4.25.
where $T_i$ is the mobilised tensile stress. Therefore, the mobilised tensile stress at each horizontal displacement can be calculated using the difference in area between the root permeated specimen and the unreinforced soil specimen. Furthermore, it can be expressed using Young’s modulus for the tree roots, as shown in Equation 4.26

$$E(t) = \frac{1}{2} T_i \varepsilon = \frac{1}{2} ke^2$$

(4.26)

where $k$ is Young’s modulus of the root and $e$ is the elongation of root. Different $k$ values calculated using the tensile test results are shown in Table 3.3 in Chapter 3. Therefore, the mobilised tensile stress at any shear displacement for an inclined root can be calculated using Equation 4.27.

$$T_i = k \left[ \frac{L^2}{\sin \beta^2} + \left(L - \frac{L_1}{\sin \beta^2} - \sqrt{((\Delta t \tan \beta + \Delta d)^2 + \Delta t^2)} \right) \right]$$

(4.27)

where $\beta^2$ is the final inclination of the deformed root with respect to the horizontal plane. $\beta^2$ can be calculated using Equation 4.28, and $\beta^2$ and $\alpha$ are illustrated using Figure 4.5a.

$$\tan \beta^2 = \frac{\Delta t}{\Delta t / \tan \alpha + \Delta d}$$

(4.28)
The actual root system considered has root in three-dimensional space, as shown in Figure 4.6(b), so Equation 4.28 was extended for three dimensions and $\alpha_1$ in Figure 4.6(b) is the projected inclination of the root on the $x$-$z$ plane and $\alpha_2$ is the projected inclination of the root on the vertical plane going through the root.

The horizontal tensile force resolution ($T_i H$) along the $x$-$z$ plane is given by Equation 4.29, and the vertical tensile force resolution ($T_i V$) along the $x$-$z$ plane is given by Equation 4.30.

$$T_i H_1 = T_i \cos \alpha_1 \quad (4.29)$$

$$T_i V_1 = T_i \sin \alpha_1 \quad (4.30)$$

The horizontal force resolution along the plane goes vertically through the root $T_i H_2$ as given by Equation 4.31, and the horizontal force resolution along the plane goes vertically through the root $T_i V_2$ as given by Equation 4.32.
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\[ T_i H_2 = T_i \cos \alpha_2 \sin \alpha_2 \]  
\[ (4.31) \]

\[ T_i V_2 = T_i \cos \alpha_2 \cos \alpha_2 \]  
\[ (4.32) \]

Then the resisting force from the shear displacement \( T \) is expressed in Equation 4.33.

\[ T = T_i \cos \alpha_2 (\sin \alpha_1 + \cos \alpha_1 \tan \varphi) \]  
\[ (4.33) \]

The tensile force resisting shear displacement can be obtained by substituting \( T_i \) from Equation 3.27 into Equation 4.33, as shown in equation 4.34.

\[ T = k \left[ \frac{L_2}{\sin \beta} + \left( L - \frac{L_1}{\sin \beta} \right) \cos \alpha_2 (\sin \alpha_1 + \cos \alpha_1 \tan \varphi) \right. \]  
\[ \left. - \sqrt{((\Delta t \tan \beta + \Delta d)^2 + \Delta \tau^2)} \right] \cos \alpha_2 (\sin \alpha_1 + \cos \alpha_1 \tan \varphi) \]  
\[ (4.34) \]

Furthermore, the bond force resisting the shear displacement can be obtained for the three dimensional system by adopting \( \alpha_1 \) and \( \alpha_2 \) from Figure 4.5 b into Equation 4.17. The yielded equation is as follows:

\[ R_r = c' + [(\sigma_N - u_a) \cos \alpha 2 + (k_0((\sigma_N - u_a) + \gamma h) \sin \alpha 2) \tan \lambda \cos \alpha_1 \]  
\[ + (u_a - u_w) \left[ \tan \delta' \left( \frac{\theta - \theta_r}{\theta_e - \theta_r} \right) \right] \]  
\[ - (\sigma_N - u_a) \sin \alpha 2 \cos \alpha_1 \]  
\[ - (k_0((\sigma_N - u_a) + \gamma h) \sin \alpha 2 \cos \alpha_2) \cos \alpha_1 \]  
\[ (4.35) \]
4.2.5 **Logical test to check the root failure methods**

To check the occurrence of the failure pattern logical test was carried out using a MATLAB programme. Here, S1, S2 and S3 parameters are introduced to the model, as shown in equation 4.36, to check the possibility of a failure pattern. S1, S2 and S3 have the value of 1 and 0 according to the following conditions.

\[
\Delta \tau_T = \sum_{i=0}^{n_1} S1_i. Ari(T_i (\cos \beta + \sin \beta \tan \phi)) / A_T \\
+ \sum_{i=0}^{n_2} S2_i. Ari(R_{ri} (\cos \beta + \sin \beta \tan \phi)) / A_T \\
+ \sum_{i=0}^{n_3} S3_i. leff. \pi eff. (Rsi (\cos \beta + \sin \beta \tan \phi)) / A
\]

(4.36)

Where \( A_T \) is the total effective area of the shear plane. If the tensile stress \( T_i \) is greater than the tensile strength \( T_s \) of the root, it has already broken, and has no contribution to the increase of the shear strength. For this condition \( S1 = 0, S2 = 0 \) and \( S3 = 0 \).

If the generated tensile stress \( T_i \) is less than or equal to the tensile strength \( T_s \) of the root, then the following two conditions must be followed. Roots with \( T_i \) less than or equal to the bond stress \( R_{ri} \) of the root experience stretching and \( S1 = 1, S2 = 0 \) and \( S3=0 \). Roots with a generated tensile stress \( T_i \) which is greater than the bond stress \( R_{ri} \) have slipped out and \( S1 = 0, S2 = 1 \) and \( S3 = 0 \). However, a percentage of these roots must be taken as stretching and this must be added to the stretching condition. The reason is that some small roots develop abundant fibrous roots, so they do not just pull out as expected. In fact, it was found that the percentage of roots which
fall into this category is low and may be considered negligible. However, it is a viable and safe assumption in geotechnical engineering design because the calculated increase in the shear strength (\( \Delta \tau_T \)) is less than the value calculated using the other method. Equation 4.36 yields to Equation 4.37, when a root is considered in three-dimensional space, as shown in Figure 4.6(b).

\[
\Delta \tau_T = \sum_{i=0}^{n_1} S1_i \cdot Ari(T_i \cos \alpha_{1i} (\cos \alpha_{2i} + \sin \alpha_{2i} \tan \phi)) / A_T \\
+ \sum_{i=0}^{n_2} S2_i \cdot Ari \cos \alpha_{1i} R_{ri} (\cos \alpha_{2i} + \sin \alpha_{2i} \tan \phi)) / A_T \\
+ \sum_{i=0}^{n_3} \text{left}. \frac{\pi \text{eff}. \cos \alpha_{1i} \cdot R_s i (\cos \alpha_{2i} + \sin \alpha_{2i} \tan \phi)) / A}{A_T}
\] (4.37)

The flow chart in Figure 4.7a shows the summarised logical test and Figure 4.7b shows the GUI developed for the logical test.

![Flow chart and GUI](image)

Figure 4.7 (a) Summarized logical test (b) MATLAB GUI.
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Figures 4.8 and 4.9 are a comparison of the experimental results and the developed model results. The model predictions were obtained using the MATLAB program developed for the proposed model and MATLAB codes are shown in Appendix I. The root system shown in Table 3.3 in Chapter 3 is the most suitable representation of the root system relevant to the time of plant growth, and it was obtained after analysing the entire root mapping data reported during the test. This root system was used as the input matrix for the MATLAB programme.

Figure 4.8. Variation of increase in shear strength with initial matric suction and applied normal stress. (a) Surface created using experimental values and (b) Surface created using model predicted values.
The experimental $\Delta \tau$ value in Figures 4.8a and 4.8b is the increase in the shear strength due to root reinforcement at the peak of the stress displacement graph. The bond strength was calculated using Equations 4.15, 4.17, and 4.20. The tensile strength developed in each broken root at peak shear was calculated using Equation 4.22. The bond coefficient for roots pulled with soil annuls was taken as equal to $\tan \phi$, where $\phi$ is the friction angle of the soil. Even though the results predicted by the model were calculated using much of the assumptions and other experimental parameters, they agreed with the experimental results, as shown in Figures 4.8 and 4.9. The experimental results shown in Figure 4.8a have the same trend as Figure 4.8b which shows the results predicted by the model. The increase in shear strength in root
permeated soil which is generated due to root reinforcement tends to increase with the soil suction and applied normal stress. Therefore, the mechanical effect of root reinforcement can be clearly identified as a function of suction and the applied normal stress.

