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Control of servo systems in the presence of motor-load inertia mismatch

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Abstract

There are a number of performance limiting factors that concern the designers of machine tool servo systems. One such factor is the mismatch that often exists between motor and load inertias. This paper briefly discusses the results of a previous study on the factors that interact to introduce stability problems in the presence of a high motor-load inertia mismatch. The effects of such an inertia mismatch, on servo system performance, are then discussed and analysed using closed loop frequency responses. Various methods of improving the system response using fast feedback control are presented, including traditional PI control and modern methods incorporating full order state observation. The effectiveness of these control methods are then compared after comprehensive testing on a specially constructed experimental rig.

Keywords

closed loop systems, feedback, frequency response, machine control, machine tools, observers, servomotors, stability, two-term control

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Control of Servo Systems in the Presence of Motor-Load Inertia Mismatch

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Abstract–**There are a number of performance limiting factors that concern the designers of machine tool servo systems. One such factor is the mismatch that often exists between motor and load inertias. This paper briefly discusses the results of a previous study on the factors that interact to introduce stability problems in the presence of a high motor-load inertia mismatch. The effects of such an inertia mismatch, on servo system performance, are then discussed and analysed using closed loop frequency responses. Various methods of improving the system response using fast feedback control are presented, including traditional PI control and modern methods incorporating full order state observation. The effectiveness of these control methods are then compared after comprehensive testing on a specially constructed experimental rig.**

I. INTRODUCTION

As a result of the tightening of machined product tolerances, there is constant and increasing pressure on industrial machine tool manufacturers to improve the accuracies of their machines. This push for increased machine tool accuracy is also often coupled with a push for higher response speeds. Although there have been advances in machine tool technology over recent years, a number of well known factors continue to limit the accuracy and closed loop bandwidth of machine tool servo systems. Two such factors are backlash and the inertia mismatch that often exists between the motor and load of a system. Whilst not losing sight of the important role backlash continues to play in machine tool design and performance, this paper particularly focuses on investigating the effects of motor-load inertia mismatch.

From the perspective of a control systems designer, a low motor inertia is often desirable as the resultant high torque to inertia ratio produces faster accelerations and subsequently, a better transient response. However, there is an interaction between the motor to load inertia ratio, flexing of the motor-load coupling and positioning of the feedback elements, which can introduce a torsional vibration. This vibration results in underdamping and even instability.

While various "rules of thumb" are widely used in industry, to define acceptable ratios of motor-load inertia, the precise reasons behind the associated stability problems are often unclear to system designers and the accepted solution for a load with high inertia is the costly one of using a larger motor. For instance, Baldor (a servo motor manufacturer) state in their product catalogue that inertia matching is a critical motor selection criteria and although motor-load inertia ratios of higher than 1:5 may be possible, they are not recommended because of possible stability problems [1]. Another issue for machine tool servo system designers is that such "rules of thumb" can vary between manufacturers, resulting in more confusion [2].

This paper briefly discusses the results of a previous study [3] that analysed the interacting factors causing such stability problems, through the development and investigation of accurate theoretical servo system models. The effects of inertia mismatch on servo system performance are then quantified and analysed using the closed loop frequency response method. Throughout this analysis a number of control solutions are compared and subsequently tested on a specially constructed experimental rig. The importance of load response information in the feedback loop is discussed, and control solutions using both direct load feedback and load estimation are included in the presented comparison. Limitations of the specially constructed experimental rig are also discussed.

II. SERVO SYSTEM MODELLING

A. *A Traditional Model*

Designers of closed loop servo systems traditionally treat motor and load inertias as a single 'lumped' inertia. Along with this, the feedback element is usually attached to the motor. In treating a system in this manner there is an inherent assumption that the coupling device connecting motor and load is infinitely stiff. However, in reality, all devices used to form the motor-load coupling are 'elastic' in nature, that is they have finite stiffness. Under these conditions the load response is not identical to that of the motor [4]. It is also worth noting that some authors [2] have used an infinitely stiff coupling when discussing why motor and load inertias should be 'matched'. However, it should be clear that an infinitely stiff coupling results in one single inertia and as such, an inertia mismatch cannot exist. For these reasons, the accurate analysis of a system under motor-load inertia mismatch requires a model incorporating an 'elastic' coupling device.

