Effect of particle angularity and size distribution on the deformation and degradation of ballast under cyclic loading

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EFFECT OF PARTICLE ANGULARITY AND SIZE DISTRIBUTION ON THE DEFORMATION AND DEGRADATION OF BALLAST UNDER CYCLIC LOADING

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CERTIFICATION

I, Yifei Sun, declare that this thesis, submitted in fulfilment of the requirements for the award of Doctor of Philosophy, in the School of Civil, Mining, and Environmental Engineering, University of Wollongong, is wholly my own work unless otherwise referenced or acknowledged. The document has not been submitted for qualifications at any other academic institution.

The following publications are related to the research work conducted in this study:


Yifei Sun

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ABSTRACT

Ballasted railroad tracks are still used all around the world because of their resiliency to repeated wheel loads, low cost construction and ease of maintenance, but current ballast gradations worldwide vary in particle size and uniformity for reasons that are not always clear. This research aims to investigate the angularity (shape) of ballast aggregates and the influence of particle size distribution on its deformation and degradation. To achieve this aim, a three-dimensional laser scanner and the triaxial apparatus designed and built at the University of Wollongong were used respectively to carry out a three dimensional characterisation of ballast aggregates and cyclic large-scale triaxial tests. The deformation and degradation of ballast under high and low track speeds was investigated by using two different loading frequencies, and then a fractional order constitutive model was proposed to predict the long term deformation of ballast by using the fractional rate for strain accumulation.

These experiments indicated that shape of ballast particles varied according to their size, such that as the particles increased in size, ballast became more rounded and less columnar. Moreover, a two-dimensional characterisation would underestimate particle sphericity, so a new index called ‘ellipsoidness’ was suggested to facilitate a three dimensional analysis of particle shape. This ellipsoidness decreased as the particles increased in size, which indicated relatively more irregularity in coarser ballast. The parent ballast aggregates became increasingly regular after cyclic tests while smaller aggregates created from ballast degradation seemed to have a lower ellipsoidness.

Ballast deformation was greatly influenced by its particle size distribution which means that larger ballast assembly experienced smaller permanent axial and radial strains, while smaller aggregates underwent dilation and then compression as the number of load cycles increased; this transition from dilation to compression was influenced by the loading frequency. Resistance to plastic strain under cyclic loading can be improved by increasing its relative density, while the permanent stains of ballast decreased initially and then increased with the
coefficient of uniformity. The rates of permanent axial (shear) and volumetric strain accumulation under cyclic loading obeyed a power law with regard to load cycles.

The resilience of ballast was also influenced by its particle size and coefficient of uniformity. For samples with the same coefficient of uniformity, the resilient modulus was observed to decrease with the increasing size span between maximum and minimum particle sizes. However, it increased with the decrease of the coefficient of uniformity for ballast with the fixed minimum and maximum particle sizes. More serious ballast breakage was found in samples tested under higher loading frequency, where at the same time a low resilient modulus was observed.

Ballast degradation can be reduced significantly by increasing the sample density. For ballast samples tested under similar initial densities, an initial decrease followed by a slight increase in the extent of breakage was observed. However, for ballast with similar initial relative densities, there was more breakage of larger ballast with a lower coefficient of uniformity. Since field ballast was more likely to be well compacted before operation, ballast degradation was divided into two zones depending on the range of the coefficient of uniformity, i.e., a high breakage zone and a reduced breakage zone. Based on these test results, a broader particle size distribution of railroad ballast was proposed to reduce deformation and degradation under high frequency cyclic loading.

The cumulative deformation of ballast is determined by the current loading step and by its previous loading history. A cyclic constitutive model was developed to predict the cumulative deformation of ballast by incorporating fractional calculus into a traditional plasticity model. This model deduced to the classical plasticity model when the fractional order approached unity. A decrease in the fractional order with increasing fractal dimension of a given material was observed. As the fractional order decreased, the model exhibited increasing densification ability. The model was calibrated and validated against a series of independent laboratory test results. Unlike other constitutive models, this model can efficiently simulate the cumulative deformation of ballast from the onset of load to a high number of load cycles.
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# TABLE OF CONTENTS

CERTIFICATION ................................................................. i
ABSTRACT .............................................................................. ii
ACKNOWLEDGEMENTS ........................................................ iv
TABLE OF CONTENTS ............................................................ v
LIST OF FIGURES ................................................................. viii
LIST OF TABLES ................................................................... xiii
LIST OF NOTATIONS ............................................................ xiii

CHAPTER 1 INTRODUCTION ................................................ 1

1.1 Research Background ................................................... 1
1.2 Statement of the Problem .............................................. 2
1.3 Objectives and Scopes ................................................... 3
1.4 Thesis Outline ............................................................... 5

CHAPTER 2 LITERATURE REVIEW ....................................... 7

2.1 Introduction .................................................................... 7
2.2 Particle Breakage ......................................................... 7
2.3 Mechanical Behaviour of Ballast under Static and Cyclic Loading ......... 15
   2.3.1 Static Behaviour of Ballast ......................................... 15
   2.3.2 Cyclic Behaviour of Ballast ........................................ 22
2.4 Particle Shape and Its Influence on the Physical and Mechanical Properties of Granular Soils ................................................................. 26
   2.4.1 Particle Shape Indices ............................................... 27
   2.4.2 Effect of Particle Shape on the Physical and Mechanical Properties of Granular Soils ................................................................. 34
2.5 Particle Size Distribution and its Effect on Physical and Mechanical Behavior of Granular Soils ........................................................................... 40
   2.5.1 Particle Size Distribution .......................................... 41
   2.5.2 Effect of PSD on Physical and Mechanical Properties of Granular Soils ............... 42

CHAPTER 3 3D CHARACTERISATION OF PARTICLE SIZE AND SHAPE ........... 52

3.1 Introduction .................................................................... 52
3.2 Experimental Investigations ........................................... 54
3.2.1 Sample Preparation ................................................................. 54
3.2.2 Procedures for 3D Measurement ................................................. 55
3.3 Characterisation of Particle Shape and Size ..................................... 57
  3.3.1 Arithmetic Mean of the Shape and Size Indices............................ 57
  3.3.2 Scanning Results ................................................................. 58
3.4 Discussions on Distribution of Shape Indices ................................... 63
3.5 Conclusions ................................................................................. 73
CHAPTER 4 STATIC AND CYCLIC TRIAXIAL TESTS OF RAILROAD BALLAST .... 75
  4.1 Introduction ............................................................................... 75
  4.2 Material Properties and Testing Procedures ................................. 76
    4.2.1 Material Properties .......................................................... 76
    4.2.2 Testing Procedures .......................................................... 78
  4.3 Static Test Results ................................................................. 81
    4.3.1 Stress and Strain ............................................................. 81
    4.3.2 Dilation and Peak Friction Angles ...................................... 83
    4.3.3 Critical State Line .......................................................... 86
  4.4 Cyclic Test Results ................................................................. 87
    4.4.1 Effect of Maximum Particle Size ......................................... 88
    4.4.2 Effect of Coefficient of Uniformity ...................................... 90
    4.4.3 Strain Accumulation Rates .............................................. 96
    4.4.4 Discussion ..................................................................... 104
  4.5 Conclusions ............................................................................. 109
CHAPTER 5 RESILIENT BEHAVIOUR OF RAILROAD BALLAST ...................... 112
  5.1 Introduction ............................................................................. 112
  5.2 Test Results ........................................................................... 114
    5.2.1 Effect of Coefficient of Uniformity ...................................... 115
    5.2.2 Effect of Particle Size ...................................................... 117
  5.3 Discussion ............................................................................. 120
  5.4 Conclusions ........................................................................... 123
CHAPTER 6 DEGRADATION OF RAILROAD BALLAST ............................ 125
  6.1 Introduction ............................................................................. 125
  6.2 Degradation of Ballast ........................................................... 126
    6.2.1 Test Results with Fixed Initial PSD ................................... 127
LIST OF FIGURES

Figure 1.1. Relationship between the dynamic stress and train speed (modified after Kempfert and Hu, 1999)............................................................................................................................. 2
Figure 2.1. Definitions of different breakage indices (modified after Indraratna et al., 2005)........ 9
Figure 2.2. Variation of $BBI$ with the $\varepsilon''$ (modified after Indraratna et al., 2014)............ 12
Figure 2.3. Influence of the loading frequency on the particle breakage behavior of ballast (modified after Indraratna et al., 2010)............................................................................................................. 13
Figure 2.4. Variation of $BBI$ with the loading frequency (modified after Sun et al., 2015).... 14
Figure 2.5. Influence of the confining pressure on the particle breakage of ballast (modified after Lackenby et al., 2007) ............................................................................................................. 15
Figure 2.6. Shear strength envelopes (modified after Indraratna et al., 1998)......................... 16
Figure 2.7. Normalized relationship between the shear strength and normal strength (modified after Indraratna et al., 1998)............................................................................................................. 17
Figure 2.8. Schematic diagram of the characteristic state in drained triaxial compression test (modified after Lade and Ibsen, 1997)............................................................................................................. 18
Figure 2.9. Correlation between the mean effective principal pressure and dilation angle..... 18
Figure 2.10. Stress dilatancy relationship for ballast (modified after Indraratna et al., 1998) 20
Figure 2.11. Critical state line of ballast in the $p' - q$ plane (modified after Indraratna et al., 2014).............................................................................................................................. 21
Figure 2.12. Critical state line of sand in the $p' - q$ plane (modified after Bandini and Coop, 2011)........................................................................................................................................ 21
Figure 2.13. Axial strain as a function of the number of cycles loading (modified after Lackenby et al., 2007).............................................................................................................................. 24
Figure 2.14. Rate of axial strain to the number of cycles (modified after Lackenby, 2006).... 24
Figure 2.15. Three dimensions of a particle (Fernlund, 1998)................................................ 28
Figure 2.16. Zingg diagram (modified after Blott and Pye, 2008)......................................... 29
Figure 2.17. Definition of roundness ..................................................................................... 30
Figure 2.18. Initial void ratio $e_0$ as a function of the elongation ratio (modified after Nouguier-Lehon, 2010)......................................................................................................................... 35
Figure 2.19. Relationship between the extreme void ratios and (a) particle sphericity; (b) particle regularity (modified after Cho et al., 2006)......................................................................................... 36
Figure 2.20. Particle size distribution by mass .................................................................... 42
Figure 2.21. Relationship between the permeability and void ratio ........................................ 44
Figure 2.22. Typical discretized CSD curve of a granular soil ................................................. 46
Figure 2.23. Influence of the PSD on the shear strength (modified after Sitharam and Nimbkar, 2000) .................................................................................................................. 48
Figure 2.24. Effect of the PSD on the permanent deformation of granular soils (modified after Thom and Brown, 1988) ............................................................................................................. 51
Figure 3.1. Flow chart for the overall experimental study .......................................................... 54
Figure 3.2. Schematic diagram of test setup ............................................................................... 55
Figure 3.3. Typical ballast particles: (a) original particles, (b) scanned results ......................... 56
Figure 3.4. Relationship between the real mass of ballast and its scanned volume ................... 57
Figure 3.5. Elongation ratio and flatness ratio of ballast aggregates in Zingg diagram ............. 60
Figure 3.6. Relationship between the true sphericity $\psi$ and: (a) sphericity $\psi_1$; (b) sphericity $\psi_2$; (c) 3D roundness $X_s$ (Hayakawa and Oguchi, 2005) ................................................................. 61
Figure 3.7. Relationship between ellipsoidness $E$ and: (a) elongation ratio $I/L$; (b) true sphericity $\psi$; (c) regularity $\rho$ ..................................................................................................................... 62
Figure 3.8. (a) Distribution of the elongation ratio and (b) mean elongation ratio vs particle size ........................................................................................................................................... 64
Figure 3.9. (a) Distribution of the flatness ratio and (b) mean flatness ratio vs particle size .... 64
Figure 3.10. (a) Distribution of the true sphericity and (b) mean true sphericity vs particle size ........................................................................................................................................... 65
Figure 3.11. (a) Distribution of the 2D sphericity and (b) mean 2D sphericity vs particle size .......... 66
Figure 3.12. (a) Distribution of the ellipsoidness and (b) mean ellipsoidness vs particle size 66
Figure 3.13. (a) Distribution of the roundness and (b) mean roundness vs particle size .... 67
Figure 3.14. Evolution of shape and size indices with particle size ........................................... 68
Figure 3.15. Predictions of (a) PSDs and (b) CSDs by mass, by surface area and by number based on geometrical average of the sieve interval ........................................................................ 70
Figure 3.16. Predictions of (a) PSDs and (b) by mass, surface area and number based on $d_n$ 72
Figure 4.1. Particle size distributions of ballast ........................................................................ 78
Figure 4.2. Static triaxial behaviour of ballast at $\sigma^\prime_3 = 30 \text{kPa}$ : (a) deviator stress, (b) volumetric strain ........................................................................................................................................... 82
Figure 4.3. Static triaxial behaviour of ballast with $C_u = 1.9$ and $d_M = 45 \text{ mm}$ .................. 83
Figure 4.4. Dilation angle of ballast with varying PSDs ............................................................ 84
Figure 4.5. Variation of the peak friction angle with the coefficient of uniformity ................. 85
Figure 4.6. Dilation and peak friction angles of ballast under different $\sigma^\prime_3$ .................... 86
Figure 4.7. Critical state lines of ballast in the (a) $p^\prime - q$ plane and (b) $e - \ln p^\prime$ plane ...... 87
Figure 4.8. Permanent deformations of ballast with the same coefficient of uniformity and void ratio: (a) axial strain, (b) volumetric strain ........................................................................................................... 89
Figure 4.9. Permanent deformations of ballast with varying coefficient of uniformity but similar density at 20 Hz: (a) axial strain, (b) volumetric strain .............................................. 91
Figure 4.10. Permanent deformations of ballast with varying coefficient of uniformity but similar density at 30 Hz: (a) axial strain, (b) volumetric strain .............................................. 93
Figure 4.11. Permanent deformations of ballast with varying coefficient of uniformity but constant relative density at 20Hz: (a) axial strain, (b) volumetric strain ............................... 94
Figure 4.12. Permanent deformations of ballast with varying coefficient of uniformity but constant relative density at 30Hz: (a) axial strain, (b) volumetric strain ............................... 95
Figure 4.13. Permanent strains of ballast under different confining pressures: (a) axial strain and (b) volumetric strain .......................................................... 96
Figure 4.14. Relationship between axial strain rate and number of load cycles for ballast in (a) log-linear scale and (b) log-log scale ........................................................... 98
Figure 4.15. Relationship between volumetric strain rate and number of load cycles for ballast in (a) log-linear scale and (b) log-log scale ........................................................... 99
Figure 4.16. Relationship between strain rate and number of load cycles for ballast and subballast (data sourced from Suiker and de Borst, 2003): (a) volumetric strain rate (b) shear strain rate ............................................................... 101
Figure 4.17. Relationship between strain accumulation rate and number of load cycles for ballast (data sourced from Sevi and Ge, 2012): (a) volumetric strain rate (b) axial strain rate ............................................................... 102
Figure 4.18. Relationship between strain accumulation rate and number of load cycles for sand (data sourced from Wichmann et al., 2009): (a) volumetric strain rate (b) shear strain rate ............................................................... 103
Figure 4.19. Relationship between strain rate and number of load cycles for grained subgrade: (a) \( w = 33.1\% \); (b) \( w = 31\% \); (c) \( w = 28.7\% \); (d) \( w = 27.6\% \) (modified after Li and Selig, 1996) ...................................................................................... 104
Figure 4.20. Effect of the particle size on the permanent strain of ballast ........................................ 106
Figure 4.21. Variation of permanent strains with varying coefficient of uniformity: (a) similar initial void ratio, (b) constant initial relative density, (c) similar initial relative density ..... 107
Figure 4.22. Variation of the final axial strain with the PSD parameter of ballast ................. 109
Figure 5.1. Variation of the resilient strain of ballast with the number of load cycles at different coefficients of uniformity ........................................................................ 116
Figure 5.2. Effect of the coefficient of uniformity on the resilient modulus of ballast .......... 117
Figure 5.3. Variation of the resilient strain of ballast with the number of load cycles at different maximum particle sizes ................................................................. 119
Figure 5.4. Effect of the maximum particle size on the resilient modulus of ballast .......... 120
Figure 5.5. Variation of resilient modulus with median particle size \( d_{50} \) ...................... 122
Figure 5.6. Variation of resilient modulus with particle breakage ratio \( B \) ...................... 122
Figure 6.1. PSDs of ballast after monotonic loading (modified after Indraratna et al., 2014a) ................................................................. 127
Figure 6.2. Breakage extent of ballast under monotonic loading ................................................................. 128
Figure 6.3. Representation of the breakage extent of ballast under monotonic loading: (a) area $A$, (b) entropy increment, (c) relative base entropy, and (d) modified $B_g$ .......................... 129
Figure 6.4(a). PSDs of ballast after cyclic loading at linear-log scale ..................................................... 130
Figure 6.4(b). PSDs of ballast after cyclic loading at log-linear scale ..................................................... 130
Figure 6.5. Breakage extent of ballast with varying $d_M$ ...................................................................... 132
Figure 6.6. Representation of the breakage extent of ballast with varying $d_M$ by different indices: (a) area $A$, (b) entropy increment, (c) relative base entropy, and (d) modified $B_g$ ... 133
Figure 6.7. Breakage extent of ballast with similar void ratio: (a) $f = 20$ Hz and (b) $f = 30$ Hz ............................................................................................................. 135
Figure 6.8. Variation of particle breakage extent with the coefficient of uniformity of Dog’s Bay sand .................................................................................................................. 136
Figure 6.9. Representation of the breakage extent of ballast with similar void ratio by different indices .......................................................... 137
Figure 6.10. Breakage extent of ballast with similar relative density .......................................................... 138
Figure 6.11. Representation of the breakage extent of ballast with similar relative density by different indices ............................................................................................................. 141
Figure 6.12. Broken particles of ballast ..................................................................................................... 142
Figure 6.13. Variation of particle shape under cyclic loading: (a) Elongation ratio $e_r$, (b) Flatness ratio $f_r$, (c) Aspect ratio $a_r$, and (d) Ellipsoidness $E_e$ ..................................................... 145
Figure 6.14. PSDs with varying deformation and degradation ...................................................................... 146
Figure 6.15. Recommended PSD for railroad ballast .................................................................................. 148
Figure 7.1. Relationship between the fractional order and the fractal dimension ........................................... 159
Figure 7.2. Schematic show of the cyclic loading and unloading .................................................................. 165
Figure 7.3. Schematic representation of the effect of the fractional derivative order on soil densification: (a) stress strain response and (b) accumulated strain vs number of load cycles ............................................................................................................. 167
Figure 7.4(a). Determination of the critical state parameters in the $e - \ln p'$ space (data sourced from Salim and Indraratna, 2004) .......................................................................................................... 169
Figure 7.4(b). Determination of the critical state parameters in the $p' - q$ space (data sourced from Salim and Indraratna, 2004)............................................................................................................. 169
Figure 7.5. Determination of the hardening parameters ............................................................................. 170
Figure 7.6. Determination of the fractional order ......................................................................................... 171
Figure 7.7. Flow chart for model implementation ......................................................................................... 172
Figure 7.8. Model simulation for drained compression tests on ballast ...................................................... 175
Figure 7.9. Model predictions of (a) axial strain and (b) volumetric strain of ballast under different confining pressures ............................................................................................................. 176
Figure 7.10. Model simulation for drained compression tests on ballast (data sourced from Salim and Indraratna, 2004) .......................................................................................................................... 177

Figure 7.11. Model predictions of (a) shear strain and (b) volumetric strain of ballast (data sourced from Indraratna et al., 2012b) under different load frequencies ............................................ 178

Figure 7.12. Model predictions of (a) axial strain and (b) volumetric strain of ballast (data sourced from Sun et al., 2015) under different confining pressures .................................................. 180

Figure 7.13. Model predictions of (a) axial strain vs deviator stress and (b) axial strain vs volumetric strain of ballast (data sourced from Anderson and Fair, 2008) ......................... 181

Figure 7.14. Model predictions of (a) axial strain and (b) volumetric strain of ballast under different confining pressures (data sourced from Anderson and Fair, 2008) .................... 182

Figure 7.15. Model predictions of (a) axial strain vs deviator stress and (b) axial strain vs volumetric strain of ballast (data sourced from Aursudkij et al., 2009) ............................. 183

Figure 7.16. Model predictions of (a) axial strain and (b) volumetric strain of ballast under different confining pressures (data sourced from Aursudkij et al., 2009) .......................... 184

Figure 7.17. Model predictions of (a) axial strain vs deviator stress and (b) axial strain vs volumetric strain (data sourced from Fu et al., 2014) ................................................................. 185

Figure 7.18. Model predictions of (a) axial strain and (b) volumetric strain under different loading amplitudes (data sourced from Fu et al., 2014) .......................................................... 186
LIST OF TABLES

Table 4.1. Characteristics of ballast ........................................................................................ 76
Table 4.2. Physical attributes of ballast ............................................................................... 77
Table 4.3. Triaxial testing program ...................................................................................... 80
Table 6.1. Shape characteristics of broken particles ............................................................. 143
Table 7.1. Material properties .............................................................................................. 173
Table 7.2. Model parameters ............................................................................................... 174
LIST OF NOTATIONS

\( A \) is the shifted area between initial and current grading
\( \overline{A} \) is the constant of proportionality
\( A_0 \) is the measured area of the particle projection
\( AI \) is the angularity index
\( ar \) is the elongation and flatness ratio
\( B \) is the fractal breakage ratio
\( B_g \) is the Marsal’s breakage ratio
\( B_{gm} \) is the modified Marsal’s breakage ratio
\( Br \) is the relative breakage ratio
\( BBI \) is the ballast breakage ratio
\( b \) is the fitting parameter
\( C \) is the shifted area between initial and fractal grading
\( C_u \) is the Coefficient of uniformity
\( C_{KC} \) is the empirical coefficient
\( C^e \) is the elastic compliance matrix
\( CCD \) is the charge coupled device
\( CSD \) is the constriction size distribution
\( c \) is the coefficient that depends on the shape of grains
\( D \) is the shifted area between initial and ultimate grading
\( \overline{D} \) means derivation
\( Dc \) is the actual constriction size
\( Ds \) is the fractal dimension of the aggregates
\( D_{cm} \) is the mean constriction size
\( D_{CD} \) is the constriction size at densest state
\( D_{CL} \) is the constriction size at loosest state
\( d \) is the particle size
\( dM \) is the maximum particle size
\( dm \) is the minimum particle size
$d_{m0}$ is the minimum particle size within an untested sample

$d_{mf}$ is the minimum particle size counted for granular aggregates (0.074 mm)

$d_i$ is the discretized diameter

$d_n$ is the equivalent diameter

$d_{60}$ is particle size at 60% passing

$d_{50}$ is the median particle size

$d_{30}$ is particle size at 30% passing

$d_{10}$ is particle size at 10% passing

$d$ is the constant that depends on the degree of densification

$E$ is the shifted area between initial grading and arbitrary boundary

$\hat{E}$ is the characteristic modulus of the material

$\overline{E}$ is the material constant

$E_e$ is the 3D ellipsoidness

$e$ is the void ratio

$e_0$ is the initial void ratio

$e_f$ is the final void ratio

$e_r$ is the parameter accounting for the particle shape effect of soils

$e_{\text{max}}$ is the maximum void ratio

$e_{\text{min}}$ is the minimum void ratio

$e_{\text{c}}$ is the critical state parameter

$e_r$ is the elongation ratio

$F$ is the external normal force

$f_{i-1,j}^n$ is the contact force between $i$-th and $(i-1)$-th interacting particles

