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Brief paper

Controller design for time-delay systems using genetic algorithms

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ABSTRACT

This paper presents a new approach to design controllers for time-delay systems by using genetic algorithms (GAs) together with the solvability of linear matrix inequalities (LMIs). Both of the state-feedback controller and the static output feedback controller can be designed with this approach. It is confirmed by numerical examples that this approach achieves less conservative results than previously existing methods on the given examples.

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1. Introduction

Recently, much efforts have been done on the problem of controller design for time-delay systems and various approaches have been proposed to reduce the conservatism of delay-dependent conditions by using new bounding for cross terms or choosing new Lyapunov–Krasovskii functional. In particular, a new inequality was introduced for bounding cross terms and a new delay-dependent stabilisation condition using a memoryless controller for state-delayed systems were presented in Moon et al. (2001). It was shown that the proposed stabilisation method can be less conservative than previously existing results. In addition, a less conservative delay-dependent H_∞ control was proposed in Lee et al. (2004) for linear systems with a state-delay based on a new Lyapunov–Krasovskii functional. It was also shown that the proposed method is much less conservative than previously existing results presented in Fridman and Shaked (2002a). Further improving conditions for the delay-dependent stabilisation problem and the solvability of the delay-dependent H_∞ control are given in Zhang et al. (2005), Palhares et al. (2005) and Xu et al. (2006), etc., where the newly less conservative results are shown.

However, in above-mentioned research works, controller synthesis conditions are always presented in terms of nonlinear matrix inequalities in order to reduce the conservatism. Although an iterative algorithm has been developed to solve the nonlinear matrix inequalities due to the nonconvex feasibility problem, the conservativeness still exists since the iterative algorithm can only find the suboptimal solution. On the other hand, for neutral systems with time-delays, genetic algorithm (GA) has been used to find the feasible solutions for controller design (Chen, 2004, 2006, 2007; Lien, 2007). But, the potential of GA in finding solutions to time-delay systems has not been emphasised in these works. In addition, the above-mentioned works only focused on the memoryless state-feedback control. Less efforts have been made in designing the static output feedback controllers for time-delay systems in spite of its importance in real-world applications.

This paper develops an algorithm to design both of the state-feedback and the static output feedback controllers for time-delay systems. The GA is used to search for the possible solutions due to its high potentialities in global optimisation, and hence, the nonlinear matrix inequalities problem is avoided, where convex optimisation algorithm can be used. Numerical examples show that the presented approach obtains less conservative results than the previously existing results on the given examples.

2. Problem formulation

Consider the following state-delayed systems:

$$\begin{aligned}
\dot{x}(t) &= Ax(t) + B_w w(t) + A_1 x(t-\tau) + Bu(t),\\
z(t) &= \begin{bmatrix} C_0 x(t) + D_w w(t) \\ C_1 x(t-\tau) \\ Du(t) \end{bmatrix},\\
y(t) &= Cx(t),\\
x(t) &= \phi(t), \quad t \in [-\tau, 0],
\end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^m$ is the control input, $w(t)$ is the disturbance input that belongs to $L_2[0,\infty)$, $z(t)$ is the controller

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output, $\tau > 0$ is the delay of the system state, $\phi(t)$ is the initial condition, A, A_1, B, B_w, C_0, C_1, D, D_w and C are known real constant matrices with appropriate dimensions. In this paper, we are considering the following two problems:

Problem 1 (*Stabilisation of state-delayed system*). Assume $B = 0$, $C_0 = 0$, $D_w = 0$, $C_1 = 0$, $D = 0$ in (1), we are interested in designing a memoryless controller

$$u(t) = Ky(t) = KCx(t), \tag{2}$$

where $K \in \mathbb{R}^{m\times n}$ is a constant gain matrix to be designed, such that the closed-loop system is stable for any time-delay τ satisfying $0 \leqslant \tau \leqslant \bar{\tau}$, where $\bar{\tau}$ is the designed upper bound of the delay.

