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Abstract

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Study of micro flexible rolling based on grained inhomogeneity

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Abstract: This paper shows an analytical, numerical and experimental investigation to comprehend the role of grained inhomogeneity which plays in micro flexible rolling in terms of the average rolling force and the thickness directional springback of the workpiece after it exits the roll bite zone. Miniature tensile tests and micro hardness tests are accomplished to identify the scattered stress-strain curves for 500 μm thick aluminium alloy 1060 samples with grain size of approximately 23-71 μm and to determine the weighted heterogeneity coefficient for each sample separately, according to which the theoretical calculations and numerical simulations based upon 3D Voronoi tessellation technique have been performed under actual experimental conditions where reductions of 25 to 50 % are selected. The scattering effect associated with the anisotropic nature of single grains has been perceived in the micro flexible rolling process and both the analytical and finite element models developed have been validated via experimental data to hold promise for predicting the rolling force and the thickness directional springback of the workpiece, as well as boosting the thickness profile control performance of the micro flexible rolling mill.

Keywords: Micro flexible rolling; Grained inhomogeneity; Rolling force; Thickness directional springback; Weighted heterogeneity coefficient; 3D Voronoi tessellation

1. Introduction

Rolling process, which reduces the thickness as well as enhances material properties of the metal by passing it between rolls, has been developed worldwide over the past decades mostly by means of analytical and numerical methods [1, 2]. One relatively common approach to describe the stress state in the roll bite is to deploy the two-dimensional differential equation derived by von Karman under condition of equilibrium [3], for which a bundle of solutions exist, depending upon the assumptions adopted by the individuals, and whereof Tselikov [4] proposed a typical one with Coulomb's law of friction employed and the arc of contact substituted for its chord. Orowan [5] presented a graphic-analytical model to calculate the rolling pressure taking into account the variation of friction coefficient along the arc of contact, in the light of which Bland and Ford [6] and Sims [7] obtained their respective models via various simplifications. Stone [8] established a mathematical model for cold rolling of thin strip, including the effects of friction, strip tension and roll flattening. Several semi-empirical formulas were constructed by Ekelund [9] and Misaka and Shida [10] to compute the rolling force in hot rolling mills in terms of material and geometrical parameters. More recently Johnson and Smelser [11] applied the asymptotic method to incorporate shear effects in plane-strain

rolling into earlier models to improve the forecast accuracy of rolling force, whose work had been extended by Domanti and McElwain [12] through scalings to a special case of the friction causing the roll pressure to seriously exceed the flow stress in the roll gap. Fleck et al. [13] treated the rolls as elastic half-spaces in their modelling of cold rolling of thin strip and related the distribution of roll pressure in the no-slip zone to the elastic roll deformation by a conversion equation. Le and Sutcliffe [14] later added a new element to the modelling of the neutral zone involved in the work by Fleck et al. [13], and wherein they combined the flow continuity for the strip with the change in roll shape to link the pressure gradient and shear stress with the roll slope. Larkiola et al. [15] integrated the neural network theory into a physical model to obtain better agreement between predicted and measured rolling force so as to indirectly optimise the capacity and efficiency of a tandem cold rolling mill.

Nowadays, finite element method, a power numerical technique to provide more intuitive and precise solutions to complex scientific and engineering problems, is increasingly being used to simulate and investigate the rolling process. Jiang et al. [16, 17] employed this approach to describe the strip rolling process considering the friction variation in the roll bite, during which the numerical calculations, such as rolling force and rolling torque, showed a good agreement with the experimental results. A rigid-plastic finite element code combined with a realistic friction model was developed by Hsu et al. [18] for analysis of lubricant flow and determination of hydrodynamic friction stress within the billet-roll interface, which had given accurate estimates of the rolling force, rolling torque as well as the outlet velocity ratio during lubricated cold rolling process. Shahani et al. [19] evaluated the influence of process parameters inclusive of initial slab thickness, rolling speed, thickness reduction and friction coefficient on the rolling force, temperature history, strain history and contact pressure distribution in hot rolling of aluminium slab by coupled thermo-viscoplastic finite element analysis. Devarajan et al. [20] similarly described a 2D elastic-plastic finite element model to investigate the effect of roll speed and roll diameter on the contact pressure and residual stress in cold rolling of steel plate. Liu et al. [21] focused their analysis on the metal flow in slab rolling based upon a three-dimensional elastic-plastic finite element method and the comparison of forecasted metal flow with experimental data, including the profile evolution of the workpiece, grid distortion and velocity fields, exhibited a very good agreement over a wide range of workpiece width/thickness ratios and reductions.

Over recent years, an innovative technique named flexible rolling has been devised by Kopp et al. [22, 23] to keep pace with the increasing demand for lightweight construction, of particular significance to the automotive industry. In this process, the roll gap is selectively altered during the pass to produce a workpiece with variable thickness along its length to allow an exact adaptation to the respective loads under which it is to be placed [24]. Liu et al. [25, 26] performed parametric studies of the influences of reduction, friction coefficient and blank horizontal velocity on the rolling force, forward slip and final thickness profile utilising the nonlinear transient dynamic explicit type of finite element analysis in order to manipulate process parameters in actual flexible rolling. Current research in flexible rolling technology deals predominantly with thick metals; nonetheless, these laws may differ when the thickness of the material reaches from the millimeter down to the micrometer range, the other two dimensions whereof are to be reduced accordingly, because the effect of each grain rises to prominence and plays a more critical role in the overall material deformation behaviour.

This phenomenon, known as size effect, is being investigated for various metal forming processes at micro scale. Joo et al. [27] designed and assembled a micro punching press to make micro-holes on brass and stainless steel foils during which shear-dominated hole wall and relatively large burr were formed as the grain number got decreased along the foil thickness. Lee et al. [28] conducted micro deep drawing experiments with 304 stainless steel foils and the results revealed that both the blank holder force and the limit drawing ratio increased with the increasing ratio of foil thickness to the average grain size (T/D ratio). Chen and Jiang [29] discussed the material behaviour of thin iron sheets in micro V-bending; they concluded that the maximum punch force decreased with the decrease in the T/D ratio whereas the springback amount decreased with the increase of T/D ratio, and the spring-forward was detected when the T/D ratio fell below 2. Rosochowski et al. [30]

executed a set of backward cup extrusion experiments using both the coarse-grained and the ultrafine-grained aluminium billets, in which more uniform material flow, better shape capability and smaller surface discontinuities on the edge of the cup were observed for the ultrafine-grained materials. Shimizu et al. [31] carried out the micro coining tests on the pure copper sheets with different average grain sizes, the results whereof provided evidence that the coarse-grained material helped increase the maximum transfer height, suggested a smaller coining load for the same transfer height and received higher assistance from the ultrasonic vibration on this forming process to create a smoother surface.

With respect to the finite element modelling for materials with size effect, Voronoi tessellation is a prevailing technique exploited to construct a polycrystalline microstructure comprising grains that have different properties equipped with them. Cao et al. [32, 33], Jiang et al. [34] and Luo et al. [35, 36] produced numerical studies on the micro extrusion, micro hydroforming, micro cross wedge rolling, micro deep drawing and micro hydro deep drawing processes of polycrystalline materials generated by the Voronoi algorithm, respectively; all these results showcased that the mechanical properties and formability of the materials as well as the final product quality had strong affiliations with the properties of individual grains such as its size, morphology and orientation. In the wake of the uneven distribution of heterogeneous grains within these materials, the scatter effect of their general flow curves has been recognised and modelled with a normal bell-shaped distribution [37]. A scattering effect of this kind was also examined and found to be greater at larger grain size and smaller workpiece dimension by Miyazaki et al. [38, 39], Gau et al. [40] and Diehl et al. [41].

