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Abstract

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BOUNDARY FILTERS WITH MAXIMUM CODING GAIN AND IDEAL DC BEHAVIOR FOR SIZE-LIMITED PARAUNITARY FILTER BANKS

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ABSTRACT

This paper presents boundary optimization techniques for the processing of arbitrary-length signals with paraunitary multirate filter banks. The boundary filters are designed to maximize the coding gain while providing an ideal DC behavior. Thus, all filters except the lowpass filter are designed to have zero mean. The proposed methods give direct solutions to the problem of finding optimal boundary filters and do not require numerical optimization.

1. INTRODUCTION

Multirate filter banks are usually designed to process ongoing signals, but it is also of significant interest to use them for the processing of finite-length signals. Applications include segmentation-based audio [1–3] and region-based (shape adaptive) image coding. From a compression point of view it is desirable to carry out a filter bank analysis of a finite length signal in a non-expansive way. This means that the total number of subband samples produced from a size-limited signal should be equal to the number of samples of the signal. Achieving this goal with filter banks, however, requires some additional steps, because the filter impulse responses are overlapping and the transient behavior at the signal boundaries must be taken into account.

Various techniques have been proposed to process finite-length signals, including circular convolution, symmetric reflection, and the use of boundary filters [4–12]. This paper concentrates on boundary filters and presents novel methods for their optimization. Using boundary filters means that the original filters of the filter bank are replaced by special filters at the boundaries of the signal which ensure that the entire information on a length-N input signal is contained in a total number of N subband samples. Circular convolution and symmetric reflection can also be interpreted as special forms of boundary filters. Throughout this paper, no restrictions on the type of the paraunitary filter bank and the signal length are imposed. Thus, the proposed methods are applicable to non-linear phase filter banks and arbitrary length signals. This is important, as the often used cosine modulated filter banks have non-linear phase.

The filters in a filter bank are often designed such that all filters except the lowpass have zero mean. This avoids leakage of a DC component of the input signal into the other bands, which might cause problems with the bit allocation. When applying a filter bank to a finite-length signal by using boundary filters, this property usually gets lost in the boundary regions. For biorthogonal filter banks, this problem had been addressed in [10, 11]. In [12] a solution for paraunitary two-channel filter banks was proposed which first optimizes the boundary filters to have desirable frequency responses and then applies a Householder transform to obtain zero-mean highpass filters. The approach for paraunitary filter banks presented in this paper considers an arbitrary number of channels. We derive solutions for the boundary filters which yield maximum coding gain under the constraint of an ideal DC behavior. Note that the coding gain has also been considered in [8,9]. In [8] numerical optimization was employed to find the boundary filters and no DC constraints were imposed. In [9] the coding gain was used to optimize the bit allocation for given boundary filters and not to optimize the filters themselves. In this paper, to control the DC behavior, a projection technique is used. Optimization is then carried out in a second step. It is shown that maximizing the coding gain through optimizing the boundary filters results in an eigenvalue problem which has a straightforward solution. Thus, in contrast to [8] no numerical optimization is required to find the optimal boundary filters.

In addition to maximizing the coding gain, a method is proposed which allows us to find boundary filters which have similar frequency responses as the original subband filters in the filter bank.

2. BOUNDARY FILTERS WITHOUT DC LEAKAGE

This section discusses the filter bank analysis of size-limited signals and the available degrees of freedom for boundary filter optimization. We consider an arbitrary signal length

$$N = KM + s \tag{1}$$

where M denotes the number of subbands and K, s are positive integers with $0 \leq s < M$. The filter bank analysis of a length-N signal $x(n)$ may be written as

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} \tag{2}$$

with $\boldsymbol{x} = [x(0), x(1), \ldots, x(N-1)]^T$ and

$$\begin{aligned}\boldsymbol{y} &= [y_0(0), \ldots, y_{M-1}(0), \ldots, y_0(K-1), \ldots \\ &\quad \ldots, y_{M-1}(K-1), y_0(K), \ldots, y_{s-1}(K)]^T.\end{aligned}$$