Theorem 1. *If there exist $P>0$, $Q>0$, $Z>0$, X, and Y such that*

$$\begin{bmatrix} (A+BKC)^{\mathrm{T}}P + P(A+BKC) + \bar{\tau}X + Y + Y^{\mathrm{T}} + Q & -Y + PA_1 & \bar{\tau}(A+BKC)^{\mathrm{T}}Z \\ * & -Q & \bar{\tau}A_1^{\mathrm{T}}Z \\ * & * & -\bar{\tau}Z \end{bmatrix} < 0, \tag{3}$$

$$\begin{bmatrix} X & Y \\ * & Z \end{bmatrix} \geqslant 0, \tag{4}$$

then Problem 1 is satisfied.

The proof of this theorem, which can be directly derived from Moon et al. (2001), is omitted here for brevity.

Problem 2 (*H_∞ control of linear system with state delay*). For system (1), we are interested in designing controller (2) such that the closed-loop system is stable, while the closed-loop system guarantees, under zero initial condition, $\|z(t)\|_2 < \gamma\|w(t)\|_2$, $\gamma > 0$ is a prescribed constant, for all $w(t) \in L_2[0,\infty)$ and any time-delay τ satisfying $0 \leqslant \tau \leqslant \bar{\tau}$.

Theorem 2. *If there exist $P_1>0$, $Q>0$, $Z>0$, P_2, P_3, X_{11}, X_{12}, X_{22}, Y_1, and Y_2 such that*

$$\begin{bmatrix} \Psi_{11} & P^{\mathrm{T}}\begin{bmatrix} 0 \\ A_1 \end{bmatrix} - \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} & P^{\mathrm{T}}\begin{bmatrix} 0 \\ B_w \end{bmatrix} + \begin{bmatrix} C_0^{\mathrm{T}}D \\ 0 \end{bmatrix} \\ * & -Q + C_1^{\mathrm{T}}C_1 & 0 \\ * & * & -\gamma^2 I + D_w^{\mathrm{T}}D_w \end{bmatrix} < 0, \tag{5}$$

$$\begin{bmatrix} X_{11} & X_{12} & Y_1 \\ * & X_{22} & Y_2 \\ * & * & Z \end{bmatrix} \geqslant 0, \tag{6}$$

where

$$\Psi_{11} = \begin{bmatrix} (A+BKC)^{\mathrm{T}}P_2 + P_2^{\mathrm{T}}(A+BKC) + \bar{\tau}X_{11} & (A+BKC)^{\mathrm{T}}P_3 - P_2^{\mathrm{T}} \\ +Q + C_0^{\mathrm{T}}C_0 + (DKC)^{\mathrm{T}}DKC + Y_1 + Y_1^{\mathrm{T}} & +P_1^{\mathrm{T}} + \bar{\tau}X_{12} + Y_2^{\mathrm{T}} \\ P_3^{\mathrm{T}}(A+BKC) + P_1 - P_2 + \bar{\tau}X_{12}^{\mathrm{T}} + Y_2 & -P_3 - P_3^{\mathrm{T}} + \bar{\tau}X_{22} + \bar{\tau}Z \end{bmatrix},$$

$$P = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix},$$

then Problem 2 is satisfied.

The proof of this theorem, which can be directly derived from Lee et al. (2004), is omitted here for brevity.

3. Algorithm

GA is a probabilistic search procedure based on the mechanism of natural selection and natural genetics. In the following, an algorithm which combines the random search of GA and the feasible solution of linear matrix inequalities (LMIs) will be proposed to find a desirable controller gain matrix K by solving the maximisation problem of

$$\max_{K\in\mathbb{R}^{m\times n}} \bar{\tau} \quad \text{subject to LMIs (3) and (4) or (5) and (6),} \tag{7}$$

where n is the number of state variables used for control, m is the number of input. In this problem, GA is used to randomly generate a matrix $K \in \mathbb{R}^{m\times n}$ initially which changes thereafter within the evolution procedure according to objective (7). If (7) is feasible for an evolved K, which has the maximum $\bar{\tau}$, then this K satisfies the specifications and thus constitutes a solution to the design problem. Note that the matrix inequalities (3)–(6) are LMIs once the control gain matrix K is known, and these LMIs can be solved efficiently by using Matlab LMI toolbox.