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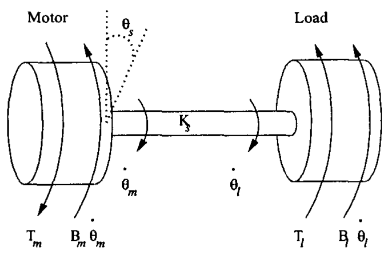

Fig. 1. Diagram of Simplified Motor-Load System

B. A Simple 'Elastic' Model

For the purpose of simplifying the analysis, the model used consists only of a motor and load connected directly by a single flexible shaft. Through minimising other factors that would normally affect performance, this simplification allows the analysis to concentrate on the effects of motor-load inertia ratios. For instance, the small number of components minimises friction terms. As no gearbox is involved, the backlash can be assumed to be negligible. Also, the connecting shaft dimensions can be chosen so that the shaft inertia is negligible (motor and load inertias dominate) and any shaft flexing can be assumed to occur over the length of the connecting shaft. It is also assumed that flexing of this shaft can be specified by a single flex angle and as such, more complex vibrational modes are negligible. A diagram of the simplified system is shown in Fig. 1.

In Fig. 1:

- T_m is the torque produced by the motor,
- T_l is a load-side disturbance torque,
- B_m and B_l are the viscous friction coefficients of the motor and load,
- K_s is the torsional stiffness of the shaft,
- $\dot{\theta}_m$ and $\dot{\theta}_l$ are the radial velocities of the motor and load shafts, and
- θ_s is the shaft flex (ie $\theta_s = \theta_m - \theta_l$).

Defining J_m and J_l as the motor and load inertias respectively, the equations of motion for the system in Fig. 1 are:

$$J_m\ddot{\theta}_m = T_m - B_m\dot{\theta}_m - K_s\theta_s \tag{1}$$

$$J_l\ddot{\theta}_l = K_s\theta_s - T_l - B_l\dot{\theta}_l \tag{2}$$

$$\theta_s = \theta_m - \theta_l \tag{3}$$

Using (1), (2) and (3), transfer functions representing the motor and load velocity responses were determined. The transfer functions given by (4) and (5) result, with (4) describing the motor response and (5) describing the load response.

$$\frac{\frac{1}{J_m}\left[s^2 + \frac{B_l}{J_l}s + \frac{K_s}{J_l}\right]}{s^3 + \left[\frac{B_l}{J_l} + \frac{B_m}{J_m}\right]s^2 + \left[\frac{K_s}{J_l} + \frac{B_mB_l}{J_mJ_l} + \frac{K_s}{J_m}\right]s + \left[\frac{K_s(B_m+B_l)}{J_mJ_l}\right]} \tag{4}$$

$$\frac{\frac{K_s}{J_mJ_l}}{s^3 + \left[\frac{B_l}{J_l} + \frac{B_m}{J_m}\right]s^2 + \left[\frac{K_s}{J_l} + \frac{B_mB_l}{J_mJ_l} + \frac{K_s}{J_m}\right]s + \left[\frac{K_s(B_m+B_l)}{J_mJ_l}\right]} \tag{5}$$

III. Development of Effective Control Solutions

A. A Brief Stability Analysis

The first step in analysing stability problems in an elastic system with high motor-load inertia mismatch is to examine flexing of the motor-load coupling [3]. Although this flexing is directly responsible for torque delivery to the load it is also directly associated with the oscillations present in motor and load responses.

Using (3), (4) and (5) the simplified transfer function given by (6) was determined to represent flex of the motor-load coupling. In determining this transfer function it was assumed that the viscous friction coefficients of the motor and load were negligible.

$$\frac{\theta_s}{T_m} = \frac{\frac{s^2}{J_m}}{s^4 + \left(\frac{K_s}{J_l} + \frac{K_s}{J_m}\right)s^2} \tag{6}$$

Applying a unit step torque and taking the inverse Laplace transform of (6), results in a description of flex as a function of time. Also if J_l is substituted with I_rJ_m (where I_r is inertia ratio $\frac{J_l}{J_m}$), this time function can be described in terms of inertia ratio:

$$\theta_s(t) = \frac{T_{step}I_r}{K_s(1+I_r)} - \frac{T_{step}I_r}{K_s(1+I_r)}\cos\left(\sqrt{\frac{K_s(1+I_r)}{J_mI_r}}t\right). \tag{7}$$