4.3 Shear modulus

The shear modulus is an important parameter in geotechnical engineering. The shear modulus of root permeated soil increases in the beginning of the test and shows a more ductile behaviour due to the root reinforcing effect, as shown in the results of the direct test presented in Chapter 3. The \( \frac{d\tau}{d\varepsilon} \) for the total shear strength can be computed by obtaining a first derivative of Equation 4.37 with respect to strain, as shown below.

\[
\frac{d\Delta\tau_T}{d\varepsilon} = \sum_{i=0}^{n_1} \frac{d}{d} S1i. Ari(T_i \cos \alpha_1 i (\cos \alpha_2 i + \sin \alpha_2 i \tan \phi))/A_T
\]

\[
+ \sum_{i=0}^{n_2} \frac{d}{d} S2i. Ari \cos \alpha_1 i R_{ri}(\cos \alpha_2 i + \sin \alpha_2 i \tan \phi)/A_T
\]

\[
+ \sum_{i=0}^{n_3} \frac{d}{d} \left( \text{eff.} \pi \text{def.} \cos \alpha_1 i . Rsi \cos \alpha_2 i + \sin \alpha_2 i \tan \phi \right)/A
\]

The value of this derivative has to be calculated for every root for the root system and the computed values are shown in Table 3.3. The MATLAB simulation for this was done by the results obtained for a suction of 50kPa and an applied normal stress of 20kPa, as shown in Figure 4.9.
The values predicted by the model and the experimental values are in good agreement as shown in this Figure 4.10, therefore the developed theoretical model can be verified by using the experimental results. The flow chart to describe the steps of calculations of the theoretical model is shown in Figure 4.11.
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START

Soil Properties.
- Friction angle
- Cohesion
- Dry density
- Suction
- Moisture Condition

Root Geometry
- Orientations in 3D space.
- Number of roots
- Root Area Ratio
- Young’s Modulus of roots

Root-soil integrated system
- Shear Modulus
- Stiffness Values
  $E_{oed}$, $E_{ref}$
- Bond coefficient

Calculations using developed equations
- Generated tensile stress on the root.
- Generated bond stress on the root.

Performing logical test to check the root failure methods

Calculation of increase of shear strength

INPUT
STEP ONE

CALCULATION PROCESS – MATA LAB SIMILATION ON THEORATICAL MODEL

Figure 4.11. Flow chart representing the calculation process.
Figure 4.11. Flow chart representing the calculation process.
4.4 Limitations of the theoretical model

Although the proposed theoretical model can predict the increase in shear strength due to root permeation by considering different methods of failure, the suction, and normal stress variations, the following limitations have been identified.

- The root system must be thoroughly defined with the orientation, diameter, and Young’s modulus. Therefore due care must be exercised while extending this study to other root systems.

  Fortunately, most of the botanical researches considered the above data for most of the root systems and therefore some correlations are available.

- The calculation process is tedious because the model was developed to calculate the resistance of every root and then obtain a summation.

  Therefore, a numerical method must be adopted to facilitate model computations, which is why MATLAB has been used in this study.

- Some of the other data which are not commonly used, such as the thickness of the shear zone and the bond coefficient between root-soil have been used in this model.

  Even though the above parameters are not commonly used, a way of measuring them has been explained in this study, and it is not a difficult task.
4.5 Summary

The increment in the shear strength of a root soil integrated system must be quantified by considering the contributions of suction variation ($\Delta \tau_S$), root reinforcement effect ($\Delta \tau_R$), and the coupling effect ($\Delta \tau_C$), as well as the failure mechanisms of the root system during shear displacement. In Equation 4.5 these terms are added $\Delta \tau_S$, $\Delta \tau_R$ and $\Delta \tau_C$ to a generalised unsaturated shear strength equation. Equations 4.8 - 4.10 represent the increase in shear strength due to three failure patterns observed during the tests. Calculating the root-soil interface friction ($B_r(\psi)$) and the sol-soil interface friction ($B_s(\psi)$) was done by considering the earth pressure and suction applied along the roots. Equation 4.27 explains how the tensile strength generated in the roots is calculated at a given displacement. The shear resistance to the shear displacement ($\Delta \tau_R$), for a root in three dimensional space is expressed in Equation 4.34. A logical test was carried out to compute the possible occurrence of root failures and this is expressed in Equation 4.36. A MATLAB simulation was done to obtain the values predicted by the model for the root system, while Figure 4.8 shows the comparative results of the values predicted by the model and the experimental results.
Chapter 5  Variations of suction moisture in vegetated ground- analysis of field results

5.1  General

The growing popularity of the use of vegetation as ground improvement method is due to the relatively small environmental footprint and the economical and alternative geotechnical engineering solutions for protecting existing infrastructure. The tree roots improve the ground via two mechanisms, i.e. physical and mechanical strengthening and moisture extraction due to root water uptake which in turn leads to an increase in soil suction, as noted in previous chapters. For instance, the root water uptake induced by evapotranspiration processes contributes to a desiccation of the soil and hence results in an increase in the soil suction around the root system (Indraratna et al. 2006). This reduction in the moisture content also induces an associated increase in the matric suction of adjacent ground and thus contributing to an overall increase in the shear
strength of soil (e.g. Fatahi et al. 2014). This phenomenon has been widely observed in field experiments and has been captured in sophisticated models in previous studies (e.g. Cameron 2001, Indraratna et al. 2006, Ng et al. 2013). In these studies, the variation in matric suction induced by transpiration has been calculated indirectly i.e. via the soil water characteristic curve (SWCC) (e.g. Fatahi et al. 2007). Furthermore, the soil matric suction is typically measured in locations considerably further away from the root zone (Cameron, 2001). The concurrent measurements of the moisture content and matric suction within the root zone have not been reported in previous field studies, to the best of the authors’ knowledge. This chapter describes a field study carried out to capture the relationship between the moisture content and suction within and in close proximity to the root zone. The role of the root water potential on the variation of matric suction is illustrated through both laboratory and field measurements carried out within and outside of the root zone.

5.2 Root water uptake, potential transpiration, and root water potential

The root water uptake of a plant or tree is typically governed by the amount of transpiration (Radcliffe et al. 1980). This process can be significant because in the summer season, trees such as Pinus Radiata can absorb amounts of water almost equal to their own weight per hour (Adam 2002). The rate of root water uptake has been related to the hydraulic conductivity of the tree canopy and the differences in water potential between the roots and the soil (Gardner 1960, Whisler et al. 1968, Molz and Remson 1970, Hillel et al. 1976). Fatahi (2007) developed an approach for predicting the root water uptake, as shown in Equation 5.1.

\[ S(x, y, z, t) = f(\psi)G(\beta)F(T_p) \]  

(5.1)
where $S(x,y,z,t)$ is the rate of root water uptake at a given point and in a given time, $F(T_p)$ is the factor related to potential transpiration, $f(\psi)$ is the factor related to soil suction, and $G(\beta)$ is the root density function.

Potential transpiration is a function of environmental parameters such as temperature, humidity, wind speed and tree physiology (Walter 1898, Nimah and Hanks 1973), and since root water uptake only takes place in active roots, it varies from point to point according to their spatial distribution, i.e. $G(\beta)$ in Equation 5.1. Potential transpiration in conjunction with other chemical and biological processes inside a tree induce water potential in the roots (i.e. the root water potential) which act as the main driving energy to extract soil water through the roots (Kramer 1995). While the root water potential is influenced by the chemical and biological processes, it may be taken as proportional to the potential transpiration for a given tree, provided the tree does not experience extreme weather conditions.