Since the standard GAs can be found in most related textbooks, an outline of our algorithm is given as:

Step 1: Use the binary string to encode the feedback gain matrix K.

Step 2: Randomly generate an initial population of N_{p} chromosomes.

Step 3: Evaluate the objective and assign fitness. Decode the initial population produced in Step 2 into real values for every controller gain matrix K_j, $j = 1, 2, \ldots, N_{\mathrm{p}}$. For every K_j, use the bisection method to search for the maximum delay τ_j such that with such a delay τ_j and K_j, LMIs (3) and (4) or (5) and (6) are feasible. Take every delay τ_j as the objective value corresponding to K_j and associate every K_j with a suitable fitness value according to rank-based fitness assignment approach, and then go to Step 4. If for a K_j, there is no feasible delay can be found such that LMIs (3) and (4) or (5) and (6) are feasible, the objective value corresponding to K_j will be assigned a large value in order to reduce its opportunity to be survived in the next generation.

Step 4: Use tournament selection approach to choose the offspring.

Step 5: Perform uniform crossover with probability p_{c} to produce new offspring.

Step 6: Do bit mutation in the population of chromosomes with a small mutation probability p_{m}.

Step 7: Retain the best chromosomes in the population using elitist reinsertion method.

Steps 3–7 correspond to one generation. The evolution process will repeat for N_{g} generations or will end when the search process converges with a given accuracy. The best chromosome is decoded into real values to produce again the control gain matrix.

Remark 1. In this paper, two problems for time-delay systems are considered and one computational algorithm is presented. For the considered problems, the systems are assumed as certain systems, that means, the systems have no parameter uncertainties. Since system uncertainties cannot be ignored in practice, robust controllers should be designed to tolerate both the time-delays and the system uncertainties. Generally, for stabilisation and control of uncertain systems with time-delays, the related matrix inequalities are firstly derived based on the use of Lyapunov–Krasovskii functional or other techniques. Then, an iterative algorithm is used to find the possible suboptimal solutions. And, the parameter uncertainties dealt with are assumed as norm-bounded or polytopic type (Moon et al., 2001; Lee et al., 2004; Xu et al., 2006; Palhares et al., 2005). In fact, for these problems, the presented algorithm can be certainly used to replace the iterative algorithm for finding solutions based on the derived matrix inequalities. For brevity, these problems will not be discussed in this paper.

4. Examples

The basic GA parameters used in this paper are as follows: $N_p = 80$, $p_c = 0.8$, $p_m = 0.01$, $N_g = 50$.

Example 1. For Problem 1, we set $A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$, $A_1 = \begin{bmatrix} -2 & -0.5 \\ 0 & -1 \end{bmatrix}$, and $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. Using our algorithm, we obtain different results as compared with the existing results in Table 1. It can be seen that our approach can obtain more larger delay for the state-feedback case than the existing methods. As expected, we can find the static output feedback controllers with appropriate delays as well.

Example 2. For Problem 1, considering the system matrices as $A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$, $A_1 = \begin{bmatrix} -1 & -1 \\ 0 & 0.9 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. The comparison results between our method and other existing methods are listed in Table 2. Clearly, our method produces much less conservative result. As another illustration, the time response of the closed-loop system subject to the constant time-delays of 6 and 33.2207 s when considering the controllers $[-70.18 \quad -77.67]$ and $[-96.1294 \quad -97.4899]$, respectively, and the initial state $x(0) = 1$, are plotted in Fig. 1. It is not hard to see the better stabilising performance of our approach.

