2008

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Publication Details

This book chapter is published as Chinnappan, M, Productive pedagogies and deep mathematical learning in a globalised world, in Kell, P, Vialle, W, Konza, D and Vogl, G (eds), Learning and the learner: exploring learning for new times, University of Wollongong, 2008, 236p. Complete book available here.

Chapter 11

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Mohan Chinnappan

Introduction

In recent years the educational community at large has been giving attention to teaching practices that would create citizenry that are capable of independent thought and innovation. As a consequence, this theme has been given prominence in reforms that are being supported by policy makers and curriculum developers. While such a move can be seen as a welcome change, less has been said about the nature of knowledge and skills that learners need to be innovative and how these would empower them to be more productive and function effectively in a globalized society. The issue has received attention in discussions about deep vs. surface level thinking in the context of learning and understanding school subjects. Specifically, levels of thinking that learners ought to demonstrate with respect to particular school subjects has been considered from the perspective of quality of knowledge that learners construct as they develop experience and expertise within that subject area.

In this paper, I explore quality of knowledge that learners could construct by drawing on the theoretical perspective of *knowledge connectedness* in order to bring greater focus to the debate about potential relations between subject matter knowledge organisation, quality of thinking and learning for the new times. In the first instance, I outline the broad range of factors that could impact on the way learners construe mathematics, and how these might constrain their repertoire of knowledge. This is followed by a general description of the major strands of the above framework with explorations about how mathematical *schemas* can be used to analyse aspects of deep thinking. The situated nature of mathematical learning and how learning in contexts would facilitate the building of meaningful schemas is considered. An illustration of schemas development that involves trigonometric knowledge reveals interconnections and features of deep learning in a problem situation. It is suggested that schemas that have a high degree of external connectedness drive innovation and creativity. The analysis poses challenges for teaching in meeting the demands of learning for the new times. I contemplate that classroom practices that are modelled on *Productive Pedagogies* (PP) appear to provide a suitable alternative approach for supporting learning for and in the new age. However, there remains the issue regarding strategies for the translation of core dimensions of PP into concrete learning actions in ways that would foster learners to deconstruct and reconstruct links that are necessary for the development of multiliteracies.

Development of mathematics concepts and understandings

Learner's everyday life abounds with mathematical objects and concepts. An awareness of the mathematics in the environment is a function of learners' ability to interpret the relevance of both the formal and informal mathematics they have been experiencing in the classroom. Regardless of the context in which learners engage with mathematics, it is generally agreed that interconnections provide teachers insight into developments in learner's understanding of the extent of their own understandings. These understandings are built on learners' personal experiences, intuitions and formal knowledge taught in the classroom.

Mathematics educators and teachers have invested considerable effort in exploring instructional strategies that would help learners to develop a better grasp of mathematical concepts. One stream of inquiry about teaching approaches has focused on teaching practices that aid in the construction of a powerful and meaningful understanding of mathematics and its utility. Developments in cognitive psychology and domain expertise have yielded significant lines of inquiry about what we mean by powerful understandings and these might be investigated.

Mathematics curriculum reforms call for teaching and learning experiences that optimise the development of a substantive understanding of mathematics (National Council of Teachers of Mathematics 2000). The question remains as to how we can characterize this type of understanding? Discussions about understandings and the manifestation of understandings within knowledge rich areas such as mathematics have focused on the structure of knowledge that is constructed by learners and its impact on meaning making and vice versa. The notion of structure implies the existence of links among strands of knowledge. Prawat (1989) argued that the building of well organised knowledge is indicative of a sophisticated understanding of a subject. It is argued, that this type of knowledge is qualitatively superior to one that is less well organized in terms of access to and use of that knowledge during problem solving.

Classroom culture and mathematical discourse

Perspectives about how learners come to know and make sense of mathematics have contributed to the emergence of socio-constructivist views. According to this framework, mathematics learning is seen as an individual, as well as, a shared activity during which learners should be encouraged to investigate, argue, justify and test conjectures (Cobb 1995). Such activities need a classroom-learning environment that supports learners to question what teachers and peers say about a particular mathematics concept.