From (7) it can be seen that increasing the torsional stiffness of the coupling also increases the frequency of oscillation in the response, while decreasing the amplitude of this oscillation. It is also worth noting that when $K_S = \infty$, flex reduces to zero (as expected). Examining the influence of inertia ratio shows that small increases above unity will result in a slight decrease in the frequency, along with a slight increase in amplitude of the oscillation. However, as the inertia ratio becomes larger it tends to cancel and have less effect on both the frequency and amplitude of this oscillation. What is evident is that torsional stiffness of the coupling has a much greater influence on flexing than does inertia ratio.

A complete understanding of the influence of inertia ratio on system stability requires an examination of the closed loop system. As mentioned in Section II-A, machine tool servo system designers usually use an infinitely stiff model with motor feedback. It should immediately be clear from (4) and (5) that there are differences between the motor and load velocity responses. As such, there are limitations associated with using a single feedback element at the motor when load velocity is the variable of interest. However, absolute stability problems arise when relying solely on load feedback [5].

For a more in-depth analysis on the interacting factors that introduce stability problems in the presence of a high motor-load inertia mismatch, the reader is referred to [3]. In this analysis it was found that stability problems are related to a band of 90 degree phase lead in the motor velocity frequency response (open loop). The position of this band, and consequently the level of stability when closing the loop, was found to be largely dependent on the torsional stiffness of the motor-load coupling. Although a reduction in the inertia mismatch between motor and load (usually achieved through using a motor with higher inertia) resulted in a reduction in bandwidth of the 90 degree phase lead, this was found to be a less effective solution when compared with choosing a more torsionally stiff coupling.

B. Closed Loop Frequency Response Analysis

To further illustrate the stability problems associated with inertia mismatch in elastic servo systems, and aid in the development of effective control solutions, closed loop frequency responses of the simplified elastic system were studied.

In order to effectively simulate both the time and frequency responses of the simplified elastic model, the mathematics package 'Matlab' was used. With the addition of 'Simulink' a block diagram approach to modelling and simulation was employed, allowing the servo drive current loop characteristics and various disturbances to be easily included in any simulations performed. Closed loop frequency responses were generated by taking fast Fourier transforms of the system transient response to an impulse. The system model, for all of the simulations presented throughout this section, consisted of a shaft with low torsional stiffness (K_S = 23.8 Nm/rad), a motor inertia of 0.00039 kgm^2, and an inertia ratio of 1:1. An example of the 'Simulink' model used is given in Fig. 2, where the system is controlled by a traditional PI (Proportional + Integral) controller.

B.1 PI Control

An initial closed loop frequency response was generated for a low gain traditional PI controller. The resulting frequency response, with load velocity as output, is shown in Fig. 3. As can be seen, the bandwidth of this system is relatively low (approx 6 Hz); however, there is a resonance at approximately 55 Hz. As expected, the frequency of this resonance is equivalent to the frequency of oscillation described by the shaft flex function

