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Ibrahim S. Raad
University of Wollongong, ibrahim@uow.edu.au

Xiaojing Huang
University of Wollongong, huang@uow.edu.au

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Abstract

This paper presents an analytical study on the new spreading matrix for block spread OFDM known as the rotation spreading matrix and simulates the BER performance in the MMSE equation. In (I. Raad et al., 2006) a new spreading matrix was presented for block spread OFDM (BSOFDM) known as the rotation spreading matrix. It was shown to greatly improve on the existing spreading matrices such as the Hadamard and the rotated Hadamard matrices in frequency selective channels such as the ultra wide band and slow fading channels. This paper studies the effect of increasing the block size in BSOFDM using this new matrix.

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ANALYTICAL STUDY OF THE ROTATION SPREADING MATRIX OF BLOCK SPREAD OFDM WITH MMSE EQUALIZATION

Ibrahim S. Raad and Xiaojing Huang

School of Electrical, Computer and Telecommunications Engineering
University of Wollongong,
Wollongong, N.S.W Australia
ibrahim.raad@gmail.com

ABSTRACT

This paper presents an analytical study on the new spreading matrix for Block Spread OFDM known as the Rotation Spreading Matrix and simulates the BER performance in the MMSE equation. In [1] a new spreading matrix was presented for Block Spread OFDM (BSOFDM) known as the Rotation Spreading Matrix. It was shown to greatly improve on the existing spreading matrices such as the Hadamard and the Rotated Hadamard matrices in frequency selective channels such as the ultra wide band and slow fading channels. This paper studies the effect of increasing the block size in BSOFDM using this new matrix.[1]

Key Words-*Rotation Spreading Matrix, Block Spread OFDM, Frequency selective channel, MMSE decoder.*

1. INTRODUCTION

Spreading matrices for OFDM based systems have been studied and have shown to greatly improve the overall system performance over frequency selective channels due to their orthogonal properties. The new systems have been called Block Spread OFDM (BSOFDM). This is where the N subcarriers of the OFDM systems are divided into M sized blocks. The M sized blocks are spread using spreading matrices such as the Hadamard matrix which in turn increases the correlation between the transmitted symbols. This ensures that in frequency selective channels the losses are minimized. Using block size of $M = 2$ for example, it can achieve a modulation scheme of 16QAM, using a block size of $M = 4$ it can achieve a modulation scheme of 64QAM when the modulation used at the transmission is QPSK. This is not the same as adaptive modulation where the data is retransmitted, this spreading increases the correlation between the transmitted symbols and therefore improves BER in frequency selective channels.

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A new spreading matrix for BSOFDM called Rotation Spreading matrix was proposed in [1] to increase the correlation between the symbols through the use of a rotation angle α, and depending on the rotation angle, α, a new and higher order modulation is achieved in the transmission of the system to increase the correlation between the transmitted symbols to improve the BER performance through frequency diversity. This matrix was shown to outperform other spreading matrices such as the Hadamard matrix and the Rotated Hadamard matrix in frequency selective channels such as the IEEE defined UWB channels. In [3] a method was proposed to obtain higher order Rotation Spreading matrix to be used with larger M sized blocks for BSOFDM. For example, when a system requires the use of a larger M sized block, this must then use a higher order Rotation Spreading matrix. The higher order Rotation Spreading matrix again showed that it outperformed the other mentioned matrices in frequency selective channels as well as slow fading channels. The reason for this was due to the Rotation Spreading matrix maintaining its orthogonality.

Its structure is as follows for $U_2 = 2 \times 2$,

$$U = \begin{bmatrix} 1 & tan(\alpha) \\ tan(\alpha) & -1 \end{bmatrix} \quad (1)$$

The angle α is chosen depending on the system requirements and a simulation studies [2],[3], [4] and [5] comparing different angles performance can be seen in Figure 1, comparing angles for $\alpha = \frac{pi}{6}, \frac{\pi}{3}, \frac{\pi}{7}$ in UWB channel $CM1$ using $N = 64$ subcarriers and ML decoding.

