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Effects of the Field Dependent J_C on the Vertical Levitation Force Between a Superconductor and a Magnet

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Abstract—The vertical levitation force between a supercon-ductor disk (SC) and a permanent magnet disk (PM) has been calculated from first principles using different $J_c(B)$ relation-ships on the magnetic field. Based upon the first principles, the current distribution inside the SC induced by the applied inhomogeneous magnetic field generated by the PM and the field profiles have been calculated with a power law $E \sim J$ relationship: $E(J) = E_c(J/J_c(B))^n$. The levitation force is highly hysteretic for the approaching and the retreating branches. The saturated current value, magnetization, and levitation force are found to depend strongly on the $J_c(B)$ relations. Features of the supercurrent distribution, the force loop, and the levitation force density are discussed.

Index Terms—Field-dependent critical current, force loop, levi-tation force, superconductor, supercurrent distribution.

I. INTRODUCTION

THE superconducting levitation system has been in-tensively studied both experimentally [1]–[3] and theoretically [4]–[8] to understand its physical properties and the prospects for application in many areas. Because of its passive and self-stabilizing features, possible applications of superconducting levitation systems include frictionless bear-ings for gyroscopes and energy storage flywheels, momentum wheels, high-speed machine tools, Mag-Lev trains and other levitated transport systems, etc.

To analyze the levitation force, the most formidable task is to determine the supercurrent distribution in superconductors of finite sizes under inhomogeneous fields. Due to the history-de-pendent electromagnetic behavior of type-II super-conductors and the complicated demagnetizing effect in an inhomogeneous field, assumptions and simplifications have to be introduced in the early works. In [9], the applied magnetic field is assumed to be constant gradient, thus the current profile can be easily obtained. Some calculations exploit the Bean model to preset a current distribution and then adjust the boundary of the cur-rent-flowing area by fitting to experimental data. Later some im-

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proved results have been made to consider the field-dependent J_c by using Kim-type $J_c(B)$ dependence [6].

Early calculations of the levitation force neglected the de-magnetizing effect, which is substantial in most of the realistic configurations in both experimental research and application prototypes. It is not proper to describe the behavior of a finite superconductor in an external field by a uniform demagnetiza-tion factor, except in very special cases, such as an ellipsoidal cylinder in a uniform applied field along its symmetrical axis. A noticeable progress in the calculation of the levitation force for a finite superconducting cylinder has been published recently [10], [11] in which the supercurrent profile is obtained by mini-mizing the energy of the magnetic field of the supercurrent and the applied field. The space-dependent demagnetization is con-sidered for both uniform and nonuniform applied fields.

An improved method to study the behavior of finite thick-ness superconductor of arbitrary cross-section under an inho-mogeneous field $B_a(r, t)$ has been developed by E. H. Brandt [12]–[14]. From first principles, the supercurrent induced by the applied field can be obtained by time integration of an integral equation without assumptions or simplifications. By using this approach, the levitation force between a superconductor disk and a permanent disk has been calculated [15]. In this paper, we present a calculation of levitation force using Kim-type $J_c(B)$ dependence as an improvement on the work in [15], in which J_c is a constant.

II. MODEL AND EXPRESSIONS

We select a simple but typical configuration that consists of a superconducting (SC) disk and a co-axially placed perma-nent magnet (PM) disk beneath the SC; such a configuration is widely used for experimental and theoretical studies. We as-sume that the magnetization of the permanent magnet is uni-form and along the axis of symmetry, and the superconductor is homogeneous. The system is in an infinitive space without any other object. In this case all physical quantities are axially sym-metrical, the induced current is circular around the axis and the levitation force is along the axis.

The symmetry of this system and its infinite boundary condi-tion allow the calculation to be simplified into a 2-D problem. All physical properties in 3-D space can be characterized by their values in a plane along the axis of symmetry. In a cylin-drical coordinate system (ρ, ϕ, z) with its z-axis going through the central axis of the PM and SC disks, the magnetic field, the magnetization vector and Ampere's force have no ϕ compo-

nent while the vector potential and the supercurrent have only ϕ components.