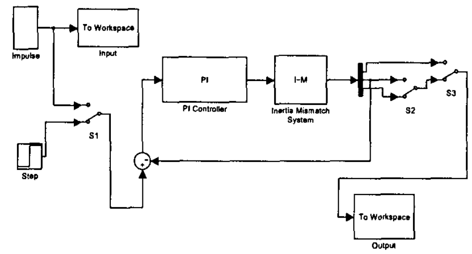

Fig. 2. Simulink Block Diagram - PI Controller

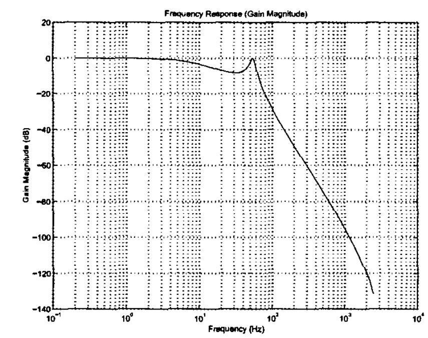

Fig. 3. System Frequency Response - Low Gain PI Controller

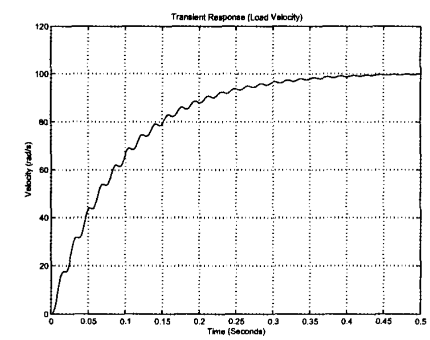

Fig. 4. Transient Response - Low Gain PI Controller

of (7). As such, this resonance will occur at higher frequencies as shaft torsional stiffness is increased. Small increases in inertia ratio (above unity) will result in the resonance occurring at lower frequencies.

The transient response of the low gain PI controller is shown in Fig. 4. As expected, a slow transient response results from the low bandwidth and a small oscillation is present at the resonant frequency.

In an attempt to achieve a faster transient response, the gains of the PI controller were increased. The resulting frequency response is shown in Fig. 5. As expected, although the higher

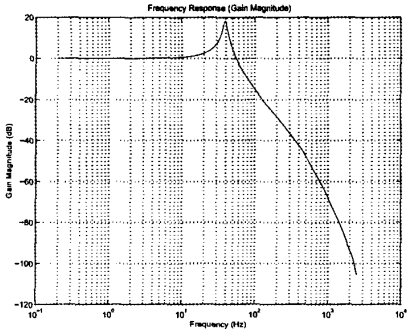

Fig. 5. System Frequency Response - High Gain PI Controller

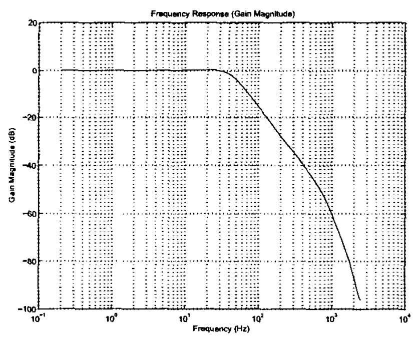

Fig. 6. System Frequency Response - Shaft Flex Feedback

gains resulted in a higher bandwidth, the magnitude of the resonance was also increased. It was found that it was not possible to concurrently achieve both an acceptable bandwidth and low resonant peak with standard PI control. Although it would be possible to reduce the resonant peak with notch filters, this is not an ideal solution.

B.2 Higher Order Controllers

It would appear from the results of the PI controller that an acceptable transient response, with low amplitude of oscillation, can only be achieved through careful choice of shaft stiffness for the particular inertia mismatch. A much better solution though, would be to use a higher order controller that takes flexing of the shaft into account. One possible solution is a pole placement (PP) controller that uses both motor and load feedback to control the flexing of the shaft. Another simpler solution is to add an extra feedback loop to the standard PI controller, so that shaft flex is taken into account (again using both motor and load feedback elements).

Closed loop frequency responses for both the pole placement and improved PI controllers were generated with identical results. Fig. 6 represents the frequency response of the

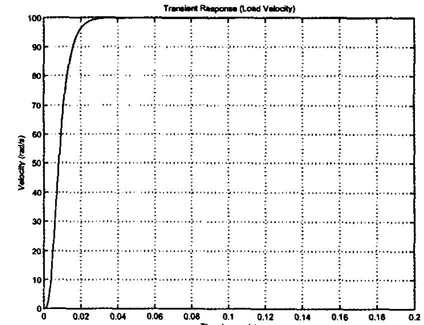

Fig. 7. Transient Response - Shaft Flex Feedback

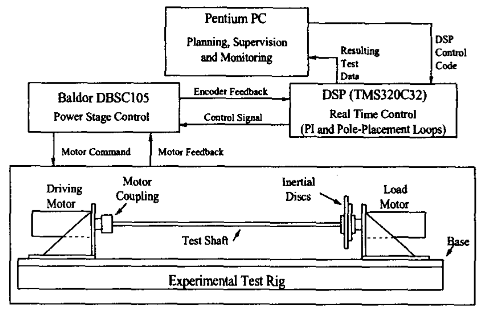

Fig. 8. Diagram of Test Rig Hardware

improved PI controller (with an additional feedback loop representing shaft flex). As can be seen the high bandwidth of Fig. 5 has been maintained, with a dramatic reduction in the resonant peak. Fig. 7 shows the transient response of the improved PI controller. As expected, the transient is both faster and smoother when compared with the transient response of the standard PI controller shown in Fig. 4 (the increased speed is particularly evidenced by the different time scales used).