$f$ is the load frequency

$fr$ is the flatness ratio

$G$ is the shear modulus

$G_0$ is the parameter related to shear modulus

$H$ is the plastic modulus

$I$ is the medium axis length

$I$ means integral

$I_{2D}$ is the irregularity
is the bulk modulus

\( \bar{K} \) is the stiffness of a column

\( K_0 \) is the parameter related to bulk modulus

\( K_t \) is the overall stiffness of the material

\( k \) is the permeability

\( k' \) is the degradation rate of minimum sized particles

\( L \) is the major axis length

\( L_c \) is the height of a column

\( L_s \) is the particle diameter

\( l_s \) is the particle size

\( M \) is the critical state friction parameter

\( M_r \) is the resilient modulus

\( \mathbf{m} \) is the plastic flow tensor

\( m \) is the number of columns

\( m_j \) is the mass of the \( j \)-th sieve interval

\( m_t \) is the total mass of the sample

\( N \) is the number of load cycle

\( N_s \) is the number of particles

\( \hat{n} \) is the fitting parameter

\( \pi \) is the total number of size ranges

\( n \) is the grading parameter

\( \mathbf{n} \) is the loading direction tensor

PSD is the particle size distribution

PSI is the particle shape index

\( P \) is the external force applied on the sample

\( P_{Mi} \) is the mass probabilities of occurrence in corresponding discretized diameter

\( P_{SAi} \) is the surface area probabilities of occurrence in corresponding discretized diameter

\( P_{Ni} \) is the number probabilities of occurrence in corresponding discretized diameter

\( P_{\alpha}(i) \) is the probability that change in angle has a value in the range of \( i \) to \( (i + 10^\circ) \)

\( P_o \) is the perimeter of the scanned particle

\( P_e \) is the perimeter of the equivalent area ellipse
$p'_i$ is the initial mean principal stress

$p'$ is the mean effective principal stress

$p'_0$ is the initial mean effective principal stress

$p(d)$ is the current density distribution function the aggregate sizes

$p_0(d)$ is the initial density distribution function of the aggregate sizes

$p_a(d)$ is the fractal density distribution function the aggregate sizes

$p'_\text{f}$ is the mean effective principal stress on bounding surface

$p'_0$ is the initial mean effective principal stress on bounding surface

$p_r$ is the unit pressure

$p_{at}$ is the atmospheric pressure

$p'_{cs}$ is the critical state stress

$q$ is the deviator stress

$q_{av}$ is the average deviator stress

$q_{max}$ is the maximum deviator stress

$q_{min}$ is the minimum deviator stress

$q$ is the deviator stress on bounding surface

$R$ is the roundness

$\bar{R}$ is the radius of a particle

$\hat{R}$ is the energy dissipation ratio

$R_d$ is the relative density

$r_i$ is the radius of each corner in the particle outline

$r_{\text{max-in}}$ is the radius of the maximum inscribed circle

$S$ is the minor axis length

$\bar{S}$ is the relative base entropy

$\Delta S$ is the entropy increment

$S_c$ is the constriction space

$S_{c,\text{max}}$ is the maximum value of the constriction space

$S_e$ is the surface area of equivalent volume ellipsoid

$S_i$ is the initial physical state

$S_o$ is the surface area of the scanned particle
\( \dot{S}_s \) is the incremental surface area of aggregates

\( \overline{SS}_j \) is the averaged shape or size index of \( j \)-th sieve interval

\( SS_i \) is the shape or size index of particle \( i \) in \( j \)-th sieve interval

\( s_0 \) is the specific surface area per unit volume of particles

\( s_n \) is the surface area of a sphere having the same volume as the scanned particle

\( U \) is the overall compressive displacement of the column

\( u_{i-1,i}^n \) is the normal displacement between \( i \)-th and \((i-1)\)-th particles

\( V \) is the real volume of the particle

\( V_s \) is the soil volume

\( w \) is the water content

\( X_s \) is the 3D roundness

\( \psi \) is the true sphericity

\( \psi_1 \) is the 2D sphericity

\( \psi_2 \) is the 2D sphericity

\( \Theta \) is the PSD parameter

\( \theta \) is the fitting parameter

\( \theta_b \) is the fitting parameter

\( \nu_b \) is the fitting parameter

\( \omega_b \) is the fitting parameter

\( \alpha \) is the fractional order

\( \alpha_0 \) is the material constant

\( \beta \) is the fitting parameter

\( \beta_b \) is the fitting parameter

\( \dot{\varepsilon} \) is the incremental strain tensor

\( \dot{\varepsilon}^e \) is the incremental elastic strain tensor

\( \dot{\varepsilon}^p \) is the incremental plastic strain tensor

\( \dot{\varepsilon}_1^p \) is the major plastic principal strain

\( \dot{\varepsilon}_3^p \) is the minor plastic principal strain
\( \varepsilon_r \) is the recoverable axial strain
\( \varepsilon_v \) is the total volumetric strain
\( \varepsilon^e_v \) is the elastic volumetric strain
\( \varepsilon_s \) is the generalised total shear strain
\( \varepsilon^e_s \) is the generalised elastic shear strain
\( \dot{\varepsilon}_v^p \) is the plastic volumetric strain
\( \dot{\varepsilon}_s^p \) is the generalised plastic shear strain
\( \eta \) is the stress ratio between deviator stress and mean effective principal stress
\( \chi \) is the unit weight of the permeant
\( \overline{\chi} \) is the plastic multiplier
\( \chi' \) is the fitting parameter
\( \delta_{in} \) is the distance from the stress origin to the image stress point
\( \delta \) is the distance from the current stress point to the image stress point
\( \Gamma(\bullet) \) is the gamma function
\( \gamma \) is the plastic flow parameter
\( \gamma_b \) is the bulk unit weight
\( \gamma_s \) is the surface shape factor
\( \rho \) is the particle regularity
\( \overline{\rho} \) is the scalar relating the image stress and the loading stress
\( \lambda \) is the gradient of the critical state line
\( \kappa \) is the gradient of the swell line
\( \mu \) is the viscosity of the permeant
\( \mu' \) is the fitting parameter
\( \Omega \) is the surface energy
\( \phi_{cr} \) is the critical state friction angle
\( \phi_d \) is the phase transformation friction angle
\( \phi_0 \) is the phase transformation friction angle at \( p' = 0 \)
\( \Delta \phi \) is the gradient of the phase transformation line
\( \dot{\sigma} \) is the incremental stress tensor

\( \sigma'_1 \) is the major effective principal stress

\( \sigma'_2 \) is the medium effective principal stress

\( \sigma'_3 \) is the minor effective principal stress

\( \sigma_d \) is magnitude of the deviatoric stress

\( \nu \) is the Poisson ratio
CHAPTER 1 INTRODUCTION

1.1 Research Background

Ballasted rail tracks are used worldwide because of their resiliency to repeated wheel loads, low construction cost and ease of maintenance. Due to the growing demand for transporting bulk commodities and passenger services, faster and heavier rail traffic has recently been introduced by many countries, including Australia, China, France and Japan (Indraratna et al., 2016). Currently, railroad transport is becoming increasingly crucial for the Australian transport network. The Australian railroad network now consists of 41,461 kilometres of rail track on three major track gauges, of which 2,940 kilometres are now electrified. Since 2010, more and more high speed railroads have been under investigation, and now the Australian government has announced a A$20 million detailed feasibility study to identify potential routes for an economically viable high-speed rail network on the east coast of Australia, linking Melbourne, Canberra, Sydney and Brisbane. The first phase of the study was completed in 2011, projecting a financial cost for high-speed rail of between A$61 billion and A$108 billion, depending on which route and station combination will be selected.

In the Australian railroad system, ballast is usually used to distribute and reduce the dynamic load transmitted from the rail to the underlying formation. To provide fast drainage and prevent the possible buildup of excess pore water pressure, a uniform particle size distribution (PSD) for ballast is preferred, but to facilitate compaction and provide minimal track deformation and degradation, broadly graded ballast is sometimes suggested (Indraratna et al., 2006). As is already known, high speed tracks, unlike low speed tracks, usually impart a higher stress onto the ballast layer and that will ultimately lead to severe deformation and degradation (Kempfert and Hu, 1999).
1.2 Statement of the Problem

There is no obvious difference in the maximum dynamic vertical stress when the speed of a passenger train is less than 150 km/h, as seen in Figure 1.2, but as the speed increases to around 300km/h, there is a significant increase in the maximum dynamic vertical stress (Kempfert and Hu, 1999), so the stress in ballasted foundation for a low speed train is totally different from a high speed train. Moreover, while heavy haul and passenger trains impart a large cyclic load which often leads to progressive deterioration of the rail track, the dominant factor leading to the deterioration of tracks is typically the deformation and degradation of ballast.

Figure 1.1. Relationship between the dynamic stress and train speed (modified after Kempfert and Hu, 1999)

Extensive laboratory experiments conducted at University of Wollongong using their large-scale triaxial equipment revealed there is an optimum confining pressure where ballast
breakage is minimal under cyclic loading. However, the influence of PSD on the deformation and degradation of railroad ballast has not been studied in depth, neither has it been investigated to find an optimum ballast PSD which provides relatively small breakage under cyclic loads with a high frequency. Since ballast gradations vary from country to country, with coefficients of uniformity ranging from 1.5 to 3.0, the reasons why these particle size distributions exist is not always explained clearly. Thom and Brown (1988) tested granular aggregates with different PSDs and found significant differences in the physical and mechanical responses but their tests were based on smaller particles with low frequency and gradations that are seldom used in practical railway engineering. Therefore, assessing the deformation and degradation of railroad ballast with different PSDs subjected to high-frequency cyclic loading is imperative for implementing a safe design strategy for high speed rail tracks.

1.3 Objectives and Scopes

The overall objective of this research is to investigate the role that the size and shape of particles play on the deformation and degradation of ballast under static and cyclic loading. To simulate high and low train speeds, 20 Hz and 30 Hz frequencies were used. Within the scope of this study, only one type of ballast aggregate was investigated. The specific objectives of this research are summarised below:

1) Laboratory investigation of particle shape of railroad ballast using a 3D laser scanner. Establish a new three dimensional (3D) index to describe the irregularity of railroad ballast with different PSDs. Explore the dependence that particle shape has on its size. Measure the change of particle shape after each test to provide details for evaluating the function of different particle fractions.
(2) Laboratory investigation of the stress-strain response of ballast with different PSDs under static loading and cyclic loading at 20 Hz and 30 Hz. Evaluate the resilient modulus and cumulative strain, and then discuss the influence the PSD curve has on the accumulation of strain, and the correlations of the material constants with grading index or granulometric properties.

(3) Comprehensively study the particle breakage of ballast using different breakage indices. Discuss the influence of PSD on particle breakage and propose a new PSD to reduce ballast breakage.

(4) Explore the effect of PSD and breakage on the resilient modulus of railroad ballast. Develop a simple mechanistic approach to explain the coupling effect of PSD and breakage on ballast resiliency.

(5) Constitutive modelling of the cyclic behaviour of railroad ballast by using fractional calculus. Evaluate model parameters and verify the model by comparing its predictions with a substantial amount of experimental data.

It should be noted that ballast in the field experiences traffic loads. However, unlike typical applications in traditional pavement engineering, Australian heavy haul trains are often more than 4 km long (sometimes exceeding 6 km in Western Australia). Therefore, for busy heavy haul tracks carrying 30 – 35 ton axle trains (e.g. catering for Australian mining hubs), one needs to recognise the significant number of loading cycles applied even by the passage of a single heavy axle long train, and appreciates the consequential effects on a ballasted track if not properly designed and constructed. In contrast, the relatively short and lightweight passenger trains do not pose the same challenges to the track designers, and often quasi-static approaches are adequate in the design for such tracks. Moreover, past UOW studies (e.g. Salim, 2004; Lackenby, 2007) have shown that the Ultimate Resilient Modulus is independent of the past loading history once a stable or shakedown void ratio is attained by
the ballast layer after being subjected to a very large number of cycles (N > 500,000). However, even at smaller number of load cycles, due to the limitation of laboratory equipment and the very lengthy time taken to conduct these large-scale tests over N > 500,000 (even longer if interim rest periods are introduced) to simulate the real track loading conditions, cyclic loading without any rest period was used in this study to investigate the (permanent and resilient) deformation and degradation of ballast, following the approach introduced by Indraratna et al., (2009). Because of limited 3-year PhD period of CSC scholarships, only one set of confining and deviator stresses is considered while varying the ballast PSDs. Cyclic triaxial tests were carried out under maximum and minimum deviatoric stresses equal to 260 kPa and 45 kPa, and at 20 Hz and 30 Hz respectively. A confining pressure of 30 kPa, that is within the optimum confining pressure provided by Lackenby (2006), was selected for most cyclic triaxial tests.

1.4 Thesis Outline

This research thesis is divided into 7 chapters, including the Introduction, and is organised as follows:

This Chapter 1 has introduced the research background, a statement of the problem, and the objectives and scopes of the research.

The following Chapter 2 provides a critical literature review of existing research into ballasted railroad tracks, and presents a general review of cumulative deformation, resilient modulus, and ballast degradation, etc. Particular attention is given to the effect that PSD and particle shape have on the mechanical response of ballast.

Chapter 3 describes the laboratory investigations carried out to study the particle shape of ballast, including the test setup and test procedure. The use of different particle shape indices
such as the elongation ratio, the flatness ratio, the aspect ratio, sphericity, roundness and so on, are fully discussed, and the evolution of these particle shape indices is analysed.

Chapter 4 describes the static and cyclic triaxial tests carried out to study the deformation of ballast with different initial PSDs, and then discusses the evolution of the permanent axial and volumetric strains with the coefficient of uniformity and particle size.

Chapter 5 explores the influence that PSDs have on the resilient modulus of ballast loaded under two different frequencies. A micromechanical representation of the variation of ballast resiliency with changing particle size and coefficient of uniformity is provided. The effect of breakage on the resilient modulus is also explained by incorporating the fractal breakage theory.

Chapter 6 provides a detailed investigation of how PSDs influence the degradation of ballast. To explain the breakage characteristics of ballast, different breakage indices are used and discussed, and variations of particle shape before and after being tested is also presented.

Chapter 7 proposes an improved constitutive model for ballast under cyclic and static loading conditions. The modification of a current bounding surface model is presented by incorporating fractional calculus. A fractional strain accumulation rate is suggested for modelling the cumulative deformation of railroad ballast.

Chapter 8 contains the Conclusions and Recommendations, followed by a list of References.
CHAPTER 2 LITERATURE REVIEW

2.1 Introduction

In recent years, ballasted railroad foundations or substructures have become increasingly overloaded due to the introduction of faster and heavier trains. The subsequent ballast degradation caused by large cyclic loading is one of the most critical problems in traditional railroad foundations, and as such has attracted the attention of more and more researchers. The costs of construction and maintenance can be reduced significantly investigating the physical and mechanical properties of the substructure under high frequency loads, particularly the ballast layer. Many past studies into the effects of confining pressure (Lackenby et al., 2010), loading condition (Ishikawa et al., 2011), loading frequency (Thakur et al., 2013; Sun et al., 2015), geosynthetics (Nimbalkar et al., 2012; Indraratna and Nimbalkar, 2013) among others have been carried out, but few attempts have been made to study the effect of particle shape and PSD. This chapter presents a general overview of literatures on particle breakage and the stress strain response of ballast under static and cyclic loading. The effect of particle shape and PSD on the physical and mechanical behaviour of granular soils is also discussed in detail.

2.2 Particle Breakage

Particle breakage usually occurs when sufficient load is applied, which can significantly degrade the original PSD and change the initial physical and mechanical characteristics of granular soils (Lade et al., 1996; Indraratna et al., 1998); this extensive particle breakage reduces the drainage and filtration efficiency and also reduces the strength of soil. The extent
of particle breakage is a function of several factors such as the compressive strength of the parent rock, the applied confining pressure, the initial compaction, and the initial PSD.

According to the research by Marsal (1967) and Carrera et al., (2011), more particle breakage would occur in soils with a relatively low coefficient of uniformity and high void ratio, whereas McDowell (1996) and Coop et al. (2004) suggested that the PSDs of different coarse granular aggregates would finally shift towards a fractal PSD if enough particle breakage were allowed. To quantify the extent of particle breakage in granular soils, many particle breakage indices have been proposed, some of which were defined based only on the change of one particular particle size within a sample; for example, the $B_{15}$ proposed by Lee and Farhoomand (1967) to assess particle breakage caused by the percentage change of particles with a size of $d_{15}$. Similarly, Lade et al. (1996) proposed $B_{10}$ to evaluate the percentage change of particles with a size of $d_{10}$. These indices were suggested specifically for evaluating the breakage induced change in permeability from previous studies, but they cannot fully assess particle breakage within the whole range of particle size.

To overcome this limitation, particle breakage ratios based on relative breakage potentials have been proposed. For instance, Hardin (1985) suggested an ultimate size of 0.074 mm for possible crushing of granular soils and defined the breakage index $B_r$, as given by:

$$B_r = \frac{A}{D}$$

(2.1)

where $A$ is the total breakage, defined as the area between the initial and current PSDs, as shown in Fig. 2.1.
Figure 2.1. Definitions of different breakage indices (modified after Indraratna et al., 2005)

$D$ is the breakage potential, defined as the area between the initial and ultimate PSDs. However, particles of a given initial PSD cannot break continuously to an ultimate size of 0.074 mm because they gradually evolve towards a self-similar distribution where the probabilities of crushing within all size ranges are the same. Therefore, Einav (2007) proposed a modified Hardin’s breakage index $B$, as:

$$B = \frac{A}{C}$$ (2.2)

where $C$ denotes the modified breakage potential, defined as the area between the initial and the corresponding fractal PSDs.
Similarly, Indraratna et al. (2005) proposed a ballast breakage index $BBI$, based on the breakage potential $E$, between the initial PSD and the arbitrary boundary for breakage:

$$BBI = \frac{A}{E} \quad (2.3)$$

where $A$ is a shift in the PSD curve after the test, and $E$ is the breakage potential or the area between the arbitrary boundary of maximum breakage and the initial PSD curve, as shown in Figure 2.1.

Apart from the breakage indices above that are defined based on breakage potentials, the grading entropy theory (Lőrincz et al., 2005) was also suggested to estimate the particle crushing of granular aggregates. The entropy of a distribution is a measure of the amount of information; for soils, it provides information about the nature of PSD or the sorting of particles. Once particle breakage occurs, the initial PSD would shift and result in a change in the grading entropy. To characterise this change of grading entropy, two entropy parameters, i.e., entropy increment and relative base entropy, were proposed where the entropy increment $\Delta S$ is defined as:

$$\Delta S = -\frac{1}{\ln 2} \sum_{i=1}^{\bar{n}} P_{Mi} \ln P_{Mi} \quad (2.4)$$

where $P_{Mi}$ is the probability of particles within the size range $i$, and $\bar{n}$ is the total number of size ranges. The relative base entropy $S$ is defined as

$$S = \frac{\sum_{i=1}^{\bar{n}} P_{Mi}(i-1)}{\bar{n}-1} \quad (2.5)$$

Note that the entropy increment is the statistical specific entropy of the PSD curve in terms of the fractions, and depends on the number of fractions ($\bar{n}$), while the relative base entropy is the mean of the reduced entropy of each fraction weighted by the relative frequencies, and is
normalised by the number of fractions. Detailed derivations of Equations (2.4) and (2.5) can be found in Lőrincz et al. (2005), so they are not repeated here.

It should be also noted that Marsal (1967) proposed a straightforward method of evaluating the extent of particle breakage by directly calculating the percentage increase of aggregates that are retained on each sieve size. The breakage index $B_g$ by Marsal (1967) is defined as follows:

$$B_g = \sum \Delta_i$$  \hspace{1cm} (2.6)

where $\Delta_i$ is the positive value of the percentage difference of each particle size fraction before and after the test. However, there seems to be no obvious upper limit for the value of $B_g$, so it provides barely sufficient information about to what extent the current breakage may take place. As a result, a modified $B_g$ index was suggested by using the ultimate value of $B_g$ for aggregates to represent a fractal PSD, as given by:

$$B_g^m = \frac{\sum \Delta_i^f}{\sum \Delta_i^f}$$  \hspace{1cm} (2.7)

where $\Delta_i^f$ denotes the positive value of the percentage difference of each particle size fraction between the initial and fractal PSDs. As expected, the modified $B_g$ ranges from 0 to 1, and the extent of particle breakage increases with the increase of $B_g$ and eventually approaches unity, where aggregates are ‘fractally’ distributed. By using the particle breakage index, the change of the physical and mechanical characteristics of granular soils such as railroad ballast can then be examined.

The effects of the confining pressure and frequency on particle breakage of railroad ballast under static and cyclic loading conditions has been addressed by Lackenby et al. (2007), Indraratna et al. (2014), and Sun et al. (2015). Figure 2.2 shows the variation of $BBI$ with the
deviatoric strain under static loading, where the extent of breakage and the $BBI$, increased drastically when shearing commenced, and then gradually approached a constant, especially under a relatively low confining pressure. A nonlinear function between the particle breakage index and the shear strain was thus suggested by (Indraratna et al., 2014a).

$$BBI = \frac{\theta_b \left[ 1 - \exp(-\nu_b \varepsilon_s^p) \right]}{\left( \omega_b - \ln p_i' \right)}$$

(2.8)

where $\theta_b$, $\nu_b$, and $\omega_b$ are the fitting parameters; $\varepsilon_s^p$ is the plastic shear strain, and $p_i'$ is the value of the initial mean principal stress.

![Graph](image)

Figure 2.2. Variation of $BBI$ with the $\varepsilon_s^p$ (modified after Indraratna et al., 2014a)

Particle breakage under cyclic loading is somehow different from that under static loading because according to Indraratna et al. (2010) and Sun et al. (2015), ballast breakage is influenced by the loading frequency $f$. As shown in Figures 2.2-2.3, the ballast breakage
index increases with an increase in the loading frequency. In fact, there was a distinct ballast degradation that corresponded to different loading frequencies. For Range I with a loading frequency of less than 30 Hz, particle degradation was mainly an attrition of asperities and corner breakage (Sun et al., 2015), but as the loading frequency increased (> 30 Hz) in Range II, particle splitting caused by fatigue and a higher degree of attrition caused by increased vibration became predominant. At a very high frequency (> 60 Hz) in Range III, the coordination number decreased, which resulted in a lot of particle splitting. Moreover, Figure 2.4 shows that the critical frequency decreased as particle breakage increased.