Table 1
Comparison results for different stabilisation controllers, Example 1

Methods	Maximum $\bar{\tau}$ allowed	Feedback gain matrix K	C matrix
Moon et al. (2001)	0.383	$[-0.8266 \quad -3.0988]$	$I_{2\times2}$
Moon et al. (2001)	0.45	$[-4.8122 \quad -7.7129]$	$I_{2\times2}$
Our method	1.0186	$[-8.0171 \quad -2.4245]$	$I_{2\times2}$
Our method	0.4999	-0.4564	$[0 \quad 1]$
Our method	0.2881	4.909	$[1 \quad 0]$

Example 3. For Problem 2, we set $A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$, $A_1 = \begin{bmatrix} -1 & -1 \\ 0 & 0.9 \end{bmatrix}$, $B_w = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $C_0 = [0 \quad 1]$, $D = 0.1$, $D_w = 0$ and compare our results with the existing results in Table 3. It can be seen that for the same performance bound γ, our method can find the state-feedback controllers that allow larger delays than the existing methods. Even for some obtained static output feedback controllers, they can allow larger delays under the same performance bound as well. Furthermore, it is noted that the state-feedback gains obtained by our method are all smaller than the corresponding ones presented in Lee et al. (2004), Xu et al. (2006) and Palhares et al. (2005) since we can naturally constrain the search range for the controller gain in our algorithm. The unit impulse time response of the closed-loop system subject to the constant time-delays of 6 and 9.8185 s when considering the controllers $[-279.35 \quad -343.63]$ and $[-261.2491 \quad -282.4425]$, respectively, are plotted in Fig. 2. From Fig. 2, it can be seen that under the constant time-delay 6 s, our controller ($[-261.2491 \quad -282.4425]$) can achieve better performance than controller $[-279.35 \quad -343.63]$, which was obtained in Palhares et al. (2005). Even under the constant time-delay 9.8185 s, our controller can still stabilise the system without much decrease in performance. The better performance of our approach is easily observed.

Table 2
Comparison results for different stabilisation controllers, Example 2

Methods	Maximum $\bar{\tau}$ allowed	Feedback gain matrix K	C matrix
Fridman and Shaked (2002b)	1.51	$[-58.31 \quad -294.9]$	$I_{2\times2}$
Gao and Wang (2003)	3.2	$[-7.964 \quad -14.77]$	$I_{2\times2}$
Zhang et al. (2005)	6	$[-70.18 \quad -77.67]$	$I_{2\times2}$
Our method	33.2207	$[-96.1294 \quad -97.4899]$	$I_{2\times2}$

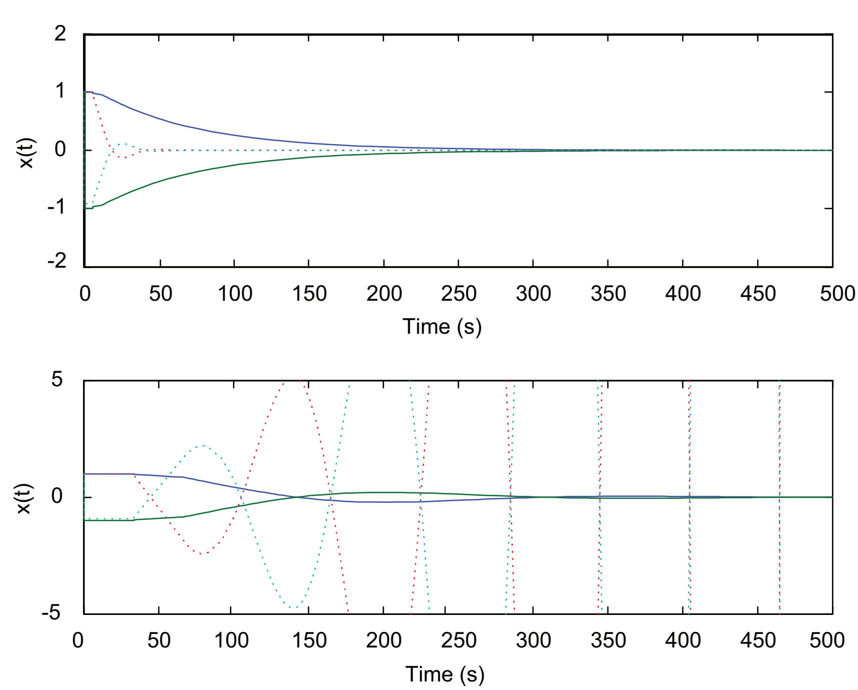

Fig. 1. Time response of closed-loop system with controllers $[-96.1294 \quad -97.4899]$ (solid line) and $[-70.18 \quad -77.67]$ (dotted line) when $x(0) = 1$ for time-delay $\tau = 6.000$ s (top) and $\tau = 33.2207$ s (bottom).