Tree roots absorb water by osmosis, which is regulated by the aforementioned factors, and therefore the root water potential is a result of osmosis inside the roots. This process also induces a pressure difference inside and outside the root, and it may also induce variations on the matric suction values measured without necessarily implying a change in the moisture content, i.e. by restructuring the pore water. Considering the above hypothesis, laboratory experiments were setup and a field investigation was carried out.
5.3 Experimental Setup and Field Investigation

5.3.1 Soil material and plant species

The typical site conditions were replicated in the laboratory. The in situ dry density was determined using sand replacement method and average dry density value for the 10m x 5m site was obtained as 1300 kgm$^{-3}$ (72% of the maximum dry density). Soil collected at field site at the University of Wollongong was used for the laboratory experiments; it was classified as low plastic (LL = 41 and PI = 7), well graded, silty-sand (SM) in accordance with the unified soil classification system. The plant used for the laboratory and field investigations is a *Eucalyptus Botryoides* (Bangalay), a native Australian evergreen tree with a good root system (type) and good tolerance to environmental changes.

5.4 Laboratory experimental setup

Laboratory experiments were set up to obtain the moisture and matric suction parameters with time in a more controlled environment (i.e. temperature and humidity) as shown in Figure 5.1. Two points were selected for concurrently measuring the moisture content and the matric suction; Point 1 is inside and close to the root zone, and Point 2 is well away from the root zone (Figure 5.1a). These two locations were used to monitor the root water potential generated by osmosis in the plant.
Two matric suction sensors (MPS2, Figure 5.1(d)) and moisture sensors (EC5, Figure 5.1(c)) were installed at each point to capture both suction and moisture with time. A sap flow meter (SFM-1, Figure 5.1(e)) was also installed in the stem of the plant to measure the sap flow of the plant using the heat ratio method (HRM). To measure the root water potential or the osmotic potential of the plant, a psychrometer (PSY, Figure 5.1(f)) was installed in the stem of the same plant, close to the roots. A tensiometer was used to verify how accurately the suction sensors (MPS2) could measure the suction of the soil, in the range of typically less than 90kPa.

Initially, the soil was compacted to replicate the field dry density of 1300kgm$^{-3}$. Then the plant was planted and sensors were installed. To eliminate the effect of
environmental changes on the moisture and suction measurements, the experiment was carried out under a controlled temperature (22 ± 2°C) and humidity (61% - 63% RH).

5.4.1 Calibration of sensors

MPS2 and EC5 sensors are factory calibrated, however cross checking while the experiments were done using tensio meters (for MPS2) and the oven dried samples (EC5). The laboratory results were in good agreement with the sensor output. Pychrometer was calibrated using Sodium Chloride (NaCl) solutions with different molarities, as shown in Figure 5.2.

Figure 5.2. (a) NaCl 1M, 0.5M, 0.4M solutions, (b) Cleaning the thermos couple using distilled water, (c) Small filter papers to use in Pychrometer, (d) Filter paper soaked in NaCl solution is inside the thermocouple and (e) Calibration is done using a data logging software.
5.5 Field site

Sensors similar to those used in the laboratory set ups were used to monitor the moisture content and suction on site; the site plan is shown in Figure 5.3.

Figure 5.3. Site Plan with the positions of sensors.
The MPS2 suction sensors and EC5 moisture content sensors were installed inside and close to the root zone to verify any variations in the measured matric suction as a result of root water potential (Figure 5.4). A total of 11 moisture sensors and 11 suction sensors were installed at 11 different points at varying depths and lateral distances from the axis of the plant (Figure 5.4). Another suction/moisture measuring point was set up outside of the root zone as a benchmark point to account for site environmental variations such as temperature, humidity, and precipitation.

Figure 5.4. Experimental set-up and locations of instrumentation (e.g. depth and distance from the tree) adopted in the field site.
To install the suction and moisture sensors, an auger was used and sensors (MPS2 or EC5) were installed at the required depth, as shown in Figure 5.4. The suction and moisture sensors were conditioned beforehand with moist soil from the site before being installed to prevent air bubbles from forming between the soil and sensors. After installation the void was backfilled and compacted to ensure the in situ density was attained. A steel pipe with an enlarged steel head (Figure 5.5b) was used to install the sensors and also to compact the soil. This method was repeated for the 11 suction and 11 moisture sensors. Figure 5.5a shows the configuration of the sensors and equipment at the field site.

Figure 5.5. (a) Field Site Configuration, (b) Re-compacting the hole after sensor installation and (c) Enlarged view of Sap flow meter and Psychrometer.
Chapter 5  Suction moisture variation in vegetated ground

The suction and moisture were measured over a two month period before the plant was planted, to check the reliability of the installation process and the response of the sensors relative to the benchmark. A Eucalyptus Botryoides (Bangalay) plant was then planted on site at the chosen locations (Figure 5.3). As with the laboratory set up, a sap flow meter (SFM-1) and a psychrometer (PSY) were installed to measure the sap flow and root water potential, or the osmotic potential of the plant, respectively. These instruments were installed 2 months after the plant was planted, so its stem was at least 10mm in diameter to ensure that the measurements would be accurate and reliable.

The instruments were also connected to data logger to enable continuous measurements over a period of six months (Figure 5.5d). To assess repeatability and take into account some degree of variability of the ground conditions, a backup plant with the same arrangement was also set up on the site.
5.6 Analysis of results

5.6.1 Variation of measured matric suction ($\psi_m$) inside and away from the root zone under constant root water potential ($\pi R$)

Figure 5.6 shows the variations of suction with volumetric moisture content (VMC) at points 1 and 2 in the laboratory study. These results represent the variations of suction and moisture at four different time periods from 01/12/2014 to 10/05/2015, and the associated soil water characteristic curve developed for soil without a plant.

![Figure 5.6. Variation of moisture content with matric suction in within and near to the root zone.](image)

Figure 5.6. Variation of moisture content with matric suction in within and near to the root zone.
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Suction moisture variation in vegetated ground

From 01/12/2014 to 24/12/2014, the variations in suction and moisture at Point 2 are very close to the soil water characteristic curve developed for equivalent soil without the plant (Original SWCC). However, the variation of suction with moisture content at point 1 moved upwards from the original SWCC, which indicates that for the same moisture content the sensor at Point 1 measured much higher matric suctions.

This is not surprising because the matric suction ($\psi_m$) measured near the root zone is influenced by the root water potential. This difference is not as obvious at points located away from the root zone where the measured matric suction values are similar to the original SWCC. The curves showing the variation in suction and moisture content at point 1 from 28/12/2014-16/01/2015 and from 16/01/2015-20/03/2015 appeared to move down towards the original SWCC over time, whereas the last curve of point 1 for the time period of 20/03/2015-10/05/2015 moved a long way away from original SWCC.

This gradual shift in the SWCC near the root zone may be due to an increase in soil densification and associated changes in the soil structure, as well as those incurred while wetting and drying. However, it can be observed that the measured matric suction ($\psi_m$) varied more than expected matric suction ($\psi_{exp}$) for the relevant moisture content close to the root zone. The root water potential was taken as constant in this study and it varied between 0.38Mpa-0.4Mpa.
5.6.2  Analysis of results- field Study

Figures 5.7a and 5.7b show the variations of matric suction ($\psi_m$), moisture content (VMC) and root water potential ($\pi_R$) profiles of field site (Figure 5.4) measured at the vertical (depth) and horizontal (distance from the plant axis) directions over time. Continuous measurements were recorded over an eight month period, including the two months prior to planting the plants. For brevity and enhanced clarity, only the most representative results are shown in Figure 5.7. Continuous results over the eight month period are shown in Appendix 2.
Figure 5.8. Suction, Volumetric moisture content and variation of root water potential over time at locations away from the tree in the: (a) horizontal direction at points P2, P5 and P8, and (b) vertical direction at points P1, P2 and P3.
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The results from points P1, P5, P8 and points P1, P2, P3 were used to describe the variations in horizontal and vertical directions, respectively, where points P2 and P5 indicate the equivalent moisture content values at “A” and “B” in Figure 5.8 a. The measured matric suction value \( \psi_m \) of both points which are relevant to the moisture content at “A” are the same, and the measured matric suction value \( \psi_m \) of point P2 at “B” varies by 125kPa more than point P5. Furthermore, the root water potential \( \pi_R \) is higher at “B” which implies the influence of the root water potential on measured matric suction which should be equal for the both points for the equivalent moisture content.

The moisture content of point P3 is higher than point P2 at “C”, as Figure 5.4b shows, whereas the suction at point P3 should be less than at point P2, according to conventional unsaturated soil mechanics. However, the measured matric suction of point P3 is higher than that at point P2 and the root water potential is also highest at this point; this also confirms the influence that the root water potential has on measured suction near the root zone. Furthermore, the results from points P2 and P3 indicate that the influence of the root water potential on the measured suction decreases with the distance away from the root zone.