The levitation force is calculated by:

$$\vec{F}_{Lev} = \int d^3\vec{r} \left(\vec{J}_S \times \vec{B}_A \right) \qquad (1)$$

with $\vec{J}_S$ the supercurrent density and $\vec{B}_A$ the applied magnetic field from a permanent magnet or other source. The total magnetic field can be written as

$$\vec{B} = \vec{B}_{SC} + \vec{B}_{PM} = \nabla \times \vec{A} = \nabla \times \left(\vec{A}_{SC} + \vec{A}_{PM} \right). \quad (2)$$

Because $\nabla \times \vec{B} = \mu_0 \vec{J}$, one can obtain

$$\mu_0 \vec{J}_S(\vec{r}) = -\nabla^2 \left(\vec{A}_{SC} + \vec{A}_{PM} \right) \qquad \vec{r}, \vec{r}' \notin V_{PM}. \quad (3)$$

The solution of this Poisson equation is [13]

$$\vec{A}_{SC} + \vec{A}_{PM} = -\mu_0 \int K(\vec{r}, \vec{r}') \vec{J}_s(\vec{r}') d^3\vec{r}', \quad \vec{r}, \vec{r}' \notin V_{PM} \quad (4)$$

in which $K(\vec{r}, \vec{r}')$ is the integral kernel [15]. This expression can also be inverted and formally written as

$$\vec{J}_S(\vec{r}, t) = -\mu_0^{-1} \int K^{-1}(\vec{r}, \vec{r}')$$
$$\cdot \left[\vec{A}_{SC}(\vec{r}', t) + \vec{A}_{PM}(\vec{r}', t) \right] d^3\vec{r}'. \quad (5)$$

Here $K^{-1}(\vec{r}, \vec{r}')$ is the inverse kernel defined as

$$\int K^{-1}(\vec{r}, \vec{r}'') K(\vec{r}', \vec{r}'') d^3\vec{r}'' = \delta(\vec{r} - \vec{r}'), \quad \vec{r}', \vec{r}'' \in V_{SC}. \quad (6)$$

According to the induction law $\nabla \times \vec{E} = -\dot{\vec{B}} = -\nabla \times \dot{\vec{A}}$, we have $\vec{E} = -\dot{\vec{A}}$. Then we have

$$\dot{\vec{J}}(\vec{r}) = \mu_0^{-1} \nabla^2 (\vec{E}_{SC} - \dot{\vec{A}}_{PM}). \quad (7)$$

That is

$$\dot{J}(\vec{r}, t) = \mu_0^{-1} \int K^{-1}(\vec{r}, \vec{r}')$$
$$\cdot \left[\vec{E} \left(\vec{J}, \vec{B}, t \right) - \dot{\vec{A}}_{PM}(\vec{r}', t) \right] d^3\vec{r}'. \quad (8)$$

Considering $\vec{J}_S(\vec{r}, t = 0) = 0$ in zero-field-cooling case, finally we can obtain constant distribution by numerical time integration of this expression using proper material equation $\vec{E}(\vec{J}, \vec{B}, t)$ and arbitrary time-varying applied field, plus the initial condition $\vec{J}_{SC}(\vec{r}, t = 0) = 0$.