Additionally, in most cases of metal forming, the products are unlikely to be manufactured exactly to the specifications like size, shape, etc., as the deformed regions are prone to a certain amount of elastic recovery after release of the loading. Material behaviour of this type is termed springback, e.g., a slight decrease in the bending angle after the V-bending process, a slight increase in the flange angle after the deep drawing process, etc., which are affected by the factors such as process parameters, material properties and tooling geometries and have a chance of being compensated by overbending the material [29, 35, 42-45]. Whilst for the strip rolling process springback may refer to the elastic recovery in the thickness after the strip exits the roll bite, and it is likely to be rectified by dint of increasing the rolling speed, adding tensions to the strip, decreasing the roll gap for a supplementary amount of reduction, and so forth [46-50]. In this context, analytical techniques are introduced firstly to construct mathematical models respecting the rolling force and the thickness directional springback of the workpiece in micro flexible rolling, integrating the influential role played by inhomogeneous grains; micro hardness tests are conducted to determine the weighted heterogeneity coefficients for the 500 μm thick aluminium alloy 1060 strips with various average grain sizes, which reflect the inhomogeneous nature of the grains in each specimen separately. Secondly, laboratory-scale micro flexible rolling experiments are implemented with reductions of 25 to 50 %; quantitative analysis is performed in regard to the variation of average rolling force and thickness directional springback in relation to the average grain size, and whereof the scatter effect is qualitatively evaluated in respect of average grain size. Finally, miniature tensile tests are executed to achieve the scattering stress-strain curves for samples of grain size of approximately 23-71 μm , which are subsequently assigned to the workpiece created on the basis of 3D Voronoi tessellation technique so as to carry out the finite element analysis of micro flexible rolling with consideration of grained heterogeneity; both analytical and numerical approaches developed to model the micro flexible rolling process are validated via quantitative comparison with experimental results, which in turn help enhance the thickness profile control performance of the micro flexible rolling mill.

2. Mathematical models of rolling force and thickness directional springback in micro flexible rolling

2.1 Von Karman unit pressure differential equation with consideration of grained inhomogeneity

In the development of mathematical models of rolling force for micro flexible rolling process, von Karman unit pressure differential equation has been used to first establish the pressure distribution along the arc of contact based on the assumptions that (1) stress is uniformly distributed over the height of the cross section of the rolled workpiece with a sufficiently small discarded lateral spreading in the deformation zone, (2) friction coefficient along the arc of contact remains constant, (3) a uniform constrained yield stress of the material occurs along the arc of contact and (4) the work rolls do not sustain any elastic deformation [51-54].

During the micro flexible rolling process, forces are considered to act on a vertical element $abcd$ with a thickness dx in the deformation area, as shown in Figure 1.

Figure 1. Forces and stresses acting on a vertical element of the workpiece in the micro flexible rolling process: a) rolling phase with thicker thickness; b) rolling phase with thinner thickness (for the deformation area below arc AC)

Suppose the workpiece is of unit width and arc ab is approximately equal to chord ab . Then the horizontal component of these forces is

$$2p_x \frac{dx}{\cos(\theta + \varphi)} \sin(\theta + \varphi) \pm 2\tau_x \frac{dx}{\cos(\theta + \varphi)} \cos(\theta + \varphi) \quad (1)$$

where p_x is the radial pressure and τ_x is the tangential pressure acting on the face ab of the element, θ is the tilt angle for thickness transition area and φ is an arbitrary angle within the deformation zone; sign “+” refers to the backward slip zone in which friction force acts in the direction of rolling and sign “-” refers to the forward slip zone in which friction force acts opposite to the direction of rolling [55].

According to assumption (1) compressive stresses produced by horizontal forces are designated $\sigma_x + d\sigma_x$ on the face ad of height $h_x + dh_x$ and σ_x on the face bc of height h_x . Solving the horizontal forces on the element of the workpiece in equilibrium yields

$$(\sigma_x + d\sigma_x)(h_x + dh_x) - \sigma_x h_x - 2p_x \frac{dx}{\cos(\theta + \varphi)} \sin(\theta + \varphi)$$

$$\pm 2\tau_x \frac{dx}{\cos(\theta + \varphi)} \cos(\theta + \varphi) = 0 \quad (2)$$

Letting $2y = h_x$ and $2dy = dh_x$, and substituting $\tan(\theta + \varphi) = \frac{dy}{dx}$ into Equation (2), simplifying and neglecting $d\sigma_x dy$ gives

$$\frac{d\sigma_x}{dx} - \frac{p_x - \sigma_x}{y} \cdot \frac{dy}{dx} \pm \frac{\tau_x}{y} = 0 \quad (3)$$

where sign “+” relates to the backward slip zone and sign “-” to the forward slip zone.

Adopting the von Mises yield criterion expressed as

$$\sigma_y - \sigma_x = 1.15\sigma_s = K \quad (4)$$

where σ_s is the yield stress and K is the constrained yield stress, and considering that the radial pressure p_x equals the normal stress σ_y , Equation (3) is rewritten as

$$\frac{dp_x}{dx} - \frac{K}{y} \cdot \frac{dy}{dx} \pm \frac{\tau_x}{y} = 0 \quad (5)$$

As a result of the scaling down of workpiece dimension, the grained inhomogeneity is no longer negligible and may indirectly cause scattering of the constrained yield stress [56]. Here, a weighted heterogeneity coefficient $\bar{\xi}$ is introduced to reflect the regulatory effect of grained inhomogeneity on constrained yield stress, which further modifies Equation (5) to

$$\frac{dp_x}{dx} - \frac{\bar{\xi}K}{y} \cdot \frac{dy}{dx} \pm \frac{\tau_x}{y} = 0 \quad (6)$$

2.2 Mathematical modelling of rolling force

Utilising Tselikov’s method a solution to Equation (6) can be obtained under conditions that (1) friction between the rolls and the workpiece obeys Coulomb’s law formulated as

$$\tau_x = \mu p_x \quad (7)$$

(2) no forward and backward tensions are applied on the workpiece and K keeps the same value at entry and exit points of roll bite, and (3) the arc of contact is approximated by linear segments [50, 57].