The structure of this paper is as follows. In Section 2 presents the overall system description of BSOFDM, which is also referred to as pre-coding. Section 3 provides equations for MMSE decoder based on the system description given in Section 2. The simulation comparing different sizes of blocks for the BSOFDM using the MMSE decoder are presented in Section 4 and finally a conclusion is given in Section 5 and some future work and recommendations.

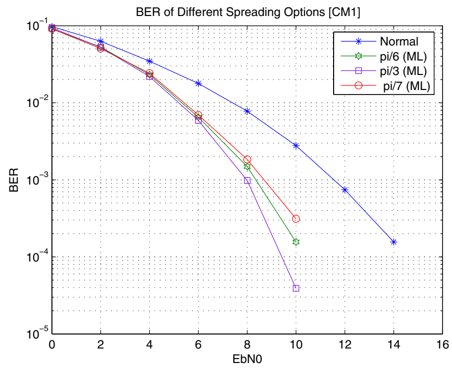

Fig. 1. Comparing the Rotation matrix with angles $\frac{pi}{6}$, $\frac{pi}{3}$ and $\frac{pi}{7}$ in UWB channel $CM1$ [2].

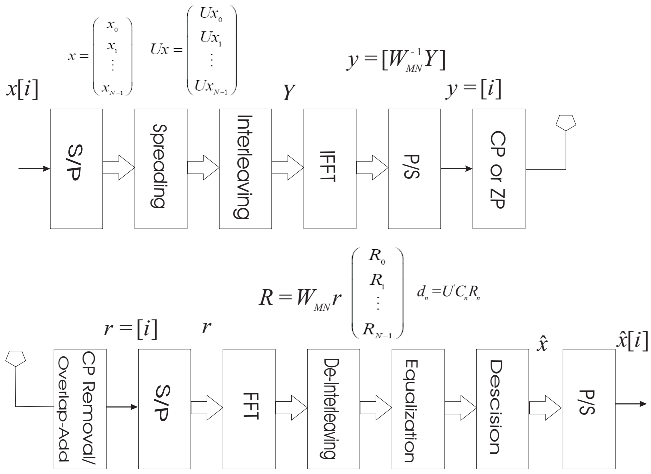

Fig. 2. Block diagram representation of the BSOFDM system

2. SYSTEM DESCRIPTION OF BSOFDM

Referring to the block diagram of the transmitter model shown in Figure 2 for BSOFDM, let $x[i], i = 0, 1, ..., MN-1$ denote MN data symbols (M and N are integer powers of 2), which are modulated from the information data bits after binary phase shift keying (BPSK), quadrature phase shift keying (QPSK) or any other quadrature amplitude modulation (QAM) constellation mapping.

Before spreading using spreading matrices (also known as precoding), the MN data symbols are firstly divided into N groups of size M with the n^{th} group denoted as a vector where $n = 0, 1, ..., N-1$ and $(.)^T$ denotes matrix transposition,

$$x_n = (x[nM], x[nM+1], .., x[nM+M-1])^T \quad (2)$$

This is then expressed as a vector after serial-to-parallel conversion (S/P),

$$x = \begin{pmatrix} x_0 \\ x_1 \\ . \\ x_{N-1} \end{pmatrix}$$

The spreading process is to apply an $M \times M$ unitary matrix U, which satisfies the following property where $(.)^{'}$ denotes transposition and complex-conjugation operation and the I is the identity matrix of order M, to each vector x_n to produce a precoded vector where each element is a linear combination of the symbols in vector x_n.

$$\begin{aligned} UU^{'} &= U^{'}U \\ &= I \end{aligned} \quad (3)$$

The precoded symbols are mapped onto subcarriers equally spaced across the transmitted bandwidth to better exploit frequency diversity. This is equivalent to a block interleaving operation among N precoded vectors Ux_n, where $n = 0, 1, ..., N-1$, and then performing IFFT of length MN on the resulting precoded and interleaved vector Y.