The vector potential and the magnetic field of the PM disk can be written as [15]:

$$A_\phi(\rho, z) = \frac{B_{rem}}{2\pi} \int_0^\pi R_{PM} \cos\phi \ln \frac{(z + t_{PM}) + L_1}{z + L_2} d\phi \quad (9)$$
$$L_1 = \sqrt{R_{PM}^2 + \rho^2 - 2\rho R_{PM} \cos\phi + (z + t_{PM})^2}$$
$$L_2 = \sqrt{R_{PM}^2 + \rho^2 - 2\rho R_{PM} \cos\phi + z^2}$$
$$B_\rho(\rho, z) = \frac{B_{rem}}{2\pi} \sqrt{\frac{R_{PM}}{\rho}} \sum_{i=0}^1 \frac{(-1)^i}{k_i}$$
$$\cdot \left[\left(1 - \tfrac{1}{2} k_i^2 \right) K(k_i) - E(k_i) \right], \quad (10)$$

$$B_z(\rho, z) = \frac{B_{rem}}{2\pi} \int_0^\pi \frac{\rho R_{PM} \cos\phi - R_{PM}^2 \cos^2\phi}{R_{PM}^2 + \rho^2 - 2\rho R_{PM} \cos\phi}$$
$$\cdot L_3 \cdot d\phi + \frac{A_\phi}{\rho} \quad (11)$$

$$L_3 = \sum_{i=0}^1 \frac{(-1)^i (z + it_{PM})}{\sqrt{R_{PM}^2 + \rho^2 - 2\rho R_{PM} \cos\phi + (z + it_{PM})^2}} \quad (12)$$

where R_{PM}, t_{PM} are the radius and thickness of PM, and $K(k)$, $E(k)$ are complete elliptic integrals

$$k_i^2 = \frac{4\rho R_{PM}}{(R_{PM} + \rho)^2 + (z + it_{PM})^2}, \qquad i = 0, 1. \quad (13)$$

The power law expression $E(J) = E_c(J/J_c)^n$ is used as a fairly good description of the E–J relation of a high-Tc superconductor. It characterizes the behavior of super-conductors from a linear E–J relation ($n = 1$) through the flux creep zone ($1 < n < \infty$) to an abrupt transition limit ($n \to \infty$).

The $J_c(B)$ dependence is a significant feature of all kinds of superconductors. In order to investigate the effect of the field-dependence of the critical current on the levitation force, we use the Kim-type $J_c(B)$ expression as $J_c(B) = C/(B + B_0)$, where C and B_0 are constants. This expression can be transformed into $J_c(B) = J_{c0}/(C_k B + 1)$, in which a coefficient $C_k = 1/B_0$ is defined to represent the sensitivity of critical current density to the magnetic field. When $C_k = 0$, $J_c(B)$ becomes a constant as in the Bean model, which is used in this work for comparison.

III. RESULTS AND DISCUSSION

One set of parameters selected for the levitation system is as follows. The radius R and the height bb of the PM disk are set equal to each other. The radius a and the height b of the SC disk is twice as large as the PM's, which are similar to the geometric parameters in some experiments [8]. An intention in the selection is that the SC disk should span the space with high magnetic fields generated by the PM. J_{c0} is between a small value that allows full penetration of the applied field and a large value that can shield about 80% of the volume of the SC disk from the applied field.

For comparison, we calculate the supercurrent J_s distribution for both Bean model ($C_k = 0$) and Kim model ($C_k = 1$). Fig. 1 shows the J_s distribution of $C_k = 0$ and Fig. 2 of $C_k = 1$ at different stages of the approaching and retreating branches. Fig. 3 shows the levitation force (F_z) loops and the magnetization (M_a) loops for the above two models.

For the Kim model ($C_k = 1$), the profile of supercurrent in the approaching branch is highly nonuniform especially when the PM is close to the SC. In the region close to the PM, where the applied field is stronger, the current density is significantly depressed. After retreating, when the PM is returned to the original position far from the SC, the final profile of the supercurrent of the Kim model is flat along the central axis but increases from the center along a radius to its cylindrical surface. That is due to the depressing of J_c by the trapped field, which is stronger in the center than outside. On the other hand, for the Bean case ($C_k = 0$) the saturated supercurrent is very flat in all stages.