2.2.1 Rolling force model for rolling phase with thicker thickness

The equation of the line passing through points A and B , as displayed in Figure 2, is assumed to be

$$y = ax + b \quad (8)$$

Figure 2. Linear approximation of the arc of contact in the rolling phase with thicker thickness

With $x = 0$, $y = a \times 0 + b = b = \frac{h}{2}$ at point B and $x = l$, $y = al + \frac{h}{2} = \frac{H}{2}$ at point A , and then $a = \frac{H-h}{2l} = \frac{\Delta h}{2l}$, Equation (8) can be reworked into

$$y = \frac{\Delta h}{2l}x + \frac{h}{2} \quad (9)$$

Differentiation then yields

$$dx = \frac{2l}{\Delta h} dy \quad (10)$$

Substituting Equations (7) and (10) into Equation (6) and rearranging, the following can be derived

$$\frac{dp_x}{\pm \delta p_x - \bar{\xi} K} = -\frac{dy}{y} \quad (11)$$

where $\delta = \frac{2l\mu}{\Delta h}$. Upon integration, this equation produces

$$\frac{1}{\pm \delta} \ln(\pm \delta p_x - \bar{\xi} K) = \ln \frac{1}{y} + C \quad (12)$$

With the aid of the boundary conditions

$$\sigma_x = 0, p_H = \bar{\xi} K \text{ at } y = \frac{H}{2} \quad (13)$$

$$\sigma_x = 0, p_h = \bar{\xi} K \text{ at } y = \frac{h}{2} \quad (14)$$

the integration constant C in Equation (12) can be fixed

for the backward slip zone

$$C_H = \frac{1}{\delta} \ln(\delta \bar{\xi} K - \bar{\xi} K) - \ln \frac{2}{H} \quad (15)$$

and for the forward slip zone

$$C_h = \frac{1}{-\delta} \ln(-\delta \bar{\xi} K - \bar{\xi} K) - \ln \frac{2}{h} \quad (16)$$

By successively substituting Equations (15) and (16) into Equation (12) and letting $h_x/2 = y$ the unit pressure can be calculated as

for the backward slip zone

$$p_H = \frac{\bar{\xi} K}{\delta} \left[(\delta - 1) \left(\frac{H}{h_x} \right)^\delta + 1 \right] \quad (17)$$

and for the forward slip zone

$$p_h = \frac{\bar{\xi} K}{\delta} \left[(\delta + 1) \left(\frac{h_x}{h} \right)^\delta - 1 \right] \quad (18)$$

The rolling force in the rolling phase with thicker thickness can therefore be computed as

$$p = \frac{B+b}{2} \int_0^l p_x dx = \frac{B+b}{2} \times \frac{l}{\Delta h} \times \frac{\bar{\xi} K}{\delta} \left\{ \int_{h_r}^H [(\delta - 1) \left(\frac{H}{h_x} \right)^\delta + 1] dh_x + \int_h^{h_r} [(\delta + 1) \left(\frac{h_x}{h} \right)^\delta - 1] dh_x \right\} \quad (19)$$

where B and b are the workpiece widths before and after rolling respectively and h_r is the thickness of the workpiece at the neutral plane. Integrating and simplifying Equation (19) gives

$$p = \frac{B+b}{2} \times \frac{l}{\Delta h} \times \frac{\bar{\xi} K}{\delta} \times h_r \left[\left(\frac{H}{h_r} \right)^\delta + \left(\frac{h_r}{h} \right)^\delta - 2 \right] \quad (20)$$

With $p_H = p_h$ at $h_x = h_r$, it expresses $\left(\frac{H}{h_r} \right)^\delta$ by the equation

$$\left(\frac{H}{h_r} \right)^\delta = \frac{1}{\delta-1} [(\delta + 1) \left(\frac{h_r}{h} \right)^\delta - 2] \quad (21)$$

Then Equation (20) can be written in the final form

$$p = (B + b) \frac{lh}{\Delta h(\delta-1)} \bar{\xi} K \left(\frac{h_r}{h} \right) \left[\left(\frac{h_r}{h} \right)^\delta - 1 \right] \quad (22)$$

2.2.2 Rolling force model for rolling phase with thinner thickness

As depicted in Figure 3, the arc of contact in the rolling phase with thinner thickness is represented by two line segments BC and AC .

Figure 3. Linear approximation of the arc of contact in the rolling phase with thinner thickness

For the forward slip zone below line BC , the unit pressure differential equation can be specified as

$$\frac{dp_x}{dx} - \frac{\bar{\xi}K}{y} \cdot \frac{dy}{dx} + \frac{\mu p_x}{y} = 0 \quad (23)$$

and the equation of the line BC is presumed to be

$$y = cx + d \quad (24)$$

At point B , using $x = 0$ and $y = \frac{h}{2}$ gives $d = \frac{h}{2}$; at point C , using $x = R \sin \theta$ and $y = \frac{h_o}{2} = \frac{h}{2} - R(1 - \cos \theta)$ gives $c = \frac{\cos \theta - 1}{\sin \theta}$, where R is the roll radius. Equation (24) is then recast as

$$y = \frac{\cos \theta - 1}{\sin \theta} x + \frac{h}{2} \quad (25)$$

Differentiation subsequently produces

$$dx = \frac{\sin \theta}{\cos \theta - 1} dy \quad (26)$$

Substituting Equations (26) into Equation (23) and rearranging finds that

$$\frac{dp_x}{\delta_1 p_x - \bar{\xi}K} = -\frac{dy}{y} \quad (27)$$

where $\delta_1 = \frac{\mu \sin \theta}{\cos \theta - 1}$. After integration this equation becomes

$$\frac{1}{\delta_1} \ln(\delta_1 p_x - \bar{\xi}K) = \ln \frac{1}{y} + C \quad (28)$$

From boundary condition

$$\sigma_x = 0, p_{h_1} = \bar{\xi}K \text{ at } y = \frac{h}{2} \quad (29)$$

the integration constant C in Equation (28) is determined

$$C = \frac{1}{\delta_1} \ln(\delta_1 \bar{\xi} K - \bar{\xi} K) - \ln \frac{2}{h} \quad (30)$$

And the unit pressure for the forward slip zone below line BC can be achieved by substituting Equation (30) into Equation (28) and letting $h_x/2 = y$

$$p_{h_1} = \frac{\bar{\xi} K}{\delta_1} \left[(\delta_1 + 1) \left(\frac{h_x}{h} \right)^{\delta_1} - 1 \right] \quad (31)$$

For the deformation area below line AC , Equation (6) remains applicable, and with $x = R \sin \theta$, $y = \frac{h_o}{2} = \frac{h}{2} - R(1 - \cos \theta)$ at point C and $x = l$, $y = \frac{H}{2}$ at point A , the line AC is defined as

$$y = \frac{\Delta h + 2R(1 - \cos \theta)}{2(l - R \sin \theta)} x + \frac{H}{2} - \frac{\Delta h + 2R(1 - \cos \theta)}{2(l - R \sin \theta)} l \quad (32)$$

Similarly, with supplementary boundary conditions

$$\sigma_x = 0, p_H = \bar{\xi} K \text{ at } y = \frac{H}{2} \quad (33)$$

$$\sigma_x = 0, p_{h_2} = \bar{\xi} K \text{ at } y = \frac{h_o}{2} \quad (34)$$

the unit pressure may eventually be described by

for the backward slip zone below line AC

$$p_H = \frac{\bar{\xi} K}{\delta_2} \left[(\delta_2 - 1) \left(\frac{H}{h_x} \right)^{\delta_2} + 1 \right] \quad (35)$$

and for the forward slip zone below line AC

$$p_{h_2} = \frac{\bar{\xi} K}{\delta_2} \left[(\delta_2 + 1) \left(\frac{h_x}{h_o} \right)^{\delta_2} - 1 \right] \quad (36)$$

where $\delta_2 = \frac{2\mu(l - R \sin \theta)}{\Delta h + 2R(1 - \cos \theta)}$.