![Figure 2.3. Influence of the loading frequency on the particle breakage behavior of ballast](modified after Indraratna et al., 2010)
Figure 2.4. Variation of BBI with the loading frequency (modified after Sun et al., 2015)

Lackenby et al. (2007) also reported how the confining pressure and loading amplitude affected the particle breakage of ballast under cyclic loading; with Figure 2.5 showing that as the loading amplitude increased, particle breakage increased monotonically, while the effect of confining pressure was not that conclusive. Particle degradation under cyclic loading can be divided into three zones, i.e., (i) dilatant, unstable degradation zone (DUDZ), (ii) optimum degradation zone (ODZ), and (iii) compressive, stable degradation zone (CSDZ). At relatively lower confining pressures, substantial particle breakage and attrition occurs in this dilatant zone due to a combination of rapid axial strains and large (expansive) radial strains. In the range of confining pressures considered (1-240 kPa), degradation was most severe at low confining pressures, although the small increases in confining pressure resulted in enhanced particle contact areas with better internal distribution of stress that caused less particle breakage. An increased in confinement actively suppresses dilation by packing the
grains together tightly, but this results in tighter a gradual increase in breakage. For each deviator stress considered, an ‘optimum’ range of confining pressures exists such that degradation is minimised. The optimum range increases in value with an increasing maximum deviator stress.

2.3 Mechanical Behaviour of Ballast under Static and Cyclic Loading

2.3.1 Static Behaviour of Ballast

The static behaviour of ballast has been studied extensively by Indraratna et al. (1998), Anderson and Key (2000), Lim and McDowell (2005), Suiker et al. (2005) and Indraratna et al. (2015). Several of their key characteristics are summarised below.
2.3.1.1 Shear Strength

The shear strength varies as the normal stress increases; as Figure 2.6 shows, the shear strength envelopes are nonlinear, especially for tests under high confining pressure. Particle breakage and the stress dilatancy of ballast can be regarded as a decisive factor in the variation of shear strength (Lim and McDowell, 2005; Suiker et al., 2005). To model the nonlinear shear strength of railroad ballast, a normalised exponential relationship was suggested by Indraratna et al. (1998), as shown in Figure 2.7, but due to continuous particle breakage a low peak friction angle was expected.

Figure 2.6. Shear strength envelopes (modified after Indraratna et al., 1998)
Figure 2.7. Normalized relationship between the shear strength and normal strength
(modified after Indraratna et al., 1998)

2.3.1.2 Phase Transformation and Stress Dilatancy

The phase transformation angle $\phi_d$ is related to the stress point $(p'_d, q_d)$ where there is a transition from compressive behaviour at lower stress levels to dilative behaviour at high stress levels. As Figure 2.8 shows, point A is the phase transformation point while point B is the peak strength point. Phase transformation angles for different confining pressures are plotted in Figure 2.9, from which it is observed that the angle of dilation decreases with an increasing mean effective principal stress. Thus the following relationship is suggested:

$$
\phi_d = \phi_0 - \Delta \phi \log\left(\frac{p'_d}{p_{at}}\right)
$$

(2.9)
where $\phi_0$ and $\Delta \phi$ are the fitting parameters 58.05 and 6.49, respectively, of railroad ballast tested by Indraratna et al. (1998), and $p_{at}$ is the atmospheric pressure (100kPa).

Figure 2.8. Schematic diagram of the characteristic state in drained triaxial compression test (modified after Lade and Ibsen, 1997)

Figure 2.9. Correlation between the mean effective principal pressure and dilation angle
Because of particle breakage within the tested sample, the stress dilatancy relationship for railroad ballast was no longer linear; as Figure 2.10 shows, all the samples were dilatant at the peak stress state. An empirical hyperbolic relationship was intuitively suggested, and after that a systematic analysis was carried out by Indraratna and Salim (2002), using a theoretical stress dilatancy equation proposed by Ueng and Chen (2000). Equation (2.10) is the ultimate form.

\[
\frac{\hat{\varepsilon}_p}{\hat{\varepsilon}_v} = \frac{9(M - \eta)}{9 + 3M - 2\eta M} + \left(\frac{B_b}{p'}\right) \left[\frac{\chi' + \mu'(M - \eta^*)}{9 + 3M - 2\eta M}\right] 
\]

where \(\hat{\varepsilon}_p\) and \(\hat{\varepsilon}_v\) are increments of the plastic volumetric strain \(\varepsilon_p\) and plastic shear strain \(\varepsilon_v\), respectively, and \(\chi'\) and \(\mu'\) are fitting parameters that define the rate of particle breakage. \(\eta\) equals the ratio between deviatoric stress \(q\) and mean principal stress \(p\).

\[\eta^* = \eta\left(\frac{p'}{p_{cs}}\right)\]

\(M\) is the slope of the critical state line in the \(p' - q\) plane. Constant \(B_b\) is defined as

\[
B_b = \frac{\beta_b}{\ln\left(\frac{p_{cs}(i)}{p'}\right)} \left[\frac{(9 - 3M)(6 + 4M)}{6 + 4M}\right] 
\]

where \(\beta_b\) is fitting parameter, correlating the energy dissipated by particle breakage.
2.3.1.3 Critical State Line

The critical state behaviour of ballast differed from clay and sand, but due to the significant breakage of ballast aggregates, the critical state lines in the $e - \ln p'$ and $p' - q$ planes had evolved. As Figure 2.11 shows, the critical state line in the $p' - q$ plane bends downwards with increasing mean effective principal stress because more ballast aggregates broke. However, particle breakage had no obvious influence on the critical state line of sand (Bandini and Coop, 2011), as shown in Figure 2.12. Similar observations were also found in rockfill as reported by Xiao et al. (2014). The critical state friction angle decreased with an increase of the mean effective principal stress.
Figure 2.11. Critical state line of ballast in the $p' - q$ plane

(modified after Indraratna et al., 2014a)

Figure 2.12. Critical state line of sand in the $p' - q$ plane

(modified after Bandini and Coop, 2011)
The critical state line in the $e - \ln p'$ plane either translated downwards due to increasing breakage caused by increasing loading stress (Chen et al., 2015; Xiao et al., 2015a), or it rotated and then translated downwards with an increase of particle breakage during loading (Bandini and Coop, 2011).

### 2.3.2 Cyclic Behaviour of Ballast

The cyclic behaviour of ballast under different loading conditions can be characterised by four main regimes. These regimes can be sorted based on the ratio of deviatoric stress $q$ to mean effective principal stress $p'$ according to Suiker and de Borst (2003). The shakedown regime defines a cyclic response of ballast which is fully elastic, while the cyclic densification regime is where ballast undergoes progressive plastic deformation under cyclic loading. In the frictional failure regime, frictional collapse occurs as the level of cyclic load exceeds the static peak strength of the ballast. The tensile failure regime, which seldom exists in a practical railroad foundation, is where ballast disintegrates because it is unable to sustain the induced tensile stresses. During the first load cycles, the permanent strains increase rapidly and the resilient strains gradually decrease with an increase in the stiffness of ballast. After this initial phase, the permanent strains tend to stabilise and the material becomes essentially elastic. This stable behaviour is generally obtained after several thousand load cycles.

#### 2.3.2.1 Resilient Behaviour

The resilient behaviour of ballast under repetitive traffic loading can be characterised by the resilient modulus $M_r$, a stress strain which can be calculated according to Equation (2.12),
where $\sigma_d$ and $\varepsilon_r$ are the magnitude of deviatoric stress and the recoverable axial strain, respectively, and where $M_r$ increases with an increase in the loading cycles; after a significant number of cycles the deformation of ballast stabilises and shakedown will commence.

\[
M_r = \frac{\sigma_d}{\varepsilon_r}
\]  

(2.12)

Note that the resilient modulus is influenced by factors such as the void ratio, the PSD, particle shape, moisture content, stress history and stress sequence, loading frequency, and the number of loading cycles (Lekarp et al., 2000a).

### 2.3.2.2 Permanent deformation

Permanent deformation is a long-term characteristic of coarse granular aggregates under vehicular (traffic) loading in the field, including ballast; it defines how well a material tolerates the traffic loading applied over the long term. However, due to the limitation of the laboratory equipment and to reduce the excessive time taken in the laboratory to simulate the real track loads over a long period of time, cyclic loading without any rest period was adopted to investigate the permanent and resilient deformations of ballast. According to Sevi and Ge (2012), the maximum applied deviatoric stress has a significant influence on the permanent deformation characteristics of granular materials under cyclic loading. When a specimen is subjected to multiple magnitudes of loading, the largest load has the greatest effect on the degree on settlement. Lackenby et al. (2007) carried out a series of high frequency cyclic triaxial tests to examine how the confining pressure and deviatoric stress affected ballast deformation. As Figure 2.13 shows, stable conditions (shakedown) occurred when $\varepsilon_s$ was below 25% during the initial few thousand cycles, while the rate of shear strain
against $N$ decreased sharply and became very small after a few hundred cycles, as shown in Figure 2.14.

Figure 2.13. Axial strain as a function of the number of cycles loading (modified after Lackenby et al., 2007)

Figure 2.14. Rate of axial strain to the number of cycles (modified after Lackenby, 2006)
Indraratna et al. (2010) calculated a series of cyclic deviatoric stress following Esveld (2001) for a train speed (frequency) with an axle load of 30 tones. Cyclic triaxial tests were carried out under different loading frequencies \( f \), and revealed that the loading frequency had a significant influence on the deformation and degradation of ballast. To comprehensively study how the loading frequency affected ballast subjected to various loading amplitudes and confining pressures, Sun et al. (2015) carried out a series of large scale cyclic triaxial tests. It was reported that there was a large increase in the axial and volumetric strains as the loading frequency increased. Note that the permanent deformation of coarse granular aggregates is not only influenced by the stress and loading frequency, but also by the PSD, particle shape, void ratio, stress history, moisture content, and principal stress reorientation (Lekarp et al., 2000b). It should be noted that the research findings reviewed here were mainly based on the cyclic triaxial tests without rest period for loads which was different from the field condition where ballast usually experienced vehicular loads with rest period. While the Writer appreciates the reality of rest periods, this can be regarded as a limitation of the study considering the available time for PhD in relation to very long testing periods for each sample of ballast. In this sense, the approach introduced by Indraratna et al. (2009) and others (Anderson and Fair, 2008; Sevi and Ge, 2012), based on the Ultimate Resilient Modulus can still be used with some caution, although it would have been more realistic to include rest periods to conduct the experiments as this may correctly reproduce fatigue effects that have not been modelled in this thesis, noting that continuous tests will lead to increased degradation of particles by fatigue and also eliminate any rebound, even if this is insignificant after a large number of cycles after attaining cyclic densification and shakedown as explained in the book by Indraratna et al (2011).
2.4 Particle Shape and Its Influence on the Physical and Mechanical Properties of Granular Soils

Every object has a particular shape, including granular particles, but it is one of the most difficult properties to characterise and quantify, for all but the simplest of shapes. Particle shape has long been recognised as an important factor with a significant influence on the engineering performance of railroad ballast (Indraratna et al., 2011). However, only limited investigations into the shape of railroad ballast and its effect have been carried out in the past (Tutumluer et al., 2000, 2005; Rao et al., 2002; Pan et al., 2006; Al-Rousan, et al., 2007; Masad et al., 2007; Anochie-Boateng et al., 2013; Wnek et al., 2013; Moaveni et al., 2013, 2014, 2016; Sun et al. 2014). Al-Rousan et al. (2007) compared different image analysis method and suggested the use of the gradient method and tracing the change in slope of a particle outline for angularity which was found to have a significant effect on the deformation behaviour of ballast under repeated loading (Tutumluer and Pan, 2008). Existing literature (Cho et al., 2006; Rousé et al., 2008; Cavarretta et al., 2010; Sezer et al., 2011; Shin and Santamarina, 2012) on sandy soils also revealed that particle shape definitely influences their physical and mechanical properties.

For instance, higher particle angularity would promote breakage and result in soil with lower stiffness and relatively higher compressibility, while randomly packed assemblages with extremely elongated particles may have higher shear strength and more dilation (Yan, 2009). Moreover, lower surface roughness may encourage particles to slide rather than rotate, eventually leading to granular soils with a smaller critical state friction angle; ultimately, the larger the particle the higher the probability of imperfections and micro-cracks (McDowell, 2002). Smaller particles usually result from larger particles being crushed, and they are stronger due to a lack of internal imperfections when suffering diametrical loading.
Unlike sandy soils, railroad ballast mainly consists of highly angular particles larger than 9.5 mm, so they are easier to break, even under a relatively smaller confining pressure (< 65 kPa), which inevitably changes its strength and deformation (Indraratna et al., 2005; Lackenby et al., 2007). This change in shape due to the breakage of angular ballast particles would change the original size distribution and thus influence the stability and duration of railroad ballast over the long term, so there is an urgent need to investigate its particle shape and size. The ability to describe railroad ballast with respect to size and shape will definitely contribute towards a better management and maintenance of railroad ballast and thus result in cost savings.

2.4.1 Particle Shape Indices

Sieve analysis has been the primary method for determining the size and often the shape of railroad ballast (AS 1141.15), but sieving is only a bulk approximation of its PSD so it cannot be used to analyse any individual particles within the ballast. To better characterise the shape and size of granular particles, several methods have recently been used to analyse their image (Fernlund, 1998; Fernlund et al., 2007; Lee et al., 2007; Muszynski et al., 2012; Altuhaﬁ et al., 2013; Ohm and Hryciw, 2013; Al-Rousan, et al., 2007; Moaveni et al., 2013). Unlike sieving analysis, an image-based analysis is relatively objective even though somewhat time consuming. Lots of particle indices have been proposed (Cho et al., 2006; Rousé et al., 2008; Altuhaﬁ et al., 2013; Le Pen et al., 2013), based on two dimension (2D) or pseudo three dimension (3D) scanning results, most of which were used to describe the form, roundness, regularity and sphericity of a particle.
2.4.1.1 Form

Following Sneed & Folk (1958), the term form is used here to describe the three-dimensional characteristics of a particle defined by the ratios of its three linear dimensions (Figure 2.15), length ($L$), width ($I$) and thickness ($S$), i.e., elongation ratio ($er$), flatness ratio ($fr$) and elongation and flatness ratio ($ar$), of which the detailed definitions are expressed below as:

\[
er = \frac{I}{L} \quad (2.13)
\]

\[
fr = \frac{S}{I} \quad (2.14)
\]

\[
ar = \frac{S}{L} \quad (2.15)
\]

The above formulae describe how a particle resembles a columnar or plate, and where once the elongation ratio and flatness ratio are obtained, a position can be found from the Zingg diagram, as shown in Figure 2.16, denoting the actual form terminology.

Figure 2.15. Three dimensions of a particle (Fernlund, 1998)
2.4.1.2 Roundness and Angularity

Conceptually, roundness is independent of the form, and relates to the relative rounding or angularity of a particle. It can be regarded as an inverse description of particle angularity used by other researchers (Tutumluer, et al., 2000; Descantes et al., 2006; Qian et al., 2014). The original definition of roundness was proposed by Wadell (1933) specifically for 2D analysis where the radius of each corner in the outline is measured, averaged, and then divided by the radius of the maximum inscribed circle. In order to describe the surface angularity of a particle of sand, i.e.,

$$ R = \frac{\sum_{i=1}^{n} r_i}{r_{\text{max-ins}}} $$  \hspace{1cm} (2.16)

where $r_i$ is the radius of each corner in the particle outline, and $r_{\text{max-ins}}$ is the radius of the maximum inscribed circle, as shown in Figure 2.17.
It should be noted that Tutumluer et al. (2000) developed a quantitative angularity index, $AI$, by using the University of Illinois Aggregate Image Analyzer. The $AI$ index is determined by tracing the slope change of the 2D particle image outline obtained from each of the top, side and front images and can be defined as:

$$AI = \sum_{i=0}^{170} i \times P_a(i)$$

(2.17)

where $i$ is the starting value for each $10^\circ$ class interval and $P_a(i)$ is the probability that change in angle $\alpha$ has a value in the range of $i$ to $(i + 10)$. Until now, no commonly accepted 3D roundness index has been proposed because most relevant indices concerning particle angularity were based on 2D analysis. To facilitate further analysis with 3D scanning, some so-called 3D indices for describing roundness were suggested. For example, Hayakawa and Oguchi (2005) might be the first to attempt to propose a 3D index $X_s$ based on an assumption that the volume/area ratio reflects particle roundness. To eliminate the influence from three representative dimensions, they suggested the following form for describing the 3D roundness of a particle:
\[ X = \frac{V}{S_{o}^{\frac{1}{3}} / ISL} \]  

(2.18)

where \( V \) is the real volume of the particle and \( S_{o} \) is the real surface area of the particle.

### 2.4.1.3 Sphericity and Equivalent Particle Diameter

Sphericity is considered here to be a measure of the degree to which the shape of a particle approximates a true sphere, i.e. a 3D form with a constant radius of curvature. It depends on form and roundness because a perfect sphere has an equal length, breadth, and thickness, and is perfectly rounded. Sphericity also represents the gross particle shape and reflects the similarity between the representative axes \( L \), \( I \) and \( S \). The quantified index of true sphericity \( \psi \) can be defined according to Wadell (1935) by:

\[ \psi = \frac{s_{n}}{S_{o}} \]  

(2.19)

where \( s_{n} \) is the surface area of a sphere having the same volume as the scanned particle. The surface area is hard to get in traditional 2D measurement, so two alternative sphericity indices specifically for 2D analysis are usually used:

\[ \psi_{1} = \left( \frac{IS}{L^{2}} \right)^{\frac{1}{3}} \]  

(2.20)

\[ \psi_{2} = \frac{d_{n}}{L} \]  

(2.21)

where \( d_{n} \) is the equivalent diameter of a sphere having the same value of particle volume \( V_{s} \) (Wadell, 1933), that can be defined as:

\[ d_{n} = \left( \frac{6V}{\pi} \right)^{\frac{1}{3}} \]  

(2.22)
Apart from the 3D scan method, the particle volume \( V \) can be measured by immersion in a fluid. The equivalent diameter is usually smaller than the major length \( L \) of the corresponding natural particle which in most cases has an irregular shape. It approaches \( L \) when the particle increasingly resembles a sphere, so the equivalent diameter reflects the effect of particle irregularity.

Another interesting attempt by Le Pen et al. (2013) should also be mentioned; instead of describing how a particle resembles the corresponding sphericity, they proposed a new definition of ellipseness to describe how a particle with a natural form resembles the corresponding ellipse. So the definition can be expressed as:

\[
\tilde{e} = \frac{P_e}{P_o}
\] (2.23)

where \( P_o \) and \( P_e \) are the perimeters of the scanned particle and the equivalent area ellipse, respectively. \( P_e \) can then be obtained from the following formula:

\[
P_e \approx \pi(a_s + b_s) \left[ 1 + \frac{\frac{3}{a_s} \left( \frac{a_s - b_s}{a_s + b_s} \right)^2}{10 + \sqrt{4 - 3 \left( \frac{a_s - b_s}{a_s + b_s} \right)^2}} \right]
\] (2.24)

in which

\[
a_s = \frac{L}{2}
\] (2.25)

\[
b_s = \frac{A_0}{\pi a_s}
\] (2.26)

where \( A_0 \) is the area of the particle projection. As pointed out by Le Pen et al. (2013), the proposed ellipseness has no correlation with sphericity and is a totally different measure of
particle shape; therefore, it may be slightly more reasonable than sphericity because in most cases, natural particles are usually elliptical rather than spherical.

2.4.1.4 Regularity

Regularity is also independent of form and relates to a deviation of the three dimensional external expression of an object from a regular body (either curved or straight sided) due to projections and indentations. One commonly accepted definition of regularity was proposed by Cho et al. (2006) as:

\[ \rho = \frac{R + \psi}{2} \]  \hspace{1cm} (2.27)

There is another index called irregularity which can be regarded as the opposite of regularity. One possible definition can be found by referring to Blott and Pye (2008) as:

\[ I_{2D} = \sum \frac{\hat{y} - \hat{x}}{\hat{y}} \]  \hspace{1cm} (2.28)

where \( \hat{x} \) is the distance from the centre of the largest inscribed circle to the nearest point of any concavity, and \( \hat{y} \) is the distance from the centre of the largest inscribed circle to the convex hull, measured in the same direction as \( \hat{x} \). The total degree of profile irregularity is indicated by the sum of all the concavities measured in the plane of projection. Where the distance to the convex hull is difficult to measure \( \hat{y} \) can be calculated by measuring the distance to the projections adjoining any concavity:

\[ \hat{y} = \frac{a_1 \cos A_1 + b_1 \cos B_1}{2} \]  \hspace{1cm} (2.29)
where \(a_1\) and \(b_1\) are the distances from the centre of the largest inscribed circle to the tip of the projections on either side of the concavity, and \(A_1\) and \(B_1\) are the angles between \(a_1\) and \(x\), and \(b_1\) and \(x\), respectively.

2.4.2 Effect of Particle Shape on the Physical and Mechanical Properties of Granular Soils

Many researchers have studied how the particle shape of coarse granular aggregates affected the physical and mechanical behaviour (Cho et al., 2006; Rousé et al., 2008; Tutumluer et al., 2005; Pan et al., 2006; Tutumluer and Pan, 2008; Boler et al., 2012; Wnek et al., 2013), from which it was found that particle shape did have a significant influence on the void ratio, shear strength, shear band, dilatancy, critical state, resilient deformation and permanent deformation.

2.4.2.1 Effect of Particle Shape on Void Ratio

Particle shape has long been recognised one of the main factors that influence the void ratio of granular aggregates. Nouguier-Lehon (2010) used a 2D discrete element method to investigate the effect of particle elongation on the physical and mechanical behaviours of granular aggregates subjected to biaxial compression. The samples were generated using the same grading. To eliminate the effect of particle roundness (angularity), all the particles were in polygonal shapes that had six edges. Figure 2.18 shows how the initial void has evolved with the elongation ratio. As Nouguier-Lehon (2010) suggested, the initial void ratio would decrease and then increase again with the increasing particle elongation; so extremely elongated aggregates or spherical particles would have a larger void ratio. A minimum void ratio was observed when the elongation ratio was around 0.7.
Moreover, Cho et al. (2006) and Rousé et al. (2008) research indicated that the maximum and minimum void ratio decreased as the particle roundness increased, which means that assemblages with more angular particles are more likely to have a higher void ratio than those with less angular particles.

Apart from the effect exerted by particle form and particle roundness, particle sphericity and regularity are also influenced by the void ratio. Figure 2.19 shows that the maximum and minimum void ratios decreased with an increasing particle sphericity and regularity, and particle regularity correlates well with the extreme void ratios. Irregular particles usually have angular or relatively long axes, which will promote larger voids between the particles, and once loaded, larger voids will quickly fill up with smaller particles breaking from larger ones. A better compaction can thus be anticipated in assemblages consisting of rounder spherical particles, but for railroad engineering, angular particles are usually used for increased drainage capacity and strength of the ballast layer. A compromise between lower settlement and higher strength should therefore be made.
2.4.2.2 Effect of Particle Shape on Soil Strength

The strength of soil described here is its peak shear strength and residual shear strength expressed by the value of the corresponding friction angle. As reported by Nouguier-Lehon (2010), the peak friction angle and the residual friction angle may change as the elongation ratio varies. At first, the peak strength appears to increase and then decrease as the particle elongation ratio increases, while the residual strength also decreases linearly as the particle elongation increases. This may be because samples with elongated particles usually have face-to-face contacts that restrict rotations, so to accommodate the increasing strain, particle rotation rather than sliding must occur. This analysis was based on the discrete-element method (DEM) by Yan et al. (2009), and it also confirmed that the strength of soil increased as the elongation ratio decreased. However, no peak was found in the relationship between the elongation ratio and shear strength, possibly because the range of the elongation ratio
studied was narrow and did not provide a full insight into the response of soil induced by different particle forms.