Variations of the definition for $\boldsymbol{y}$ are straightforward. Given the definitions for $\boldsymbol{x}$ and $\boldsymbol{y}$, the $N \times N$ matrix $\boldsymbol{H}$ can be

$$\left[\begin{array}{cccccccc}
h_0^{\ell}(2) & h_0^{\ell}(1) & h_0^{\ell}(0) & & & & & \\
h_1^{\ell}(2) & h_1^{\ell}(1) & h_1^{\ell}(0) & & & & & \\
\hline
 & h_0(3) & h_0(2) & h_0(1) & h_0(0) & & & \\
 & h_1(3) & h_1(2) & h_1(1) & h_1(0) & & & \\
 & & & h_0(3) & h_0(2) & h_0(1) & h_0(0) & \\
 & & & h_1(3) & h_1(2) & h_1(1) & h_1(0) & \\
\hline
 & & & & & h_0^{r}(3) & h_0^{r}(2) & h_0^{r}(1) \\
 & & & & & h_1^{r}(3) & h_1^{r}(2) & h_1^{r}(1)
\end{array}\right]$$

Figure 1: Example of size-limited analysis matrix $\boldsymbol{H}$; $M = 2$, $N = 8$, length-4 filters.

set up to describe the filter bank analysis. Fig. 1 shows an example.

The matrix $\boldsymbol{H}$ may be partitioned as

$$\boldsymbol{H} = \left[\boldsymbol{H}_1^T | \boldsymbol{H}_2^T | \boldsymbol{H}_3^T\right]^T \tag{3}$$

where the center part contains the original impulse responses of the analysis filters, while the upper and lower parts contain boundary filters. Using this partitioning, the analysis equation (2) can be re-written as

$$\boldsymbol{y}_k = \boldsymbol{H}_k \boldsymbol{x}, \quad k = 1, 2, 3, \tag{4}$$

so that $\boldsymbol{y} = [\boldsymbol{y}_1^T, \boldsymbol{y}_2^T, \boldsymbol{y}_3^T]^T$. Similarly, the synthesis operation can be written as

$$\hat{\boldsymbol{x}} = \sum_{k=1}^{3} \boldsymbol{G}_k \boldsymbol{y}_k \tag{5}$$

where $\boldsymbol{G}_k$ are the corresponding partitions of the synthesis matrix $\boldsymbol{G}$, such that $\hat{\boldsymbol{x}} = \boldsymbol{G}\boldsymbol{y}$. Perfect reconstruction (PR) is given if $\boldsymbol{G}\boldsymbol{H} = \boldsymbol{I}$. In particular, if the size-limited filter bank is unitary, we have PR with $\boldsymbol{G} = \boldsymbol{H}^T$. To design a matrix $\boldsymbol{H}$ which satisfies $\boldsymbol{H}^T\boldsymbol{H} = \boldsymbol{I}$, the Gram-Schmidt procedure can be used as described in [5,6]. The drawback of this method is that it does not automatically yield boundary filters with good properties. Thus, further optimization is required.

We now assume that a PR solution for $\boldsymbol{H}_k$ is known (e.g. designed via the Gram-Schmidt method). An optimized analysis can then be written as

$$\boldsymbol{v}_k = \boldsymbol{U}_k \boldsymbol{H}_k \boldsymbol{x} \tag{6}$$

with $\boldsymbol{U}_1$ and $\boldsymbol{U}_3$ being unitary matrices and $\boldsymbol{U}_2 = \boldsymbol{I}$. The synthesis operation then becomes

$$\hat{\boldsymbol{x}} = \sum_{k=1}^{3} \boldsymbol{G}_k \boldsymbol{U}_k^T \boldsymbol{v}_k. \tag{7}$$

To avoid DC leakage, the matrices $\boldsymbol{U}_1$ and $\boldsymbol{U}_3$ need to be restricted in a certain way. In [11] a direct parameterization was proposed, but this parameterization does not yield orthogonal matrices $\boldsymbol{U}_k$. In [12] the boundary filters were first optimized and then a Householder transform was applied which ensured zero-mean highpass boundary filters. In this paper, we go a different way. We first generate basis vectors which represent a DC signal in the row spaces of $\boldsymbol{H}_1$ and $\boldsymbol{H}_3$. Then we use the Gram-Schmidt procedure to complete $\boldsymbol{H}_1$, $\boldsymbol{H}_3$. The remaining optimization steps are carried out in such a way that we have control over the DC component of an input signal.