The volumes occupied by the induced supercurrent are different at all stages for both models. The Kim type SC disk is
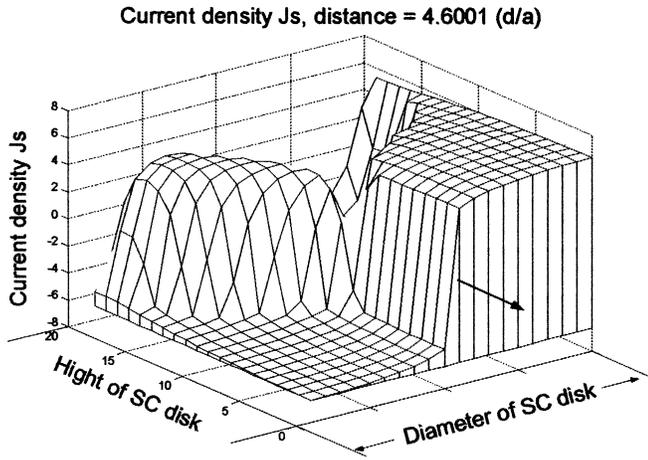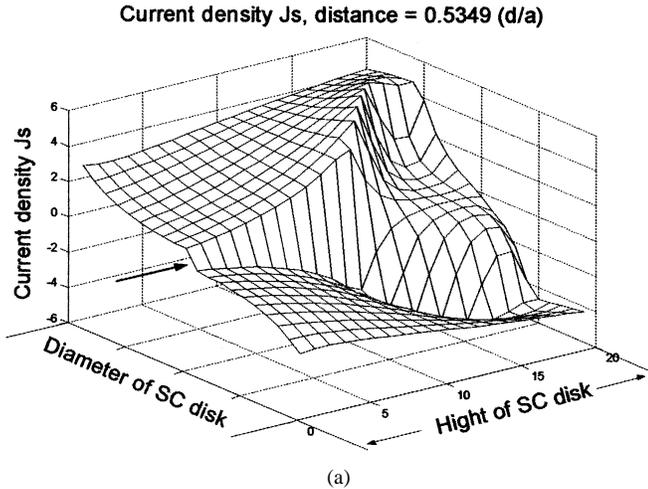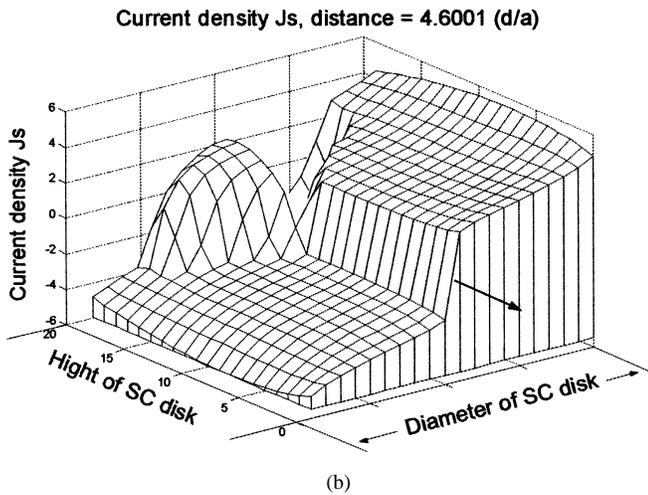
Fig. 1. Supercurrent J_s distribution according to the Bean model at the end of retreat of the PM. Arrow indicates the direction of retreat. Distance is normalized by radius a of the PM.
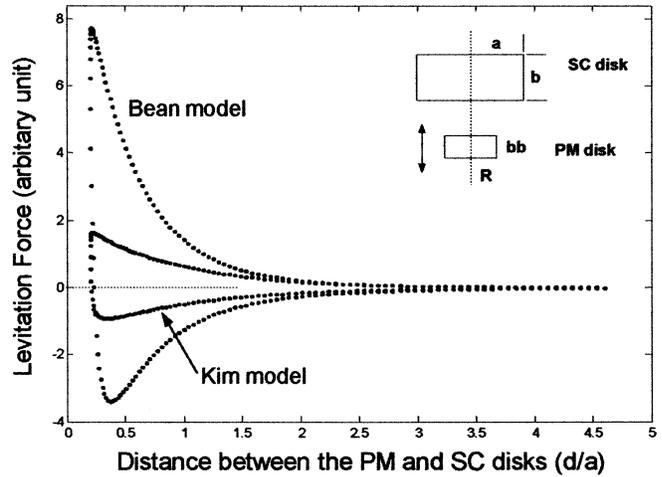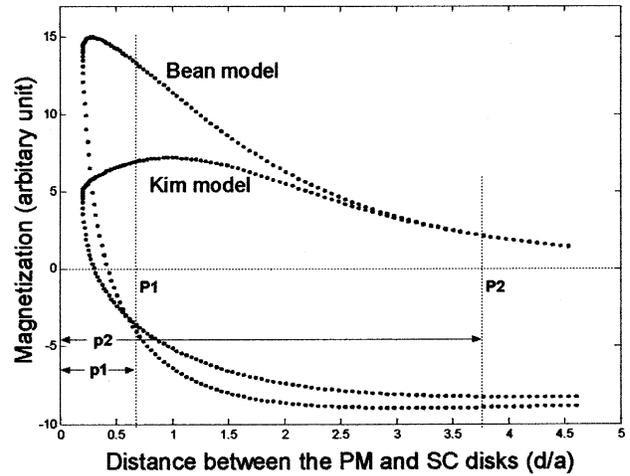