Afterwards, the rolling force in the rolling phase with thinner thickness can be calculated by the following equation

$$\begin{aligned} p &= \frac{B+b}{2} \int_0^l p_x dx = \frac{B+b}{2} \left(\int_{h_o}^h p_{h_1} dx + \int_{h_o}^{h_r} p_{h_2} dx + \int_{h_r}^H p_H dx \right) \\ &= \frac{B+b}{2} \left\{ \frac{\sin \theta}{2(\cos \theta - 1)} \frac{\bar{\xi} K}{\delta_1} h_o \left[1 - \left(\frac{h_o}{h} \right)^{\delta_1} \right] + \frac{l - R \sin \theta}{\Delta h + 2R(1 - \cos \theta)} \frac{\bar{\xi} K}{\delta_2} h \left(\frac{h_r}{h} \right) \left[\left(\frac{h_r}{h} \right)^{\delta_2} \left(\frac{h}{h_o} \right)^{\delta_2} \right. \right. \\ &\quad \left. \left. - 1 \right] + \frac{l - R \sin \theta}{\Delta h + 2R(1 - \cos \theta)} \frac{\bar{\xi} K}{\delta_2} h \left(\frac{h_r}{h} \right) \left[\left(\frac{H}{h_r} \right)^{\delta_2} - 1 \right] \right\} \end{aligned} \quad (37)$$

where $\left(\frac{H}{h_r}\right)^{\delta_2}$ is written in the form with $p_H = p_{h_2}$ at $h_x = h_r$

$$\left(\frac{H}{h_r}\right)^{\delta_2} = \frac{1}{\delta_2 - 1} [(\delta_2 + 1) \left(\frac{h}{h_o}\right)^{\delta_2} \left(\frac{h_r}{h}\right)^{\delta_2} - 2] \quad (38)$$

2.3 Springback of workpiece in thickness direction

For rolled workpiece, the thickness directional springback is an embodiment of the elastic recovery owing to the elastic compression evoked by rolling force. By decomposing the micro flexible rolling process into infinitesimal rolling phases with invariable thickness the thickness directional springback at arbitrary location along the workpiece length may be worked out on the basis of Hertz contact theory of contact between two cylinders [58, 59].

Figure 4 illustrates the elastic behaviour of the workpiece during each infinitesimal rolling phase with an invariable thickness, in which the workpiece is regarded as a cylinder with infinite radius and the roll a rigid body.

Figure 4. Elastic behaviour of the workpiece in the rolling phase with an invariable thickness

where Δ denotes the elastic springback of the workpiece in thickness direction, which can be quantitatively expressed by

$$\Delta = 2 \cdot 2x_1 \bar{p} \cdot \frac{1-\nu^2}{\pi E} \quad (39)$$

where $\bar{p}$ is the average unit rolling force, ν and E are the Poisson's ratio and elastic modulus of the workpiece, respectively.

Neglecting contact area asymmetry with respect to the central line of the rolls leads to an oval-shaped distribution of unit rolling force, as shown in Figure 5.

Figure 5. Elliptically symmetric distribution of unit rolling force along the length of arc of contact

where a is equal to half the arc length of contact and p' indicates the rolling force per unit width at arbitrary location x in the closed interval $[-a, a]$, which may be mathematically described as

$$p' = p'_0 \sqrt{1 - \frac{x^2}{a^2}} \quad (40)$$

where p'_0 means the rolling force per unit width at location $x = 0$ and numerically equals $\frac{4p}{\pi a(B+b)}$ [60].

In the light of probability theory, the average unit rolling force can be estimated as

$$\bar{p} = \int_{-a}^a p' \cdot \frac{1}{2a} dx = \int_{-a}^a p'_0 \sqrt{1 - \frac{x^2}{a^2}} \cdot \frac{1}{2a} dx = \frac{\pi}{4} p'_0 = \frac{p}{a(B+b)} \quad (41)$$

The thickness directional springback of rolled workpiece may thereupon be evaluated in the following manner

$$\Delta = 2 \cdot 2a \cdot \frac{p}{a(B+b)} \cdot \frac{1-\nu^2}{\pi E} = \frac{4p}{(B+b)} \cdot \frac{1-\nu^2}{\pi E} \quad (42)$$

3. Preparation of experimental materials and their determination of parameters

3.1 Annealing treatment

Aluminium alloy 1060 strips with initial thickness of 500 μm have been adopted for this study. In order to explore the effect of grain size on the mechanical properties and formabilities of workpieces, annealing treatment was separately employed to these materials with different heating temperatures and holding times in an argon atmosphere to obtain workpieces with differing grain sizes. Table 1 gives the annealing parameters and the achieved grain sizes along with their respective deviations from the average values for aluminium samples.

Table 1. Various grain sizes obtained from corresponding annealing parameters for aluminium alloy 1060 samples (in argon atmosphere)

Sample No.	Heating Temperature (°C)	Holding Time (min)	Average Grain Size (μm)	Standard Deviation (μm)
A ₀	-	-	9	5
A ₁	400	120	23	13
A ₂	450	120	31	14
A ₃	500	120	42	20
A ₄	550	60	51	27
A ₅	600	60	63	30
A ₆	620	60	71	36

where A₀ is the as-received rolled material therein composed of elongated columnar grains, which have an average aspect ratio of 8.94 and a standard deviation of 3.99.

3.2 Determination of stress-strain curve

A miniature tensile test scheme has been devised for a more accurate measurement of mechanical properties of experimental specimens as geometry size effect upon miniaturisation of experimental specimens leads to different material behaviour compared with the macroscale [61]. Figure 6 presents the dimensions of the test sample miniaturised in accordance with the ISO 6892-1:2009 standard. The stress-strain curve can be fitted to the stress-strain data points acquired from a miniature tensile test. Figure 7 plots the true stress-strain curves for aluminium alloy 1060 samples with different annealing conditions listed in Table 1 at the strain rate of 0.0022/s.

Figure 6. Dimensions of the miniaturised tensile sample: (a) in-plane dimension; (b) thickness dimension (unless otherwise specified, all dimensions are in millimeters)

Figure 7. True stress-strain curves for aluminium alloy 1060 samples under different annealing conditions

As is seen from Figure 7, annealing can increase the ductility of aluminium alloy 1060 samples by at least 47 % since the internal stresses that contribute to brittleness of the sample are largely alleviated during the process. For annealed samples, the yield stress and the ultimate tensile stress both increase as the average recrystallised grain size decreases, which is accounted for by the enhancement of the grain boundary strengthening effect with decreasing grain size. In addition, there exists a larger scatter of flow stress for annealed samples with greater grain size, as the property of each individual grain becomes dominant to decide the stress-strain characteristics of the sample. Moreover, an increase in strength resulting from a finer average grain size is, however, accompanied by a loss of ductility for aluminium alloy 1060 samples.

3.3 Determination of weighted heterogeneity coefficient

Firstly, micro hardness tests were implemented for grain hardness determination in annealed samples, on the grounds that different hardness measurement data equivalently evince different grain properties, i.e. grained inhomogeneity. Next, disparate heterogeneity coefficients were selected to correspond to the hardness values detected in a given area of each sample [62, 63]. Lastly, the weighted heterogeneity coefficient was determined below, revealing the importance of each of these grain properties to the heterogeneity of the whole material

$$\bar{\xi} = \frac{\xi_1\eta_1 + \xi_2\eta_2 + \dots + \xi_i\eta_i}{H}, i = 1, 2, \dots, n \quad (43)$$

where $\xi_1, \xi_2, \dots, \xi_i$ are the heterogeneity coefficients, which appear $\eta_1, \eta_2, \dots, \eta_i$ times respectively within the given area of the sample, while H is the sum of occurrences of all the heterogeneity coefficients. Table 2 reports the parameters for and the values of weighted heterogeneity coefficients for aluminium samples.