IV. Experimental Results

A. Overview

The actual performance of servo systems with varying inertia ratios and shaft stiffness was verified through application to a specially built experimental rig. This rig was constructed to match the simplified motor-load model described in Section II-B, and shown in Fig. 1. The basic layout of this test rig is illustrated in Fig. 8, along with a block diagram of the applied control scheme. The load motor is used for applying constant load torques. A Digital Signal Processor is used for real-time control of the motors and a PC for monitoring and overall supervision. A more detailed description of this experimental rig is given in [6].

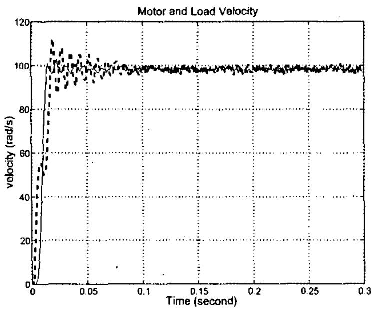

Fig. 9. Test Rig Response - PI Control (K_S = 23.8 Nm/rad, I_r = 1)

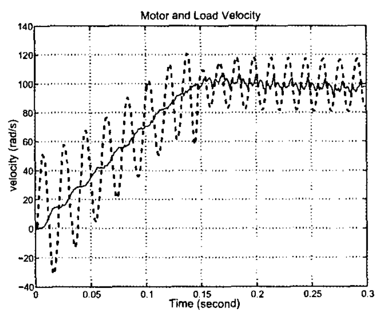

Fig. 10. Test Rig Response - PI Control (K_S = 23.8 Nm/rad, I_r = 15)

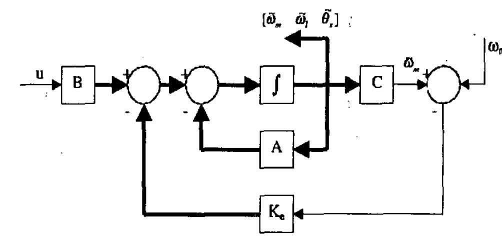

Fig. 11. Block Diagram of Full Order State Observer

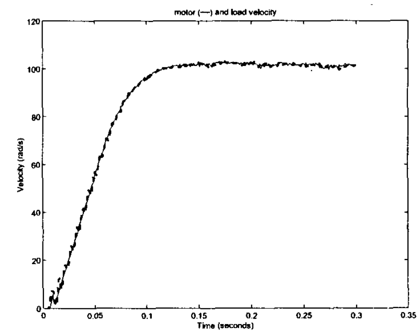

Fig. 12. Test Rig Response - PP Control with Observer (I_r = 15)

B. Results

B.1 PI Control

The transient shown in Fig. 9 represents the response of the experimental system (K_S = 23.8 Nm/rad and I_r =1) to a step input. A standard PI controller was used with motor velocity as the only feedback. The response shown in Fig. 9 is the best response achieved on the experimental rig after tuning both the proportional and integral gains. Although quite a fast response was attained, there are significant oscillations present in both the motor (dashed line) and load (solid line) responses. This result is consistent with the closed loop frequency analysis of PI controllers presented in Section III-B.1.

Fig. 10 shows the response of the experimental rig with the same shaft, but an increased inertia ratio of 1:15. Again a tuned PI controller was used with motor velocity as the only feedback. As expected, the response is slower when compared with the 1:1 case of Fig. 9. Also, the oscillations have become larger (particularly in the motor velocity).

B.2 Higher Order Controllers

As mentioned in Section III-B.2, the higher order controllers discussed require some form of load feedback. It is acknowledged that such load feedback is not always possible in industrial situations. For this reason two different pole placement controllers were implemented, and compared, on the experimental rig. The first controller used direct load feedback, while the second controller used a full order state observer to estimate the load information.