5.6.3 Increments in the measured suction \( (\psi_{add}) \) with the root water potential

Figure 5.9 shows the variation of the measured suction differences between points P5 and P2, with the root water potential for the equal moisture contents. This graph was developed using the results extracted during one drying curve of the soil.
The results in Figure 5.9 show that the relationship between the measured suction difference (matric suction difference between Points P2 and P5) with the root water potential is linear. Total soil suction is an addition of the matric suction ($\psi$) generated by the three phase system inside the small pores and osmotic suction ($\pi$) which is generated by solutes in the pore fluid (Equation 5.2).

$$\psi_{Total} = \pi + \psi$$  \hspace{1cm} (5.2)

This study has only discussed the matric suction ($\psi$) in Equation 5.2, which is generally evaluated by the SWCC developed for the relevant soil. However, it was verified using
Chapter 5  Suction moisture variation in vegetated ground

the experimental result whereby the matric suction inside and close to the root zone varies from point to point, even with the same moisture content, and this variation is directly related to the root water potential of the plant (Figure 5.9). Therefore Equation 5.3 can be used to define the measured matric suction near to the root zone since it shows the value addition rather than the computed value from the SWCC.

$$\psi_m = \psi_{exp} + \psi_{add}$$ (5.3)

In Equation 5.3, $\psi_m$ is the measured matric suction, $\psi_{exp}$ is the estimated matric suction with respect to the original SWCC of the relevant soil, and $\psi_{add}$ is the value addition to the suction due the root water potential. Therefore, $\psi_{add}$ is a function of the effective root water potential ($\pi_{Reff}$) at the relevant point. The phrase, effective root water potential was used because the root water potential on $\psi_{add}$ decreases vertically and horizontally away from the root zone, and therefore the zero effective root water potential ($\pi_{Reff}$) implies that the measured suction is equal to the estimated matric suction from SWCC. These results will be explained conceptually in the next section.
5.7 Development of the theoretical concept

The above results can be explained conceptually in following way, and since it is outside the scope of this study, they will not be proven mathematically.

Conceptual approach:

“Suction” is a potential energy related to capillary pressure (Fredlund et al. 2012). The height of water rising in a capillary tube and the radius of the curvature of the air-water interface (contractile skin) directly influences the water content and matric suction of soil. According to Fredlund et al. (2012), this capillary rise can differ during wetting and drying due to variations in the pore size, and therefore the resulting matric suction also changes. Figure 5.10 shows a physical model of capillarity and the height of the water in the capillary tube and \( h_c \) is expressed in Equation 5.4.

\[
h_c = \frac{2T_s}{\rho_w g R_s}
\]  
\( (5.4) \)

where \( T_s \) is the surface tension of the water, \( \rho_w \) is the density of the water, \( g \) is the gravitational acceleration, and \( R_s \) is the radius of the meniscus.
The radius of the meniscus tends to change if there is another significant energy exerts on the contractile skin inside the small soil pores (i.e. the value of $u_a$ in the pores which is related to Figure 5.10 can change). Therefore, if there is another energy source which can penetrate through the small pores and act on the contractile skin without damaging it, there will be a change in the soil suction for the same water content. It appears to be impossible to generate such energy which penetrates through the small pores without damaging the contractile skin of the soil water interface, e.g. due to the surface tension.

Figure 5.10. Physical model of a capillary (modified after Fredlund et al. 2012).
Similarly, “tree root water potential” could potentially deliver this external energy to restructure the pore water and alter the suction related to the particular moisture content. Wang and Fredlund (2003) observed the variation of the hyperbolic shape of the contractile due to the vapour pressure variations using the snapshots obtained from a high speed camera (9000 frames per seconds). However, any change in the radius of the contractile skin resulting from root water potential could not be observed experimentally in this study. This study does not intend to mathematically illustrate the micro-physical behaviour of the pore structure of a soil and the aforementioned concept has been developed to explain the observed results, so any numerical model outputs are beyond the scope of this study.

5.8 Summary

The strength of soil in vegetated ground is increased by the suction generated during plant processes, e.g. evapotranspiration. The soil water characteristic curve which was developed for a particular soil is important in the quantification of suction, especially matric suction, but in a vegetated environment close to the root zone, using the moisture contents to predict suction via a conventional soil water characteristic curve is unreliable. This is because the root water potential can generate additional potential energy that can alter the hydraulic state of soil and hence the free water available to generate suction is less than the values measured by the moisture sensor. A laboratory experiment in a controlled environment and a field test were used to capture true behaviour, from which the results are reported. Figure 5.9 shows the linear relationship between the root water potential and variation of suction. This effect decreases further away from the root zone, a result seen in the laboratory and field
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experiments. To explain this effect, a concept was developed using micro soil physics and the availability of water in the soil.
Chapter 6 Numerical simulation on practical applications

6.1 General

The effect of root reinforcement and suction on the shear strength of soil has been proven mathematically and experimentally in previous chapters. Chapter three illustrated the experimental observations and chapter four explained the theoretical development used to support the experimental observations. A MATLAB simulation has also been carried out to obtain the numerical results related to the theoretical development, and the accuracy of the theoretical model has been verified as stated in chapter four. The practical application of this model is explained in this chapter using finite element modelling (FEM analysis) and the data obtained by the MATLAB
simulation which is relevant to the developed model. The effect of an increase in shear strength by tree roots was simulated using PLAXIS 2D 2015 (VIP) software.

6.2 Finite element modelling (FEM) in geotechnical engineering

The FEM method is used to predict unforeseen events which could be encountered in geotechnical engineering projects and to avoid the time required to solve calculations in complex theoretical models. FEM generally provides an approximate solution to a governing mathematical equation by solving a series of algebraic equations; it does this to obtain the response of individual parts which are created by dividing the area considered into finite elements; the accuracy of the approximated solution can then be controlled by the allowable error in iterations. To compare the behaviour of unreinforced ground and root permeated ground the settlements underneath the railway ballest related to the practical applications are used. Safety factors are used for practical applications.

6.2.1 High- order elements used in PLAXIS 2D

As stated in previous section PLAXIS 2D 2015 (VIP) was used in this study because it is a widely used finite element program which can be utilised in geotechnical engineering designs. PLAXIS 2D has been developed to analyse the deformation stability and ground water flow in a two dimensional plane because most geotechnical problems require time dependant, non- linear advanced constitute modelling. PLAXIS 2D can also handle hydrostatic and non- hydrostatic pore pressures and multi- phase materials (PLAXIS manual 2015).

Geometrical inputs can also be done using a graphical interface based on CAD drawing procedures. Here the output results are calculated according to a serviceability limit state calculation which is compatible with Eurocode 7 or LFRD. Bore holes are used
to define the soil layers and multiple bore holes can be used to define non-horizontal layers. PLAXIS can generate mesh automatically, but it can be controlled according to user requirements (i.e., ‘coarse’ to ‘very fine’ options). This generated mesh divides the defined geometry into a finite number of elements and then an analysis and output are given according to the number of nodes in the elements. Quadratic 6 nodes and 4\(^{th}\) order 15 node triangular elements are used PLAXIS 2D, and then output results such as deformation and pore pressures are given for nodes in these elements. Figure 6.1 shows the Gaussian distribution of these nodes in triangular elements.

![Diagram showing 15-node and 6-node triangular elements](Figure 6.1)

Values such as deformations and stresses are continuous over the boundaries of the elements, so a polynomial interpolation is used to obtain the values within the elements of the mesh created. The number of nodes in the element determines the order of the polynomial equation such that the higher the number of nodes in a finite element mesh, the more accurate the results, but this does increase the time taken for analysis. The 15 node elements have 12 integration points for the outputs and 6 node elements have 3
integration points. Six nodes elements are used in this study to consider the two dimensional behaviour of the problem.

The material model used in PLAXIS is based on the stress strain relationship of the input material, and it is defined by the set of mathematical equations related to the stress strain rates. Plane strain analysis was used in this study considering that the strain can only takes place in x-y direction. Root system was reduced to 2 dimensional system considering the stiffness variation along the tree line, because the analysis time increases considerably in 3 dimensional system than 2 dimensional system. The output of the 2D system was checked over the 3D system and there were no any noticeable variation.