The approach towards teaching needs to move away from the transmission mode towards one that fosters free and open inquiry and debate. The social context in which learning takes place is as important as the concepts themselves in the growth of the type of understanding that is constructed by the learners (Lee 1998). The social and participative nature of learning reflects a view of knowledge that involves personal constructs as opposed to ‘received wisdom.’

Pedagogies that reflect the communal nature of learning mathematics would also practice different norms in the classroom. In these mathematics classrooms the rules of learner behaviour and mathematical discourse are negotiable. Learners are offered multiple opportunities to engage constructively and critically with mathematical ideas. Teachers who subscribe to such an open classroom environment need to display an understanding of how individual students would work in that environment, including their background and beliefs about what it means ‘to do mathematics.’ Thus, the scaffolding of learning has emerged to be a priority for teachers.

Arguments about shared mathematical ‘meaning making’ by learners places premium on the language that is used in this process. During the course of negotiated understanding, learners need to communicate their own ideas with peers and others in the community. Wertsch (1998) suggested that language is a cultural artefact or tool that mediates individual’s cognition in mathematics. The mathematical language that is used by members of the mathematical community includes special words, symbols, diagrams and representations with meanings that are different from those occurring in every-day language. The vocabulary of mathematics that is used by mathematicians, mathematics textbooks and, to some extent, mathematics teachers, tends to be at variance and conflict with that used by learners.

While mathematicians may understand each other, learners often experience difficulties with the language of mathematics and every-day language (Sfard and Kieran 2001). One of the difficulties here is that learners attempt to extract the meaning that is embedded in what teachers and textbooks say in their own mathematical language. For example, the notion of *function* and *roots* as used by the mathematical community has a special meaning in comparison to the every-day meaning of function. Through a process of acculturation learners come to discriminate the duality of meanings that is associated with these two terms. This process entails the learner engaging mathematics concepts in activities that make sense and aids in resolving the apparent tensions. The adoption of such activities to foster debate and discussion about mathematics terminology and the way these are used constitutes an important strategy for assisting learners with different cultural backgrounds to engage in knowledge building (White 2003).

The failure of pedagogies to recognize learning mathematics as a problem concerning language and communication exacerbates the situation for certain groups of learners. Learners might form the view that they have not grasped what is being taught when in fact the problem could be their difficulty to communicate their understandings and intuitions into mathematically precise statements. Many learners' problems with mathematical understandings that are related to the solution of word problems can be attributed to problems of language. The above tension between mathematical language and the learner's own language is impeding, if not preventing, the construction of important connections that are necessary for the growth of deeper levels of understanding. The gap in communication between teachers and learners is further complicated by the multiple and idiosyncratic interpretations that learners construct about a particular mathematical concept and their own linguistic backgrounds (Clarkson 1992).

Framework of connectedness

The interest in knowledge organization and performance has generated a number of theoretical frameworks including *connectedness*. Mayer (1975) examined the notion of *connectedness* as involving the accumulation of new information in long-term memory, adding new nodes to memory and connecting the new nodes with components of the existing knowledge network of the learner. He identified two types of connectedness: internal and external. *Internal connectedness* refers to the degree to which new nodes of information are connected with one another to form a single well-defined whole or schema.

Schemas refer to clusters or chunks of knowledge about a particular mathematical concept. They represent objects, contexts and prior experiences of the learner with that concept. Beyond the above elements, schemas also contain a network of interrelations. For example, a student might have developed a schema about whole numbers. This whole number schema could include information about size of numbers and operations involving this class of numbers. The nature of schemas and how these may be used to account for learning has been investigated in a number of areas, such as, composing stories (Bereiter and Scardamalia 1986), chess (Chase and Simon 1973), analogical problem solving (Gick and Holyoak 1983) and mathematics (Skemp 1987).

Connectedness refers to both the presence of nodes related to a schema and the quality of the network of interrelationships established among those nodes. The broad notion of quality of knowledge here can be related, in part, to what Anderson (2000) refers to as 'strength' of connections. In this sense the stronger the connections among the nodes in a particular schema, the better the quality of that well-defined structure. Mayer (1975) visualized *external connectedness* as the degree to which newly established knowledge structures are connected with structures already existing in the learner's knowledge base.