In respect of a given reduction, the analytical calculation is able to proceed in accordance with the average fit to the measured stress-strain data points conjointly with the determined weighted heterogeneity coefficient.

Table 2. Parameters for and values of weighted heterogeneity coefficients for aluminium alloy 1060 samples

Weighted Heterogeneity Coefficient		Sample No.					
Parameter	Value	A ₁	A ₂	A ₃	A ₄	A ₅	A ₆
ξ_1	-	0.3 [63]	0.3 [60]	0.4 [57]	0.4 [54]	0.5 [51]	0.5 [49]
η_1	-	8	7	6	7	8	7
ξ_2	-	0.5 [64]	0.5 [61]	0.6 [58]	0.6 [55]	0.7 [52]	0.7 [50]
η_2	-	12	11	13	11	11	10
ξ_3	-	0.7 [65]	0.7 [62]	0.8 [59]	0.8 [56]	0.9 [53]	0.9 [51]
η_3	-	21	19	21	17	15	10
ξ_4	-	0.9 [66]	0.9 [63]	1.0 [60]	1.0 [57]	1.1 [54]	1.1 [52]
η_4	-	36	29	25	20	17	12
ξ_5	-	1.1 [67]	1.1 [64]	1.2 [61]	1.2 [58]	1.3 [55]	1.3 [53]
η_5	-	41	26	20	16	13	11
ξ_6	-	1.3 [68]	1.3 [65]	1.4 [62]	1.4 [59]	1.5 [56]	1.5 [54]
η_6	-	28	18	15	12	11	9
ξ_7	-	1.5 [69]	1.5 [66]	1.6 [63]	1.6 [60]	-	-
η_7	-	14	13	9	9	-	-
ξ_8	-	1.7 [70]	1.7 [67]	-	-	-	-
η_8	-	7	10	-	-	-	-
H	-	167	133	109	92	75	59
-	$\bar{\xi}$	1.017	1.019	1.022	1.015	1.031	1.025

where data in brackets are the corresponding Vickers hardness values of the heterogeneity coefficients

4. Physical experiment and numerical simulation of micro flexible rolling

4.1 Micro flexible rolling experiment

A micro flexible rolling mill has been designed and developed to carry out the physical experiments. Figure 8 represents the exterior of the facility with two working rolls each of diameter = 25 mm and length = 35 mm. The rolling mill can be started when the workpiece has been found to exist between the roll pair by the laser detector. The roll gap is adjusted in real time in terms of vertical movements of the upper roll powered by the electric cylinder as per the commands from the control console to acquire the workpiece profile in demand, and the actual rolling force as well as the value of the actual roll gap is recorded at a sampling rate of 2/s using software STEP 7 installed on the laptop.

1 - Power transmission shafts	5 - Displacement sensor
2 - Working rolls	6 - Electric cylinder
3 - Laser detector	7 - Control console
4 - Force sensor	8 - Laptop

Figure 8. Exterior of the micro flexible rolling mill

A rolling speed of 20 cm/min was chosen for all the experiments, in which the specimens had been cut into 10 mm wide and 40 mm long and were rolled with no lubrication. At the beginning of each experiment, the value of the roll gap was manually set to zero under an estimated maximum rolling force applied to the rolls, to eliminate the effect of roll deflection and flattening to a largest extent. For aluminium alloy 1060 specimens, the micro flexible rolling process consisted of three phases: firstly, the rolling phase with 25 % thickness reduction, viz. that the amount of thickness reduction is 125 μm , subsequently the rolling phase with thickness reduction that rose quickly from 25 to 50 %, and then the rolling phase with 50 % thickness reduction, to wit, that the amount of thickness reduction is 250 μm .

4.2 Experimental results

Figure 9 unveils the morphology of the specimens after micro flexible rolling and the profile in thickness transition area of each specimen under microscope VHX-1000 KEYENCE at $\times 100$ magnification. As can be seen from Figure 9, the thickness transition areas of aluminium specimens can be clearly observed due to the large amount of thickness variation. Four sets of experimental data, including the average actual rolling force, the average value of the actual roll gap, the final thickness and the thickness directional springback, are summarised in Table 3 for aluminium alloy 1060 specimens, where the final specimen thickness is an average value of ten separate measurements by using a micrometer and the thickness directional springback of the specimen is calculated as the final thickness minus the average value of the actual roll gap. Thickness measurements were first taken at five locations, spaced evenly along the front to the rear side of the test area (thicker area with 25 % thickness reduction or thinner one with 50 % thickness reduction) on the left, and then at five locations on the right in a similar manner; this being the case the accuracy and precision of thickness measurement may be ensured by averaging measurements at these ten locations which were selected in such a way as to be statistically representative of the whole test area.

Figure 9. Morphology and magnified profile in thickness transition areas of specimens after micro flexible rolling

Table 3. Experimental results for the micro flexible rolling of aluminium alloy 1060 specimens: (a) 25 % reduction; (b) 50 % reduction

(a)							
Test Index	Specimen No.						
	A ₀	A ₁	A ₂	A ₃	A ₄	A ₅	A ₆
Average Actual Rolling Force (N)	4,164	3,925	3,751	3,608	3,445	3,307	3,183
Average Value of Actual Roll Gap (μm)	372.688	372.404	372.458	372.941	372.697	372.073	372.315
Final Thickness (μm)	376.300	375.900	376.000	376.300	376.100	375.600	375.800
Thickness Directional Springback (μm)	3.612	3.496	3.542	3.359	3.403	3.527	3.485
(b)							
Test Index	Specimen No.						
	A ₀	A ₁	A ₂	A ₃	A ₄	A ₅	A ₆
Average Actual Rolling Force (N)	5,914	5,693	5,436	5,047	4,791	4,549	4,290
Average Value of Actual Roll Gap (μm)	245.985	246.408	245.649	246.166	246.218	245.671	245.453
Final Thickness (μm)	251.200	251.500	250.800	251.100	251.400	251.000	250.700
Thickness Directional Springback (μm)	5.215	5.092	5.151	4.934	5.182	5.329	5.247

4.3 Numerical simulation of micro flexible rolling

4.3.1 Incremental method of elastic-plastic analysis

The micro flexible rolling process involves the elastic-plastic behaviour of the rolled material, whose strain state is dependent not only on the stress state but also on the entire stress history after entering to the plastic realm [64, 65]. The analysis of such a nonlinear elastic-plastic problem can proceed in an incremental manner. Specifically, the loading is applied in successive incremental stages and an iterative approach is adopted to find the elastic-plastic equilibrium at each increment [66].