6.3 Development of application model

Settlement was compared at the most critical point where the middle of the ballast layer sits between the vegetated and non-vegetated sections at different suction values; the results are then used to predict the effectiveness of trees planted alongside railway ballast. Furthermore, the method outlined in this chapter can be used to determine the distance from the tree to the toe of railway ballast needed to achieve the settlement values desired for future designs.

6.3.1 Material models used in the analysis

This chapter explains the FEM modelling which could be applicable to the computation of initial settlement values of the system due to trees alongside the rail corridor. As for FEM modelling of embankments, only half the system is modelled due to symmetry. Three main material components and loading condition could be identified as important in the FEM model, as shown in Figure 6.2. The Mohr-Coulomb
and soft soil models were used for the soil layer and to define the unsaturated flow properties, user defined Van Genuchten parameters related to the Van Genuchten (1980) model were used. The hardening soil model with small strain, which was available in Pl, was used for the integrated soil-root system.

6.3.2 Hardening soil model: Small strain

Two main types of hardening have been considered in PLAXIS analysis; shear hardening and compression hardening. The shear hardening model is used in this study because the large number of experimental observations showed that the shear hardening can be occurred in the root permeated system. The hardening model in...
PLAXIS can predict the behaviour of soft and stiff soil (Schanz et al 1999). The parameters used in this study are $E_{\text{oed}}$ which is the tangential stiffness of the oedometer test, and $E_{50}$ which is the secant modulus defined by $\sigma_3$, which can be measured in a triaxial test. This test includes the elastic behaviour, plasticity due to shear hardening, and plasticity due to compression hardening. The Van Genuchten (1980) model was used to define the flow parameters. Ballast is defined as a linear elastic material considering the load transfer behaviour, onto which a 30 ton axle load was applied according to the related area of the model (considering the distance between the axles as 1.435m and 2.5m between them).

6.3.3 Geometrical inputs, boundary and conditions in the model

The layers of soil in unreinforced analysis were defined using only one bore hole (Figure 6.2a), but three holes were used to define the triangular root zone, as shown in Figure 6.2b. Three boundaries were found in the defined geometry (Bc1, Bc2 and Bc3). Bc1 was set as a symmetrical boundary for deformation, and a ‘closed’ boundary for ground water flow because there is no flow through the symmetrical line. The deformation boundaries for Bc2 were fixed for vertical movement and free for lateral movement; Bc3 was fixed for lateral movement and free for vertical movements, because the Bc2 and Bc3 boundary lines were set far away (15m away from the toe of the ballets) so as not to get a boundary effect. Ground water flow boundaries for Bc2 and Bc3 were set as ‘seepage’ to allow for any possible ground water movement due to settlement.
6.3.4 Initial check on the SH small soil model.

PLAXIS can run material tests to check the behaviour of model used. Therefore the DSS (Direct simple shear test) available on PLAXIS 2D 2015 was run for input material with the soil hardening HS model for the large direct shear test results, of 50kPa suction and 20kPa applied normal stress test which were shown in Chapter 3.

Table 6.1 shows the input data for the model. All the parameters were calculated using the experimental results obtained from the large direct shear tests. Figure 6.3 shows the experimental direct shear results and the results from the model predictions, according to selected inputs.

<table>
<thead>
<tr>
<th>Input type</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deformation Model</td>
<td>HS small</td>
</tr>
<tr>
<td>Ysat</td>
<td>18kN/m3</td>
</tr>
<tr>
<td>E50ref</td>
<td>20MPa</td>
</tr>
<tr>
<td>Eoed</td>
<td>20MPa</td>
</tr>
<tr>
<td>C'</td>
<td>5</td>
</tr>
<tr>
<td>Suction</td>
<td>50kPa</td>
</tr>
<tr>
<td>Normal load</td>
<td>20kPa</td>
</tr>
<tr>
<td>ϕ</td>
<td>28</td>
</tr>
<tr>
<td>G0ref</td>
<td>15MPa</td>
</tr>
<tr>
<td>Ground water model</td>
<td>Vangnuchten</td>
</tr>
<tr>
<td>Sres</td>
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</tr>
<tr>
<td>Ssat</td>
<td>1</td>
</tr>
<tr>
<td>1/m</td>
<td>3.8</td>
</tr>
<tr>
<td>ψunsat</td>
<td>5.09m</td>
</tr>
</tbody>
</table>
In Figure 6.3 experimental results are in good agreement with the PLAXIS model predicted results. Therefore, it can be verified that HS small model in PLAXIS (2015) can be used effectively in this simulation.

![Figure 6.3. HS small material model check with PLAXIS Direct simple shear.](image)

**6.4 Simulation of a practical application**

**6.4.1 Description of the application**

The mathematical model explained in Chapter 4 can predict the increase in shear strength due to native vegetation, which is relevant to the properties of the root system and soil, as well as the hydraulic properties of the system. Therefore if these properties for any railway system are known, then the predictions can be made using FEM analysis with PLAXIS 2D. Furthermore, to design a railway system with a green corridor, this model and the FEM analysis can be used effectively, as described in the next section. The data obtained from Potter (2006) and Fatahi’s (2007) publications for the green corridor along railway line in Miram Australia, have been used in this FEM analysis. The type of tree available at this site is Black box (Eucalyptus Largiflorens), and the geometry of the site is shown in Figure 6.4.
Chapter 6  
Numerical simulation on practical applications.

Vegetation at this site was only available for one side; however in this study it was assumed there are two parallel lines of trees along the rail corridor. The soil classification according to the USCS were silty sand to the depth of 3m and silty clay to the depth of 15m. The variations of moisture in vegetated and non-vegetated ground are shown in Figure 6.5.

![Schematic diagram of the Miram site in Victoria (after Potter 2006).](image)

Vegetation at this site was only available for one side; however in this study it was assumed there are two parallel lines of trees along the rail corridor. The soil classification according to the USCS were silty sand to the depth of 3m and silty clay to the depth of 15m. The variations of moisture in vegetated and non-vegetated ground are shown in Figure 6.5.

![Suction Variation In vegetated and non-vegetated section of the Miram Victoria site (after Potter 2006).](image)

Figure 6.4. Schematic diagram of the Miram site in Victoria (after Potter 2006).

Figure 6.5. Suction Variation In vegetated and non-vegetated section of the Miram Victoria site (after Potter 2006).
The root system was modelled according to the data for the Black box tree (Eucalyptus Largiflorens) available in Fatahi (2007), which was available close to the same site. Fatahi (2007) excavated a trench to map the root system and Table 6.2 shows the root system data used for the FEM analysis. The ensile strength of the roots was obtained from the study carried out by Cheng et al (2012).

Fine mesh was used to obtain more accurate results and very fine mesh was generated at the root zone and at the toe of the ballast layer, as shown in Figure 6.6. Boundary Bc2 was 15m away from the toe of the ballast and Bc3 was 15 m deep to avoid any possible boundary effects. This was confirmed with the stress distribution shown in Figure 6.7.

### Table 6.2: Root system data

<table>
<thead>
<tr>
<th>Diameter (m)</th>
<th>Youngs Modulus kN/m²/mm</th>
<th>Length (m)</th>
<th>Tensile Strength (kN/m²)</th>
<th>Orientation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.025</td>
<td>10000</td>
<td>1</td>
<td>10000</td>
<td>0 30</td>
</tr>
<tr>
<td>0.03</td>
<td>10000</td>
<td>0.8</td>
<td>10000</td>
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<td>0.25</td>
<td>10100</td>
<td>0.6</td>
<td>10000</td>
<td>40 60</td>
</tr>
<tr>
<td>0.03</td>
<td>10000</td>
<td>0.65</td>
<td>10000</td>
<td>60 30</td>
</tr>
<tr>
<td>0.03</td>
<td>10000</td>
<td>0.75</td>
<td>10000</td>
<td>80 45</td>
</tr>
<tr>
<td>0.03</td>
<td>10000</td>
<td>0.8</td>
<td>10000</td>
<td>100 60</td>
</tr>
<tr>
<td>0.025</td>
<td>10100</td>
<td>1</td>
<td>10000</td>
<td>120 30</td>
</tr>
<tr>
<td>0.03</td>
<td>10000</td>
<td>0.8</td>
<td>10000</td>
<td>140 45</td>
</tr>
<tr>
<td>0.025</td>
<td>10000</td>
<td>0.75</td>
<td>10000</td>
<td>160 60</td>
</tr>
<tr>
<td>0.03</td>
<td>10000</td>
<td>0.65</td>
<td>10000</td>
<td>180 30</td>
</tr>
<tr>
<td>0.025</td>
<td>10100</td>
<td>0.6</td>
<td>10000</td>
<td>200 45</td>
</tr>
<tr>
<td>0.025</td>
<td>10000</td>
<td>0.75</td>
<td>10000</td>
<td>220 60</td>
</tr>
<tr>
<td>0.02</td>
<td>10100</td>
<td>0.8</td>
<td>10000</td>
<td>240 30</td>
</tr>
<tr>
<td>0.03</td>
<td>10000</td>
<td>1</td>
<td>10000</td>
<td>260 45</td>
</tr>
</tbody>
</table>
Figure 6.6. Graphical representation of generated mesh.
Figure 6.7. Distribution of the pressure bulb underneath the rail track.
6.5 FEM results and discussion