Let us examine these ideas in the context of a schema for proportion. A learner might be expected to relate a schema for proportion with schemas for ratio or fraction. These external connections between proportion and ratio or fraction will have a certain quality that would impact on learner's ability to use them in order to solve problems or provide alternative representations. In such a case, the new schema for proportion will have both a certain quality in its internal structure (internal connectedness) and a certain quality in its connections to related schemas (external connectedness). This analysis of connectedness was used to support the argument that the linking of the different pieces of knowledge of geometry and algebra reflect deeper and richer understandings (Chinnappan 1998; Chinnappan and Thomas 2003).

In classrooms where teachers support students to talk, the higher level of input from learners during their critical evaluation of mathematical concepts would help them reflect and reconstruct new understandings. The ensuing debate and problem solving that adumbrates examination of a focus concept can be expected to aid in learners building the elements that are necessary for the growth of external connectedness. For example, young learners working on the fractions will be motivated to explore meaningful contexts where part-whole relations that are embedded in the fraction number are given concrete and richer representations.

Mayer's (1975) analysis of connectedness, while useful for the exploration of organizational features of conceptual aspects of mathematical understanding is somewhat limited in that it does not explicitly consider idiosyncrasies of the individual learner and the contexts in which learning takes place. Authenticity of learning tasks contribute directly to the quality of schemas that can be constructed but this feature of the task is embedded is the context that moulds one's thinking.

Situated mathematical learning

While cognitive theory focuses on the way learner's process information, the need to examine how and why individual learners' construct different mathematical understandings have led educators to examine the situated nature of mathematical learning. There is now an emerging consensus that learning and the quality of learning is a function of the context and the activity in which learning takes place. The failure to see this important link between school mathematics and real-life mathematics has contributed to many of the difficulties experienced by learners (Ginsburg and Allardice 1984). More importantly, a situated view of learning mathematics has the potential to provide insights in to the conditions under which learners can be encouraged to construct links that would provide external connectedness to their schemas.

If learning were context dependent, it would seem reasonable to suggest that agents that encourage the learner to interact more closely with that context would optimize learning. That is, working in groups could help individual make more sense of the environment. From this perspective, mathematical knowledge is regarded as something that the learner constructs within social contexts and grows within that context as opposed to bits of information that lie inside one's head. The acquisition of knowledge of mathematical concepts, procedures and conventions, and the transformation of this knowledge is argued to occur in a complex socio-cultural environment. The social practice in which the participant engages in moulds the child's perceptions, understandings and mathematical realities.

Understanding mathematics through an activity is central to the situated view of learning. A key assumption here is that as teachers and learners engage in learning activities, the tools that are employed during these activities, structures the nature of learner participation and the meanings constructed by the participants (Lave 1988; Wertsch 1998). Any knowledge that an individual develops is seen as a product of, and anchored by, the tools that drive an activity. The communications between the members of a learning community are mediated by a variety of socio-semiotic resources which influence the meanings that individuals develop. For example, diagrams to represent a real-life problem and the conventions and symbols that are used the construction of diagram constitute such resources (Saenz-Ludlow and Walgamuth 2001). The framework of activity is consistent with that of productive pedagogies (more about this later) as it permits the teacher to understand the dynamics and patterns of social interactions that aid learners in coming to construe a concept and develop schemas that embody those understandings.

An example of contextualizing learning

In their attempts to make sense of concepts in secondary mathematics curriculum, learners are expected to establish multiple links. Figure 1 shows the range of connections that a Year 10 learner (Gary) had constructed about right-angled triangles. Being a spatial concept, one would expect to see a number of visual elements to this learner's schema about right-angled triangle.