Besides, particle roundness also has some influence on the shear strength of granular soils. Following the researches carried out by Cho et al. (2006), Miura et al. (1998), Rousé et al. (2008), Cavarretta et al. (2010) and Shin and Santamarina (2012), an increase in the strength of soil with decreasing particle roundness also occurred. Less roundness will definitely promote rotation between the particles and also indicate a higher dilatancy that contributes to soil strength (Ueng and Chen, 2002; Indraratna et al., 2011); this means a higher shear strength of assemblage with more angular particles can be expected.

There was a similar trend in the relationship between the friction angle and particle sphericity, as well as regularity, both of which have a negative effect on the shear strength of granular soils (Sezer et al., 2011). The research by Tutumluer and Pan (2008) also confirmed the influence of particle angularity on the strength and stability of unbound materials by increasing the friction between the aggregates. The particle form was found to have a minor effect on lateral stability of ballast when particle breakage was not considered (Tutumluer et al., 2006). However, even though the higher angularity would promote the shear strength of granular soils, it could also yield higher fouling and increased breakdown potential of the aggregates (Wnek et al., 2013), and this is a common phenomenon in railroad ballast that can significantly decrease the shear resistance of original granular mass, especially under higher confining pressures (Indraratna et al., 2014a; Fu et al., 2014; Xiao et al., 2015a).

Moreover, a shear band usually occurs prior to the peak strength in sands, as do factors such as the stress path, drainage condition, confining pressure, and boundary friction and particle shape. Zhuang et al. (2014) investigated how particle shape affects the plane strain compression of granular materials using the PSC method and the PSCD method. Here, PSC means the conventional Plane Strain Compression (PSC) test, where the specimen was
sheared under a constant axial displacement rate (displacement-controlled test) and a constant
effective confining stress, while PSCD means the Plane Strain Compression test by
Decreasing (PSCD) the effective confining pressure and a constant effective axial stress. The
roundness of Ube No.6A ($R_u$) sand is higher than Toyoura sand ($R_T$) and Glassbead1 ($R_1$), as
well as Glassbead2 ($R_2$), with a sequence of $R_u > R_T > R_2 > R_1$. Here the widths of the shear
band in these two sands were clearly smaller than in Glassbead1, even though they had
similar mean particle sizes. The width of the shear band became thinner when the particle
shape was more angular.

2.4.2.3 Effect of Particle Shape on Dilatancy and Critical State

Samples with spherical particles can still exhibit dilatancy when the confining pressure is low,
but in general, an assemblage formed by longer particles gives higher shear strength and
more dilation (Kozicki et al., 2012). In fact, due to high dilatancy, elongated particles also
have much higher shear strength while the samples with less spherical and regular particles
would also dilatant more.

A critical state is where soil continues to deform at a constant stress and constant volumetric
strain (Roscoe et al., 1958), and the critical state line is the loci of critical state conditions in
the $e \sim p'\sim q$ space, and its projection on the $p'\sim q$ plane defines the critical state angle $\phi_{cs}$
as follows:

$$M = \frac{6 \sin \phi_{cs} \sin^3 \phi_{cs}}{3 - \sin \phi_{cs}}$$

where $M$ is the slope of the critical state line in the $p'\sim q$ plane. Moreover, a projection of
the critical state line onto the $e \sim \ln p'$ space defines the slope $\lambda$ and intercept $e_f$ as follows:
\[ e_{cs} = e_r - \lambda \log \left( \frac{p_a^*}{p_a} \right) \] (2.31)

where \( e_{cs} \) is the critical state void ratio and \( p_a \) is the reference pressure, usually taken as the atmospheric pressure. According to research by Cho et al. (2006) and Shin and Santamarina (2012), particle shape does influence the critical state of granular soils because all the critical state parameters, i.e., \( e_{cs}, \lambda \) and \( e_r \) decrease with the increasing particle roundness and sphericity, as well as particle regularity. A higher critical state line should be observed in soils with more angular particles. Moreover, as Cho et al. (2006) suggested, particle roundness is more relevant to the critical state friction angle \( \phi_{cs} \) and intercept \( e_r \) than particle sphericity, which means that particle roundness is extremely important when determining the stress strain response of granular soils and ballast under static and cyclic loading conditions.

### 2.4.2.4 Effect of Particle Shape on Resiliency and Permanent Deformation

A number of researchers (Allen, 1973; Barksdale and Itani, 1989; Thom and Brown, 1988) have reported that crushed aggregates consisting of angular to sub-angular shaped particles, have better load spreading properties and a higher resilient modulus than uncrushed gravel with sub-rounded or rounded particles; while a rough particle surface also resulted in a higher resilient modulus. Pan et al. (2006) tested the resilient modulus of a total of 21 unbound specimens with different particle shapes in a laboratory triaxial setup. They found that as the particle angularity and surface roughness increased, the resilient modulus was improved considerably due to the increased shear strength attributed to better aggregate interlock and intrinsic frictional properties (Tutumluer and Pan, 2008). Barksdale and Itani (1989) investigated several types of aggregates and observed that the resilient modulus of rough and
angular crushed materials was higher than rounded gravel by about 50% at low mean normal stress, and about 25% at high mean normal stress. Although an increasing particle angularity and surface roughness could result in a higher resilient modulus, studies show that Poisson’s ratio decreased under the same conditions (Allen, 1973). Allen (1973) related the difference in plastic strains between different types of aggregates with the same density as the surface characteristics of the particles. He argued that angular materials such as crushed stone underwent smaller plastic deformation than materials such as gravel with rounded particles. This behaviour was said to be the result of a higher angle of shearing resistance in angular materials due to better particle interlock. Barksdale and Itani (1989) investigated the influence that the shape and surface characteristics of aggregate would have on aggregate rutting and concluded that a blade-shaped crushed aggregate was slightly more susceptible to rutting than other types of crushed aggregates. Moreover, cube-shaped, rounded river gravel with smooth surfaces was much more susceptible to rutting than crushed aggregates.

2.5 Particle Size Distribution and its Effect on Physical and Mechanical Behavior of Granular Soils

Particle size distribution statistically explains how different size particles are distributed and are usually displayed as either mass based PSD, surface area based PSD, or number based PSD. Mass based PSD is usually utilised because it is easier to determine, but when doing some particular research such as constriction size distribution (CSD) based filtration analysis (Kenny et al., 1985; Trani and Indraratna, 2010), the other two PSDs are preferred (Raut and Indraratna, 2004; Locke et al., 2001; Indraratna et al., 2007). The distribution of particle sizes has an obvious and significant influence on the physical and mechanical behaviour of granular soils, including sand and railroad ballast. Many researches (Thom and Brown, 1988;
Åberg, 1992; Åberg, 1996; Lade and Yamamuro, 1997; Boadu, 2000; Sitharam and Nimblekar, 2000, Cubrinovski and Ishihara, 2002; Wichtmann and Triantafyllidis, 2009; Tutumluer et al., 2009; Carrera et al., 2011; Ueda et al., 2011; Yan and Dong, 2011; Cunningham et al., 2013; Zhang and Buscarnera, 2014) have been carried out by researchers on the void ratio (dry density), permeability, elastic modulus (resilient modulus), dilatancy, shear strength, particle breakage, critical state, and permanent deformation.

2.5.1 Particle Size Distribution

The PSD of a sample can be displayed in three ways, but the first and also the most common is mass based PSD, which can be directly determined from the sieve results, as follows:

\[
P_{Mt} = \frac{\sum_{i=1}^{n} m_j}{m_t}
\]

(2.32)

where \( m_j \) is the mass of the corresponding sieve interval and \( m_t \) is the total mass of the sample; \( i \) ranges from 1 to \( n \) where \( n \) is the total number of the sieve intervals. However, as Locke et al. (2001) explained, the use of PSD by mass may introduce errors in well graded soils because large particles with a high individual mass but low in number will be over represented because it is unlikely that these few large particles will meet together to form a large constriction. To overcome this limitation, particle surface area-based and particle number-based PSDs are used. The transformation from mass based PSD to surface area based PSD or number based PSD can be obtained by the following equations:

\[
P_{Sd} = \frac{P_{Mt}}{d_j} / \sum_{i=1}^{n} \frac{P_{Mt}}{d_j}
\]

(2.33)

\[
P_{Ni} = \frac{P_{Mt}}{d_j^3} / \sum_{i=1}^{n} \frac{P_{Mt}}{d_j^3}
\]

(2.34)
where $P_{Mi}$, $P_{Ni}$ and $P_{Si}$ are the mass, number and surface area probabilities of occurrence in a corresponding discretised diameter $d_i$, respectively (Figure 2.20). Note that the discretised diameter $d_i$ is usually treated as the geometrical average of two neighbouring sieve sizes (Humes, 1996; Trani and Indraratna, 2010; Vincens et al., 2014).

![Particle size distribution by mass](image)

Figure 2.20. Particle size distribution by mass

2.5.2 Effect of PSD on Physical and Mechanical Properties of Granular Soils

2.5.2.1 Effect of PSD on Void Ratio

Uniformly graded granular aggregates usually have a larger void ratio than a well graded one. For one sample with a given PSD, the void ratio is determined if the compaction effort is according to Åberg (1992, 1996). However, the void ratio can continuously decrease by adding more and more smaller particles which increasingly fill the voids created by larger particles. Moreover, the void ratio usually decreases with an increasing ratio of maximum
particle size \( d_M \) to minimum particle size \( d_m \). According to Åberg (1992), a void ratio relative to a given PSD can be obtained by following formula:

\[
e = 2c \left[ \int_0^1 \frac{F}{dF} \right] + 2\bar{d}
\]

where \( F \) is the percentage passing, \( c \) denotes the coefficient that depends on the shape of grains (\( c \approx 0.6 \) for spheres, \( c \approx 0.75 \) for sand and gravel, and \( c \approx 1.0 \) for crushed rock); and \( \bar{d} \) is the constant that depends on the degree of densification (\( \bar{d} \approx 0.18 \) for loosest possible packing or relative density of approximately 0, \( \bar{d} \approx 0 \) for heavily compacted sand and gravel or relative density of approximately 1.0). Equation (2.35) can be used to calculate the void ratio of soils at extreme states, i.e., the loosest and the densest. Following Indraratna et al. (2011), the following summation form of Equation (2.35) can be adopted for practical use:

\[
e = 2c \sum_{i=1}^n \frac{F_{Mi}/d_i F_{Mi}}{\sum_{i=1}^n (F_{Mi}/d_i)} + 2\bar{d}
\]

where \( F_{Mi} \) is the mass frequency as shown in Figure 2.20.

### 2.5.2.2 Effect of PSD on Permeability

According to the Kenny and Carman equation below, a change of PSD results in a change in the distribution of the void ratio which directly influences the hydraulic conductivity of granular soils:

\[
k = \left( \frac{\chi}{\mu} \right) \left( \frac{1}{C_{KC}} \right) \left( \frac{1}{s_0^2} \right) \left( \frac{e^2}{1+e} \right)
\]

where \( k \) is the soil permeability, \( e \) is the void ratio, \( \chi \) is the unit weight of the permeant, \( \mu \) is the viscosity of the permeant, \( C_{KC} \) is the empirical coefficient, and \( s_0 \) is the specific surface.
area per unit volume of particles. For a given PSD, the soil property is defined so that all the parameters in Equation (2.30) except the void ratio are fixed. Figure 2.21 shows the evolution of permeability with the void ratio; it also shows that permeability gradually increases when the void ratio is relatively small, and then rapidly increases when the void ratio is large.

\[ k' = kC_s \frac{2}{\varepsilon \chi} \]

**Figure 2.21. Relationship between the permeability and void ratio**

However, soils consist of solid particles with voids that enable water to pass through the medium, and these pore constrictions form the smallest link between voids. Water flowing through a void network encounters constrictions that are randomly distributed along the flow path, and whose sizes are expected to govern hydraulic conductivity. There is no doubt that hydraulic conductivity correlates to the size of the void network of a granular medium rather than its particle sizes (Kenney et al., 1985; Humes, 1996; Trani and Indraratna, 2010; Indraratna et al., 2012a). Recent studies by Indraratna et al. (2007) as well as Raut and Indraratna (2008) discussed the advantages of applying the constriction sizes of the void
network for granular soil, with clear implications for the constriction-size distribution (CSD) on hydraulic conductivity and associated computational procedure.

Raut and Indraratna (2008) proposed theoretical concepts to calculate the CSD from particle gradation and the initial density. In granular soil, particles exist in a group of three or four, representing the densest and loosest arrangements, respectively. Humes (1996) assumed that at the densest arrangement, the constriction size \(D_{cD}\) is defined as the diameter of the largest circle that can fit within three tangent soil particles (Indraratna et al., 2012a), such that

\[
\left( \frac{2}{D_1} \right)^2 + \left( \frac{2}{D_2} \right)^2 + \left( \frac{2}{D_3} \right)^2 + \left( \frac{2}{D_{cD}} \right)^2 = \frac{1}{2} \left[ \left( \frac{2}{D_1} \right) + \left( \frac{2}{D_2} \right) + \left( \frac{2}{D_3} \right) + \left( \frac{2}{D_{cD}} \right) \right]^2
\]

(2.38)

in which \(D_1, D_2\) and \(D_3\) are diameters of particles forming the densest arrangement.

Following Silveira et al. (1975), the constriction size \(D_{cL}\) corresponding to the loosest state can be defined as follows:

\[
D_{cL} = \sqrt{\frac{4S_{c,\text{max}}}{\pi}}
\]

(2.39)

where \(S_{c,\text{max}}\) is the maximum value of the constriction space \(S_c\) between four particles. To calculate the constriction size distribution of granular soils, number-based and surface area-based PSDs are usually used, but the PSD by number can sometimes over represent the finer constrictions because small particles with a high number will be over represented because these small particles are unlikely to meet together to form a finer constriction. Although there are only a small number of large particles, they impose significant contact with other small particles due to their larger surface area. In real granular aggregates, small particles are more likely to surround larger particles, so the PSD by particle surface area seems to be the best option for CSD analysis. To account for the effect of soil compaction, a surface area based constriction size was proposed by Locke et al. (2001) as follows:
\[ D_c = D_{cD} + P_c\left(1 - R_d\right)\left(D_{cL} - D_{cD}\right) \]  

(2.40)

where \( D_c \) is the actual constriction size for a given value of the percentage finer \( (P_c) \) and \( R_d \) is the relative density of the granular soil. Indraratna et al. (2012a) suggested a correlation between the mean constriction size and the permeability of granular soil, where the mean constriction size \( D_c^m \) can be determined from the CSD curve as follows:

\[ D_c^m = \frac{\sum_{i=1}^{n} P_{ci} D_{ci}}{\sum_{i=1}^{n} P_{ci}} \]  

(2.41)

where \( D_{ci} \) is the constriction size consisting of a given CSD, and \( P_{ci} \) is the corresponding values of probability of occurrence, as shown in Figure 2.22. By accepting the mean constriction size as the controlling factor that determines the hydraulic behaviour of granular soils, a power law function for predicting permeability was defined according to Indraratna et al. (2012a).

Figure 2.22. Typical discretized CSD curve of a granular soil
2.5.2.3 Effect of PSD on Dilatancy, Shear Strength, Particle Breakage and Critical State

Different PSDs undergo different particle breakage during loading and this can directly affect the dilatancy of soils; indeed, according to research by Ueng and Chen (2000), a combination of particle breakage and stress dilatancy can also influence the shear strength of granular soils.

A triaxial analysis of a granular assemblage using 3D DEM (Yan and Dong, 2011) indicated that an assemblage with a wider particle grading gives more contractive response and acts towards strain hardening upon shearing. The narrower the PSD, the more dilatancy occurred at the same stress condition, and since dilatancy would increase the shear strength of granular soils, a higher peak strength was observed in the PSD with a smaller $C_u$.

A ring shear test on carbonates and by Coop et al. (2006) revealed that particles would continuously break and shift the PSD until they reached an ultimate fractal grading. Based on this research, Altuhafi and Coop (2001) further observed a large amount of breakage in poorly graded samples; however, a significant reduction in particle breakage could be anticipated by changing the initial grading to a better graded sample. For very well-graded samples no significant particle breakage can be measured which means that soils with wider PSDs will have less particle breakage under the same stress conditions than other narrow graded soils, given they have the same relative densities.

A DEM analysis by Sitharam and Nimbkar (2000) revealed that a change to a wider gradation (by maintaining the minimum grain size) would result in a decrease in the angle of internal friction and a large increase in volumetric strain. Figure 2.23 shows a plot of the internal friction angle with maximum particle size for all the tests, showing that for exactly parallel gradations ($C_u$ and $d_M/d_m$ are almost the same) there was a marginal increase in the angle of internal friction as the maximum particle size increased. This reflects the obvious
effect of $d_{50}$ on the shear strength, as was also observed by Wang et al. (2013) who conducted comprehensive laboratory research on the accumulation soils with different PSDs. This small increase in the angle of internal friction can be attributed to an increase in the contact area at the grain scale level as the particle size increases. However, in PSDs with the same minimum grain size, even though there was an increase in the average particle size in the sample, there was a decrease in the angle of internal friction as the PSD became wider. This occurred because the dilatancy of grains and the development of induced anisotropy in contact forces play a major role in gradations with the same minimum grain size.

Figure 2.23. Influence of the PSD on the shear strength (modified after Sitharam and Nimbkar, 2000)

According to laboratory triaxial tests on Dog’s Bay sand by Bandini and Coop (2011), the critical state friction angle did not change with PSDs, but the critical state line in the $e - \ln p'$ plane did evolve with the changing PSD. Muir Wood and Maeda (2008) suggested that as the
PSD shifted towards the ultimate fractal grading, the critical state line moved downwards in the $e - \ln p'$ plane (Kikumoto et al., 2010).

2.5.2.4 Effect of PSD on Resilient Modulus and Permanent Deformation

Unlike the elastic modulus which describes the behaviour of materials in elastic region, the resilient modulus captures the strain bounce characteristic of the plastically deformed material during unloading. Previous researches in this area showed that the resiliency of the material depends to some degree on the particle size and its distribution. Research carried out by Jorenby and Hicks (1986) showed an initially increasing stiffness (elasticity) and then a large reduction as small particles were added to the crushed aggregates. This initial improvement in stiffness was attributed to increased contact as the pore spaces were filled by excess fines gradually separating forcing the coarse particles apart so that stiffness decreased. For aggregates with the same amount of fines and a similar shape PSD, the resilient modulus increases with an increasing maximum particle size (Gray, 1962; Thom and Brown, 1988; Kolisoja, 1997). According to Kolisoja (1997), the particulate explanation of this response is that most of the load acting on a granular assembly is transmitted by particle queues, so when a load is transmitted via coarser particles, the smaller number of particle contacts results in less total deformation and consequently higher stiffness.

The PSD of granular materials seems to have some influence on material stiffness, although it is generally considered to be of minor significance. Thom and Brown (1988) studied the behaviour of crushed limestone with different initial PSDs and concluded that uniformly graded aggregates were only slightly stiffer than well-graded aggregates. Similar results were reported by Brown and Selig (1991) and Raad et al. (1992). Plaistow (1994) argued that when moisture is introduced to well-graded materials, the effect of PSD can be increased quite
significantly because these materials can hold water in the pores. Moreover, they can also achieve higher densities than uniformly graded materials because the smaller grains fill the voids between the larger particles. Plaistow (1994) therefore concluded that the PSD has an indirect effect on the elastic/resilient behaviour of granular aggregates by controlling the impact of moisture and density in the system. Heydinger et al. (1996) compared the effect of PSD on the resilient modulus for limestone, gravel, and slag and found that limestone had a higher resilient modulus when open-graded, but there was no trend for the variation of modulus in gravel. With slag, the results were opposite, so a denser gradation tended to give a higher stiffness. Lackenby (2006) investigated the cyclic triaxial behaviour of railroad ballast and observed an increase in the resilient modulus with increasing loading cycles; possibly due to a continuous shifting of ballast PSD from internal particle breakage that decreases the void ratio \( e \) among particles. This increasing density within the samples contributes to an increasing resilient modulus, according to the following formulas by Hardin and Richart (1963):

\[
G = G_0 \left( \frac{e_r - e}{1 + e} \right)^{\hat{n}} \left( \frac{p'}{p_{at}} \right)
\]

\[
K = K_0 \left( \frac{e_r - e}{1 + e} \right)^{\hat{n}} \left( \frac{p'}{p_{at}} \right)
\]

where \( G \) and \( K \) are the elastic shear modulus and volumetric modulus, respectively; \( G_0 \) and \( K_0 \) are parameters that decrease with an increasing amount of fines; \( p_{at} \) is the atmospheric pressure (101kPa) and \( \hat{n} \) is the fitting parameters; \( e_r \) accounts for the particle shape effect of soils that equals to 2.97 for angular sands and 2.17 for round sands. However, Wichtmann and Triantafyllidis (2009) showed that for a constant void ratio, while \( G_{max} \) was not influenced by any variation of the mean grain-size \( d_{50} \) in the range investigated, it decreased significantly as the coefficient of uniformity \( C_u \) of the PSD increased.
The effect of PSD on permanent deformation was studied initially by Thom and Brown (1988), who found that the evolutionary trend of permanent deformation depended mainly on the level of compaction, as shown in Figure 2.24. When uncompacted, those specimens with a uniform PSD exhibited the least permanent strain but for lightly compacted specimens, the least amount of permanent deformation existed in widely graded aggregates. Moreover, the resistance to plastic strain was similar for all PSDs when the specimens were heavily compacted, but the extremely wide range of densities and PSDs adopted by Thom and Brown far exceeded the range likely to be experienced in railroad engineering.
CHAPTER 3 3D CHARACTERISATION OF PARTICLE SIZE 
AND SHAPE

3.1 Introduction

The size and shape of aggregates has long been recognised as the two major factors that affect the performance of railroad ballast (Indraratna et al., 2011), but only limited investigations have been carried out (Thom and Brown, 1988; Anochie-Boateng et al., 2013; Le Pen et al., 2013). Previous studies on sandy soils revealed that particle shape and size definitely influence the physical and mechanical properties of sandy soils (Cho et al., 2006; Rousé et al., 2008; Cavarretta et al., 2010; Sezer et al., 2011; Shin and Santamarina, 2012). Higher particle angularity promotes particle breakage and results in lower strength and relatively higher compressibility. Randomly packed assemblages with highly elongated particles have higher shear strength and undergo more dilation (Yan, 2009), whereas lower surface roughness may encourage particles to slide rather than rotate; which eventually leads to granular soils with a smaller critical state friction angle. Fundamentally, the larger the particle is, the higher the probability of imperfections and micro-cracks would be (McDowell, 2002). Smaller particles usually come from the crushing of larger particles and are stronger due to a lack of internal imperfections under diametrical loading. Unlike sandy soils, railway ballast mainly consists of very coarse angular particles that are usually larger than 9.5 mm. These particles break even under relatively small confining pressure ($\sigma_3' \leq 60$ kPa), and this inevitably changes the strength and deformation of ballast (Indraratna et al., 2005; Lackenby et al., 2007). This resulting change in particle shape as angular ballast particles break would change the original PSD and thus influence the stability and duration of ballast over the long
term, therefore there is an urgent need to investigate the size and shape of ballast particles. The ability to correctly describe the size and shape of ballast will definitely contribute to better management and maintenance of railway ballast and result in substantial cost savings.