(a)

(b)

Fig. 2. Supercurrent J_s distribution of the Kim model in the late stage of: (a) approach and (b) end of retreat of the permanent magnet (PM). Arrows indicate the direction of approach and retreat of the PM.

fully penetrated by the magnetic field, while the Bean type SC sustains larger supercurrents, which shields the magnetic field from its inner volume.

(a)

(b)

Fig. 3. Levitation force loops and magnetization loops for both Bean-type $J_c(B)(C_k = 0)$ and Kim-type $J_c(B)(C_k = 1)$.

The magnetization loop for $C_k = 1$ shows that the maximum magnetization does not appear at the end of the approach but midway through the approach. This is due to the suppression of J_c by the applied field when the PM is close to the SC.

From Figs. 1 and 2, it is found that the negative current generated during approach cannot be fully reversed by decreasing the applied field. The volume of positive current induced during retreat does not cover the whole volume of negative current even when the applied field is very strong. This phenomenon can be explained as follows.

Consider any two positions P_1 and P_2 of the PM in Fig. 3. Let the distances of the PM and SC be p_1 and $p_2(p_1 < p_2)$. When the PM goes from P_2 to P_1 during approach, the induced supercurrent $\Delta J_{app\,max}$ at a certain point in the SC changes from zero to $J_c(B_{P1})$; while the PM goes from P_1 to P_2 during retreat, the induced supercurrent $\Delta J_{ret\,max}$ at the same point in the SC changes from $J_c(B_{P1})$ to $-J_c(B_{P2})$, the possible maximum change $\Delta J_{ret\,max} = J_c(B_{P1}) + J_c(B_{P2})$. For the Bean-type cases, $J_c(B_{P1}) = J_c(B_{P2})$, but for Kim-type cases, $J_c(B_{P1}) < J_c(B_{P2})$, so we get $\Delta J_{ret\,max} = J_c(B_{P1}) + J_c(B_{P2}) \geq 2J_c(B_{P1})$. In general the amplitude of the induced supercurrent change during retreat is at least two times larger
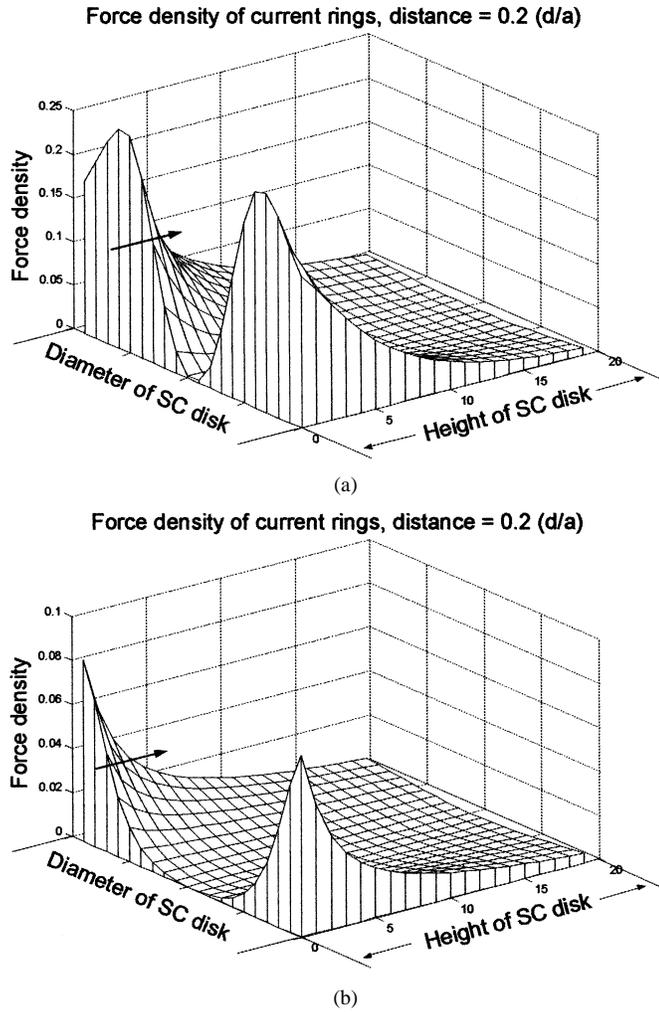
Fig. 4.   Force densities for: (a) the Bean model ($C_k = 0$) and (b) the Kim model ($C_k = 1$), each at the end of retreat, shown in arbitrary units. Arrows indicate the direction of approach of the PM.

