

Efficiency and equity analysis of toll pricing on Sydney Harbour Bridge with heterogeneous travellers

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Abstract: Sydney Harbour Bridge is a key transport infrastructure that connects North Sydney and Sydney Central Business District (CBD). To alleviate the congestion on Sydney Harbour Bridge, NSW Roads and Maritime Services imposes a time of day tolling between \$2.5 and \$4 on the southbound traffic to Sydney CBD. This study develops mathematical models for formulating the toll pricing problem on Sydney Harbour Bridge considering that different travellers may have different value-of-times (VOTs). The models examine quantitatively the effect of different toll levels on the efficiency (in terms of the total generalized travel time and generalized travel cost of all travellers) and equity (in terms of the ratio of generalized travel cost among different traveller classes). The proposed models can serve as a useful decision-support tool for NSW Roads and Maritime Services.

Key words: Toll pricing; Traffic equilibrium; Heterogeneous value-of-time; Sydney Harbour Bridge; Optimization.

I. Introduction

In view of the importance of the Sydney Harbour Bridge on transport efficiency in Sydney, this paper develops mathematical models for formulating the toll pricing problem on Sydney Harbour Bridge considering that different travellers may have different value-of-times (VOTs). The models examine quantitatively the effect of different toll levels on the efficiency and equity. The objective is to provide a useful decision-support tool for NSW Roads and Maritime Services.

II. Toll pricing with homogeneous travellers

We assume that there are two roads from North Sydney to CBD of Sydney: one via Sydney Harbour Bridge (link a) and the other via other bridges (link b), see Figure 1. Link b is longer and has a larger capacity than link a . The link travel time functions are assumed to be of the Bureau of Public Roads (BPR) form:

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$$t_i(v_i) = t_i^0 \left[1 + \alpha_i \left(\frac{v_i}{C_i} \right)^{\beta_i} \right], i \in \{a, b\} \quad (1)$$

where t_i^0 is the free-flow travel time (minutes), v_i (vph) is the traffic flow, C_i (vph) is the link flow capacity, and $\alpha_i > 0$ and $\beta_i > 1$ are two parameters.

We represent by q (vph) the travel demand from North Sydney to Sydney CBD, and we denote by v_a and v_b the flow on link a and b , respectively. The link flow must satisfy flow conservation equations. For simplicity, we define a set:

$$\Omega = \{(v_a, v_b) \mid v_a + v_b = q, v_a \geq 0, v_b \geq 0\} \quad (2)$$

containing all the possible link flow vector $v := (v_a, v_b) \in \Omega$.

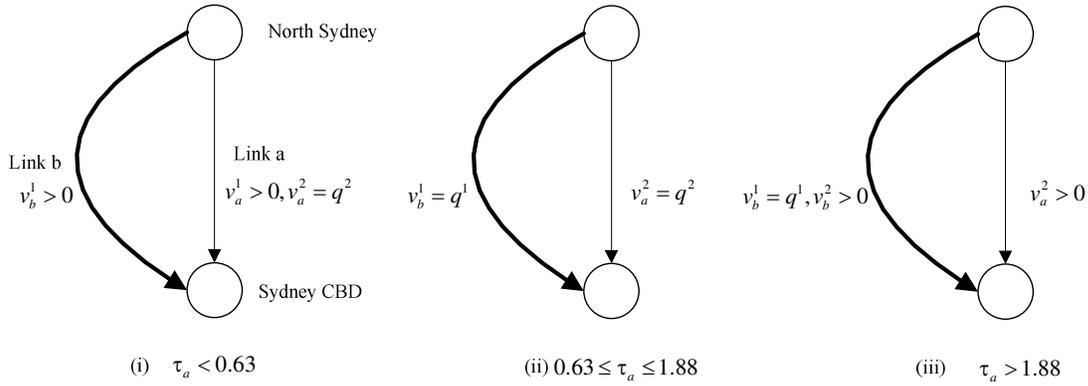


Figure 1. Transport flow with different tolls.