PSD analysis has been one of the main methods used to determine the size and sometimes the shape of ballast (AS 1141.15, 1999). Sieving is just a bulk approximation and cannot accurately analyse the shape of particles, whereas an image-based analysis is relatively objective and less time consuming. Several 2D image-analysis based methods have been used to characterise the size and shape of granular particles (Fernlund, 1998; Fernlund et al., 2007; Lee et al., 2007; Muszynski et al., 2012; Altuhafi et al., 2013; Ohm and Hryciw, 2013), but most existing studies (Cho et al., 2006; Rousé et al., 2008; Altuhafi et al., 2013; Le Pen et al., 2013) are based on two dimensional (2D) or pseudo 3D scanning which cannot provide comprehensive information about the form of individual particles. As will be shown later in this paper, 2D analysis actually underestimates true sphericity. Anochie-Boateng et al. (2013) used the University of Illinois Aggregate Image Analyzer (UIAIA) system to characterise the shape of ballast based on the flakiness index. Moaveni et al., (2013, 2014, 2016) studied the effect of particle shape on the breakage, abrasion, and polishing characteristics of ballast. Le Pen et al. (2013) studied the dependence of shape on particle size based on 2D image analysis, but they did not address the roundness of ballast particles very well.

The aim here is to carry out a 3D assessment of particle shape and size by utilising a 3D laser scanner. Particle shape and size indices such as the flatness ratio, the elongation ratio, true sphericity, and roundness will be presented, and a 3D ellipsoidness index will be proposed instead of a conventional sphericity and roundness index to characterise the regularity of ballast particles. The PSD of aggregates is represented by the mass, the surface area, and the number of particles. Modified formulations for accurately transforming conventional PSD based on mass, into PSDs based on the surface area and the number are proposed. The role of
the constriction size distribution (CSD) in lieu of PSD is also discussed, based on the surface area and the number of particles. A flow chart showing the overall experimental program can be found in Figure 3.1.

![Flow chart for the overall experimental study](image)

Figure 3.1. Flow chart for the overall experimental study

### 3.2 Experimental Investigations

#### 3.2.1 Sample Preparation

In the current study, railway ballast was collected from a designated quarry at Bombo, New South Wales, Australia, which supplies ballast for NSW railroads. Particles were divided into 7 intervals by standard sieving method (ASTM 2006): (i) 13.2 mm - 19 mm, (ii) 19 mm - 26.5 mm, (iii) 26.5 mm - 31.5 mm, (iv) 31.5 mm - 37.5 mm, (v) 37.5 mm – 40 mm, (vi) 40 mm – 45 mm, and (vii) 45 mm – 53 mm. About 30 particles were randomly selected from each sieve interval (similar to the approach adopted by Cho et al. (2006) and Rousé et al. (2008)). All the particles were carefully washed, dried, and painted white before being tested.
and then marked by small dots to facilitate different alignments during the scanning process (Figure 3.2).

3.2.2 Procedures for 3D Measurement

The scanning equipment was a non-contact 3D Laser Scanner VIVID 910 with an accuracy of 0.22 mm horizontally, 0.16 mm vertically, and 0.1 mm longitudinally, as shown in Figure 3.2. This 3D laser scanner uses the triangulation light block method where a sample is placed onto a black pedestal and then scanned by a plane of laser light coming from the VIVID's source aperture. The plane of light is swept across the field of view by a mirror rotated by a precise galvanometer, and then this laser light is reflected from the surface of the scanned sample. Each scan line was observed by a single frame and then captured by the CCD camera. The surface contour was derived from the shape of the image of each reflected scan line and then converted into a lattice of over 300,000 vertices, after which a polygonal-mesh is created from all the connected information. During processing, different tools (including the focus,
align, merge, and registration tools) integrated in the laser scanner controlling system are used to bring the scanned surfaces together, and then the noise and small holes on the merged model which originated from an improper reflection of the laser due to local surface conditions, were corrected using a spatial interpolation tool in the image analysis software Geomagic Qualify 12 (version: 15.0, 2010). Note that only those particles with a sieve size larger than 13.2 mm were used in this paper because imaging or mapping the surface boundary of particles less than 13.2 mm in diameter would not be accurate enough for this particular laser scanner. Particle shape before and after the test was also analysed to quantify the actual size and shape of aggregates that changed due to cyclic loading.

![Figure 3.3. Typical ballast particles: (a) original particles, (b) scanned results](image-url)
Figure 3.3 shows the representative particles of railway ballast and the corresponding scanned results, from which a good capture of the particle shape and surface characteristics can be observed. Geomagic Qualify 12 divides the surface mesh of the scanned ballast particle into triangular sub-surfaces called poly-faces that make up the particle. The total surface area $S$ can be obtained by the sum of all the poly-faces. Similarly, the total volume, $V$, can be computed by the sum of the sub-volumes of all the tetrahedral mesh. An excellent linear correlation between the real particle mass and its corresponding scanned volume ($R^2 = 0.99$) can be seen in Figure 3.4, which means this 3D laser scan system can give accurate scan results of ballast particles. Moreover, the density of the ballast aggregate, which equals the gradient of the fitting line, was determined to be precisely 2.66 g/cm$^3$.

![Graph showing the relationship between scanned volume and real particle mass](image)

Figure 3.4. Relationship between the real mass of ballast and its scanned volume

### 3.3 Characterisation of Particle Shape and Size

#### 3.3.1 Arithmetic Mean of the Shape and Size Indices

According to the probability theorem, the law of large numbers and the average of a sample approaches the mean of the population as the size of a sample increases. So to obtain a
statistically accurate evaluation of the shape and size of railroad ballast, the shape and size indices of 210 individual particles are measured and then averaged by the following formula:

\[
\overline{SS^j} = \frac{1}{n} \sum_{i=1}^{n} SS^j_i
\]  

(3.1)

where \( \overline{SS^j} \) is the averaged index of \( j \)-th sieve interval, and \( SS^j_i \) is the shape or size index of particle \( i \) in \( j \)-th sieve interval. Note that \( i \) ranges from 1 to 30 while \( j \) ranges from 1 to 7 in the current research.

### 3.3.2 Scanning Results

Particle size and shape are characterised by eight existing indices; the elongation ratio \( I/L \) (Zingg, 1935), the flatness ratio \( S/I \) (Zingg, 1935); true sphericity \( \psi \) (Wadell, 1935); sphericity \( d_{w}/L \) (Wadell, 1933); sphericity \( (IS/L^2)^{1/3} \) and roundness \( R \) (Krumbein, 1941); roundness \( X_s \) (Hayakawa and Oguchi, 2005), and regularity \( \rho \) (Cho et al., 2006). Note that estimating roundness as well as regularity depends on determining the curved corners of the particle, which is somewhat subjective because the outline of an angular particle is different at different viewing scales (Hayakawa and Oguchi, 2005; Sun et al., 2014). Most importantly, it cannot be used in 3D analysis because the definition of roundness is based on a 2D image. To overcome this limitation Le Pen et al. (2013) proposed a new index called ellipseness, but it is specifically defined for 2D analysis, so rather than projecting 3D scanning results onto 3 orthogonal planes to determine particle ellipseness or roundness, a relatively simple 3D measure called ellipsoidness \( E_e \) is proposed in this study, which was defined by (Sun et al., 2014) as:

\[
E_e = \frac{S_r}{S_o}
\]  

(3.2)
where $S_e$ denotes the surface area of the equivalent volume ellipsoid (with a major radius $a = L/2$ and two minor radii $b$ and $c$). For simplicity, $a > b = c$ is assumed. Once the representative particle lengths and volume are known, the minor radius $b$ (or $c$) as well as $S_e$ can be obtained from:

\begin{align}
    b &= \sqrt[3]{\frac{3V}{2\pi L}} \quad \text{(3.3a)} \\
    S_e &= 4\pi \left[ b^2 + a^2 \frac{\arccos(b/a)}{\tan(\arccos(b/a))} \right] \quad \text{(3.3b)}
\end{align}

Note that $S_o$ would decrease with decreasing angularity or an increasing surface regularity of a ballast particle for the same value of the actual volume $V$ and $S_o$ would approach $S_e$ with $E$ approaching unit when the surface of a particle becomes rounder. It will be shown later that the ellipsoidness $E_e$, has a relatively good correlation with the regularity $\rho$.

Figures 3.5 - 3.8 are the scattering results of different size and shape indices of individual particles from different sieve intervals obtained by using a laser scanner. The elongation and flatness ratios are plotted in the modified Zingg diagram (Figure 3.5) proposed by Blott and Pye (2008), and indicate that most of the ballast particles are slightly elongated and moderately flat, while only a small fraction of ballast remains moderately elongated and very flat. The elongation and flatness ratios seem to have no connection with each other, whereas the elongation ratio $I/L$ and the flatness ratio $S/I$ and true sphericity $\psi$ do increase with an increasing particle size, indicating that a more elongated and flatter shape should be observed in smaller particles of ballast. This accords with Altuhafi and Coop (2011)’s work where the aspect ratio (elongation ratio) was less in smaller particles being created.
Figure 3.5. Elongation ratio and flatness ratio of ballast aggregates in Zingg diagram
Figure 3.6. Relationship between the true sphericity $\psi$ and: (a) sphericity $\psi_1$; (b) sphericity $\psi_2$; (c) 3D roundness $X_s$ (Hayakawa and Oguchi, 2005)
Figure 3.7. Relationship between ellipsoidness $E$ and: (a) elongation ratio $I/L$; (b) true sphericity $\psi$; (c) regularity $\rho$

Besides, as Figures 3.6(a) and 3.6(b) show, the true sphericity (3D) $\psi$ mainly ranges between 0.7 ~ 0.8 while the 2D sphericity indices $\psi_1$ and $\psi_2$ mainly range between 0.6 ~ 0.8 and 0.5 ~ 0.7, respectively. The sphericity indices $\psi_1$ and $\psi_2$ shows a general increase with increasing true sphericity $\psi$, but their correlations are not strong. The 2D sphericity indices...
can therefore be used as a useful qualitative evaluation of particle sphericity, even though they both underestimate the true sphericity of the scanned particles. The 3D roundness index $X_s$ in Figure 3.6(c) was suggested by Hayakawa and Oguchi (2005) based on the idea that the volume/area ratio reflects particle roundness. As Figure 3.6(c) shows, $X_s$ is related to particle sphericity and increases as particle sphericity increases, but as Cho et al. (2006) pointed out, roundness must be independent of sphericity, which means that the 3D roundness $X_s$ is not appropriate for evaluating particle roundness. Fig. 3.7(a) and 3.7(b) are estimations by 3D ellipsoidness, from which there was a weak correlation with elongation ratio and true sphericity. Ellipsoidness seems to decrease with an increase in the elongation ratio, as shown in Figure 3.7(a), but Figure 3.7(c) shows that ellipsoidness increases with increasing regularity, which considers the combined effect of roundness and sphericity.

3.4 Discussions on Distribution of Shape Indices

The distribution of various size and shape indices of individual particles from different sieve intervals are plotted in Figures 3.8 - Figures 3.13. To show the overall trend of evolution, the best fitted straight lines are also drawn along with the average values of particle shape indices for ballast in Figure 3.14. Note that the ordinates for particle size in Figures 3.8 - Figures 3.15 are the geometrical averages of two neighbouring sieve sizes. The distributions of the elongation and flatness ratios are plotted in Figure 3.8(a) and 3.9(a). The average values of the elongation ratios from each sieve interval are plotted on Figures 3.8(b) and 3.9(b), which means the elongation ratio $I/L$ and flatness ratio $S/I$ increase with increasing particle size. This differed from the results reported by Le Pen (2013) where most of the ballast particles were found to be slightly flat and elongated, possibly due to the different techniques that quarries use to crush stone.
Figure 3.8. (a) Distribution of the elongation ratio and (b) mean elongation ratio vs particle size

Figure 3.9. (a) Distribution of the flatness ratio and (b) mean flatness ratio vs particle size
Figures 3.10 and 3.11 are the distributions of the estimated true (3D) sphericity $\varphi$ and its corresponding 2D sphericity $\varphi_1$. A comparison between Figures 3.10(a) and 3.11(a) indicates that the 2D analysis underestimated the true sphericity of the scanned particles. This was also reported by Fonseca et al. (2012) who used CT images to study the particle shape of sands. Note that the 3D sphericity and 2D sphericity are different, considering their geometrical definition; with 2D sphericity actually measuring the projection of a particle while 3D sphericity measures the real particle shape. If a particle almost spherical, the 3D and 2D values would approach unit. However, natural particles are more likely to be irregular so there are different values of 2D sphericity from different particle projections. Therefore, a 3D investigation must be carried out to accurately evaluate particle shape. Moreover, the average values of true sphericity (Figure 3.10(b)) and 2D sphericity (Figure 3.11(b)) increase with an increasing particle size, which is different from the result by Cunningham et al. (2013) who reported a decrease of sphericity with an increasing particle size.

Figure 3.10. (a) Distribution of the true sphericity and (b) mean true sphericity vs particle size
Figure 3.11. (a) Distribution of the 2D sphericity and (b) mean 2D sphericity vs particle size

Figure 3.12. (a) Distribution of the ellipsoidness and (b) mean ellipsoidness vs particle size
The distribution of ellipsoidness shows more variation than roundness, as Figures 3.12(a) and 3.13(a) show. The average values of ellipsoidness (Figure 3.12(b)) and roundness (Figure 3.13(b)) show a slight decrease with increasing particle size, which implies there should be more angular particles in larger particles. A similar observation of shape depending on particle size was reported by Le Pen et al. (2013). To show the overall size dependence of particle shape, best-fitted straight lines along with the averaged shape indices are drawn in Figure 3.14. Note that even though the particle shape of each sieve interval only shows a small variation, it may still have a significant influence on the mechanical response of granular soils (O'Sullivan et al. 2002). Traditional railroad ballast is usually graded uniformly and mainly consists of larger particles, so there should be relatively higher angularity than a well graded one due to the size dependence of the particle shape of railroad ballast. Thus traditional railroad ballast may undergo more settlement as well as a loss of strength and drainage capacity due to particle breakage under continuous traffic loading over the long term.
(Indraratna et al., 2011). To optimise its strength and stability, ballast PSD with a reduced particle breakage and sufficient strength and stability can be obtained by adding a proper amount of smaller particles which are less angular according to the current research. This addition of smaller particles will not only decrease general particle angularity but also improve compaction between particles because samples with lower angularity are more likely to be compacted into smaller void ratios, according to Cho et al. (2006). However, if these particles are too small, they will migrate in the original ballast. Proper small particles should be larger than the constriction size formed by traditional railroad ballast to avoid fouling caused by particle migration. This means that further investigations into determining the sizes and amount of small particles to add to traditional railroad ballast are needed.

![Figure 3.14. Evolution of shape and size indices with particle size](image)

Conventional (i.e. mass-based) PSD is usually determined by sieve analysis, but as Raut and Indraratna (2004) pointed out, large particles with a high individual mass but low in number
will be over represented if a mass-based PSD is used. To overcome this limitation, the representation of PSD by number or surface area (Locke et al. 2001; Indraratna et al. 2007) was sometimes used, as shown in Equations (2.33) and (2.34), however, using geometrical averages of the sieve interval may introduce errors, as shown in Figure 3.15. By recalling the definition of Equations (2.33) and (2.34), predictions of two randomly selected PSDs and the corresponding CSDs by surface area and number are plotted in Figure 3.15(a). Note that Equations (2.33) and (2.34) over predicted the PSD by surface area and by number, especially the predictions of PSD curves by number. This means that predictions based on a traditional transforming method can still over represent the function of larger particles even if a mass-based PSD is not used. This is possibly because Equations (2.33) and (2.34) intrinsically treated each particle as a sphere, which departs from the fact that natural particles are mainly of an irregular form. At this point the corresponding CSD may have also been over estimated, as shown in Fig 3.15(b), which could lead to a report of higher drainage capacity from the view of a CSD-based hydraulic conductivity analysis (Indraratna et al., 2012a). A number-based CSD obviously overestimated the fraction of smaller sized particles while the CSD predicted by the surface area only deviated slightly from the experimental results. Trani and Indraratna (2010) suggested using the particle surface area based PSD instead of the mass-based PSD to determine the effective diameter in the Kozeny–Carman equation (Carman, 1938). A good prediction performance was observed.
Figure 3.15. Predictions of (a) PSDs and (b) CSDs by mass, by surface area and by number based on geometrical average of the sieve interval
It seems that a surface area-based CSD can automatically weaken the effect of particle shape and size when calculating the CSD, so to completely eliminate the effect of particle size and shape, an improved prediction was suggested by substituting the geometrical average in Equations (2.33) and (2.34) by the equivalent diameter $d_n$ of each sieve interval. Equations (3.4) and (3.5) are the modified formulae:

\[
P_{Ni} = \frac{P_{Mi}}{d_n^3} / \sum_{i=1}^{n} \frac{P_{Mi}}{d_{ni}^3}
\]

\[
P_{SAi} = \frac{P_{Mi}}{d_n^3} / \sum_{i=1}^{n} \frac{P_{Mi}}{d_{ni}}
\]

In the above, $d_{ni}$ is the average equivalent diameter of each sieve interval. $d_n$ is slightly larger than geometrical average $d_i$ and should approach $d_i$ with increasing particle regularity. As Figure 3.16 shows, the equivalent diameter-based transforming method can be used to accurately transform the PSD and its corresponding CSD. The use of the equivalent diameter considers the shape of the particles of each sieve interval, so the effect that the shape and size of individual particles have on calculating the particle number and particle surface area is safely addressed.
Figure 3.16. Predictions of (a) PSDs and (b) by mass, surface area and number based on $d_n$
3.5 Conclusions

The mechanical response of railroad ballast is largely based on particle size distribution as determined from sieve analysis, but two assemblages with completely different shape characteristics can still have the same sieve size. Recycled ballast cannot function well without any geosynthetic reinforcement, even if it strictly fulfills the PSD requirements, because, the macro-mechanical behaviour of railroad ballast is also sensitive to shape variation apart from grading. However, the shape and size of ballast were usually characterized qualitatively beforehand. To quantify the shape and size of railroad ballast, a thorough image-based investigation has been carried out in this paper by 3D laser scanning and an analysis of more than 200 particles of fresh ballast. Concrete values of relevant particle shape and size indices such as the elongation ratio, the flatness ratio, sphericity, and roundness, were also given. A new shape index called ellipsoidness to describe the overall surface regularity of railroad ballast was suggested, and a modified method for transforming mass-based PSD into a corresponding particle surface area-based or particle number-based one was successfully proposed. Six major findings from this study are listed below:

1. A 3D laser scan system can give reliable results of the size and shape of railroad ballast. Based on the measured values of volume and mass, an accurate density of ballast particle was 2.66 g/cm³.

2. The elongation and flatness ratios of ballast particles do change as the particle diameter changes. With the increase of particle size, ballast becomes less platy (more rounded) and less columnar.

3. Particle sphericity increases with increasing particle size, which indicates there was more similarity between the representative lengths of L, I and S in the larger particles. Moreover, a comparison between the true sphericity index and the two alternative 2D
sphericity indices showed that the 2D sphericity index would underestimate particle sphericity.

(4) The roundness of railroad ballast varied slightly as the particles changed in size, and there was small increase in angularity in larger ballast particles.

(5) Ellipsoidness decreased with increasing particle size, which means that larger particles of railroad ballast would be relatively more irregular.

(6) By using the equivalent diameter instead of the geometrical average in the transforming equation, the PSD and CSD by surface area and by number can be simulated successfully. The modified transforming method considers the size and shape of particles and therefore allows for an accurate representation of the PSD and its corresponding CSD of coarse granular aggregates. A better performance in constriction based analysis can be anticipated by using the proposed equivalent diameter based gradation transforming method.

Further studies are needed to determine more comprehensively how the shape and size of railroad ballast change after loading over a large number of cycles.
CHAPTER 4 STATIC AND CYCLIC TRIAXIAL TESTS OF RAILROAD BALLAST

4.1 Introduction

Increasing population densities and volumes of traffic have led to a rapid expansion of urban transportation infrastructure, including railroads. Due to their relatively low cost construction and flexible foundation design, ballasted tracks are still preferred in many countries where train speeds exceeding 300 km/h are regularly reached, for example, in France and Germany. However, a common problem with these tracks is the progressive deterioration of ballast as the traffic load increases where the sharp (angular) corners break off and larger sized aggregates split (Indraratna et al., 2005). Ballast can also suffer from significant lateral spreading leading to instability unless it is suitably confined, and indeed, severe deformation and degradation has occurred at higher loading frequency (≥ 30 Hz) (Sun et al., 2015). To minimise particle breakage and lateral deformation under cyclic loading, increased lateral confinement (Lackenby et al., 2007) and more broadly graded particle size distribution (PSD) (Indraratna et al., 2006, 2016) have been suggested to provide better particle interlock. The mechanical response of railroad ballast is often complicated and is influenced by factors such as the stress history (Suiker et al. 2005), confining pressure (Lackenby et al., 2007), PSD (Barksdale, 1972; Indraratna et al., 2004, 2006; Cunningham et al., 2013), and loading frequency (Sun et al., 2015), as well as particle shape (Le Pen et al., 2013; Sun et al., 2014). In this study two sets of drained cyclic triaxial tests were carried out on ballast with different maximum particle sizes and coefficients of uniformity using the large-scale triaxial apparatus designed and built at the University of Wollongong. One part of the study examined the influence of maximum particle size on the permanent deformation and degradation of ballast
by testing five PSDs, while the other evaluated the effect of $C_u$ and density on the mechanical response of ballast by testing nine different PSDs having the same minimum and maximum size particles. Here, the variations of particle size and shape before and after cyclic loading were examined using a three dimensional (3D) laser scanner.

### 4.2 Material Properties and Testing Procedures

#### 4.2.1 Material Properties

Ballast (crushed basalt) is dark volcanic latite basalt that contains the primary minerals feldspar, plagioclase, and augite (Lackenby et al., 2007). Its physical attributes and durability were evaluated using the standard test procedures shown in AS 2758.7 (1996) and in Table 4.1. Figure 4.1 illustrates the PSDs used in the present study, while Table 4.2 lists the values of the $C_u$, $d_m$, and $d_M$ among others, for each PSD.