It is postulated that the displacement, strain and stress at time t are obtained as $u_i(t)$, $\varepsilon_{ij}(t)$ and $\sigma_{ij}(t)$, separately, and the increments of Δu_i , $\Delta \varepsilon_{ij}$ and $\Delta \sigma_{ij}$ are allowed respectively in their values with the time increment of Δt . Consequently, the displacement, strain and stress at time $t + \Delta t$ are computed as [67]

$$\begin{cases} u_i(t + \Delta t) = u_i(t) + \Delta u_i \\ \varepsilon_{ij}(t + \Delta t) = \varepsilon_{ij}(t) + \Delta \varepsilon_{ij} \\ \sigma_{ij}(t + \Delta t) = \sigma_{ij}(t) + \Delta \sigma_{ij} \end{cases} \quad (44)$$

which meet the following conditions for equilibrium, and the strain-displacement relation (in V)

$$\varepsilon_{ij}(t) + \Delta \varepsilon_{ij} = \frac{1}{2} \left(u_{i,j}(t) + u_{j,i}(t) \right) + \frac{1}{2} \left(\Delta u_{i,j} + \Delta u_{j,i} \right) \quad (45)$$

the linearised stress-strain relation (in V)

$$\Delta \sigma_{ij} = D_{ijkl}^{ep}(\tau) \Delta \varepsilon_{kl} \quad (t \leq \tau \leq t + \Delta t) \quad (46)$$

In accordance of the principle of virtual displacements, if the material is in equilibrium under the action of all forces (including stress $\sigma_{ij}(t) + \Delta \sigma_{ij}$, volume load $F_i(t) + \Delta F_i$ and boundary load $T_i(t) + \Delta T_i$ at time $t + \Delta t$) the total work done by these forces for a virtual displacement is zero [68]. This can be formulated as

$$\int_V (\sigma_{ij}(t) + \Delta \sigma_{ij}) \delta \Delta \varepsilon_{ij} dV - \int_V (F_i(t) + \Delta F_i) \delta \Delta u_i dV - \int_{S_\sigma} (T_i(t) + \Delta T_i) \delta \Delta u_i dS = 0 \quad (47)$$

Substituting Equation (46) into Equation (47) produces (in matrix form)

$$\begin{aligned} & \int_V \delta \Delta \boldsymbol{\varepsilon}^T \mathbf{D}_{ep}(\tau) \Delta \boldsymbol{\varepsilon} dV - \int_V \delta \Delta \mathbf{u}^T \Delta \mathbf{F} dV - \int_{S_\sigma} \delta \Delta \mathbf{u}^T \Delta \mathbf{T} dS \\ & = - \int_V \delta \Delta \boldsymbol{\varepsilon}^T \boldsymbol{\sigma}(t) dV + \int_V \delta \Delta \mathbf{u}^T \mathbf{F}(t) dV + \int_{S_\sigma} \delta \Delta \mathbf{u}^T \mathbf{T}(t) dS \end{aligned} \quad (48)$$

In order to formulate the incremental finite element equilibrium equation, the incremental displacement within a finite element is expressed in terms of the interpolation for the incremental nodal displacement [16, 69]

$$\Delta \mathbf{u} = \mathbf{N} \Delta \hat{\mathbf{u}} \quad (49)$$

By employing the appropriate strain-displacement transformation matrix $\mathbf{B}$ the incremental strain within each element can be reckoned by

$$\Delta \boldsymbol{\varepsilon} = \mathbf{B} \Delta \hat{\mathbf{u}} \quad (50)$$

Substituting Equations (49) and (50) into Equation (48) and using the fact that the components of the virtual displacement are arbitrary yield the finite element system equilibrium equation:

$$\mathbf{K}_{ep}(\tau) \Delta \mathbf{U} = \Delta \mathbf{Q} \quad (51)$$

where $\mathbf{K}_{ep}(\tau)$, $\Delta \mathbf{U}$ and $\Delta \mathbf{Q}$ define the elastic-plastic stiffness matrix, the incremental displacement vector and the unbalanced force vector of the system, respectively.

By utilising the Newton-Raphson iterative method Equation (51) may be reformulated as

$$\mathbf{K}_{ep}^{(n)}(t + \Delta t) \Delta \mathbf{U}^{(n)} = \Delta \mathbf{Q}^{(n)}, n = 0, 1, 2, \dots \quad (52)$$

in which n is the number of iterations. The n th correction of the displacement increment can thereafter be computed by

$$\Delta \mathbf{U}^{(n)} = (\mathbf{K}_{ep}^{(n)}(t + \Delta t))^{-1} \Delta \mathbf{Q}^{(n)} \quad (53)$$

Then the displacement is quantified at the $(n+1)$ th iteration by

$$\mathbf{U}^{(n+1)}(t + \Delta t) = \mathbf{U}^{(n)}(t + \Delta t) + \Delta \mathbf{U}^{(n)} \quad (54)$$

Further, the strain increment and the correction of the stress increment of the n th iteration may be calculated in each element as

$$\Delta \boldsymbol{\varepsilon}^{(n)} = \mathbf{B} \Delta \mathbf{U}^{(n)} \quad (55)$$

$$\Delta \boldsymbol{\sigma}^{(n)} = \omega \mathbf{D}_e \Delta \boldsymbol{\varepsilon} + \int_0^{(1-\omega)\Delta \boldsymbol{\varepsilon}^{(n)}} \mathbf{D}_{ep} d\boldsymbol{\varepsilon} \quad (56)$$

in which ω varies between 0 and 1 and $\omega \mathbf{D}_e \Delta \boldsymbol{\varepsilon}$ denotes the elastic state of the material. The stress at the n th iteration is then updated in each element as

$$\boldsymbol{\sigma}^{(n+1)}(t + \Delta t) = \boldsymbol{\sigma}^{(n)}(t + \Delta t) + \Delta \boldsymbol{\sigma}^{(n)} \quad (57)$$

The analysis can be deemed converged for this increment step if the convergence criteria for force and displacement are met [70]. The iteration procedure is repeated for each increment step until the final time has been reached.

4.3.2 Grained inhomogeneity

The influence of grained inhomogeneity on micro flexible rolling has been numerically modelled by the use of three-dimensional Voronoi tessellation method, which takes into account explicitly the topological and geometrical features of the microstructure [71].

The realisation process starts with creating M Voronoi nuclei at random positions inside the workpiece volume with M unit cubes, to wit, that each cube contains a single nucleus. After this step,

Voronoi cells may be constructed by defining each cell as the region consisting of all points closer to a particular nucleus than the other, namely

$$\text{Vor}(S_i) = \{v \in V | \text{dist}(v, S_i) < \text{dist}(v, S_j) \forall j \neq i\} \quad (58)$$

where $\text{dist}(v, S)$ is the distance from point v to nucleus S [72, 73].

In the following, the Voronoi polyhedra formed to represent the individual grains are associated with a variety of properties so as to manifest the grained heterogeneity in real material, whereof the grain properties are classified by their hardness and further quantified according to the stress-strain relationship identified in the miniature tensile test. Taking aluminium alloy 1060 sample A_1 as an example, eight kinds of grain properties are referred depending on the discovery of grains with eight levels of hardness in the sample. The scattering stress-strain data for A_1 are correspondingly fitted into eight representative stress-strain curves and each grain is stochastically assigned one of these stress-strain properties, whose total probability distribution follows that of grain hardness in this sample.

The entire course is codified in MATLAB, which generates a Python file containing the information about the simulated grains after the code is executed. The output Python file is later imported into ABAQUS/CAE to complete the establishment of diverse grains, whose average size can be modified with changing the edge-length of the unit cube of the workpiece. For instance, Figure 10 discloses the mimicked microstructures of aluminium alloy 1060 samples.

Figure 10. Mimicked microstructures of aluminium alloy 1060 samples: (a) A_1 ; (b) A_2 ; (c) A_3 ; (d) A_4 ; (e) A_5 ; (f) A_6

4.3.3 Simulation conditions

The top half of the 3D micro flexible rolling finite element model has been set up taking into consideration the deformation symmetrical with respect to the x - z plane, together with higher computational efficiency. As can be noted in Figure 11, the upper roll was conceived to be an elastic hollow cylinder to further save the computational time, while its outer diameter, length, density, etc., were kept identical to those of the real one.