A. User equilibrium, system optimum, and optimal toll pricing

Since each traveller aims to minimize her/his travel time, at the user equilibrium (UE), if no toll is imposed, the travel times on the two links are identical $t_a(v_a) = t_b(v_b)$. We assume that all travelers are homogeneous with value of time (VOT) γ (\$/min). If the government imposes toll on both links denoted by $\tau := (\tau_a, \tau_b)$, the equilibrium is:

$$\gamma t_a(v_a) + \tau_a = \gamma t_b(v_b) + \tau_b \quad (3)$$

We can compute the resulting link flow v_a and v_b denoted by v^{UE} .

At system optimum (SO), the total travel time of all road users is minimized, and the SO link flow is denoted by v^{SO} . The optimal SO toll denoted by τ^* can be computed by the marginal social cost theory. Alternatively, the government may impose a non-zero toll on only one link by requiring that:

$$\tau_a = \tau_a^* - \min\{\tau_a^*, \tau_b^*\} \quad (4)$$

$$\tau_b = \tau_b^* - \min\{\tau_a^*, \tau_b^*\} \quad (5)$$

B. An example using Sydney Harbour Bridge

We use the case of Sydney Harbour Bridge to demonstrate the above models. The free-flow travel time of Sydney Harbour Bridge is calculated as follows. It is 1.149 km long and we assume a free-flow travel speed of 80 km/h. Hence, $t_a^0 = 0.86$ min. It has 8 traffic lanes controlled by Electronic Lane Changing System (ELCS). Therefore we assume that 6 lanes are used in the southbound direction during morning peak. We further assume that the capacity of one lane is 2000 vehicles/h. Consequently, $C_a = 12,000$. The coefficients $\alpha_a = 0.15$ and $\beta_a = 2$. We assume that the length of link b is 5km, its free-flow travel speed is 80 km/h, and it has 20 lanes. Therefore, $t_b^0 = 3.75$ and $C_b = 40,000$. The coefficients $\alpha_b = 0.15$ and $\beta_a = 4$. We assume $\beta_b > \beta_a$ because the traffic on Sydney Harbour Bridge is well controlled and regulated. We assume that the demand in the peak hour is 120,000 vph. The value of time $\gamma = 0.5$ \$/min. The results are reported in Table 1, where the row “Toll (\$4)” means a toll of \$4 is charged on Sydney Harbour Bridge and the column “TTT” means the total travel time of all travelers as defined below:

$$TTT = v_a t_a(v_a) + v_b t_b(v_b) \quad (6)$$

When \$4 is charged (which is the actual toll charge during morning and evening peak hours on weekdays), the flow on Sydney Harbour Bridge is 45,050. To achieve the SO traffic flow, the optimal toll charge on Sydney Harbour Bridge should be \$0.36. The toll charge \$4 is apparently too high, and the resulting TTT is not only larger than that under SO, but also significantly larger than that without any intervention (UE).

Table 1. Results with homogeneous travellers.

	v_a	v_b	$t_a(v_a)$	$t_b(v_b)$	τ_a	TTT
No toll (UE)	69,422	50,578	5.188	5.188	0	622,550
SO	66,480	53,520	4.829	5.553	0.36 ^c	618,219
Toll (\$4)	45,050	74,950	2.684	10.684	4	921,642

^c This is the optimal τ_a to achieve SO with $\tau_b = 0$.

III. Toll pricing with heterogeneous travelers

We examine heterogeneous travellers in this section. We consider M traveller classes, and the VOT of class $m = 1, 2, \dots, M$ is denoted by γ^m (\$/min). The demand of class m is q^m . The number of travellers from class m on link i is denoted by v_i^m . For simplicity, we define a set:

$$\hat{\Omega} = \left\{ (v_a^m, v_b^m, m = 1, 2, \dots, M) \mid v_a^m + v_b^m = q^m, v_a^m \geq 0, v_b^m \geq 0, \forall m \right\} \quad (7)$$

containing all the possible link flow vector $v := (v_a^m, v_b^m, m = 1, \dots, M) \in \hat{\Omega}$.

C. User equilibrium, system optimum, and optimal toll pricing

With heterogeneous travellers, there are two types of SO: SO in cost units and SO in time units. The SO model in cost units can be formulated as:

$$\min \sum_{m=1}^M \gamma^m [v_a^m t_a(v_a) + v_b^m t_b(v_b)] \quad (8)$$

According to Ref. 1, the optimal toll can be computed.