A block diagram of a full order state observer is shown in Fig. 11. This observer estimates system states (ω_m, ω_l and θ_s) using a model of the system (A, B and C), the system input (u) and the standard output (motor velocity ω_m). The difference between the motor velocity and estimated motor velocity is multiplied by a gain ($K_e(\omega_m - \tilde{\omega}_m)$) and used as feedback to provide convergence of the estimated states ($\tilde{\omega}_m$, $\tilde{\omega}_l$, $\tilde{\theta}_s$) to the actual states (ω_m, ω_l, θ_s). These estimated states are then used for controller feedback.

The gains used in both the pole-placement controller and the full order state observer were determined using the well known Ackermann's formula. To ensure that the estimator response was quicker than the actual controller response, the observer poles were chosen to be 5 times faster than the slowest poles of the pole placement controller. Using this design, the performance of the pole placement controller with estimated feedback was found to be equal to that of the standard pole placement controller.

The transient shown in Fig. 12 represents the response of the experimental system using a pole placement controller and full order state observer. Once again the system consisted of a shaft with low torsional stiffness (K_S = 23.8 Nm/rad) and an inertia ratio of 1:15. When compared with the transient response of the standard PI controller, shown in Fig. 10, it can be seen that the pole placement controller is both faster and smoother. Also, the motor and load velocities have almost identical responses. This result is consistent with the closed loop frequency analysis presented in Section III-B.2.

V. OUTCOMES

As mentioned in Section I, there is a common thought in industry that high load inertias should be avoided in order to limit vibrational stability problems in servo systems. This thought often results in machine tool designers choosing larger intertia motors. Although the choice of a larger inertia motor does often improve performance, it is clear from the results shown in Sections III and IV that the main performance improvement is a result of the larger motor having a larger diameter and therefore stiffer shaft. A more cost effective solution would be to place more emphasis on increasing stiffness of the motor-load coupling, rather than using a larger motor.

Another possible solution is to use a control strategy that takes flexing of the motor-load coupling into account. There are a number of simple higher order control schemes that result in fast, smooth transients even in the presence of very high inertia mismatches and couplings of low torsional stiffness. Although these control schemes require load feedback, which is not always practical, they also perform very well when the load response is estimated. Load response estimation is possible when an accurate mathematical model of the system, that is completely state observable, can be determined. In many situations this would be a much cheaper and more effective solution than over-engineering the mechanical components of the machine tool servo system.

VI. CONCLUSIONS AND FUTURE RESEARCH

A brief discussion of the work presented in [3] was given in Section III-A. This work included an analysis of the interacting factors that introduce stability problems in the presence of a high motor-load inertia mismatch. The analysis was based on the development of an accurate servo system model, simplified in order to concentrate on the direct effects of motor-load inertia ratios in elastic two mass systems. It was found that when a unit step torque is applied in open loop, there is a frequency of oscillation in the motor-load coupling. While both torsional stiffness of the coupling and motor-load inertia ratios were found to affect the frequency and amplitude of this oscillation, torsional stiffness of the coupling was found to have a much greater influence. In particular, with increased torsional stiffness of the motor-load coupling the frequency of oscillation increased, while the amplitude decreased.

The results of this analysis were then verified using closed loop frequency responses, with a number of control solutions also analysed. These control solutions were then compared through both simulation and application to a specially constructed experimental rig. It was found that oscillations were exhibited in systems with low torsional stiffness, even when the motor-load inertia ratio was 1:1. These oscillations could not be adequately controlled using a standard PI controller. Higher order controllers were found to provide far superior performance, which did not appear to degrade when state estimation techniques were used. Although a number of vibration suppression methods have previously been proposed for two mass systems, the comparison presented showed that good performance can be achieved using simple conventional methods as long as flexing of the coupling is taken into account.

It is recognised that there are limitations with the simple model and experimental rig used for this research. For instance, friction terms have been minimised and backlash totally eliminated. As such, the model does not completely describe a real machine tool servo system. It is also acknowledged that accurate mathematical models may be difficult to determine for more complicated industrial systems. For these reasons, current research is focused on extending this model, along with developing self-tuning and system identification algorithms for real industrial machine tool servo systems.

VII. ACKNOWLEDGMENT

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