**Table 4.1. Characteristics of ballast**

<table>
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<th>Characteristics</th>
<th>Index</th>
<th>Test value</th>
<th>AS recommendations</th>
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<td>Durability</td>
<td>Aggregate crushing value</td>
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<td>Los Angeles abrasion</td>
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<td>Wet attrition value</td>
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<td>&lt;30%</td>
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<tr>
<td>Attributes</td>
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Table 4.2. Physical attributes of ballast

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<th>No.</th>
<th>$e_0$</th>
<th>$e_{min}$</th>
<th>$e_{max}$</th>
<th>$R_d$</th>
<th>$\gamma_b$ (kN/m³)</th>
<th>$C_u$</th>
<th>$d_M$ (mm)</th>
<th>$d_{50}$ (mm)</th>
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<th>$d_m$ (mm)</th>
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<td>33.7</td>
<td>19.5</td>
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</table>
4.2.2 Testing Procedures

A large scale triaxial apparatus (Indraratna et al. 1998) was used to investigate the effect of particle size distribution. Aggregates were selected from each size range, carefully washed, air-dried, and then weighed separately and mixed together, and then divided into four equal parts. Each part was then placed inside a lubricated rubber membrane in four separate layers to achieve the initial void ratio and relative density set out in Table 4.2. Compaction was via a split cylindrical mould which could easily be removed, before placing the sample (height = 600 mm, diameter = 300 mm) inside the pressure chamber. Before being tested, the sample was saturated by passing water through the base of the triaxial cell under a back pressure of 10 kPa and then through a top drainage system to remove any air voids. The samples were
then isotropically compressed at an effective confining pressure of 30 kPa before an axial load was applied. The confining pressure was increased in several steps and the corresponding change in the volume of the specimen was recorded. For static testing, fully drained compression was performed at an axial strain rate of 3 mm/min to prevent any excess pore water pressure from developing. A load cell, pressure transducers, and a linear variable differential transformer (LVDT) were connected to a computer-controlled data acquisition system. Shearing continued until the vertical strain reached about 30%. Cyclic tests were carried out with an input maximum deviator stress $q_{\text{max}}$ of 230 kPa, which represented the stress imposed by a 25-ton axle load, and a minimum deviator stress $q_{\text{min}}$ of 45 kPa, which is typical of an unloaded track superstructure. A typical harmonic cyclic load (sinusoidal waveform) was used while testing at two different frequencies, i.e., 20 Hz and 30 Hz. However, it should be pointed out that this harmonic cyclic loading pattern does not exactly simulate the real-life traffic. But, due to the difficulties in simulating the real track moving load in the laboratory, repeated sinusoidal loading with different frequencies was used as a simplification. Similar approaches are found in Suiker et al. (2005), Anderson and Fair (2008), and Sevi and Ge (2012) to study the deformation behaviour of railroad ballast. A conditioning phase of 1 Hz was applied at the first 20 cycles of the loading regime to prevent any impact loading and loss of actuator contact with the specimen. Permanent deformation data were collected at regular time intervals. Membrane correction was applied to the current stress measurements according to the ASTM (2011). At a confining pressure $\sigma'_3 = 30$ kPa, the maximum correction for a 7 mm thick rubber membrane was less than 5%. Cyclic Loading was applied up to 500000 cycles, and each sample was passed through a set of 12 standard sieves (2.36 mm – 53 mm) (ASTM 2006) to estimate particle breakage before and after testing. Details of the static and cyclic triaxial testing program are summarised in Table 4.3.
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<th>Initial state</th>
<th>Test series</th>
<th>Maximum particle size (mm) / Coefficient of uniformity ( (d_M / C_u) )</th>
<th>Confining pressure (kPa)</th>
<th>Loading frequency ( f ) (Hz)</th>
<th>Minimum deviator stress ( q_{\text{min}} ) (kPa)</th>
<th>Maximum deviator stress ( q_{\text{max}} ) (kPa)</th>
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<td>-</td>
<td>-</td>
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<td></td>
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<td>30</td>
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4.3 Static Test Results

4.3.1 Stress and Strain

The results of the static triaxial test are shown in Figures 4.2 - 4.3. Figure 4.2 shows the results of ballast tested with varying coefficients of uniformity but fixed minimum and maximum particle sizes. To provide mechanical details for conducting cyclic triaxial tests in order to study the influence that PSD has on the deformation and degradation of railroad ballast, tests were carried out on samples under a constant confining pressure. Here the stress increased rapidly at the initial loading stage and then gradually stabilised, but unlike smaller aggregates such as sand, there was no distinct failure plane shear or shear band in the ballast specimens. Instead, specimen failure is usually accompanied by bulging, as reported by Indraratna et al. (1998) and Xiao et al. (2015b). Specimen failure is defined at the peak shear stress of the specimen, and here the highest peak shear stress was in samples with a coefficient of uniformity equal to 3.0. All the samples were dilated after the test, as shown in Figure 4.2(b). The highest volumetric dilation was found in samples with the lowest coefficient of uniformity. Even though shearing continued until the axial strain reached 25%, no critical state occurred in the samples tested at \( \sigma_3' = 30\text{kPa} \). Figure 4.3 shows the results of ballast tested with varying confining pressure but a fixed PSD; here the stress became more stable as the axial strain increased, which means that the higher the confining pressure is, the higher the shear strength will be. A critical state was almost attained in samples tested under relatively higher stress, but the sample tested under \( \sigma_3' = 50\text{kPa} \) did not reach the critical state.
Figure 4.2. Static triaxial behaviour of ballast at $\sigma_3' = 30$ kPa: (a) deviator stress, (b) volumetric strain
4.3.2 Dilation and Peak Friction Angles

The dilation angle evaluated here is the friction angle at the phase transformation point, as illustrated in Figure 2.8. The dilation and peak friction angles of ballast with varying PSD and confining pressure are shown in Figures 4.4 - 4.6. The dilation angle of ballast with varying PSD depends mainly on the coefficient of uniformity and sample density; the dilation angle of sample with similar void ratio generally decreased with an increasing coefficient of
uniformity (Figure 4.4), implying that less dilation occurs in well graded ballast. The uniform distribution of angular aggregates in samples with a low coefficient of uniformity restrains frictional sliding between particles and thus promotes dilation; this is in accordance with Li et al. (2014). However, the dilation angle of samples with a constant relative density increased with an increasing coefficient of uniformity.

The peak friction angle of ballast with varying PSD also depends mainly on the coefficient of uniformity and sample density, with Figure 4.5 showing that the peak friction angle of ballast increases and then decreases with an increasing coefficient of uniformity, while the peak friction angle of samples with the same PSD increased with an increasing density.

![Figure 4.4. Dilation angle of ballast with varying PSDs](image)
The influence of confining pressure on the dilation and peak friction angles of ballast was also evaluated, and is shown in Figure 4.6. Note that the abscissa in Figure 4.6 is in a logarithmic scale while the ordinate is in arithmetic scale. Here, the peak friction angle decreased significantly as confining pressure increased, while the dilation angle only decreased slightly with the confining pressure due to the increasingly significant particle breakage that occurred in tests under a higher confining pressure. A good linear correlation between $\phi$ and $\log p'$ can be expected, as shown in Equation (2.9).
4.3.3 Critical State Line

Due to the significant amount of particle breakage under a higher confining pressure, the critical state line and the critical state line of granular soils in the $p' - q$ plane bent downwards effective mean stress increased (Indraratna et al., 2014a; Xiao et al., 2014). However, since field ballast is more likely to experience a relatively low confining pressure (Sun et al., 2014), the critical state lines in this research are reasonably assumed to be linear rather than the curving (Figure 4.7). Note that the results obtained from tests held under a low confining pressure of 50 kPa are not represented here because the critical state was not reached, but as expected, the critical state points can be well fitted by using straight lines in the $p' - q$ and $e - \ln p'$ planes.

Figure 4.6. Dilation and peak friction angles of ballast under different $\sigma'_3$
4.4 Cyclic Test Results

The cyclic behaviour of ballast is represented by the permanent deformation resulting from a series of drained cyclic triaxial tests under constant confining pressure of 30 kPa and two loading frequencies of 20 Hz and 30 Hz. The main objective was to investigate the effects of coefficient of uniformity, particle size, and loading frequency on the permanent axial and
volumetric strain of railroad ballast. Experimental data and the corresponding theoretical analysis in relation to resilient modulus and breakage can be found in Chapters 5 and 6.

### 4.4.1 Effect of Maximum Particle Size

The permanent deformation of ballast under cyclic loading can be found in Figures 4.8 - 4.13, while the effect of the maximum particle sizes \( d_m = 31.5, 37.5, 40, 45, \text{ and } 53 \text{ mm} \) on the permanent axial and volumetric strains of ballast can be found in Figure 4.8. As expected, the axial and volumetric strains \( (\varepsilon_1, \varepsilon_v) \) increased with an increasing number of load cycles \( (N) \), and smaller aggregates \( (d_M \leq 37.5 \text{ mm}) \) tended to dilate slightly during the first few hundred load cycles, but then compressed quickly as the load cycles increased. The transition from dilation to compression was mainly influenced by the loading frequency, so the final volumetric strain was lower in smaller aggregates because of the initial dilation. The permanent axial strain of ballast against the number of load cycles can be divided into four zones (Sun et al. 2015): (i) ‘elastic shakedown’ exhibited by no accumulation of plastic strain, (ii) ‘plastic shakedown’ characterised by a steady-state response with a small accumulation of plastic strain, (iii) a ‘ratcheting’ zone that shows a constant accumulation of plastic strain, and (iv) a ‘plastic collapse’ zone where plastic strains accumulate rapidly and failure occurs in a relatively short time (Sloan et al. 2008). Those specimens subjected to a frequency of 20 Hz reached elastic shakedown at large number of load cycles, whereas plastic shakedown was evident at a higher frequency of 30 Hz.
Figure 4.8. Permanent deformations of ballast with the same coefficient of uniformity and void ratio: (a) axial strain, (b) volumetric strain
4.4.2 Effect of Coefficient of Uniformity

Permanent axial and volumetric strains of samples with different coefficients of uniformity but similar densities are shown in Figures 4.9 and 4.10. Here the axial and volumetric strains increase with the increasing number of load cycles, irrespective of the loading frequency. As shown in Figure 4.9, the axial and volumetric strains of samples tested at 20Hz stabilise after $N > 10^3$, which indicates a state of elastic shakedown. The axial strain of ballast with the coefficient of uniformity ranging between 2.0 and 4.5 stabilises quickly within the first few hundred loading cycles whereas ballast with the coefficient of uniformity less than 2.0 still suffers from permanent deformation until a large number of load cycles is reached. A slight dilation also occurs in ballast with the smallest coefficient of uniformity ($Cu = 1.2$), while permanent deformation decreases with the coefficient of uniformity when $Cu < 2.5$, but it increases again as the coefficient of uniformity increases further ($Cu \geq 2.5$). This is possibly due to the fact that specimen with the coefficient of uniformity equal to 1.2 had the largest void ratio and the lowest placement density that resulted in smaller inter-particle contacts. As the coefficient of uniformity further increases, i.e., $2.5 > Cu > 1.2$, an optimum packing arrangement of the assembly is attained thus exhibiting lowest deformations. At higher values of the coefficient of uniformity, i.e. $Cu \geq 2.5$, an increasing number of smaller-sized particles occupy the empty voids space around larger sized aggregates which facilitates the movement of the skeleton particles and therefore results in the increased deformation of the overall granular assembly.
Figure 4.9. Permanent deformations of ballast with varying coefficient of uniformity but similar density at 20 Hz: (a) axial strain, (b) volumetric strain
As Figure 4.10 shows, the samples tested at 30 Hz tended towards plastic shakedown due to particle fatigue (Sun et al., 2015) after $N > 10^5$, whereas the axial strain increases faster in ballast with a higher coefficient of uniformity but similar initial density. Note that the four tests carried out on samples with a coefficient of uniformity larger than 4.0 failed quickly, within hundreds of load cycles (as shown in Figures 4.9 and 4.10); this can be characterised as the zone of ‘plastic collapse’ due to the reduced shear strength of ballast (as shown in Figure 4.5) that caused the specimen to suddenly fail.

Figures 4.11 - 4.12 show the permanent axial and volumetric strains of samples with different coefficients of uniformity but constant relative densities. Here, the resistance to plastic deformation under cyclic loading was improved considerably by increasing the sample density. Unlike those samples with a similar void ratio, samples with initially constant relative densities exhibited elastic shakedown after thousands of loading cycles when loaded under 20 Hz (Figure 4.11), but there was also a slight plastic shakedown in samples tested at 30 Hz (Figure 4.12). Similar to those tests with the same initial density, samples with a coefficient of uniformity equal to 1.2 also exhibited slight dilation at the initial loading stage, whereas the permanent strain decreased with the coefficient of uniformity when $Cu \leq 2.5$, but it increased again as the coefficient of uniformity increased further ($Cu > 2.5$). The assembly also attained an optimum packing arrangement with a coefficient of uniformity of around 2.0, and thus exhibited reduced deformation.
Figure 4.10. Permanent deformations of ballast with varying coefficient of uniformity but similar density at 30 Hz: (a) axial strain, (b) volumetric strain
Figure 4.11. Permanent deformations of ballast with varying coefficient of uniformity but constant relative density at 20Hz: (a) axial strain, (b) volumetric strain
Figure 4.12. Permanent deformations of ballast with varying coefficient of uniformity but constant relative density at 30Hz: (a) axial strain, (b) volumetric strain

Figure 4.13 represents the permanent axial and volumetric strains of ballast with a coefficient of uniformity equal to 1.9 and maximum particle size equal to 45 mm. Here, the axial strain rapidly increased and then began to stabilise with the increasing number of load cycles,
whereas the axial strain increased with an increase in the confining pressure (Figure 4.13(a))
while the volumetric strain decrease with the increasing confining pressure (Figure 4.13(b)).
This can be attributed to an increasing amount of ballast degradation with an increasing
confining pressure which causes the specimen to be compressed (Sun et al., 2015).

4.4.3 Strain Accumulation Rates

The rate of permanent strain accumulation offers an alternative way of analysing the
mechanical response of the specimen subjected to cyclic loading. Figures 4.14 - 4.19 show
the strain accumulation rate for different granular soils, including railroad ballast, subballast, and sand. The axial and volumetric strain accumulation rates of ballast with $C_u = 1.9$ and $d_M = 45$ mm are shown in Figures 4.14 - 4.15. The rate of axial strain accumulation is highest during the initial loading stage and then becomes relatively insignificant and stable after only 1000 loading cycles. As shown in Figure 4.13, the axial strain accumulation rate increases with the decreasing confining pressure. A similar observation of the volumetric strain accumulation rate can be found in Figure 4.15 where the rate of accumulated volumetric strain decreased rapidly until it became insignificant as the number of load cycles increased. However, the rate of volumetric strain accumulation increased with an increasing confining pressure. Further rephrasing of the strain rate and the number of load cycles in the log-log scale instead of the log-linear scale, as shown in Figures 4.14(b) and 4.15(b), means that a linear relationship by Equation (4.1) can be obtained.

$$\log \dot{\varepsilon} = -\beta \log N + \log b$$

(4.1)

where $\beta$ and $b$ are the material constants for a given loading state, and $\dot{\varepsilon}$ denotes the strain accumulation rate which can be considered as scale invariant with regard to load cycles $N$. 


Figure 4.14. Relationship between axial strain rate and number of load cycles for ballast in (a) log-linear scale and (b) log-log scale.

$log\varepsilon_1^p = -\beta \log N + \log b, 0 < \beta < 1$
Figure 4.15. Relationship between volumetric strain rate and number of load cycles for ballast in (a) log-linear scale and (b) log-log scale

Similar observations can also be found in Figure 4.16 where the evolutions of the shear and volumetric strain accumulation rates of ballast and subballast (Suiker and de Borst, 2003) with the number of loading cycles are reported. Note that the ballast used was uniformly graded with a maximum particle size $d_M = 38$ mm while the subballast was well graded with $d_M = 20$ mm, and the cylindrical specimen was compacted layer by layer in a dry condition. Cyclic triaxial tests were carried out at 5 Hz and two different confining pressures, as shown in Figure 4.16. Ballast is observed to have a lower strain accumulation rate than that of the
subballast. Here the volumetric and shear strain accumulation rates decreased with the increasing number of load cycles where linear variations were also indicated.

Figure 4.17 shows the results of the axial and volumetric strain accumulation rates of two types of uniform ballast (Sevi and Ge, 2012) with different maximum particle sizes ($d_M = 38$ mm and $d_M = 19$ mm). These tests were carried out under dry conditions at 1 Hz and with a confining pressure of 20 kPa. Note that the axial and volumetric strain accumulation rates varied linearly with the increasing number of load cycles when represented in the coordinates with the log-log scale. Figure 4.18 shows the strain accumulation rates of sand subjected to cyclic tests with a different average deviator stress $q_{av}$. Here the sand had a median particle size of 0.35 mm and a coefficient of uniformity of 1.9. All the specimens were prepared using the pluviation technique so that each sample was 100 mm in diameter by 200 mm high; each specimen was then saturated and tested at 1 Hz and with an average mean principal pressure of $p_{av}$, which was equal to 200 kPa. Figure 4.18 shows that there exists a power law between the strain accumulation rates and the number of load cycles, a phenomenon that also occurs in grained subgrade materials with different initial water contents (Li and Selig, 1996) as shown in Figure 4.19.
Figure 4.16. Relationship between strain rate and number of load cycles for ballast and subballast (data sourced from Suiker and de Borst, 2003): (a) volumetric strain rate (b) shear strain rate
Figure 4.17. Relationship between strain accumulation rate and number of load cycles for ballast (data sourced from Sevi and Ge, 2012): (a) volumetric strain rate (b) axial strain rate
Figure 4.18. Relationship between strain accumulation rate and number of load cycles for sand (data sourced from Wichtmann et al., 2009): (a) volumetric strain rate (b) shear strain rate

\[ \log \varepsilon_p^\alpha = -\beta \log N + \log b, \ 0 < \beta < 1 \]

\[ \log \varepsilon_p^\nu = -\beta \log N + \log b, \ 0 < \beta < 1 \]
4.4.4 Discussion

The PSD has a large influence on the permanent strain behaviour of railroad ballast. To quantify the effect of the maximum particle size and coefficient of uniformity, variations of the final permanent strain of railroad ballast are discussed as shown in Figures 4.20 - 4.21. To account for the combined effects of $C_u$ and $d_M$, a normalised PSD parameter is proposed and discussed in Figure 4.22.
Note that as the maximum particle size increased, the ballast exhibited a lower axial strain regardless of the loading frequency, as shown in Figure 4.20(a). Similar observations were also reported by Janardhanam and Desai (1983) and Sevi and Ge (2012), as shown in Figure 4.20(b).

As Figure 4.21 shows, the final axial and volumetric strains decrease with the increasing coefficient of uniformity when $C_u \leq 2.5$, but they increase again as the coefficient of uniformity increases further ($C_u > 2.5$). However, note that the reported variation of permanent strain with the varying coefficient of uniformity also depends on the relative density, as shown in Figures 4.21(a) and 4.21(b). The higher the relative density is, the lower the final permanent strain will be. Thom and Brown (1988) reported similar observations where the sample with more fines would have a higher $C_u \geq 6$ and could still experience sudden failure (Figure 4.21(c)) due to excessive fines ($d < 13.2$ mm) caused by increasing the coefficient of uniformity which in turn decreases the interlocking among angular skeleton particles, which then facilitates the movement of aggregates under cyclic loading.
Figure 4.20. Effect of the particle size on the permanent strain of ballast
To incorporate the effects of the maximum particle size \(d_M\) and the coefficient of uniformity \(C_u\), a dimensionless PSD parameter \(\Theta\) is proposed as follows:

Figure 4.21. Variation of permanent strains with varying coefficient of uniformity: (a) similar initial void ratio, (b) constant initial relative density, (c) similar initial relative density
\[ \Theta = \frac{d_M}{d_{m0}} e^{-c_c} \]  \hspace{1cm} (4.2)

where \( d_{m0} \) is used to make the PSD parameter dimensionless and its value can be set at 2.36 mm, which is the minimum particle size counted in ballast. Figure 4.22 shows the relationship between the final axial strain and the PSD parameter \( \Theta \) for samples with similar initial void ratios and relative densities. The axial strain decreases and then increases with \( \Theta \), irrespective of the loading frequency, thus implying the existence of minimum settlement. A regression analysis can be performed to describe this trend of evolution; the results of this regression analysis and the polynomial expression are shown in Figure 4.22 and describe the performance of railroad ballast with a given PSD. For example, ballast with \( d_M = 63 \) mm and \( C_u = 1.8 \) has a calculated PSD parameter of \( \Theta \) is 4.41 where the corresponding axial strain is lowest. However, if \( C_u \) increased to 2.8, \( \Theta \) will change to 1.62 where significant axial strain would occur (Figure 4.22). Moreover, the \( \Theta \) values of the upper and lower bounds of PSD recommended by the Australian rail industry (AS 2758.7, 1996) are determined to be 4.5 and 6.6, respectively, which falls outside the optimum range (2.7 ~ 4.2) defined by \( \frac{\partial e_a}{\partial \Theta} = 0 \). Therefore, the current Australian standard PSD should be optimised, as discussed later. However, the PSD parameter \( \Theta \) proposed here is based on the large triaxial results to specifically evaluate the performance of railroad ballast whose \( d_M \) ranges between 31.5 mm to 63 mm.
Figure 4.22. Variation of the final axial strain with the PSD parameter of ballast

### 4.5 Conclusions

The influence of PSD on the permanent deformation of railroad ballast under cyclic loading was investigated by drained triaxial tests conducted using a large-scale cylindrical triaxial apparatus. Ballast specimens with a varied range of PSDs were tested under two different cyclic loading frequencies where \( f = 20 \) and \( 30 \) Hz. Five PSDs with the same coefficient of uniformity were tested first, and then a number of PSDs \( (1.2 \leq C_u \leq 10) \) with the same
minimum and maximum size particles were tested. The major findings of this study are summarised below:

(1) Larger ballast aggregates experienced smaller axial and radial strains, while smaller particles underwent dilation within the first few hundred loading cycles followed by compression as the number of load cycles increased. This transition from dilation to compression was influenced by the loading frequency.

(2) Resistance to plastic strain under cyclic loading was greatly improved by an increase in the relative density of ballast.

(3) The permanent axial strain and volumetric strain of ballast decreased initially and then increased as the coefficient of uniformity increased. Ballast with a larger coefficient of uniformity deformed rapidly during the first few hundred loading cycles.

(4) There was a power law between the rates of permanent axial (shear) and volumetric strain accumulation under cyclic loading and the number of load cycles.

(5) Based on these test results, an optimum PSD with $1.8 \leq C_u \leq 2.0$ for ballast was proposed to minimise deformation under high frequency cyclic loading.

It should be noted that the experimental results and the related research findings were based on triaxial tests with fixed principal axes and isotropic (fluid) confining pressure as well as loading amplitudes and frequencies that are not exactly the same as real-life train loading patterns. Also, cyclic loads without any rest period were used for simplicity even at relatively low number of loading cycles, in certain cases before the final shakedown level is attained at the critical state where the ultimate resilient modulus becomes stable. The adaptability of the current research outcomes in improving the field behaviour of railroad ballast layers still needs further investigation, in view of the different and non-uniform loading patterns that are encountered in complex railroad conditions. In particular, the effect of fatigue due to
continual testing may have been increased, hence the results of this study are expected to be conservative.
CHAPTER 5 RESILIENT BEHAVIOUR OF RAILROAD BALLAST

5.1 Introduction

The cyclic behaviour of railroad ballast usually consists of permanent deformation and degradation and transient deformation characterised by resiliency and damping. The permanent deformation of ballast under different loading conditions has been discussed in the previous chapter and it can also be found in the following references (Suiker et al., 2005; Lackenby et al., 2007; Indraratna et al., 2010; Sevi and Ge, 2012; Sun et al., 2014; Okonta, 2015). However, the resilient behaviour of railroad ballast is also a critical issue, especially for high speed rail tracks where the ballast layer usually experiences high frequency vibration. Previous studies indicate that the resilient behaviour of railroad ballast is affected by the loading stress and number of load cycles (Suiker et al., 2005), the loading frequency (Sun et al. 2015), confining pressure (Thakur et al., 2013), particle breakage (Indraratna et al., 2009), and particle size (Janardhanam and Desai, 1983; Sevi and Ge, 2012), etc. It has been proved that the resilient modulus increased with the loading cycles and the bulk stress (the sum of all principal stresses) due to continuous compaction by the cyclic load. However, the effect of PSD on the resilient modulus of ballast and other granular aggregates still remains inconclusive and open to discussion; in fact, some results even conflict with each other. For instance, a generally higher resilient modulus in larger sized ballast was reported (Kolisoja, 1997; Sevi and Ge, 2012), whereas a slight decrease in the resilient modulus with increasing particle size was observed by Indraratna et al. (1998), which is consistent with other granular aggregates such as quartz sand (Patel et al., 2008) and glass beads (Bartake and Singh, 2007). Moreover, Wichtmann and Triantafyllidis (2009) carried out a series of resonant column tests
to examine the effect of PSD on the stiffness of sand and concluded that particle size had no obvious influence on the shear modulus when the median particle size varied from 0.1 mm to 6 mm, but there was a remarkable decrease in the shear modulus with an increasing coefficient of uniformity. Similar conclusions can also be found elsewhere (Iwasaki and Tatsuoka, 1977; Yang and Gu, 2013).