Figure 11. Top half of 3D micro flexible rolling finite element model

The finite element model involved one contact pair including the outer surface of the roll and the top surface of the workpiece, which was defined as surface-to-surface contact utilising finite sliding formulation and surface-to-surface discretisation method, and modelled with frictional interface adopting a friction coefficient of 0.15 [74]. The outer surface of the roll in the contact pair was set as master surface, in view of higher stiffness and consequently less deformation of the roll [75], which was meshed with linear brick elements (C3D8R) with an element size of 0.3 mm, whilst the top surface of the workpiece was selected as the corresponding slave surface with the whole workpiece meshed with 10-node modified quadratic tetrahedron elements (C3D10M), with a relatively small element size of 0.125 mm, for the sake of precise description of the complex geometry of grain boundaries as well as non-penetration of the nodes at the outer surface of the roll [76, 77].

The simulation procedure comprised two major phases, during which no front and back tensions were exerted on the workpiece to maintain consistency with the experimental conditions. The first phase was concerned with advancing the workpiece toward the roll at a given initial speed in the negative X direction. No sooner did the roll fully engage the workpiece than the second phase came into play, where the auxiliary speed was removed from the workpiece, which was then continuously advanced by the friction between the two instances. The second phase proceeded with the Y-directional translational motion control of the roll to obtain the predefined thickness profile over the total workpiece length, where a supplementary amount for the reduction had been defined to compensate for the thickness directional springback of the workpiece along with the elastic deformation of the roll. The simulations were performed on the local computer whose specifications include (1) processor: Intel Core i7-4770, (2) processor speed: 3.40 GHz, (3) installed memory (RAM): 8.00 GB, and (4) system type: 64-bit Operating System. It took 52 hours on average to complete one calculation.

4.4 Results and discussion

Figure 12 displays one exemplary simulation result of rolling of sample A₁ with reduction of 25 to 50 %, where the left side was procured from the model with grains of uniform properties, whereas the right side was obtained from that with grains of distinctive properties. It is seen from Figure 12 (a) that the equivalent stress concentrates along the edges and diminishes towards the centre of the workpieces as the maximum elastic deflection takes place at the roll centre, which induces more compression at the edges than in the centre of the workpiece. It can be also noticed from Figure 12 (b) that the effective stress has a heterogeneous distribution in the non-uniform granular workpiece material because when stress is applied and gradually increased plastic deformation might be expected to occur in those grains with low yield strength whereas the other grains still behaved as elastic, making the stress state inhomogeneous throughout the entire workpiece. Additionally, a

magnified view of the thickness transition area of sample A₁ is uncovered in Figure 13, through which the equivalent plastic strain gets distributed in a non-uniform manner as well when grains are endowed with an assortment of properties.

Although the thickness transition area of each sample can be located by observing the variation of the stress-strain state, along with measuring the thickness distribution in the rolling direction, it is quite a tiny part in comparison to the whole workpiece, and in which transient changes take place in the rolling force as well as the workpiece thickness. Thus the discussion in following subsections has been raised in respect of rolling phases with invariable thickness, where relatively stable data were collected and arranged.

Figure 12. Distribution of von Mises stress in the rolled sample A₁: (a) grains with homogeneous properties; (b) grains with differing properties

Figure 13. Distribution of equivalent plastic strain through thickness transition area of rolled sample A₁: (a) grains with homogeneous properties; (b) grains with differing properties

4.4.1 Rolling force during micro flexible rolling of aluminium alloy 1060 strips

The aluminium alloy 1060 strips were divided into seven groups with each composed of fifteen samples depending upon the annealing conditions listed in Table 1, and those numbered A₀-A₆ are considered to be the typical samples as they respectively hold the central position of the data set for each group. Note that the simulated rolling force was produced by summing the reaction forces acting on the reference points located at both ends of the roll.

First, Figure 14 shows the change of average rolling force relative to percentage reduction for Group A₀. It is apparent that the unannealed samples can be rolled up to 50 % reduction under the force of nearly 6,000 N, whereas a smaller force of around 4,000 N is necessary for the thicker areas with 25 % thickness reduction; besides, the simulation results are slightly higher than the theoretical values both reductions, inasmuch as a larger force was applied to produce a further reduction in such a way so as to offset the thickness directional springback of the samples together with the inevitable deflection in the rolls. Moreover, the actual rolling force is minutely bigger than the simulated one, possibly on account of the rough workpiece surface as well as the roll wear in real situation, which brings about an increase in the friction coefficient, followed by an increase in the average rolling force. A further point worthy of note pertains to a small increase in the scatter of average rolling force, notwithstanding a large increase in reduction, thanks to the relatively homogeneous columnar grain structures in the as-received aluminium samples, which exhibit a narrow range of mechanical properties, thereby reducing the range of variation for the average rolling force [78].

Following, the variations of both average rolling force and grain size are graphed against each other in Figure 15, for Groups A₁-A₆ of differently annealed samples. Note that (1) to each group there corresponds just one average grain size value, whose offsets among different types of data are deliberately set to ensure that the scatter of average rolling force for each data category is clearly recognised, and (2) Series 1 of simulation results was obtained from the models with grains of uniform properties while Series 2 was produced by those with grains of dissimilar properties. The Series 2 of simulation models, as indicated in Figure 15, yields closer results to the experimental ones as they better mimic the morphology and the properties of the crystalline strips at different grain sizes. In regard, nonetheless, to the whole data set, increasing average grain size decreases the average rolling force, in view of the fact that more dislocations propagate to the grain boundaries as the grains grow bigger, creating stronger repulsive forces to promote dislocations; hence smaller applied forces are needed to move dislocations across grain boundaries, allowing further deformation in the workpieces.

Furthermore, for either reduction a larger average grain size results in a higher scatter of average rolling force by reason of fewer grains dispersed in the workpieces thanks to their increased size, which highlights the properties of single grains and exerts a greater effect of each single grain on the overall mechanical properties of the workpieces. Meantime, for each type of data the bigger reduction exacerbates the decline in average rolling force with increasing average grain size, and amplifies the scattering of the average rolling force at each average grain size, which is related to the stress-strain characteristics of the annealed samples; namely larger discrepancies between the flow curves for samples with different average grain sizes emerge, as well as bigger scatter around each curve, when the materials undergo greater deformations (Figure 7).

Finally, the theoretical calculations are compared with the experimental data with a view to evaluating the performance of the established mathematical model of rolling force with samples A₀-A₆. From Table 4 it can be observed that the derived analytical expression suggests an appropriate form to represent the true rolling force in the micro flexible rolling mill, whose error lies between 3.05 and 8.91 % and generally decreases with the decreasing flow stress of the sample which reduces

the force needed for the supplementary amount of reduction, bringing the experimental and theoretical values closer to each other.