A time domain sequence $y[i], i = 0, 1, ..., MN-1$ is produced after IFFT and parallel to serial conversion (P/S). Either a cyclic prefix (CP) or zero padding (ZP) of sufficient length (longer than the maximum channel multipath delays in samples) are added to $y[i]$ to form a precoded OFDM symbol. This is done to avoid interference between adjacent precoded OFDM symbols and turn the linear convolution of the transmitted signal with the channel impulse response into a circular one. The precoded OFDM signal is then transmitted over a frequency selective multipath fading channel and received at the receiver baseband. By removing the CP or performing an overlap-add operation, $MN-point$ received precoded OFDM samples $r[i], i = 0, 1, ..., MN-1$, will be produced. After FFT and de-interleaving, the discrete-time received signal can be expressed in the frequency domain as, where $n = 0, 1, ..., N-1$

$$R_n = H_n U x_n + V_n \quad (4)$$

and

$$R_n = (R[n], R[N+n], .., R[(M-1)N+n])^T \quad (5)$$

is a vector of M elements which are decimated from $R[k]$, the $MN-point$ discrete Fourier transform (DFT) or $r[i]$, by a down-sampling factor N.

$$\begin{aligned} H_n = \; & diag(H[n], H[N+n], .., \\ & H[(M-1)N+n]) \end{aligned} \quad (6)$$

is an $M \times M$ diagonal matrix with diagonal elements decimated from $H[k]$, the $MN - point$ DFT of the normalized discrete channel impulse response $h[i]$, and V_n is a zero-mean Gaussian noise vector with covariance matrix $E\{V_n V_n^{'}\} = \sigma_v^2 I$, where $E\{\}$ denotes ensemble average.

To recover the transmitted data vector x_n, equalization and detection must be performed on the received signal R_n. Due to the complexity of Maximum Likelihood Decoder (ML) which is not considered to be practical in many system, only the Minimum Mean Square Error decoder (MMSE) is considered since it can simply use a one tap equalizer for each subcarrier in the frequency domain.

The equalization and detection process can be described as follows. Let $C[k]$ denote the one tap equalizer coefficient to be applied to the received signal $R[k]$ on the subcarrier k and the following denotes an $M \times M$ diagonal matrix,

$$\begin{aligned} C_n &= diag(C[n], C[N+n], .., \\ &\quad C[(M-1)N+n]) \end{aligned} \tag{7}$$

with diagonal elements $C[lN+n]$, where $l = 0, 1, .., M-1$. First, applying C_n to R_n produces the equalized precoded data vector $C_n R_n$. Secondly, using the $U^{'}$ (which is the inverse of the spreading matrix) to remove the precoding yields the decision variable vector $d_n = U^{'} C_n R_n$. Finally, an estimate of the transmitted data vector x_n is obtained after hard decision. Repeating the above process for $n = 0, 1, .., N-1$, all the transmitted data symbols are retrieved.

3. PERFORMANCE OF MMSE EQUALIZATION

The first step is to derive the post-equalization signal to noise ratio (SNR) as a function of the equalizer coefficients $C[lN+n]$ for the received signal vector R_n. The decision variable vector, according to the equalization process described above, can be expressed as,

$$\begin{aligned} d_n &= U^{'} C_n R_n \\ &= U^{'} C_n H_n U x_n + U^{'} C_n V_n \end{aligned} \tag{8}$$

Assume that the data symbols in x_n are independent with average power σ_x^2 so that $E\{x_n x_n^{'}\} = \sigma_x^2 I$. The covariance matrix of d_n can be derived as follows,

$$\begin{aligned} E\{d_n d_n^{'}\} &= \sigma_x^2 U^{'} C_n H_n H_n^{'} C_n^{'} U + \\ &\quad \sigma_v^2 U^{'} C_n C_n^{'} U \end{aligned} \tag{9}$$