Let $\bar{\boldsymbol{H}}_1$ be a matrix which contains a basis for the row space of $\boldsymbol{H}_1$. It does not need to be an orthogonal matrix, but it must have maximum rank, so that its rows span the entire subspace of left boundary filters. Further, let $\boldsymbol{t}$ be a length-N vector of ones: $\boldsymbol{t} = [1, 1, \ldots, 1]^T$. We now compute the orthogonal projection of $\boldsymbol{t}$ onto the row space of $\bar{\boldsymbol{H}}_1$:

$$\hat{\boldsymbol{t}}_1 := \bar{\boldsymbol{H}}_1^T [\bar{\boldsymbol{H}}_1 \bar{\boldsymbol{H}}_1^T]^{-1} \bar{\boldsymbol{H}}_1 \boldsymbol{t}. \tag{8}$$

The first row of the matrix $\boldsymbol{H}_1$ is then chosen as $\hat{\boldsymbol{t}}_1^T$. All further rows of $\boldsymbol{H}_1$ can be found via the Gram-Schmidt procedure, using the rows of $\bar{\boldsymbol{H}}_1$ as a given basis for the subspace in question. Note that one of the rows of $\bar{\boldsymbol{H}}_1$ will not be needed, because $\hat{\boldsymbol{t}}_1^T$ has been included, which already is a linear combination of the rows of $\bar{\boldsymbol{H}}_1$. For more details on the Gram Schmidt technique, the reader is referred to [5,6].

The matrix $\boldsymbol{H}_1$ constructed with the above algorithm has the property that all its rows, except the first one, have zero mean. This property is easily kept by choosing $\boldsymbol{U}_1$ as

$$\boldsymbol{U}_1 = \begin{bmatrix} 1 & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{V}_1 \end{bmatrix} \tag{9}$$

where $\boldsymbol{V}_1$ is orthogonal. The same concept can be used for the right boundary.

3. BOUNDARY FILTER OPTIMIZATION

In this section, we derive solutions for $\boldsymbol{U}_1$ and $\boldsymbol{U}_3$, and thus for the boundary filters, which maximize the coding gain. Regardless of the actual number of bands, we interpret the subband decomposition according to (6) as a unitary transform that maps N input values into N transform coefficients. Under the assumption of a high bit rate and uncorrelated quantization errors the coding gain may then be expressed as [13, 14]

$$G = \sigma_x^2 \prod_{\ell=0}^{N-1} (\sigma_{v_\ell}^2)^{-1/N} \tag{10}$$

where $\sigma_{v_\ell}^2$ are the variances of the subband samples computed via (6). Thus, optimizing the boundary filters to yield maximum coding gain turns out to be equivalent to minimizing the products of the diagonal elements of

$$\boldsymbol{R}_{v_k v_k} = \boldsymbol{U}_k \boldsymbol{H}_k \boldsymbol{R}_{xx} \boldsymbol{H}_k^T \boldsymbol{U}_k^T, \quad k = 1, 3. \tag{11}$$

The matrices $\boldsymbol{R}_{v_k v_k}$ are the autocorrelation matrices of the subband samples $\boldsymbol{v}_k$, generated from an input process $\boldsymbol{x}$ with autocorrelation matrix $\boldsymbol{R}_{xx}$. Minimizing the product of the diagonal elements is accomplished by the Karhunen-Loève transforms (KLT's) of the processes $\boldsymbol{y}_k$. In other words, the rows of the optimal matrices $\boldsymbol{U}_k$, $k = 1, 3$ are the transposed eigenvectors of

$$\boldsymbol{R}_{y_k y_k} = \boldsymbol{H}_k \boldsymbol{R}_{xx} \boldsymbol{H}_k^T. \tag{12}$$