Figure 6.8 shows the settlement results obtained at the Miram site in the vegetated and non-vegetated sections. Maximum settlement occurs at the centre of the ballast, which is marked as ‘X’; settlement at X in non-vegetated ground is 36mm and 23mm in vegetated ground. Both settlements are acceptable with regards to the serviceability of the rail track, and since this reduction is more than 50%, it is a considerable reduction in the settlement due to the presence of tree roots. Figures 6.9 (a) and 6.9(b) show the distribution of the displacement vector of the non-vegetated and vegetated ground.
Figure 6.8. Settlement Distribution of the railway line: (a) non-vegetated and (b) vegetated.
Figure 6.9. Displacement vector (a) non-vegetated ground and (b) vegetated ground (shows less displacement).
Figure 6.10 shows the possible variations of initial settlement with the subgrade suction; this range of suction varied from 1000kPa to 3500kPa, which was the seasonal variation at a depth of 1m reported at the Miram site where the maximum penetration of most roots systems was from 1m to 1.5m deep. The initial settlement of vegetated and non-vegetated ground increased with suction because the shear strength increased, as shown in Table 6.2. However, it tends to decrease at a certain suction because the increment in the shear strength of the soil decreases.
Chapter 6  Application to case history

6.6 Summary

Finite element modelling is commonly used in geotechnical engineering designs to avoid rigorous calculations and time consumption. This chapter explains the simplified usage of the results obtained from the mathematical model and MATLAB simulation in PLAXIS 2D, a commonly used, user friendly numerical modelling software. The field results at the Miram site of Victoria obtained by Potter (2006) and Fatahi (2007) were used to perform a more realistic simulation. section 6.3 explains the development of the application model and the material used in this simulation. A simple direct shear test was simulated to verify the usability of the material model chosen for the root permeated section and input parameters; the results are shown in section 6.3.4. Figure 6.8 shows the variation of settlement due to root permeation, while Figure 6.10 shows the variations of settlement with the initial suction values.
Chapter 7 Conclusions and recommendations

7.1 General summary

This study aimed at developing a mathematical-analytical model which can predict improvements in the shear strength of soil due to the mechanical strengthening and hydraulic behaviour of vegetated ground. The Abstract is followed by Chapter 1 which introduced and described the nature of this study. Chapter 2 was a literature review which presented a comprehensive and insightful explanation of previous studies related to this study. Most of the previous predictive models found in literature focused either on the mechanical behaviour of roots or their hydraulic behaviour. Past experimental works have been conducted using substitutes for natural roots such as wood anchors and polymer fibres because conducting experiments and developing models for naturally grown roots are time-consuming and complex. Chapter 3 described the experiments and observations carried out on naturally grown roots to
Chapter 7

Conclusion and recommendations

capture the true behaviour of a root system during shearing. Chapter 4 educated a mathematical model developed considering the experimental results observed during laboratory experiments, and the MATLAB simulation to conduct a rigorous analysis. Chapter 5 included the results of field experiments which were analysed to capture variations in the moisture and suction in vegetated ground, and Chapter 6 described the finite element simulation of a practical application using the commercially available software, PLAXIS 2D.

7.2 Specific observations and outcomes

7.2.1 Identified components of the increase in the shear strength of root permeated soil, during direct shear testing

Three main components were identified with respect to the increased shear strength of naturally grown roots during direct shear tests using a large shear box. In this analysis, the results of thirty successful tests on vegetated and non-vegetated specimens were carried out at five different levels of suction and three applied stresses for each level of suction. The following three components related to root permeated soil were identified and presented in a modified Mohr-Coulomb analysis.

\[ \Delta \tau_R = \text{increase in shear strength due to the effect of root reinforcement only (in a saturated condition)}, \]

\[ \Delta \tau_C = \text{increase in shear strength due to an integrated root-suction system, and} \]

\[ \Delta \tau_S = \text{increase in shear strength due to increased in soil suction from tree transpiration}. \]
Chapter 7

Conclusion and recommendations

The influence of $\Delta \tau_C$ on increasing the shear strength of soil was proven using the experimental results which had not been captured in previous studies.

7.2.2 Development of mathematical analytical results over the experimental observations

The following three root failure methods were observed during the laboratory tests and then analysed both qualitatively and quantitatively; they are as follows:

- Roots fail under tension (broken roots),
- Roots completely pulled out (pure slipping) and
- Roots pulled out with a soil lump.

A mathematical expression for the increase in shear strength of root permeated soil was developed to consider these failure patterns; a MATLAB simulation carried out to obtain results related to the root system was also evaluated quantitatively via the direct shear experiments. The experimental results and the results obtained by the MATLAB simulation were in good agreement and therefore this mathematical model could be considered as an acceptable method for predicting the increase in soil shear strength due to root permeation. However, this model needs a large number of parameters related to the root type and properties and their failure modes, which is unavoidable in analysing systems with naturally grown roots. Figure 4.10 in Chapter 4 provided the flow chart showing the calculation process and the associated parameters.

The root system must be accurately defined with the orientation, diameter, and Young’s modulus. This means exercising due care if this system is extended to study
other root systems. Fortunately, correlations available in the botanical research could be used to evaluate most of the relevant parameters. Some of the other data which are not commonly used, such as the thickness of the shear zone and the bond coefficient between root-soil have been used in this model, and therefore methodologies for estimating them have been explained in this study.

7.2.3 Analysis of variations in suction and moisture in vegetated ground using field results

Concurrent measurements of soil suction and moisture content were obtained over a period of ten months, along with the root water potential values in a field site situated on the premises of the University of Wollongong. It was observed that the soil suction measured near the root zone differs from the suction measured away from the root zone for the same moisture condition. The suction measured away from the root zone is almost equal to the values obtained from the soil water characteristic curve while the values measured within the root zone show variations. Therefore, it can be concluded that the soil suction near the root zone deviates from the value related to the equivalent soil moisture content presented in SWCC.

This variation from the equivalent values obtained using the SWCC was strongly correlated with the distance from the root zone. Furthermore, a linear relationship between the difference of the two suction values (i.e. SWCC deviation) and the root water potential values was observed. This relationship was explained using the micro-physical behaviour of contractile skin, as expounded by Fredlund et al (2012). However, within the scope of study it was not possible to measure variations in the radius of contractile skin.
Chapter 7 Conclusion and recommendations

7.2.4 Finite Element modelling with PLAXIS

The PLAXIS finite element package (PLAXIS 2D 2015) was used to simulate the vegetated and non-vegetated sections of the rail corridor in Miram, Victoria, Australia. The field data available from Potter (2005) and Fatahi (2007) related to the Miram field site was used for the PLAXIS simulation. The developed mathematical model was verified using the MATLAB simulation described in Chapter 4. The improved shear strength parameters were used in the root permeated section of the simulation.

- Mohr-Coulomb and soft soil models were used in the non-vegetated soil sections and “Hardening small strain” model (HS small in PLAXIS) was used for the root permeated section with the values of stiffness (\(E_{oed}, E_{ref}\)) calculated according to the stiffness variation pattern observed during the experiments.

- To verify the accuracy of the usage of this Hardening soil small strain model (HS small in PLAXIS) in the root permeated section, the direct shear test simulation available in PLAXIS was used for the parameters related to the laboratory direct shear test, and then the experimental results and the model predictions were compared. They were in good agreement, as described in Chapter 6.

- To simulate vegetated ground in a plane strain condition it was assumed that the trees were in close proximity to each other and the stiffness parameters were spread equally over the third dimension (along the rail corridor) as explained in Chapter 6, because a 3D simulation increases the time of analysis considerably and the output results do not change noticeably in 3D simulation compared to the equivalent 2D simulation with modified parameters.