Figure 14. Average rolling force versus percentage reduction for Group A₀

(a)

(b)

Figure 15. Average rolling force versus average grain size for Groups A₁-A₆: (a) 25 % reduction; (b) 50 % reduction

Table 4. Comparison between theoretical and experimental rolling forces for aluminium alloy 1060 samples: (a) 25 % reduction; (b) 50 % reduction

(a)							
Average Rolling Force	Sample No.						
	A ₀	A ₁	A ₂	A ₃	A ₄	A ₅	A ₆
Theoretical Value (N)	3,880	3,604	3,475	3,382	3,289	3,136	3,068
Experimental Value (N)	4,164	3,925	3,751	3,608	3,445	3,307	3,183
Error-I (%)	7.32	8.91	7.94	6.68	4.74	5.45	3.75
(b)							
Average Rolling Force	Sample No.						
	A ₀	A ₁	A ₂	A ₃	A ₄	A ₅	A ₆
Theoretical Value (N)	5,582	5,326	5,109	4,860	4,585	4,391	4,163
Experimental Value (N)	5,914	5,693	5,436	5,047	4,791	4,549	4,290
Error-I (%)	5.95	6.89	6.40	3.85	4.49	3.60	3.05

where the error-I is estimated by

$$Error - I = \frac{|Theoretical\ Value - Experimental\ Value|}{Theoretical\ Value} \times 100\% \quad (59)$$

4.4.2 Thickness directional springback after micro flexible rolling of aluminium alloy 1060 strips

The elastic springback of the workpiece in thickness direction is a vital factor which influences the accuracy and mostly determines the supplementary amount of the reduction in micro flexible rolling. In conformity with Equation (42), the amount of the springback is generally directly proportional to the rolling force and inversely proportional to the elastic modulus of the material.

Firstly, as can be read off from Figure 16, for Group A₀ the overall level of springback starts at approximately 3.5 μm in the rolling phase with 25 % thickness reduction and ascends almost to 5.0

μm when the rolling reduction escalates to 50 %, whereof the middle value of the numerical results still lags slightly behind that of the experimental data and moves slightly beyond that during the theoretical calculation for each reduction, with a small variation in the scatter centred around it attributable to the microstructures of comparable uniformity in the unannealed strips. Note that the simulated springback is measured by the average distance between the node pairs from the top and bottom surfaces of the workpiece at the exit of the roll bite minus the minimum average distance between those in the roll bite zone.

In subsequence, Figure 17 presents the tendency of thickness directional springback for each class of data to display an undulating movement with varying average grain size, for the reason that in spite of a decrease in both average rolling force and elastic modulus of the material after annealing, the magnitude of the decrease in each parameter remains uncertain across disparate groups of average grain size; in consequence, a rise is seen in the value of thickness directional springback as the magnitude of the decrease in elastic modulus may exceed that in average rolling force, and vice versa. On the other hand, there exists a mild tendency towards declining level of disagreement between the theoretical and experimental results, which is associated with that of average rolling force when the value of elastic modulus remains unique within a specific group.

At last, Tables 5 and 6 provide the quantified comparison between theoretical predictions and experimental data of the thickness directional springback and the difference between the target and actual thicknesses for exemplary samples A_0 - A_6 , individually. As is obvious from these two tables, the desired thickness profiles can be created with negligible errors in the longitudinal direction of the strips by allocating compensation for the thickness directional springback reasonably estimated via the analytical model, which satisfies the requirements for thickness accuracy and declares the practicability of the micro flexible rolling system.

Figure 16. Thickness directional springback versus percentage reduction for Group A_0

(a)

(b)

Figure 17. Thickness directional springback versus average grain size for Groups A₁-A₆: (a) 25 % reduction; (b) 50 % reduction

Table 5. Comparison between theoretical and experimental results of thickness directional springback for aluminium alloy 1060 samples: (a) 25 % reduction; (b) 50 % reduction

Thickness Directional Springback	Sample No.						
	A ₀	A ₁	A ₂	A ₃	A ₄	A ₅	A ₆
Theoretical Value (μm)	3.435	3.284	3.339	3.198	3.220	3.352	3.341
Experimental Value (μm)	3.612	3.496	3.542	3.359	3.403	3.527	3.485
Error-I (%)	5.15	6.46	6.08	5.03	5.68	5.22	4.31

(b)

Thickness Directional Springback	Sample No.						
	A ₀	A ₁	A ₂	A ₃	A ₄	A ₅	A ₆
Theoretical Value (μm)	4.994	4.813	4.894	4.672	4.937	5.106	5.049
Experimental Value (μm)	5.215	5.092	5.151	4.934	5.182	5.329	5.247
Error-I (%)	4.43	5.80	5.25	5.61	4.96	4.37	3.92

Table 6. Thickness errors of micro flexible rolled aluminium alloy 1060 strips: (a) 25 % reduction; (b) 50 % reduction

(a)			
Specimen No.	Specimen Thickness		
	Target Value (μm)	Actual Final Value (μm)	Error-II (%)
A ₀	375.000	376.300	0.35
A ₁	375.000	375.900	0.24
A ₂	375.000	376.000	0.27
A ₃	375.000	376.300	0.35
A ₄	375.000	376.100	0.29
A ₅	375.000	375.600	0.16
A ₆	375.000	375.800	0.21

(b)			
Specimen No.	Specimen Thickness		
	Target Value (μm)	Actual Final Value (μm)	Error-II (%)
A ₀	250.000	251.200	0.48
A ₁	250.000	251.500	0.60
A ₂	250.000	250.800	0.32
A ₃	250.000	251.100	0.44
A ₄	250.000	251.400	0.56
A ₅	250.000	251.000	0.40
A ₆	250.000	250.700	0.28

where the error-II is reckoned as follows

$$Error - II = \frac{|Target Value - Actual Final Value|}{Target Value} \times 100\% \quad (60)$$

5. Conclusions

This paper deals with the exploration into the micro flexible rolling process wherein the dimensions of the grains are comparable to the workpiece thickness, in an attempt to disclose the pattern of the grained heterogeneity influencing the average rolling force and the thickness directional springback of the workpiece employing analytical, numerical and experimental methodologies. The principle conclusions of this research are as follows:

1. The 2D analytical models are worked out to analyse the forces acting within the roll bite region as well as the elastic recovery in the thickness after the workpiece exits this area during micro flexible rolling, which take into account the impact of grain size and property anisotropy by incorporating the weighted heterogeneity coefficient.
2. Miniature tensile tests are performed on aluminium alloy 1060 samples, whereby the effect of annealing condition on the ductility and strength are quantitatively evaluated, and whereby the scatter effect of their flow stresses is qualitatively elucidated with regard to the average grain size; whereas micro hardness tests are conducted for the identification of the distribution of grain hardness in order to determine the weighted heterogeneity coefficients for the respective samples.

3. Laboratory-scale experiments are implemented utilising 500 μm thick aluminium alloy 1060 strips with various average grain sizes which correspond to those considered in the tensile tests, and reductions of 25 to 50 % are chosen for these materials in accordance with their strength and ductility; a quantitative assessment of the average rolling force and thickness directional springback, followed by a qualitative estimation of their scatter effect, is carried out in respect of the reduction and the average grain size.
4. The 3D finite element models employing the Voronoi tessellation technique are built to mimic the crystalline structure of the materials, inclusive of their grain size and shape, while the anisotropic properties of single grains which are represented by the scattered stress-strain curves are afterwards embedded in conformity with the pattern in the distribution of grain hardness identified within the individual samples; simulations are fulfilled under the experimental conditions and the superiority of the finite element models with grains of inhomogeneous properties over those with grains of homogeneous properties is confirmed by the experimental evidence.
5. The viability of the established mathematical models is proved through the experimental validation of their analytical predictions of the average rolling force and the thickness directional springback of the workpiece during the micro flexible rolling process, whereby the thickness profile control performance of the micro flexible rolling mill is strengthened.

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Declaration of conflicting interests

The authors declare that there is no conflict of interests.

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