Suppose that we want to decide the m^{th} data symbol $x[nM+m]$ in x_n from the m^{th} element in d_n. The useful signal component can be found from the first term on the right hand side of Equation 8 as,

$$\sum_{l=0}^{M-1} C[lN+n] H[lN+n] |u_{l,m}|^2 \cdot x[nM+m] \tag{10}$$

where $u_{l,m}$ is an element of U at the l^{th} row and the m^{th} column, and thus the useful signal power after equalization is as follows,

$$\left| \sum_{l=0}^{M-1} C[lN+n] H[lN+n] |u_{l,m}|^2 \right|^2 \sigma_x^2 = q_0[n,m] \tag{11}$$

The average power of the m^{th} element in d_n can also found from Equation 9 as follows,

$$\begin{aligned} &\sigma_x^2 \sum_{l=0}^{M-1} |C[lN+n] H[lN+n] u_{l,m}|^2 \\ &\quad + \sigma_v^2 \sum_{l=0}^{M-1} |C[lN+n] u_{l,m}|^2 \qquad (12) \\ &= q_0[n,m] \qquad (13) \end{aligned}$$

Therefore, the output SNR after equalization can be expressed as follows,

$$\gamma[n,m] = \frac{q_0[n,m]}{q_1[n,m] - q_0[n,m]} \tag{14}$$

According to the MMSE criterion, C_n should be designed so that the following,

$$\begin{aligned} E\{(d_n - x_n)^{'} (d_n - x_n)\} &= \\ E\{(U d_n - U x_n)^{'} (U d_n - U x_n)\} &= \\ E\{(C_n R_n - U x_n)^{'} (C_n R_n - U x_n)\} & \end{aligned} \tag{15}$$

is minimized. Using the orthogonality principle, we have

$$E\{(C_n R_n - U x_n) R_n^{'}\} = 0 \tag{16}$$

and consequently,

$$\begin{aligned} C_n &= E\{Ux_nR_n'\}(E\{R_nR_n'\})^{-1} \\ &= UE\{x_nx_n'\}U'H_n'(H_nUE\{x_nx_n'\} \\ &\quad U'H_n' + E\{V_nV_n'\})^{-1} \\ &= H_n'\left(H_nH' + \frac{1}{\gamma_{in}}I\right)^{-1} \end{aligned} \tag{17}$$

where $\gamma_{in} = \frac{\sigma_x^2}{\sigma_v^2}$ is the input SNR before equalization.

From Equation 17, the diagonal element is found to be,

$$C[lN+n] = \frac{H^*[lN+n]}{|H[lN+n]|^2 + \frac{1}{\gamma_{in}}} \tag{18}$$

Substituting Equation 18 into Equation 11 and 12 and using Equation 14,

The output SNR after MMSE equalization is finally expressed as,

$$\gamma[n,m] = \frac{\sum_{l=0}^{M-1} \frac{|H[lN+n]u_{l,m}|^2}{|H[lN+n]|^2+\frac{1}{\gamma_{in}}}}{1-\sum_{l=0}^{M-1} \frac{|H[lN+n]u_{l,m}|^2}{|H[lN+n]|^2+\frac{1}{\gamma_{in}}}} \tag{19}$$

We see that the output SNR is determined by the channel frequency response $H[k]$, or equivalently, the channel impulse response $h[i]$. Assuming QPSK modulation for data symbols and making a Gaussian distribution approximation for ISI, the bit error probability of the equalizer for a realization of the channel impulse response can be evaluated as,

$$\frac{1}{N}\sum_{n=0}^{N-1} Q(\sqrt{\gamma[n]}) \tag{20}$$

where the Q-function is defined as

$$Q(x) = \frac{1}{\sqrt{(2\pi)}}\int_x^{\infty} e^{-\frac{t^2}{2}}dt \tag{21}$$

and also assuming that the channel impulse response has L independent paths, each of which is modelled as an independent complex Gaussian process, the average BER for such frequency-selective fading channel can be evaluated as, where $E_h\{.\}$ denotes the ensemble averaging over all possible $h[i]$. It can be seen that Equation 22 is a function of the data group size M and the multipath length L.