- The initial settlement values were compared in vegetated and non-vegetated ground considering suction, from which it was noted that the initial
settlement values in vegetated ground were less than the non-vegetated ground; this trend continued with descending values over the increase in soil suction.

### 7.3 Recommendations for future work

- Root failure patterns were observed from the direct shear experiments with small root systems; it is therefore suggested that the failure methods of relatively large trees planted in the field should be observed during shearing.

- The stiffness values were extrapolated using the experimental values and the root area ratio of the large root system; it is therefore suggested that a method to predict the stiffness values according to the spatial distribution of the large root systems should be developed.

- The finite element simulation was verified at the beginning of the simulation using the direct shear results, however there were no data sets available to check the variations of initial settlement values over the variations of suction in vegetated and non-vegetated ground; it is therefore suggested that a large scale field experiment be carried out with a rail load applied, that would allow the monitoring of initial settlement values at different levels of suction.

- The relationship between the monitored soil suction, moisture content and the root water potential could not be proved or properly established with a sound mathematical model. Therefore, a conceptual theory using the micro-physics related to the contractile skin was introduced. It is strongly suggested that a comprehensive study be carried out with a proven experimental result (i.e. measuring the variations of contractile skin with applied root water potential) to capture the relationship between the root water potential of the tree and variations of the measured soil suction.
References

REFERANCES


References


References


Fatahi, B. (2007). Modelling of Influence of Matric suction induced by Native vegetation on sub-soil Improvement, University of Wollongong.


References


References


References


**References**


References


References


References


References


References


Appendices

APPENDIX 1

MAT LAB CODES FOR THE THEORATICAL MODEL
function tensile(handles)

global c N q y d M1 Mr Ms T epath P

inp=load(epath);

% tensile strength of root T
T=pi*inp(:,1).^2/4.*inp(:,9);
a=size(T);

% Moved length of the root Lm
Lm=inp(:,8);

% tensile force generated (kN) at any root Ts
Ts=inp(:,2).*inp(:,8);

% Addhesive Force (F1) calculation (kN)
c=5; %Cohesion of soil
N=10; %Normal load kN/m2
n=a(1); %number of roots
q=pi()/6; %Soil friction angle Rad
k0=(1-sin(q))*ones(n,1);
b=pi*inp(:,10)/180; % inclination of the root to horizontal rad
y=0.22; % coefficient of adhesion.
d=18; %soil density kN/m3
P=50; %Matric suction kN/m2
M1=0.20; % Moisture content at test
Mr=0.03; % residual moisture content
Ms=0.6; % saturated moisture content
h=inp(:,11);
F1=c*ones(n,1)+((N*cos(b)+ k0*N+d*h).*sin(b).*y+(P*ones(n,1)*y*(M1-Mr)/(Ms-Mr))-(N*sin(b).*cos(b))-(k0*N+d*h).*sin(b));

for i=1:a(1)
    if T(i)>Ts(i)
        s1(i)=0;
    elseif T(i)<=F1(i);

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\[ s1(i)=1; \]
\[ S2(i)=0; \]

else
\[ S2(i)=1; \]

end

end

s1
S2

Leff=0.03; \text{ %effective length of the soil annulus} \\
Deff=0.01; \text{ %effective diameter of the soil annulus} \\
n2=3; \text{ % number of soil annulus observed} \\
Z=\pi*30/180 \text{ %Coefficient of} \\
Rsoil=c*\text{ones} (n,1)+((N*cos(b)+k0*N+d*h).*\text{sin}(b))*\tan Z+(P*\text{ones} (n,1)*y*(M1-Mr)/(M2-Mr))- \(N*\text{sin}(b).*\text{cos}(b))-\text{(k0*N+d*h)}. \text{sin}(b); \text{ %addhesive strength of soil} \\
Aadd=\pi*Leff*Deff*Deff*Rsoil \\

function varargout = ROOT(varargin) 
% ROOT MATLAB code for ROOT.fig 
% ROOT, by itself, creates a new ROOT or raises the existing singleton*. 
% H = ROOT returns the handle to a new ROOT or the handle to the existing singleton*. 
% ROOT('CALLBACK', hObject, eventData, handles,...) calls the local function named CALLBACK in ROOT.M with the given input arguments. 
% ROOT('Property','Value',...) creates a new ROOT or raises the
Appendices

% existing singleton*. Starting from the left, property value pairs
% are
% applied to the GUI before ROOT_OpeningFcn gets called. An
% unrecognized property name or invalid value makes property application
% stop. All inputs are passed to ROOT_OpeningFcn via varargin.
% *See GUI Options on GUIDE's Tools menu. Choose "GUI allows only one
% instance to run (singleton)".
%
% See also: GUIDE, GUIDATA, GUIHANDLES

% Edit the above text to modify the response to help ROOT

% Last Modified by GUIDE v2.5 11-Sep-2016 11:36:38

% Begin initialization code - DO NOT EDIT

gui_Singleton = 1;

gui_State = struct('gui_Name',       mfilename, ...
                     'gui_Singleton',  gui_Singleton, ...
                     'gui_OpeningFcn', @ROOT_OpeningFcn, ...
                     'gui_OutputFcn',  @ROOT_OutputFcn, ...
                     'gui_LayoutFcn',  [] , ...
                     'gui_Callback',   []);

if nargin && ischar(varargin{1})
    gui_State.gui_Callback = str2func(varargin{1});
end

if nargout
    [varargout{1:nargout}] = gui_mainfcn(gui_State, varargin{:});

else
    gui_mainfcn(gui_State, varargin{:});
end

% End initialization code - DO NOT EDIT

% --- Executes just before ROOT is made visible.
function ROOT_OpeningFcn(hObject, eventdata, handles, varargin)
% This function has no output args, see OutputFcn.
% hObject    handle to figure
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    structure with handles and user data (see GUIDATA)
% varargin   command line arguments to ROOT (see VARARGIN)

% Choose default command line output for ROOT
handles.output = hObject;

% Update handles structure
guidata(hObject, handles);

% UIWAIT makes ROOT wait for user response (see UIRESUME)
% uiwait(handles.figure1);

% --- Outputs from this function are returned to the command line.
function varargout = ROOT_OutputFcn(hObject, eventdata, handles)
% varargout    cell array for returning output args (see VARARGOUT);
% hObject      handle to figure
% eventdata    reserved - to be defined in a future version of MATLAB
% varargout    cell array for returning output args (see VARARGOUT)
Appendices

% handles structure with handles and user data (see GUIDATA)

% Get default command line output from handles structure
varargout{1} = handles.output;

function c_Callback(hObject, eventdata, handles)
% hObject handle to c (see GCBO)
% eventdata reserved - to be defined in a future version of MATLAB
% handles structure with handles and user data (see GUIDATA)

% Hints: get(hObject,'String') returns contents of c as text
%       str2double(get(hObject,'String')) returns contents of c as a double

% --- Executes during object creation, after setting all properties.
function c_CreateFcn(hObject, eventdata, handles)
% hObject handle to c (see GCBO)
% eventdata reserved - to be defined in a future version of MATLAB
% handles empty - handles not created until after all CreateFcns called

% Hint: edit controls usually have a white background on Windows.
%       See ISPC and COMPUTER.
if ispc && isequal(get(hObject,'BackgroundColor'), get(0,'defaultUicontrolBackgroundColor'))
    set(hObject,'BackgroundColor','white');
end
function N_Callback(hObject, eventdata, handles)
% hObject    handle to N (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    structure with handles and user data (see GUIDATA)

% Hints: get(hObject,'String') returns contents of N as text
% str2double(get(hObject,'String')) returns contents of N as a double

% --- Executes during object creation, after setting all properties.
function N_CreateFcn(hObject, eventdata, handles)
% hObject    handle to N (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    empty - handles not created until after all CreateFcns called

% Hint: edit controls usually have a white background on Windows.
% See ISPC and COMPUTER.
if ispc && isequal(get(hObject,'BackgroundColor'),
    get(0,'defaultUicontrolBackgroundColor'))
    set(hObject,'BackgroundColor','white');
end

function q_Callback(hObject, eventdata, handles)
% hObject    handle to q (see GCBO)
Appendices

% eventdata reserved - to be defined in a future version of MATLAB
% handles structure with handles and user data (see GUIDATA)