In order to provide an alternative insight into the effect of PSD on the resiliency of granular soils, Yang and Gu (2013) used a micromechanics-based approach to interpret the effect of size from the grain scale. They suggested that soil resiliency was independent of its internal particle size, but noted that they only evaluated monodisperse aggregates representing a uniformly graded granular soil, and the force chains in their model were distributed more equally than in a polydisperse material (Wichtmann and Triantafyllidis, 2009). The complexity of the effect of particle size or PSD on soil resiliency can be partly attributed to the variation of internal stress distribution provided by the different spatial, size and shape distributions of the granular aggregates. In fact, different types of PSDs, including uniform and the non-uniform PSDs, were used by different researchers. The internal particle configuration and shape (Sun et al., 2014) may vary widely, which should more or less result in a discrepancy in the above conclusions, so the role of non-uniformity should be considered appropriately when studying the effect of PSD on the resiliency of granular soils.

In this chapter the experimental results of the resilient modulus of railroad ballast obtained from the drained large scale triaxial tests are analysed, and the influence of the maximum and minimum particle sizes, as well as the coefficient of uniformity on the resilient modulus of ballast are discussed. Furthermore, a micromechanical representation of the effect that PSD and particle breakage has on ballast resiliency is also provided. This proposed approach is quantitatively validated by comparing the experimental results and the corresponding theoretical predictions. It is noted that the resilient modulus calculated in this chapter was
obtained from cyclic triaxial tests with no rest period for load. This method (without having to use rest periods) was first introduced by Indraratna et al. (2009) in Géotechnique to evaluate the ULTIMATE state of resiliency of railroad ballast for Australian conditions, and now also adopted by ARTC and Sydney Trains, which is quite different from the pavement engineering methods used in North America where a load rest period is applied between loading cycles. Through several past PhD theses (Salim 2004; Lackenby 2006), UOW test results have shown that the Ultimate Resilient Modulus (URM) is found to be relatively constant, and independent of the stress history (with or without rest periods) once the ballast is subjected to a large number of loading cycles. This is further supported by a few past studies (e.g. Hicks and Monismith, 1971; Brown and Hyde, 1975; Lackenby et al., 2007; Indraratna et al., 2014b). In addition, the resilient modulus evaluated here is used for studying the effect of PSD. Since all the values of the resilient moduli of different samples were obtained under the same loading condition, comparisons of the resilient moduli between different PSDs can be conducted.

5.2 Test Results

The evolution of resilient strain with the increasing number of load cycles is shown in the following sections. The effect of PSD on the long-term resilience of ballast is further elaborated by calculating the resilient modulus of samples with different coefficients of uniformity and maximum particle sizes. It can be obtained by using the Equation (2.12) shown previously.
5.2.1 Effect of Coefficient of Uniformity

Figure 5.1 shows the variation of the long-term resilient strain of ballast with different PSD curves. As expected, the resilient strain decreases with an increasing number of load cycles, implying a potential increase of ballast resiliency under repetitive traffic load. When loaded under a relatively low frequency (20 Hz) (Figure 5.1(a)), the resilient strain decreases rapidly at the initial loading stage and then approaches constant after a high number of load cycles ($N > 10^4$), indicating a state of elastic shakedown. However, the resilient modulus of samples tested under higher frequency (30 Hz) decreased continuously with the increasing number of load cycles, as shown in Figure 5.1(b). With this increase in the coefficient of uniformity, the resilient strain of different samples at the same number of load cycles also increases.

Figure 5.2 shows the effect of the coefficient of uniformity and loading frequency on the resilient modulus of ballast. The resilient modulus rapidly increases with the number of load cycles and then progressively stabilises within the number of loading cycles tested. This initial rapid increase in $M_r$ can be attributed to cyclic densification where a fast rearrangement and breakage of ballast aggregates occur in order to achieve a stable configuration (Indraratna et al., 2016). It also shows that the increment of resilient modulus with the number of load cycles was larger for samples tested under a lower loading frequency. A higher resilient modulus for a lower loading frequency occurred, which is in accordance with that by Thakur et al. (2013). The resilient modulus decreased with an increasing $C_u$, indicating that uniformly graded ballast has a better resilient capacity; this can be attributed to the decreasing number of particle contacts caused by a decreasing $C_u$ that makes the sample stiffer. All the resilient moduli obtained under higher loading frequency are shown to increase continuously until a large number of loading cycles ($N > 3 \times 10^5$) is reached, whereas the resiliency of ballast tested under low frequency stabilised after a high number of loading cycles ($N > 10^5$).
Figure 5.1. Variation of the resilient strain of ballast with the number of load cycles at different coefficients of uniformity
5.2.2 Effect of Particle Size

Figure 5.3 shows the variation of the long-term resilient strain of ballast with different maximum particle sizes, and as expected, the resilient strain decreases with the increasing number of load cycles. When loaded under a relatively low frequency (20 Hz) (Figure 5.3(a)), the resilient strain decreases rapidly at the initial loading stage and then progressively approaches constant after a high number of load cycles ($N > 10^4$), indicating a state of elastic
shakedown. However, the resilient modulus of samples tested under higher frequency (30 Hz) decreased continuously with the increasing number of load cycles, as shown in Figure 5.3(b), while the resilient strain of ballast with the same coefficients of uniformity and minimum particle size generally increased with an increasing maximum particle size.

Figure 5.4 shows the effect of particle size and frequency on the resilient behaviour of ballast. Similar to the test results of samples with varying coefficients of uniformity, a higher resilient modulus also occurred in ballast loaded under lower frequency. Therefore, the original ground for low speed rail tracks should be improved to provide better resiliency before the operation of high speed tracks. For ballast with the same coefficient of uniformity, the resilient modulus appears to depend mainly on the variation in particle size, because there was a general increase in the resilient modulus with a decreasing maximum particle size under the current loading conditions. Similar observations with regards to the effect of the particle size of granular soils were also reported by Bartake and Singh (2007) and Patel et al. (2008) after carrying out bender element tests on sand and glass beads with different PSD curves. There was an increase of the shear modulus and a decrease of the mean particle size that was possibly due to the variation in particle configuration which changes simultaneously with the PSD.
Figure 5.3. Variation of the resilient strain of ballast with the number of load cycles at different maximum particle sizes
5.3 Discussion

The influence of the coefficient of uniformity and particle size, including the median, maximum, and minimum particle sizes, is complicated because analysing the effect of any one of the above factors seems to be only conducted by keeping the other factors unchanged. As Figure 5.5 shows, the variation of the resilient modulus with median particle size is shown without considering the influence of the coefficient of uniformity because it generally
decreased and then the resilient modulus increased with an increasing particle size. However, the resilient modulus was reported to generally decrease with increasing particle size, given that the coefficients of uniformity were the same. As the PSDs reveal in Figure 4.1, even though the coefficient of uniformity remains the same for ballast with varying $d_M$, the particle configurations can be different. For example, larger sized particles are usually surrounded by a substantial amount of small particles in a broadly graded PSD (e.g., No. 1) when compared to a narrowly graded one (e.g., No. 5) where the deviation between particle sizes is small. Assuming the smallest homogeneous cubic element that contains enough particles to represent a given PSD, fewer particle contacts in a representative load carrying queue should be expected in ballast with a lower difference in values between $d_M$ and $d_m$ because the contact forces between particles are much higher which therefore increases the resilient modulus in smaller (narrowly graded) ballast (Tong and Wang, 2014). However, with a further increase in the loading stress, aggregates are increasingly held together. Higher stiffness was also reported in coarser blends (Cunningham et al., 2013) where the samples tested had the same $d_M$ and $d_m$ but different $Cu$. In their study, it appeared to be the change in the coefficient of uniformity rather than particle size alone that influenced the resilient modulus.
Figure 5.5. Variation of resilient modulus with median particle size $d_{50}$

Figure 5.6. Variation of resilient modulus with particle breakage ratio $B$
To study the relationship between the resilient modulus and the extent of particle breakage, a fractal breakage ratio $B$ proposed by Einav (2007) was used. As Figure 5.6 shows, the extent of breakage is generally higher in samples loaded under high frequency due to significant corner breakage and internal aggregates being fatigued (Sun et al., 2015). With this increase in the particle breakage ratio, the resilient modulus decreased due to an increase in the coefficient of uniformity caused by particle breakage. As mentioned above, the higher the coefficient of uniformity, the lower the resilient modulus.

5.4 Conclusions

The PSD dependence of ballast resiliency under cyclic loading was investigated by experimentally and theoretically studying different PSDs under two different loading frequencies; it was found that PSD had a significant influence over the resilient modulus of railway ballast. Several main conclusions are summarised below:

(1) For ballast with the same coefficient of uniformity, the resilient modulus appeared to decrease as the particle size increased, but it increased with a decrease in the coefficient of uniformity in ballast with fixed maximum and minimum particle sizes.

(2) The combined effects of deviation between the maximum and minimum particle sizes and the coefficient of uniformity influenced the resilient modulus of granular aggregates, such that the highest resilient modulus was in the samples with almost the same maximum and minimum particle sizes.

(3) Apart from the PSD, particle breakage can also be regarded as a critical factor that influenced the resiliency of granular aggregates because more serious breakage occurred in samples tested under a higher loading frequency where a low resilient modulus was observed. Particle breakage increased under a higher loading frequency,
which shifted the initial PSD towards a broader distribution; this may be regarded as a potential reason for an overall decrease in the resilient modulus with an increasing loading frequency.
CHAPTER 6 DEGRADATION OF RAILROAD BALLAST

6.1 Introduction

Particle breakage usually occurs in angular aggregates such as ballast, even at low confining pressures (Lade et al., 1996; Lackenby et al., 2007; Agustian and Goto, 2008; Liu and Zou, 2013; Sun et al., 2015). Aggregates become increasingly rounded during monotonic and cyclic loading because of particle breakage. It has been shown that particle breakage affects the strength and deformation of granular soils, including railroad ballast (Indraratna et al., 1993, 1998; Anderson and Key, 2000), rockfill (Xiao et al., 2014; Fu et al., 2014), and sand (Luzzani and Coop, 2002; Vilhar et al., 2013). In response to which many theoretical and experimental researches have been carried out to investigate how particle breakage affects the deformation and degradation of granular soils. For samples with a given PSD, particle breakage shifted the initial PSD of granular soil towards an ultimate PSD where particles were distributed fractally (McDowell and Bolton, 1998; Einav, 2007). Moreover, since particle breakage increases monotonically with an increasing loading stress and strain, less breakage could be expected when the PSD increasingly approaches the ultimately fractal distribution. Aggregates with lower angularity tend to experience less internal breakage, but until now, most research on particle breakage were carried out on samples with the same fixed initial PSD. There were some past research studies (Sitharam and Nimbkar, 2000; Kim et al., 2007; Sevi and Ge, 2012; Cunningham et al., 2013) into the influence of PSD on the mechanical response of granular soils, but only limited research has been carried out to study the variation of particle breakage with varying initial PSDs (Indraratna et al., 2006; Carrera et al., 2011, Indraratna et al., 2016; Li et al., 2014). Moreover, the mechanisms for particle
breakage and the applicability of different breakage indices were not comprehensively studied.

In this chapter, the degradation of railroad ballast with different initial PSDs under both monotonic and cyclic triaxial tests is studied, along with ability of eight different breakage indices ($B_r$, $BBI$, $B$, $B_g$, $B_g^m$, area $A$, $\Delta S$, $\bar{S}$) to evaluate the breakage mechanism for railroad ballast with different initial PSDs. Particles with a variety of shapes are evaluated by studying the shape of ballast aggregates before and after testing, using the three dimensional (3D) laser scanner described in Chapter 3.

### 6.2 Degradation of Ballast

The breakage indices, i.e., $B_r$, $BBI$, $B$, $B_g$, $B_g^m$, area $A$, $\Delta S$, $\bar{S}$, were initially suggested to quantify the extent of particle breakage of a single initial PSD, which differs from the present study where multiple initial PSDs are under investigation. So in order to comprehensively evaluate the extent of particle breakage of railroad ballast, all the above breakage indices are used and discussed. For the purpose of an initial comparison, those breakage indices used to assess the breakage extent of ballast with a fixed initial PSD and then subjected to monotonic triaxial tests are presented first, and then the extent of ballast breakage with multiple initial PSDs subjected to cyclic loads are evaluated. The effect of PSD and density on the breakage of railroad ballast is discussed. As will be demonstrated, the breakage indices do work well with the same initial PSD because only $B_g$ and area $A$ can provide an appropriate evolution of the extent of breakage when multiple initial PSDs are used.
6.2.1 Test Results with Fixed Initial PSD

Figure 6.1 shows the PSDs of ballast with a fixed initial grading after monotonic triaxial tests (Indraratna et al., 2014a). As expected, with the increase of the increasing confining pressure, the PSDs of ballast shift towards left, which indicated that smaller ballast aggregates had been generated.

To further quantify the particle breakage of ballast, breakage indices such as the entropy increment, and the modified Marsal’s breakage ratio, etc., were used. Figure 6.2 shows the extent of particle breakage represented by $BBI$, $B$, $B_r$, and $B_g$, and the figure shows how they all generally increased with the increasing confining pressure, except for one test carried out under a confining pressure equal to 180 kPa.

![Figure 6.1. PSDs of ballast after monotonic loading (modified after Indraratna et al., 2014a)](image_url)

Figure 6.1. PSDs of ballast after monotonic loading (modified after Indraratna et al., 2014a)
Figure 6.2. Breakage extent of ballast under monotonic loading

Figure 6.3 shows the extent of particle breakage represented by area $A$, the entropy increment, the normalised base entropy and modified $B_g$, and as expected, the initial PSD shifted more as the confining pressure increased, which led to an increase of area $A$ (Figure 6.3(a)). The grading entropy measures the amount of statistical information in a given PSD such that the broader the PSD, the more entropy it should contain, and therefore the entropy increment $\Delta S$ increased with the increasing extent of particle breakage, as shown in Figure 6.3(b). However, unlike the other breakage indices, the relative base entropy $\bar{S}$ generally decreases with the increasing confining pressure, as shown in Figure 6.3(c). The modified $B_g$ (Figure 6.3(d)) also shows a general increase with the increasing confining pressure, which is in accordance with the predictions given by using $B_g$, $BBI$, $B$, $B_r$, area $A$, and $\Delta S$.
Figure 6.3. Representation of the breakage extent of ballast under monotonic loading: (a) area $A$, (b) entropy increment, (c) relative base entropy, and (d) modified $B_g$

6.3.2 Test Results of Multiple Initial PSDs

The effects of PSD and density on the breakage characteristics of railroad ballast are investigated in this section by analysing the PSDs of each sample of ballast after cyclic
triaxial tests. Since the relative density varied significantly with the varying coefficient of uniformity, the results of this test series were prepared based on the similar initial void ratio and the similar initial relative density, as described previously. PSDs where the maximum particle size and coefficients of uniformity varied after each test are shown in Figure 6.4. Note that Figure 6.4(a) is illustrated in a linear-log scale to distinctly show the shifting of each PSD, while Figure 6.4(b) is plotted in a traditional log-linear scale for comparison. As expected, after a cyclic test the PSDs shifted towards the left, which indicated aggregate breakage within the ballast sample, whereas a larger shift occurred in samples tested under a higher load frequency. Further quantification of the extent of particle breakage of samples with varying maximum particle size can be found in Figures 6.5 - 6.6, while Figures 6.7 - 6.11 presents the breakage characteristics of ballast with varying coefficients of uniformity.

Figure 6.4(a). PSDs of ballast after cyclic loading at linear-log scale
6.2.2.1 Effect of Particle Size

Figure 6.5 shows the variations of the extent of particle breakage as the particle size increased. As expected, more ballast breakage occurs at a higher loading frequency, but the evolutionary trends relative to the effect of particle size differ when different breakage indices are used. Note that the extent of breakage represented by $BBI$, $B$, and $Br$ decreased monotonically as the particle size increased, whereas the value of $Bs$ increased, irrespective of the loading frequency. This difference is caused by an inappropriate use of the breakage potentials.
between multiple initial PSDs. As Indraratna et al. (2016) pointed out, the calculation of $BBI$, $B$, and $B_r$ depends intrinsically on the measurement of their breakage potentials, and they can differ completely with different initial PSDs. For instance, a well-graded PSD should have a low breakage potential (area $C$ or $E$, if $B$ or $BBI$ were used) when compared to a uniform one, but in this situation, even a small shift in the initial PSD (area $A$) could result in a higher value of $\frac{A}{C}$ or $\frac{A}{E}$, as shown in Figure 2.1. Therefore, the use of $BBI$, $B$, and $B_r$ may not provide a good comparison of the breakage results between multiple initial PSDs. Since ballast degradation is mainly attributed to attrition and the corner breakage of larger sized angular particles (Indraratna et al., 2005; Lu and McDowell, 2006), larger sized angular ballast would produce more breakage, as indicated by $B_g$ in Figure 6.5; this is consistent with Varadarajan et al. (2003).

![Figure 6.5. Breakage extent of ballast with varying $d_M$](image_url)
Figure 6.6 shows the extent of breakage as represented by area $A$, the increment of entropy, the relative base entropy, and a modified $B_g$. It can be observed that the area $A$ increases monotonically with an increasing particle size, which indicates there is more particle breakage from larger ballast. The entropy increment and modified $B_g$ also increased as the particle size increased, but the evolutionary trend of relative base entropy was not conclusive because there was an initial decrease followed by an increase in the relative base entropy is observed. Therefore, the use of $B_g$, a modified $B_g$, area $A$, and the entropy increment seemed to assess the extent of ballast breakage quite well with a varying maximum particle size.
6.2.2.2 Effect of Coefficient of Uniformity

The effect of the coefficient of uniformity on ballast degradation under a similar initial void ratio is shown in Figures 6.7 - 6.9. It can be observed from Figure 6.7 that the extent of breakage as represented by $B_{BI}$, $B$, and $B_r$ generally increased with an increasing coefficient of uniformity. However, there was an initial decrease followed by an increase in the coefficient of uniformity when $B_g$ was used. This difference in prediction between $B_g$ and the other three indices ($B_{BI}$, $B$, and $B_r$) can also be attributed to an inappropriate use of the breakage potentials between multiple initial PSDs. Therefore, to assess the extent of particle breakage of samples with multiple initial PSDs, the use of $B_g$ is suggested. Indeed, this variation of $B_g$ with the coefficient of uniformity can be attributed to initially unstable packing resulted from maintaining a constant initial void ratio across all the specimens with varying coefficients of uniformity (Indraratna et al., 2016). Moreover, the high concentration of stress between the aggregates caused significant corner breakage and splitting which resulted in an increase in the extent of breakage $B_g$. 
Figure 6.7. Breakage extent of ballast with similar void ratio: (a) $f = 20$ Hz and (b) $f = 30$ Hz

Similar observations can also be found in Altuhafi and Coop (2011) where the breakage characteristics of Dog’s Bay sand with changing PSDs were evaluated. Figure 6.8 shows the representative breakage of Dog’s Bay sand when $B_g$ and $B_r$ were used, as it varied from a general decrease followed by an increase with an increasing coefficient of uniformity. However, it cannot be denied that as the coefficient of uniformity increases further, larger particles can still be cushioned better by smaller particles even without being well compacted and therefore hard to break, which obviously reduces the extent of breakage. However, ballast in these size ranges should be discarded where fast drainage is concerned.
Figure 6.8. Variation of particle breakage extent with the coefficient of uniformity of Dog’s Bay sand

Fig. 6.9 shows the extent of particle breakage as represented by area $A$, $\Delta S$, $\bar{S}$, and a modified $B_g$. It shows an initial decrease followed by an increase with the increasing coefficient of uniformity when area $A$ and a modified $B_g$ were used. However, the entropy increment still exhibits a monotonic increase with an increasing coefficient of uniformity, regardless of the actual extent of particle breakage within the sample. The evolutionary trend of the relative base entropy is not obvious.
Figure 6.9. Representation of the breakage extent of ballast with similar void ratio by different indices
The breakage extent of railroad ballast under a similar initial relative density is shown in Figures 6.10 and 6.11. As can be observed, the breakage extent of ballast tested under higher load frequency is generally larger than that under a lower load frequency. With the increase of the coefficient of uniformity, the extent of breakage as represented by $BBI$, $B$, and $B_r$ increase. It is found that the breakage potential only had a slight effect on the calculation of
$B_r$. However, the extent of breakage by $B_r$ decreased significantly when the coefficient of uniformity is small ($C_u < 1.8$), it then decreased gradually as the coefficient of uniformity increased further. This is because of the different aggregate arrangements in different ballast samples. When the coefficient of uniformity is small ($C_u < 1.8$), ballast consists mainly of larger aggregates ($d > 37.5 \text{ mm}$) with high angularity and relatively large pore volumes, and this results in a significant concentration of stress at the contacts. To facilitate particle rearrangement, larger ballast experiences significant corner breakage, surface attrition, and splitting due to the concentration of tensile strength and fatigue as the number of load cycles increases (Sun et al., 2015). As the coefficient of uniformity increases, more and more small particles ($d \leq 31.5 \text{ mm}$) are added, and an increasingly better particle configuration is attained during compaction, which reduces the extent of breakage by a more uniform distribution of internal stresses. Similar results can be also found in the DEM study carried out by Xiao et al. (2012) and Xiao and Tutumluer (2016) where the packing density and coordination number of unbound aggregate materials can be increased by adequately broadening the material PSD to have a gravel-to-sand ratio of about 1.5. A comparison of the results plotted in Figures 6.7 and 6.10(a) indicates that the ratio of particle breakage decreases when the sample density increases, which amply substantiates the role that relative density plays on particle breakage. Since field ballast is likely to be well compacted before operation, ballast breakage under constant initial relative density can be placed into two different categories based on the range of uniformity coefficients adopted, i.e., a high breakage zone and a reduced breakage zone, as shown in Figure 6.10(b).