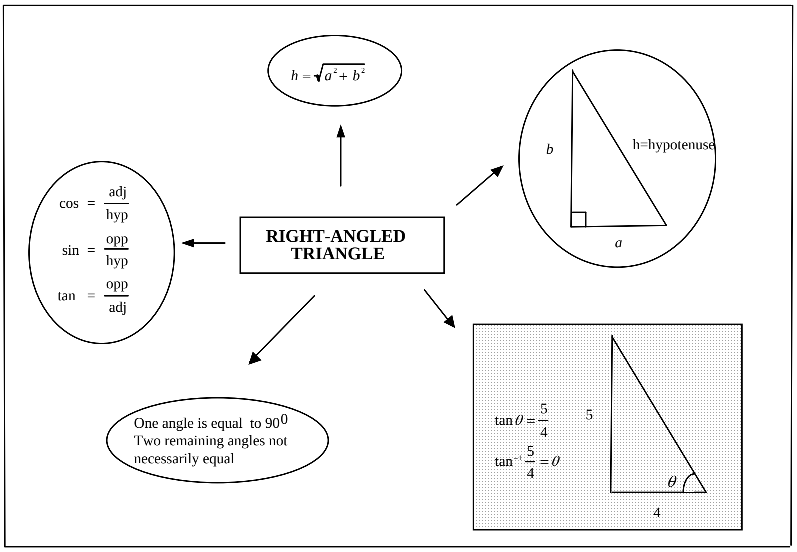

Figure 1: Connectedness for right-angled triangles

Gary had built up a number of concepts about the properties of right-angled triangles. This is indicated by concepts that appear within the oval shapes. The bottom right-hand corner shows an additional link about right-angled triangles that was constructed after a trigonometry lesson. The concepts and links that appear in Figure 1 reflect internal connectedness of the learner's schema for right-angled triangles. In fact the internal connectedness displayed here could be argued to be reasonably robust as he was able to identify the sides, the relationship between angles measures, the Pythagoras' theorem and associated trigonometric ratios. Collectively, these components are necessary for working out unknown angles and sides of a given triangle that belongs to the above category.
However, what appears to be missing are elements that one can consider to be external to the right-angle schema such as the notion of gradient, symmetrical properties of this class of triangles and the type of tessellations that allow this shape to be used in tiling work. This, the external connectedness seems to be relatively limited in comparison to his internal connectedness. Perhaps the lesson was limited in talking about real-life contexts where this class of triangles appears. Learning for the new times needs to demonstrate a greater level of external connectedness than has been demonstrated here by situating the concepts in grounds that are familiar to the learner.

A schema-based analysis of connectedness helps one disentangle the idiosyncrasies of individual's personal experiences and their developing multiple understandings. For example, the absence of connections involving gradient in Gary's schema could influence and be influenced by the constructions made from the classroom instruction, as well as, experienced outside the school. There is a widely accepted view that learners do attempt to make sense of school mathematics by examining real, practical or hypothetical problems beyond the classroom. A solid understanding of core concepts could better facilitate this integration of classroom mathematics with real-life problems – an important requirement for learners to become numerate (Willis 2000).

Implications for deep learning

The foregoing analysis of the quality of learning that can be expected from future learners of mathematics raises questions about the adequacy of current models of teaching in the classroom. The challenge facing future reforms of classroom practice is that we need to be explicit about strategies that would improve the external and internal connectedness of learner's mathematical schema. It would seem that a situated view of learning has the highest potential to empower learners to engage mathematics in meaningful and powerful ways resulting in sophisticated schemas that supports deeper and substantive understanding of mathematics and its use in real life.

The need for the assimilation and accommodation of new mathematical knowledge with prior knowledge also raises questions about the nature of links teachers could and should expect their learners to construct. The building of connections among the various conceptual strands of mathematics suggests that teachers have to be creative in providing the learner with activities and tools that encourage the investigation of potential relations. Research on classroom practices that emphasize reflection and application is beginning to redirect our efforts to this important aspect of the learner's cognition (Carpenter and Lehrer 1999). Recently, the work on best classroom practices has contributed to the emergence of an influential model of teaching called *Productive Pedagogies* (Gore 2001). This model (PP) identified a series of four core dimensions that have been shown to be important for learners' active involvement in the learning process and to attain the type of understanding envisaged by proponents of deep learning: Intellectual Quality, Relevance, Supportive classroom environment, Recognition of difference. While discussions continue about the efficacy of this model, it is acknowledged that productive pedagogies do provide a useful framework for teachers. It allows them to place a greater level of emphasis on the provision of learning opportunities which assist learners to extend their prior knowledge in ways that enriches their quality of understanding and interaction with other members of the community.