Zero-Mean Constraint

To obtain boundary filters with maximum coding gain under the zero-mean constrain, we use the parameterization (9). Again, the key to the solution is the KLT. We partition $\boldsymbol{H}_k$, $k = 1, 3$ as

$$\boldsymbol{H}_k = \begin{bmatrix} \hat{\boldsymbol{t}}_k^T \\ \hat{\boldsymbol{H}}_k \end{bmatrix} \quad (13)$$

and, following the same ideas as above, we find the rows of the optimal matrices $\boldsymbol{V}_k$ to be the transposed eigenvectors of

$$\hat{\boldsymbol{R}}_{y_k y_k} = \hat{\boldsymbol{H}}_k \boldsymbol{R}_{xx} \hat{\boldsymbol{H}}_k^T.$$

Frequency Response Approximation

The above described design methods, although optimal, usually do not lead to boundary filters which have similar frequency responses as the original filters. Typically, the design results in narrowband boundary filters with different passbands in the frequency range $[0, \pi]$. By linearly combining previously constructed boundary filters it is possible to design new ones which have similar time-frequency resolutions as the original filters. This allows for the use of the same bit allocation at the boundaries as in the center of a signal. For a brief explanation, let us assume that the number of boundary filters is given by $L_k = \nu_k M$ where ν_k is an integer. Let $\hat{\boldsymbol{h}}_{i,k}^T$, $i = 1, 2, \ldots, L_k - 1$ denote the ith row of $\hat{\boldsymbol{H}}_k = \boldsymbol{U}_k \boldsymbol{H}_k$. Let us assume that the rows of $\boldsymbol{U}_k$ are ordered according to the corresponding eigenvalues of $\boldsymbol{R}_{y_k y_k}$ or $\hat{\boldsymbol{R}}_{y_k y_k}$, depending on the method used. We assume that the first row corresponds to the largest eigenvalue. Let $\boldsymbol{A}_k$ be orthogonal matrices of size $\nu_k \times \nu_k$. The new filters are constructed as

$$\begin{bmatrix} \tilde{\boldsymbol{h}}_{(i-1)\nu_k+1,k}^T \\ \vdots \\ \tilde{\boldsymbol{h}}_{i\nu_k,k}^T \end{bmatrix} = \boldsymbol{A}_k \begin{bmatrix} \hat{\boldsymbol{h}}_{(i-1)\nu_k+1,k}^T \\ \vdots \\ \hat{\boldsymbol{h}}_{i\nu_k,k}^T \end{bmatrix} \quad (14)$$

for $i = 1, 2, \ldots, M$, where $\tilde{\boldsymbol{h}}_{i,k}^T$ forms the ith row of the final optimized analysis matrix. For $\nu_k = 2$ we choose

$$\boldsymbol{A}_k = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}. \quad (15)$$

For $\nu_k > 2$ the $\nu_k \times \nu_k$ DCT-II matrices are possible choices. Note that related methods have been described in [15] for the design of time-varying filter banks without transition filters and in [16] for the design of non-uniform filter banks.

4. DESIGN EXAMPLES

We consider a paraunitary, cosine-modulated 32-band filter bank with ELT prototype according to [17]. In this filter bank, the subband filters have non-linear phase. ELT filters have filter length $4M$, and the total number of boundary filters for the left-hand side turns out to be $L_1 = 2M$. On the right-hand side, their number depends on the parameter s used to describe N in (1).

Table 1
Coding gain of left boundary filters for 32-band ELT and AR(1) process with correlation coefficient ρ.

	$\rho = 0.9$	$\rho = 0.95$
Gram Schmidt	6.919 dB	9.515 dB
Optimal, unconstrained	7.341 dB	10.404 dB
Optimal under DC constraint	7.337 dB	10.395 dB
With boundary filter recombination	7.327 dB	10.376 dB
ELT for unlimited signal	7.166 dB	10.011 dB

