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by

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Can Negative EV Gambling be Rational?
An Analysis of a Doubling Scheme for Roulette

David Edelman

It is well-known (e.g., Wald (1947)) that complicated betting strategies and stopping times cannot turn unfavourable games into favourable ones. While economists tend to attribute individuals' willingness to play such games to irrationality (as might be modeled by increasing marginal utility, for example) the present paper presents an example which suggests an alternative model, one which suggests that negative expected-value (EV) gamblers may in some instances be behaving rationally after all.

It is a practical fact that with regard to personal finances, individuals' effective time value of money differs greatly, depending primarily on their level of personal wealth. An individual with no equity who needs to borrow might be able to obtain an unsecured loan for a limited amount, at an interest rate which is somewhat higher than current short-term interest rates. By contrast, an individual with equity, on the other hand, will generally be able to obtain secured credit at a rate just above the current interest rate. An individual who is wealthy enough to not need borrowing, but rather has money to invest will generally be able to earn a rate just below the current interest rate. Thus, it appears that generally the less well-off an individual is, the higher that individual's effective discount rate characterising time value of money.

In what follows below, an analysis of a particularly popular 'doubling scheme' will be analysed from this point of view, and it will be shown that for a sufficiently high discount rate such a scheme, while having a decidedly negative EV, will have a positive net present value (NPV), and may be thought of as a method of 'time leveraging'.

An Example

Consider the doubling strategy in which a gambler plays Roulette repeatedly, betting on 'Black', doubling bets until a win occurs. Let $p < \frac{1}{2}$ be the probability of 'Black' occurring on a single play, and suppose cash flows are discounted at rate r per play. For notational ease, let $a = \frac{1}{1+r}$.

If the gambler has enough capital for a maximum of N plays, the Expected Net Present Value of Return is given by

$$p \cdot a \cdot 1 + (1-p)p \cdot a^2 \cdot 1 + (1-p)^2 p \cdot a^3 \cdot 1 + \cdots + (1-p)^{N-1} p a^N \cdot 1 - (1-p)^N a^N (2^N - 1)$$

$(1 + 2 + 4 + \cdots + 2^{N-1} = 2^N - 1$ being the loss incurred in the case of N straight losing plays).

If p in the above equation were $\frac{1}{2}$ and a were 1, the expectation would reduce to

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^N} - \frac{2^N - 1}{2^N}$$

$$= \frac{\frac{1}{2} - \frac{1}{2}^{N+1}}{1 - \frac{1}{2}} - (1 - \frac{1}{2^N})$$

$$= (1 - \frac{1}{2^N}) - (1 - \frac{1}{2^N}) = 0,$$

regardless of N.

Returning to the more general expression, the NPV equals

$$\frac{p}{1-p}\{(a(1-p)) + (a(1-p))^2 + \cdots + (a(1-p))^N\} - (a(1-p))^N(2^N - 1)$$

$$= \frac{p}{1-p}\frac{a(1-p) - (a(1-p))^{N+1}}{1 - a(1-p)} - (a(1-p))^N(2^N - 1).$$

If $a(1-p) < \frac{1}{2}$, then as the Horizon N increases, the above expression tends to

$$\frac{p}{1-p}\frac{a(1-p)}{1-a(1-p)} = \frac{ap}{1-a(1-p)}$$

Thus, it appears that the NPV is positive even if $p < \frac{1}{2}$, provided $a < 2(1-p)$.

In this case, instead of exhibiting increasing marginal utility, it is plausible that gamblers might be optimising NPV, but exhibiting 'impatience', as modeled by a discount factor a which could be significantly less than 1. This might also explain why gambling as a phenomenon is more prevalent among lower income groups, whose effective personal discount rate is the highest.

References

Wald. A. (1947) *Sequential Analysis.* New York: Wiley

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