Table 3
Comparison results for different H_∞ controllers, Example 3

Methods	Given γ	Maximum $\bar{\tau}$ allowed	Feedback gain matrix K	C matrix
Lee et al. (2004)	0.1015	0.999	$[3.6828 \quad -827.0898]$	$I_{2\times 2}$
Our method	0.1015	1.4125	$[-0.0069 \quad -611.2532]$	$I_{2\times 2}$
Our method	0.1015	1.4131	-570.7319	$[0 \ \ 1]$
Fridman and Shaked (2002a)	0.1287	0.999	$[0 - 1.0285 \times 10^6]$	$I_{2\times 2}$
Lee et al. (2004)	0.1287	1.25	$[0.6407 \quad -89.1149]$	$I_{2\times 2}$
Xu et al. (2006)	0.1287	1.25	$[0.1789 \quad -45.8572]$	$I_{2\times 2}$
Our method	0.1287	1.4137	$[-0.0002 \quad -19.8295]$	$I_{2\times 2}$
Our method	0.1287	1.4137	-29.1439	$[0 \ \ 1]$
Palhares et al. (2005)	19.12	6	$[-279.35 \quad -343.63]$	$I_{2\times 2}$
Our method	19.12	9.8185	$[-261.2491 \quad -282.4425]$	$I_{2\times 2}$

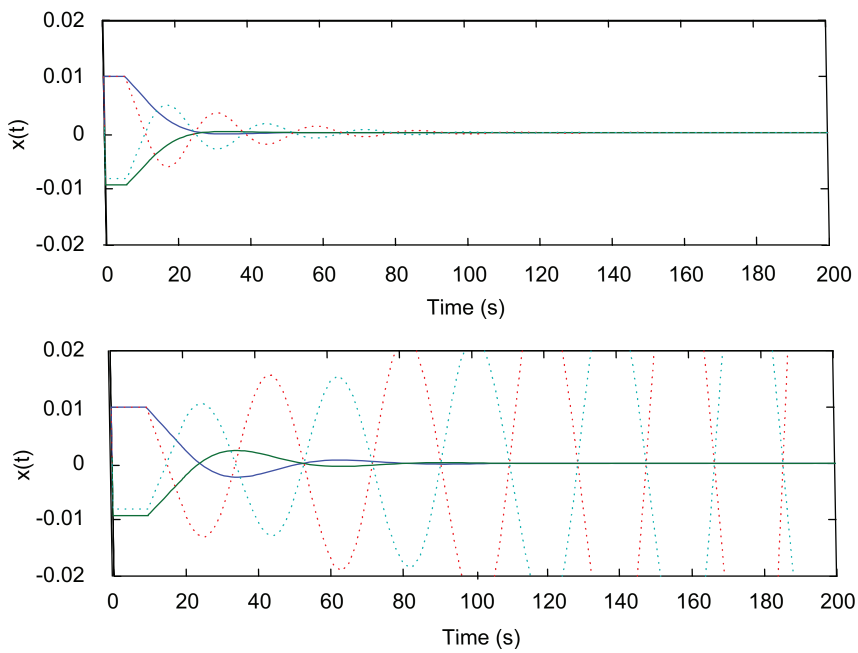

Fig. 2. Unit impulse time response of closed-loop system with controllers $[-261.2491 \quad -282.4425]$ (solid line) and $[-279.35 \quad -343.63]$ (dotted line) for time-delay $\tau = 6.000\,\text{s}$ (top) and $\tau = 9.8185\,\text{s}$ (bottom).

5. Conclusion

This paper presents an algorithm in designing delay-dependent memoryless controllers for time-delay systems. By using GAs to search for the possible controller gain and solving a set of LMIs, the required controller gain matrix can be determined. Some structure-specified controllers can be obtained as well. Although somewhat computational efforts are required to obtain such controllers, the obtained results on the numerical examples are less conservative than those obtained by previously existing methods in that the obtained controllers allow larger delays bound under the same performance requirements.

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