We consider the left boundary. A first set of boundary filters was designed via the Gram-Schmidt procedure. The frequency responses of the left boundary filters are depicted in Fig. 2. As the plot shows, in this example, the Gram-Schmidt procedure directly yields boundary filters with relatively good frequency selectivity. The filters divide the frequency range $[0, \pi]$ into M bands, and there are always two filters with the same passband, but different time localizations. A weakness of the method is that several boundary filters, in addition to the two lowpass ones, have large non-zero mean. A second set of boundary boundary filters was designed to maximize the coding gain under the zero-mean constraint. The input process was considered to be an AR(1) process with correlation coefficient $\rho = 0.9$. The frequency responses of the filters are shown in Fig. 3. These filters not only maximize the coding gain, they also have good frequency selectivity. It can be seen that the $2M$ boundary filters have $2M$ disjoint passbands, which can be expected from filters that maximize the coding gain. Finally, the designed filters were converted into filters with only M passbands by taking linear combinations of the previously designed filters with $\boldsymbol{A}_k$ as in (15). The frequency responses are depicted in Fig. 4. These filters have similar frequency responses as the original filters and allow for the use of the same bit allocation in the center and at the boundaries of a signal. The results in Table 1 show that the drop in coding gain due to this manipulation is only marginal. The highest coding gain is obtained when the filters are not restricted to have no DC leakage. Note that when using the coding gain as the optimality criterion without further constraints, filters with relatively little DC leakage may be found by assuming a correlation coefficient very close to one. Further note that the coding gains of all optimized filters are higher than for the plain ELT for unlimited signals.

5. CONCLUSIONS

The methods presented in this paper enable the design of orthogonal, perfect reconstruction boundary filters with ideal DC behavior and maximum coding gain. All methods presented provide direct solutions and need no cost intensive numerical optimization. Thus, they are applicable to systems with a large number of subbands and/or very long filter impulse responses. The signal lengths can be chosen independent of the number of channel of the filter bank. This allows for segmented coding where the segmentation can take place at arbitrary points.

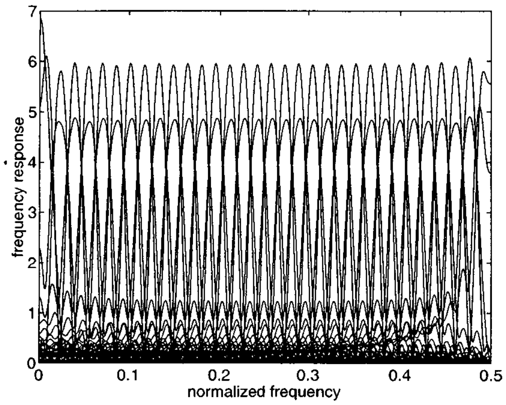

Figure 2: Frequency responses of left boundary filters designed via Gram-Schmidt method.

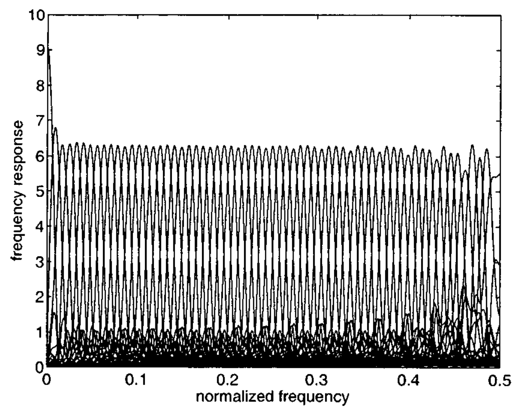

Figure 3: Frequency responses of left boundary filters with maximum coding gain under the zero-mean constraint.

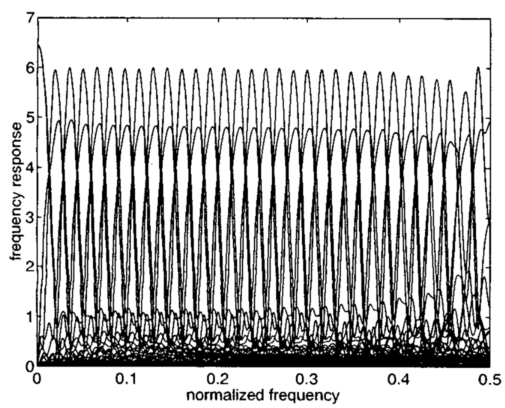

Figure 4: Frequency responses of left boundary filters which resemble the frequency responses of the original filters.

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