The use of PP to guide teachers' decisions about lesson aims and their implementation would prompt teachers to think more deeply about the mathematics and the teaching of mathematics. Additionally, teachers would have to consider the different ways learners could utilise mathematics discussed in the classroom to solve problems at work and beyond. Teachers could draw on the various dimensions of productive pedagogies in order to evaluate their lessons and examine learners' engagement with mathematics and understandings that are constructed as a consequence of learning experiences provided in the classroom. These reflections could provide fruitful directions concerning future strategies to help shift learners to higher planes of understandings. These richer and more complex interpretations of mathematical ideas could be gleaned from the width and density of learner's mathematical schemas.

Multiliteracies and productive pedagogies

Practices that are based on the dimensions that are specified in PP could motivate learners to extend their mathematical constructions in different directions. Focus on the dimension of Intellectual Quality, for instance, would assist learners to create different representations of a particular mathematics concept. Activities that aim to get learners to think about Recognition of Difference would help attend to cultural and linguistic factors involved in understanding and communicating mathematics. Thus, teaching that promotes PP has the potential to provide a holistic meaning of the nature of mathematical knowledge and how this knowledge evolved over the centuries. This holistic approach to mathematical learning really captures the spirit and structure of externally connected schemas.

Learners of the future must be able to relate to and work with peers from diverse linguistic and cultural differences. Citizens from different nationalities working together to solve complex problems will populate the global work place of the future. The use of mathematics in these contexts requires that individuals be better informed about linguistic barriers to sharing understandings. Further, these learners will have to communicate via a range of technologies. This duality in knowledge construction and communication has prompted researchers to develop the framework of multiliteracies. According to this view, in order to function effectively in a global world, individuals have to think about how one comes to comprehend and share one aspect of their environment and how this might differ from other cultures. Cazden, Cope, Kalantzis, Luke, Luke, and Nakata (1996) explored this notion further in their argument that the 'critical engagement' that is necessary in the private and public life of individuals should be based on pedagogies that support multiliteracies.

I have argued that a mathematical schema that can be characterized as having a high degree of external connectedness will have multiple and more complex relations with other mathematical and related concepts. The assumptions underlying framework of multiliteracies can be seen as being consistent with proponents of externally connected schemas. This line of argument suggests that teaching approaches that are based on PP and the resulting increased focus on the building of externally connected mathematics will help learners become multiliterate. The point here is that a deep understanding of a mathematical concept allows the learner to appreciate the different ways peers from other cultures would make sense of and communicate that concept. This ability to cross the cultural barriers in learning and sharing understanding, according to (Shulman and Shulman 2004: 263), constitutes 'intelligent and adaptive action.'

Summary and Conclusion

In this chapter, I set out to examine the characteristics of mathematical understanding that learners need in order to participate in the activities of the global community. These characteristics were analysed from the perspective of connectedness which suggests that the building of deep understandings of mathematics for numeracy for the new times can be analysed in terms of the quality and robustness of links that form a schema. For learners to build strong and well organized mathematical links, learning experiences need to focus not only on the development of multiple representation of mathematics but also the construction of links among these representations that are anchored in meaningful and problematic contexts. Mathematics learning that is embedded in an activity is a powerful step in reinforcing the links among representations. With suitable scaffoldings learners can be assisted to explore mathematical problems and communicate new ideas.

The framework of Productive Pedagogies provided the core dimensions along which the above scaffoldings could be constructed. Moreover, the advantages conferred by teachers who embrace the principles of PP have been given further impetus by proponents of multiliteracies. Multiliteracies place emphasis on learners having to appreciate the complexities involved not only in individual's understanding mathematics but also how this sense-making is accomplished and communicated by members of other cultures. The challenge for teachers of mathematics is how to recast schools mathematics concepts and conventions in ways that would help learners in the new times come to terms with the communication of their understanding with others cultures.

While the connectedness view of the nature of mathematics and its understanding helps us to explain the phenomenon in terms of construction of powerful relations, teachers are left with the task of deciding the nature of such relations. The idea of shared meaning making, while it is consistent with the framework of productive pedagogies, also raises further questions about how we can make judgments about what constitutes ‘good’ links. Here is a second front in the teaching enterprise that requires debate in our quest to describe mathematical learning for the new times.

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