then that during approach at any point in the SC. Larger response amplitude means better field shielding. The "new current," which is induced during retreat by the total field consisting of both the applied field and self-field, can never reach the far frontier of the "old" current if only the frontier is not limited by the boundary of the superconductor.

The sharp drop of F_z from the maximum repulsive force to an attractive force can be explained by the different ranges of supercurrent change. During approach, the range of supercurrent changes is from zero to $J_c(B)$, when the applied field decreases, the supercurrent changes from $J_c(B)$ to $-J_c(B)$, or even to a $J_c(B')$, which is larger in absolute value than $J_c(B)$ according to the Kim model.

In order to analyze the levitation force distribution, we define the relative force density D_F as the levitation force acting on supercurrent rings with the same cross-section but different radii: $D_F = 2\pi R(B_{ar} * J_s) \cdot \Delta S$. Here B_{ar} is the radial component of the applied field; ΔS is the cross-section of the current ring, R the ring radius. In Fig. 4 the force densities for Bean model

($C_k = 0$) and Kim model ($C_k = 1$) are shown. Negative D_F represents attractive forces. For $C_k = 0$ high force density is found along the circumference of the PM, while for $C_k = 1$ high force density is found along the circumference of the SC. It is important to increase the radius of the SC disk for achieving high repulsive or attractive force in the disk-disk configuration with Kim-type $J_c(B)$.

IV. CONCLUSION

Based upon first principles, the levitation force between a superconductor disk and a permanent magnet disk is calculated by using a field-dependent $J_c(B)$ according to the Kim model. The saturated supercurrent is suppressed by the magnetic field and becomes highly nonuniform. The maximum levitation force is just about 1/4 of that in the Bean model. The supercurrent induced during approach cannot be fully reversed after retreat. The levitation force density of current rings is calculated. In the Kim model the high force density region is localized in the outer section of the superconductor disk.

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