% Hints: get(hObject,'String') returns contents of q as text
%        str2double(get(hObject,'String')) returns contents of q as a double

% --- Executes during object creation, after setting all properties.
function q_CreateFcn(hObject, eventdata, handles)
% hObject handle to q (see GCBO)
% eventdata reserved - to be defined in a future version of MATLAB
% handles empty - handles not created until after all CreateFcns called

% Hint: edit controls usually have a white background on Windows.
% See ISPC and COMPUTER.
if ispc && isequal(get(hObject,'BackgroundColor'),
    get(0,'defaultUicontrolBackgroundColor'))
    set(hObject,'BackgroundColor','white');
end

function y_Callback(hObject, eventdata, handles)
% hObject handle to y (see GCBO)
% eventdata reserved - to be defined in a future version of MATLAB
% handles structure with handles and user data (see GUIDATA)

% Hints: get(hObject,'String') returns contents of y as text
%        str2double(get(hObject,'String')) returns contents of y as a double

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% --- Executes during object creation, after setting all properties.
function y_CreateFcn(hObject, eventdata, handles)
% hObject    handle to y (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    empty - handles not created until after all CreateFcns called

% Hint: edit controls usually have a white background on Windows.
% See ISPC and COMPUTER.
if ispc && isequal(get(hObject,'BackgroundColor'),
                  get(0,'defaultUicontrolBackgroundColor'))
    set(hObject,'BackgroundColor','white');
end

function d_Callback(hObject, eventdata, handles)
% hObject    handle to d (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    structure with handles and user data (see GUIDATA)

% Hints: get(hObject,'String') returns contents of d as text
% str2double(get(hObject,'String')) returns contents of d as a double

% --- Executes during object creation, after setting all properties.
function d_CreateFcn(hObject, eventdata, handles)
Appendices

% hObject    handle to d (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    empty - handles not created until after all CreateFcns called

% Hint: edit controls usually have a white background on Windows.
%       See ISPC and COMPUTER.
if ispc && isequal(get(hObject,'BackgroundColor'),
                get(0,'defaultUicontrolBackgroundColor'))
    set(hObject,'BackgroundColor','white');
end

function P_Callback(hObject, eventdata, handles)
% hObject    handle to P (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    structure with handles and user data (see GUIDATA)

% Hints: get(hObject,'String') returns contents of P as text
%        str2double(get(hObject,'String')) returns contents of P as a double

% --- Executes during object creation, after setting all properties.
function P_CreateFcn(hObject, eventdata, handles)
% hObject    handle to P (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    empty - handles not created until after all CreateFcns called

% Hint: edit controls usually have a white background on Windows.
Appendices

% See ISPC and COMPUTER.

if ispc && isequal(get(hObject,'BackgroundColor'),
    get(0,'defaultUicontrolBackgroundColor'))
    set(hObject,'BackgroundColor','white');
end

function M1_Callback(hObject, eventdata, handles)

% hObject handle to M1 (see GCBO)
% eventdata reserved - to be defined in a future version of MATLAB
% handles structure with handles and user data (see GUIDATA)

% Hints: get(hObject,'String') returns contents of M1 as text
%        str2double(get(hObject,'String')) returns contents of M1 as a double

% --- Executes during object creation, after setting all properties.
function M1_CreateFcn(hObject, eventdata, handles)

% hObject handle to M1 (see GCBO)
% eventdata reserved - to be defined in a future version of MATLAB
% handles empty - handles not created until after all CreateFcns called

% Hint: edit controls usually have a white background on Windows.
% See ISPC and COMPUTER.
if ispc && isequal(get(hObject,'BackgroundColor'),
    get(0,'defaultUicontrolBackgroundColor'))
    set(hObject,'BackgroundColor','white');
end
Appendices

function Mr_Callback(hObject, eventdata, handles)

% hObject handle to Mr (see GCBO)
% eventdata reserved - to be defined in a future version of MATLAB
% handles structure with handles and user data (see GUIDATA)

% Hints: get(hObject,'String') returns contents of Mr as text
% str2double(get(hObject,'String')) returns contents of Mr as a double

% --- Executes during object creation, after setting all properties.
function Mr_CreateFcn(hObject, eventdata, handles)

% hObject handle to Mr (see GCBO)
% eventdata reserved - to be defined in a future version of MATLAB
% handles empty - handles not created until after all CreateFcns called

% Hint: edit controls usually have a white background on Windows.
% See ISPC and COMPUTER.
if ispc && isequal(get(hObject,'BackgroundColor'),
    get(0,'defaultUicontrolBackgroundColor'))
    set(hObject,'BackgroundColor','white');
end

function Ms_Callback(hObject, eventdata, handles)
Appendices

% hObject    handle to Ms (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    structure with handles and user data (see GUIDATA)

% Hints: get(hObject,'String') returns contents of Ms as text
%        str2double(get(hObject,'String')) returns contents of Ms as a double

% --- Executes during object creation, after setting all properties.
function Ms_CreateFcn(hObject, eventdata, handles)
% hObject    handle to Ms (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    empty - handles not created until after all CreateFcns called

% Hint: edit controls usually have a white background on Windows.
%       See ISPC and COMPUTER.
if ispc && isequal(get(hObject,'BackgroundColor'),
    get(0,'defaultUicontrolBackgroundColor'))
    set(hObject,'BackgroundColor','white');
end

function inputmat_Callback(hObject, eventdata, handles)
% hObject    handle to inputmat (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    structure with handles and user data (see GUIDATA)

% Hints: get(hObject,'String') returns contents of inputmat as text
Appendices

% str2double(get(hObject,'String')) returns contents of inputmat as a double

% --- Executes during object creation, after setting all properties.
function inputmat_CreateFcn(hObject, eventdata, handles)
    % hObject    handle to inputmat (see GCBO)
    % eventdata  reserved - to be defined in a future version of MATLAB
    % handles    empty - handles not created until after all CreateFcns called

% Hint: edit controls usually have a white background on Windows.
% See ISPC and COMPUTER.
if ispc && isequal(get(hObject,'BackgroundColor'),
    get(0,'defaultUicontrolBackgroundColor'))
    set(hObject,'BackgroundColor','white');
end

% --- Executes on button press in pushbutton1.
function pushbutton1_Callback(hObject, eventdata, handles)
    % hObject    handle to pushbutton1 (see GCBO)
    % eventdata  reserved - to be defined in a future version of MATLAB
    % handles    structure with handles and user data (see GUIDATA)
    global epath
    [afile,apath,~]=uigetfile({'*.dat','DAT-files (*.mat)';'*.mat','MAT-files (*.mat)';},'Select root data file','e.dat');
    epath=strcat(apath,afile);
    set(handles.inputmat,'String',strcat(apath,afile));
Appendices

% --- Executes on button press in pushbutton2.
function pushbutton2_Callback(hObject, eventdata, handles)
% hObject    handle to pushbutton2 (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    structure with handles and user data (see GUIDATA)
global c N q y d M1 Mr Ms P
texts=findobj('Style','edit');
[n,~]=size(texts);
for i=1:n
    tt=get(texts(i),'String');
    if isempty(tt)
        messagebox('Some fields are empty')
        return
    end
end
c=str2num(get(handles.c,'String'));
N=str2num(get(handles.N,'String'));
q=degtorad(str2num(get(handles.q,'String')));
y=str2num(get(handles.y,'String'));
d=str2num(get(handles.d,'String'));
M1=str2num(get(handles.M1,'String'));
Mr=str2num(get(handles.Mr,'String'));
Ms=str2num(get(handles.Ms,'String'));
P=str2num(get(handles.P,'String'));
tensileF(handles)
% -------------------------------------------------------------------
function mfile_Callback(hObject, eventdata, handles)
% hObject    handle to mfile (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    structure with handles and user data (see GUIDATA)

% function mexit_Callback(hObject, eventdata, handles)
% hObject    handle to mexit (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    structure with handles and user data (see GUIDATA)
chk=questdlg('Do you want to exit the programme?','Information','Yes','No','Yes');
if strcmp(chk,'Yes')
    close all
end
APPENDIX 2

FIELD SUCTION, MOISTURE AND WATER POTENTIAL DATA
Horizontal variation of suction, moisture and water potential in the Field.
Comparison of the field test result of point 2 with the guiding point
Vertical variation of suction, moisture and water potential in the field.