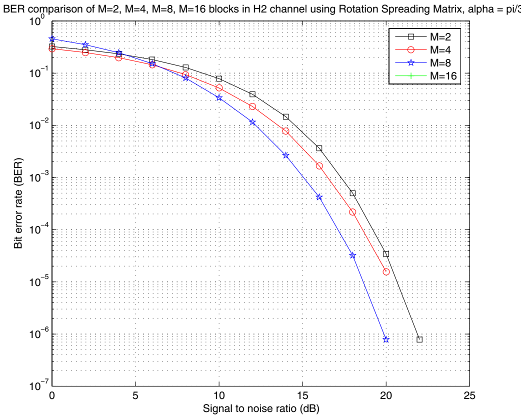

Fig. 3. Simulation results comparing different block sizes, using Rotation Spreading Matrix with $\alpha = \frac{\pi}{3}$ $N = 64$, $H2$ channel,

$$P_e(M,L) = E_h\left\{\frac{1}{N}\sum_{n=0}^{N-1} Q(\sqrt{\gamma[n]})\right\} \tag{22}$$

4. RESULTS

The following simulation results shown below use 10000 BSOFDM packets, using for the spreading matrix the Rotation Spreading matrix with rotation angle $\alpha = \frac{\pi}{3}$, subcarriers $N = 64, 128, 512$ and two ray fading channel.

As can be seen from the results in Figures 3, 4 and 5, as the block sizes increased the system performance in terms of BER improved. As an example of better results using larger blocks for the block size $M = 16$ using $N = 64$ subcarriers yields BER of zero, which was also true for $N = 128$ and $N = 512$ subcarriers.

5. CONCLUSION

This paper presented equations which represent the Rotation Spreading matrix using a MMSE decoder at the receiver in a BSOFDM system. It can be concluded from the results that with the increase in the group size M the performance improved in terms of BER improved. The MMSE decoder is a practical solution to decoding due to its performance and less complexity, since this decoder utilizes one tap equalization in the frequency domain. This paper again shown the excellent properties that the new spreading matrix Rotation Spreading matrix shows when used for systems such as BSOFDM. Future work would be to study this spreading matrix for all linear decoders.

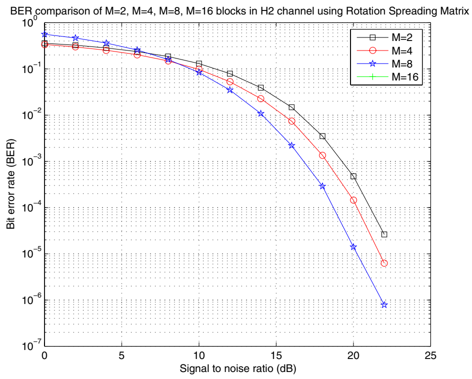

Fig. 4. Simulation results comparing different block sizes, using Rotation Spreading Matrix with $\alpha = \frac{\pi}{3}$ $N = 128$, $H2$ channel,

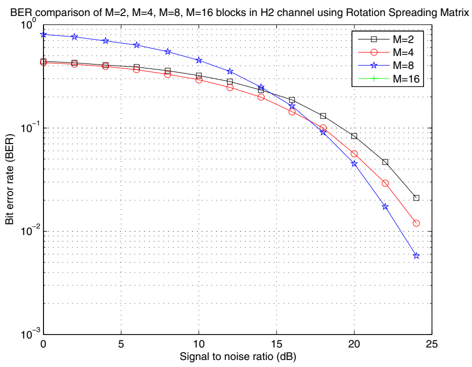

Fig. 5. Simulation results comparing different block sizes, using Rotation Spreading Matrix with $\alpha = \frac{\pi}{3}$ $N = 512$, $H2$ channel,

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