D. An example using Sydney Harbour Bridge

We look at the Sydney Harbour Bridge example again, and assume two classes: $\gamma^1 = 0.25$, $\gamma^2 = 0.75$, $q^1 = 60,000$ and $q^2 = 60,000$. Therefore the total demand and the average VOT are the same as sub-section II.B.

The results are reported in Table 2, where the column “*Toll (\$4)*” means a toll of \$4 is charged on Sydney Harbour Bridge. The column “*TTT*” means the total travel time of all travellers defined by Eq. (6). The column “*TTC*” means the total travel cost of all travellers defined below:

$$TTC = \sum_{m=1}^M \gamma^m [v_a^m t_a(v_a) + v_b^m t_b(v_b)] \quad (9)$$

We can see that to minimize total travel cost, a toll of \$1.92 should be imposed on Sydney Harbour Bridge, and only travellers in class 2 use Sydney Harbour Bridge. Both travellers in class 1 and class 2 use other bridges, and hence their generalized travel time is the same, and their generalized travel cost is proportional to their VOTs. If we minimize the total travel time, then slightly fewer people use Sydney Harbour Bridge than the case of no toll (UE). To this end, a toll of \$0.18 is imposed and therefore all travellers in class 2 use Sydney Harbour Bridge. Travellers in class 1 use both links. Since travellers in class 1 and class 2 have the same travel time and pay the same toll on Sydney Harbour Bridge, the ratio of the generalized travel cost between class 2 and class 1 is reduced from 3 to 2.740.

When a toll of \$4 is imposed on Sydney Harbour Bridge, then only travellers in class 2 use it. This case is similar to the case of Cost SO and since both travellers in class 1 and class 2 use the untolled link, their generalized travel time is identical.

Table 2. Results with heterogeneous travellers.

	No toll (UE)	Cost SO	Time SO	Toll (\$4)
v_a^1	34,711	0	6,480	0
v_a^2	34,711	59,819	60,000	51,531
v_b^1	25,289	60,000	53,520	60,000
v_b^2	25,289	181	0	8,469
$t_a(v_a)$	5.188	4.074	4.829	3.245
$t_b(v_b)$	5.188	6.632	5.553	8.579
τ_a	0	1.92 ^d	0.18 ^e	4
Generalized time for class 1	5.188	6.632	5.553	8.579
Generalized time for class 2	5.188	6.632	5.070	8.579
Generalized cost for class 1	1.297	1.658	1.388	2.145
Generalized cost for class 2	3.891	4.974	3.803	6.434
Ratio of cost of class 2 over cost of class 1	3.000	3.000	2.740	3.000
TTT	662,550	642,824	618,219	754,636
TTC	311,276	283,153	308,421	308,607

E. Impact analysis of different toll levels

We further analyse the flow pattern under different toll levels, and the results are shown in Figure 1. When $\tau_a < 0.63$, travelers in class 1 and all travellers in class 2 use Sydney Harbour Bridge and experience the same travel time and toll; some travellers in class 1 use other bridges. Therefore, the ratio of the generalized travel cost of class 2 and class 1 is between 3 and 2.24 (less than the ratio of their VOTs). When $0.63 \leq \tau_a \leq 1.88$, all travellers in class 2 use Sydney Harbour Bridge and all travellers in class 1 use other bridges. The ratio of the generalized travel cost is between 2.24 and 3.00. When $\tau_a > 1.88$, some travellers in class 2 use Sydney Harbour Bridge; all travellers in class 1 and the other travellers in class 2 use other bridges. Therefore, on link b , the two classes of travellers experience the same travel time and zero tolls. The ratio of the generalized travel cost is 3.00.

IV. Conclusions

This paper has developed mathematical models for formulating the toll pricing problem on Sydney Harbour Bridge considering that different travellers may have different value-of-times. The models examine quantitatively the effect of different toll levels on the efficiency and equity.

^d This is the optimal τ_a to achieve Cost SO with $\tau_b = 0$.

^e This is the optimal τ_a to achieve Cost SO with $\tau_b = 0$.

The proposed models can serve as a useful decision-support tool for NSW Roads and Maritime Services.

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