(1) High breakage zone (Zone A)

When the coefficient of uniformity is small ($C_u < 1.8$), ballast mainly consists of larger aggregates ($d > 37.5 \text{ mm}$), and because of their high angularity and relatively large pore
volume, the inter-particle area in this zone is low, and this results in a significant concentration of stress at the contacts that can be confirmed by the DEM study of the ballast packing density by Tutumluer et al. (2009) where the more uniformly graded angular materials tend to have larger void space (porosity), hence more unstable. To facilitate particle rearrangement, larger ballast experiences significant corner breakage and surface attrition, and they also split due to the concentration of tensile strength and fatigue as the number of load cycles increases. However, the fine particles generated by breakage accumulate inside the pores of the larger particles and experience no further significant breakage. Particle breakage in this zone is rapid and occurs mainly at the onset of cyclic loading, but as the coefficient of uniformity increases, ballast aggregates interlock (better packing) more and the subsequent number of inter-particle contacts increases; this then reduces the concentration of stress and the extent of particle breakage. For the range of uniformity coefficients studied here, breakage in this zone was the most significant.

(2) Reduced breakage zone (Zone B)

As the coefficient of uniformity increased, more and more small particles ($d \leq 31.5$ mm) were added and an increasingly better particle configuration (packing arrangement) and reduced air voids were attained during compaction, which helped to reduce the overall breakage by a more uniform distribution of internal stresses. This behaviour is shown in Figure 6.10(b) for $C_u \geq 1.8$. In this zone a better internal distribution of contact stress was attained, which in turn increased the inter-particle contact areas that reduced the tensile stresses within the granular assembly, and thereby reduced the extent of breakage. The number of inter-particulate contacts also increased compared to the high breakage zone.
Figure 6.11 shows the extent of particle breakage as depicted by the remaining breakage indices. Here area $A$ and the relative base entropy show a general decrease with an increasing coefficient of uniformity, indicating a decreasing trend of the extent of breakage with the coefficient of uniformity. However, the entropy increment increased with the increasing coefficient of uniformity, irrespective of the actual extent of breakage that occurred inside the sample. Figure 11(d) shows that there was an initial decrease followed by an increase of the modified $B_g$ with the coefficient of uniformity and the highest extent of breakage by a modified $B_g$ occurred in samples with a low coefficient of uniformity ($C_u = 1.2$).

Figure 6.11. Representation of the breakage extent of ballast with similar relative density by different indices
6.2.2.3 Change of Particle Shape of Ballast

In addition to particle size, particle shape also plays a significant role in the mechanical response of railroad ballast. To investigate the variations in particle shape due to cyclic loading, the aggregates \(d_M = 53 \text{ mm}\) were scanned with a 3D laser. Table 6.1 shows the variation in particle shapes of the typical particles shown in Figure 6.12 before and after the tests; their elongation, flatness, and aspect ratios increased after cyclic loading, indicating that ballast becomes increasingly less platy and columnar during loading. An apparent increase in ellipsoidness indicates that severe surface abrasion has occurred and turned the aggregates into a more regular form. There was also a significant difference in the shapes of larger and smaller aggregates, such that the ellipsoidness of larger particles increased while smaller particles (No. 3, 4, 5) produced by corner breakage exhibited lower ellipsoidness. Moreover, particles that experienced surface abrasion also underwent elongation and increased aspect ratios while smaller particles (No. 3, 8, 9) which experienced corner breakage also exhibited a lower elongation ratio.

Figure 6.12. Broken particles of ballast
Table 6.1. Shape characteristics of broken particles

<table>
<thead>
<tr>
<th>Particle No.</th>
<th>Elongation ratio $er$</th>
<th>Flatness ratio $fr$</th>
<th>Aspect ratio $ar$</th>
<th>Ellipsoidness $E_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Before</td>
<td>After</td>
<td>Before</td>
<td>After</td>
</tr>
<tr>
<td>1</td>
<td>0.730</td>
<td>0.837</td>
<td>0.478</td>
<td>0.746</td>
</tr>
<tr>
<td>2</td>
<td>0.730</td>
<td>0.826</td>
<td>0.478</td>
<td>0.676</td>
</tr>
<tr>
<td>3</td>
<td>0.832</td>
<td>0.563</td>
<td>0.512</td>
<td>0.681</td>
</tr>
<tr>
<td>4</td>
<td>0.832</td>
<td>0.985</td>
<td>0.512</td>
<td>0.630</td>
</tr>
<tr>
<td>5</td>
<td>0.587</td>
<td>0.983</td>
<td>0.538</td>
<td>0.978</td>
</tr>
<tr>
<td>6</td>
<td>0.738</td>
<td>0.525</td>
<td>0.692</td>
<td>0.787</td>
</tr>
<tr>
<td>7</td>
<td>0.738</td>
<td>0.654</td>
<td>0.692</td>
<td>0.549</td>
</tr>
<tr>
<td>8</td>
<td>0.866</td>
<td>0.483</td>
<td>0.271</td>
<td>0.686</td>
</tr>
<tr>
<td>9</td>
<td>0.866</td>
<td>0.621</td>
<td>0.271</td>
<td>0.422</td>
</tr>
<tr>
<td>10</td>
<td>0.592</td>
<td>0.867</td>
<td>0.522</td>
<td>0.446</td>
</tr>
<tr>
<td>11</td>
<td>0.867</td>
<td>0.859</td>
<td>0.778</td>
<td>0.512</td>
</tr>
<tr>
<td>12</td>
<td>0.690</td>
<td>0.742</td>
<td>0.589</td>
<td>0.752</td>
</tr>
<tr>
<td>13</td>
<td>0.654</td>
<td>0.808</td>
<td>0.477</td>
<td>0.615</td>
</tr>
<tr>
<td>14</td>
<td>0.782</td>
<td>0.771</td>
<td>0.590</td>
<td>0.775</td>
</tr>
<tr>
<td>15</td>
<td>0.855</td>
<td>0.856</td>
<td>0.634</td>
<td>0.821</td>
</tr>
</tbody>
</table>

To quantify the overall variations in shape of all the aggregates within the sample, the overall particle shape indices ($PSI$) were obtained as follows:

$$PSI = \frac{\sum P_{Mi}SS_i}{\sum P_{Mi}}$$

where $P_{Mi}$ and $SS_i$ are the mass probabilities of occurrence and the particle shape index in $i$th sieve interval, respectively. In this study, three particle shape indices, i.e., elongation ratio ($er$), flatness ratio ($fr$), aspect ratio ($ar$), and 3D ellipsoidness ($E_e$), are used.

Figure 6.13 shows the mean values of the particle shape indices of each fraction from PSD No. 1 before and after the test. Note that larger ballast generally has a lower ellipsoidness (higher angularity) than smaller ballast, although the elongation ratio, flatness ratio, and aspect ratio generally increases as the particles increased in size, which indicated that larger ballast aggregates are less platy and columnar. Therefore, using a scaled PSD in a triaxial test is problematic because of inconsistency in particle shape and the associated coordination number. As ellipsoidness increases the particles become increasingly regular which improves...
their compaction Cho et al. (2006); as a consequence, the contact surface area between particles increases and the probability of stress concentration that promotes breakage diminishes. Apart from a change in the coordination number caused by particle size, the variation of ballast breakage with aggregate size may also be related to overall variations of particle shape. It can be also observed that the elongation, flatness and aspect ratios increased after cyclic loading. Moreover, there was a slightly higher value of each particle form index for ballast tested under higher loading frequency, indicating that ballast becomes increasingly less platy and columnar with an increase in the loading frequency. An apparent increase in ellipsoidness indicates that severe surface abrasion has occurred and turned the aggregates into a more regular form. Fundamentally, the higher the loading frequency the higher the value of ellipsoidness, and therefore a higher degree of attrition for larger and smaller particles is expected in tests with a higher loading frequency.
Figure 6.13. Variation of particle shape under cyclic loading: (a) Elongation ratio $er$, (b) Flatness ratio $fr$, (c) Aspect ratio $ar$, and (d) Ellipsoidness $E_e$.

### 6.3 Recommended PSD for Ballast

The test results indicated that even a small change in particle size can substantially influence the deformation and degradation of ballast. Railroad ballast is often designed using uniformly graded material to mainly satisfy the drainage requirement. However, uniformly graded
aggregates tend to give more settlement than broadly or densely graded materials. A good ballast design needs to consider both its void space and structural stability (Tutumluer et al., 2009). The PSD currently used by rail industries in Australia has a low coefficient of uniformity $C_u$ of around 1.4 ~ 1.6 (AS 2758.7, 1996) and therefore falls within Zone A which results in severe ballast breakage. To reduce breakage, a modified PSD with higher $C_u$ was suggested (Indraratna et al., 2006), albeit it did not include the deformation aspect. In view of this, the PSD should be optimised to include the effect of particle size on both breakage and deformation.

Figure 6.14. PSDs with varying deformation and degradation

Figure 6.14 shows upper and lower bound that can be obtained in ballast with a coefficient of uniformity where $C_u = 2.0$ and a maximum particle size where $d_M = 53$ mm. A minimum particle size where $d_m = 9.5$ mm was chosen for drainage considerations. The lower bound
contains mainly finer aggregates while the upper bound contains mainly coarser aggregates. Those PSDs approaching the upper bound exhibited smaller deformation (Figure 4.20), but this was usually accompanied by more breakage after compaction (Figure 6.5). However, the lower bound PSD exhibited less breakage but higher deformation, so the optimum PSD should fall between these upper bound (low deformation) and lower bound (reduced breakage) limits that resemble less ballast degradation and deformation (Figure 6.14). This is also validated by the DEM study on ballast with different PSDs by Tutumluer et al. (2009). When PSDs became more uniform, the corresponding ballast samples would produce more and more settlement. However, very well-graded ballast aggregates also yielded higher settlements. Therefore, an optimised PSD should ideally be placed between a uniform gradation and a well-graded distribution that produces less ballast deformation. By applying real track load on ballast with different particle gradations, a PSD with $C_u$ around 2.1 (AREMA No. 24) was found to provide the least deformation of the ballast layer (Tutumluer et al., 2009). In view of the above, a new PSD can be proposed for ballast that incorporates a value of $C_u$ which provides the least particle breakage and deformation, as shown in Figure 6.15. This proposed PSD has a moderate degradation as well as deformation and there are less than 1% of finer aggregates ($d < 9.5$ mm) to ensure sufficient drainage capacity. The $C_u$ value is around $1.8 \sim 2.0$ which belongs to Zone B that provides denser packing, better interlocking, and hence less settlement and reduced breakage, as indicated in Figures 4.21, 4.22 and 6.10. The corresponding values of the PSD parameter $\Theta$ for the upper and lower bounds of the recommended PSD were determined to be 3.0 and 4.4, respectively, which represents less ballast deformation than the Australian standard PSD ($4.5 \sim 6.6$), as shown in Figure 4.22.
Figure 6.15. Recommended PSD for railroad ballast
6.4 Conclusions

Particle breakage occurs even at low confining pressures which significantly influences the mechanical behaviour of railroad ballast. To investigate particle breakage and the associated change in shape, monotonic and cyclic tests were carried out on ballast using a large-scale cylindrical triaxial apparatus. Variations in the shape of ballast before and after the test were evaluated with a 3D laser scanner. The effect of particle size distribution and density on the breakage of ballast was studied by using different particle breakage indices. The main findings of this chapter are summarised below:

(1) The PSD and density of railroad ballast significantly influenced its degradation, but the extent of breakage was reduced by increasing the sample density. All the breakage indices captured the breakage of ballast with a fixed initial PSD very well, but the extent of breakage defined by the breakage potential and grading entropy may not be the best way to evaluate the breakage of ballast with multi-initial PSDs.

(2) The extent of breakage as defined by the breakage potential indicated an increase in the extent as the particle size decreased. However, there was an increase in Marsal’s breakage ratio and the grading entropy as the particle size increased, which implied that larger ballast had a higher extent of breakage.

(3) The breakage indices defined by the breakage potential and grading entropy failed to characterise the extent of breakage with different initial coefficients of uniformity. However, Marsal’s breakage ratio generally decreased as the coefficient of uniformity of ballast with similar initial relative densities increased. Ballast degradation was thus divided into two zones depending on the range of the coefficient of uniformity, i.e., a high breakage zone, and a reduced breakage zone.
(4) Ballast samples tested under similar initial void ratios experience an initial decrease followed by a slight increase of Marsal’s breakage ratio, which was attributed to the initially unstable packing which induced a high concentration of stress and subsequently increased extent of breakage between aggregates with an increasing coefficient of uniformity.

(5) The extent of ballast breakage increased with an increasing load frequency; so too did the elongation ratio, flatness ratio, aspect ratio, and ellipsoidness of samples tested under a higher load frequency. Particle breakage led to parent ballast aggregates becoming rounder, whereas smaller aggregates created by particle breakage seemed to have a lower ellipsoidness and elongation ratio.

(6) Based on these test results, an optimum PSD (1.8 \( \leq C_u \leq 2.0 \), \( \Theta = 3.0 – 4.4 \), \( E_e = 0.375 – 0.376 \)) for ballast was proposed to minimise deformation and degradation under high frequency cyclic loading.
CHAPTER 7 FRACTIONAL ORDER MODELLING OF PERMANENT DEFORMATION OF BALLAST

7.1 Introduction

Ballast usually serves as a construction material that forms a track bed to withstand the load from railroad ties (sleepers) and to facilitate the free drainage of water. During the whole period of operation, rail tracks usually suffer from a large number of train passages which leads to a cumulative deformation of the underlying railroad ballast. An accurate prediction of the corresponding maintenance periods necessitates the development of an advanced constitutive model. Although the traditional plastic constitutive models have been widely investigated and successfully applied in many fields, there is still a lot of work to be done in constructing models that can realistically describe the cumulative deformation of railroad ballast subjected to long-term loads. While the traditional plasticity approaches such as the elasto-plastic models (Carter et al., 1982; Liu and Carter, 2000), the generalised plasticity models (Pastor et al., 1990; Liu et al., 2014), and the bounding surface plasticity models (Khalili et al., 2005; Österlöf et al., 2014; Xiao et al., 2014a; Chen et al., 2015), reveal the deformation mechanism of soils subjected to cyclic loading, they were just able to reasonably simulate the cyclic behaviour of granular soils for very limited cycles of probably less than a hundred, so for cumulative deformation under a large number of load cycles ($\geq 10^3$). Owing to the unintentional accumulation of numerical errors and huge calculation effort, these models usually failed. The empirical models usually target the problem and are therefore flexible enough for engineering applications (Lekarp and Dawson, 1998; Kargah-Ostadi and Stoffels, 2015), but they did not account for the underlying mechanism for deformation of granular soils, whereas semi-empirical models usually provide an alternative way to model
cumulative deformation. For example, Suiker and de Borst (2003) proposed an elasto-plastic methodology for simulating the cyclic deterioration of railway tracks by assuming that permanent deformation is caused by frictional sliding and volumetric compaction, so the growth of each deformation component is empirically simulated by a power law. Niemunis et al. (2005) and Wichtmann et al. (2010) suggested using a cyclic flow rule to model a high cycle model for sand under low amplitude cyclic loading, but some of their parameters are physically meaningless and can only be determined by extensive laboratory tests.

Whether or not different modelling techniques are chosen to describe experimentally observed stress strain behaviour a fundamental question is, do we use correct mathematical tools to describe the deformation of material? More precisely, are commonly used increments in a particular model correctly assumed to be an integer order or should a more general one be chosen, such as the operators of a fractional order? The answer to these questions is not clear because the the cumulative deformation of granular soils should be determined by both the current loading step and loading history (López-Querol and Coop, 2012). This is indeed a memory-intensive phenomenon which can be elaborated by fractional calculus (Sun et al., 2011, 2013; Sumelka, 2014; Sumelka and Nowak, 2015). Fractional calculus has been used to model many geotechnical issues, including the creep and relaxation behaviour of composite soil (Sun et al., 2011), the strain hardening and softening behaviour of sand and clay under static loading (Yin et al., 2013), the vibration of rail pads (Fenander, 1997), and even the anomalous diffusion of underground water (Sun et al., 2009; Chen et al., 2010). Fractional calculus has a lot of potential in modelling the static response of soils and can be easily applied in commercial software due to its simplicity. However, a lot more work is needed to determine the cumulative deformation of ballast subjected to a large number of load cycles.
The aim of this chapter is to develop a more rigorous model for predicting the cumulative
deformation of ballast subjected to high loading cycles. Traditional plasticity theory is used
and modified by incorporating fractional calculus, and then the model is validated against a
series of laboratory tests. Monotonic and cyclic test results from Chapter 4 are used herein.
Specifically, under monotonic loading, Sections 4.3.1 and 4.3.3 provide the stress-strain and
critical state behaviour of ballast, respectively; Cambridge based Critical State Theory allows
certain pertinent geotechnical parameters to be obtained from static or monotonic tests as
they can still be applied under cyclic load testing as widely conducted these days, for instance
in Offshore Geotechnics (e.g. EH Davis Lecture by Prof Mark Cassidy, 2012). These
experimental data are then used to obtain the crucial model parameters and validate the model
performance in predicting the virgin loading behaviour of ballast. Section 4.4.3 provides the
experimental evidence to propose the fractional strain accumulation rate used for cyclic fractional order constitutive modelling.

7.2 Critical Concepts of Fractional Calculus

The theory of fractional calculus consists of both the fractional derivative and fractional
integral. So in current study the common definitions known as the Riemann-Liouville
fractional calculus will be used.

The Riemann-Liouville fractional derivative of the function $z(x)$ can be formulated as
(Podlubny, 1998):

$$\begin{align*}
\frac{\mathcal{D}_x^\alpha z(x)}{dx^\alpha} &= \frac{d}{dx} \int_0^x \frac{z(\tau)d\tau}{(x-\tau)^{\alpha}}, \\
&= \frac{d}{dx} z(x), \quad x > 0
\end{align*}
\tag{7.1}$$

where $\mathcal{D}$ indicates performing differentiation and $\alpha \in [0,1]$, denotes the fractional order.
Obviously, the fractional derivative is defined on an interval, unlike the integer order
differential operators defined in a single point. Thanks to the integral form of the fractional
derivative, it has an inherently strong memory of its variable $x$, which results in a critical
difference between the integer order derivative and the corresponding fractional order
derivative. The integer order derivative of a constant is 0, whereas the Riemann-Liouville
(Podlubny, 1998) fractional order derivative of a constant $\tilde{C}$ is not, and is given as:

$$\tilde{0}_D_\alpha^\alpha \tilde{C} = \frac{\tilde{C} x^{-\alpha}}{\Gamma(1-\alpha)} \quad (7.2)$$

where $\Gamma(x)$ is the gamma function and can be defined as:

$$\Gamma(x) = \int_0^\infty e^{-\tau} \tau^{-x-1} \, d\tau \quad (7.3)$$

$$\Gamma(x+1) = x\Gamma(x) \quad (7.4)$$

The Riemann-Liouville fractional integral is defined by:

$$0_\alpha^\alpha I^\alpha z(x) = \frac{1}{\Gamma(\alpha)} \int_0^x \frac{z(\tau) \delta \tau}{(x-\tau)^{1-\alpha}}, \quad x > 0 \quad (7.5)$$

where $I$ means integral. Note there are two important properties for the Riemann-Liouville
definition, and one is:

$$0_\alpha^\alpha I^\alpha z(x) = 0_\alpha^\alpha D_\alpha^\alpha \left(0_\alpha^\alpha I^\alpha z(x) \right) \quad (7.6)$$

which means that the Riemann-Liouville derivative operator is a left inverse to the Riemann-
Liouville operator, and the other is:

$$0_\alpha^\alpha I^\alpha \left(0_\alpha^\alpha D_\alpha^\alpha z(x) \right) = \left. \frac{\delta^{\alpha-1} z(x)}{\delta \kappa^{\alpha-1}} \right|_{\kappa=0} \frac{x^{\alpha-1}}{\Gamma(\alpha)} \quad (7.7)$$
7.3 Fractional Strain Accumulation Rate

In the model presented here the ballast is assumed to be homogeneous and isotropic, tension is negative and compression is positive. Triaxial stress notations are used in the formulation.

Many mathematical models on the constitutive behaviours of granular soils under triaxial loading conditions have been proposed (Xu and Xia, 2006; Russell, 2011; McDowell et al., 2013). By regarding the soil as an intermediate material lying between ideal solids (e.g. steel, concrete) which obey Hooke’ law of elasticity, and the Newtonian fluids (e.g. water, lubricant oil) which satisfy Newton’s law of viscosity, Yin et al. (2013) developed a fractional order constitutive modelling for geomaterials under monotonic loading. For clarification, their work is briefly introduced here, and the basic constitutive law is defined by:

\[
\sigma = \bar{E} \theta^\alpha \bar{D}^\alpha \varepsilon, \quad 0 \leq \alpha \leq 1 \tag{7.8}
\]

where \(\sigma\) and \(\varepsilon\) are the stress and strain, respectively, and \(\bar{E}\) and \(\theta\) are two parameters. Note that Hooke’s law where \(\alpha = 0\) and Newton’s law where \(\alpha = 1\) are just special cases of Equation (7.8), which has been shown as efficient and simple way to model the static behaviour of geomaterials (Yin et al., 2013; Meng et al., 2016). However, the fractional order models available for soils, including Equation (7.8), are all phenomenological models where viscoelasticity is used to simulate the stress strain response of soils which are actually elastoplastic. Therefore, a fractional order elastoplastic model which has the fractional order and elastoplastic approach for geomaterials should be proposed. The cyclic behaviour of ballast depends on its history, so its current deformation is often influenced by its previous loading history (Indraratna et al., 2012b). The fractional derivative of a variable, as shown in Equation (7.1), is determined by the state of the current differentiation point and by its performance during the whole loading period, from 0 to \(x\). As will be demonstrated, using the
fractional rate for strain accumulation is an alternative approach for simulating the cumulative deformation of ballast.

In the context of cyclic tests, the number of load cycles \( N \) rather than the time \( t \) is used (Wichtmann et al., 2010; François et al., 2010) as the strain rate \( (d\varepsilon/dN) \) during repeated loading and unloading. According to Equation (4.1), the strain accumulation rate \( \dot{\varepsilon}^p \), with respect to the number of load cycles \( N \), can be considered to obey the power law with regard to \( N \), which can be described by the following fractional rate for strain accumulation:

\[
\overline{D}_N^\alpha \varepsilon^p = b\Gamma(\alpha)
\]  

(7.9)

where \( \varepsilon^p \) denotes the plastic strain. The fractional derivative order \( \alpha = 1 - \beta \). \( \beta \) and \( b \) are material constants, as shown previously in Figures 4.14(b) and 4.15(b), for a given loading state. Therefore, a constant strain accumulation rate can be suggested if a fractional scale of differentiation is used. However, the physical origin of \( \alpha \) has not been revealed in the references available (Fenander, 1997; Yin et al., 2013; Sumelka, 2014; Meng et al., 2016), so in the next section, one possible connection between \( \alpha \) and fractal dimension of the tested material will be explored.

### 7.4 Fractional Order and Fractal Dimension

An attempt is made here to show how the fractal dimension of the aggregates influences the fractional derivative order during cyclic loading. To begin with, the modified Cam Clay energy equation that considers dissipation by particle breakage, and the associated particle rearrangement, is used (Russell, 2011):

\[
p'\dot{\varepsilon}_s^p + q\dot{\varepsilon}_s^p = Mp'\dot{\varepsilon}_s^p + \frac{\Omega S_s}{V_s(1 + e_0)}(1 + \dot{\hat{R}})
\]  

(7.10)
where the mean effective principal stress $p' = (\sigma'_i + 2\sigma'_3)/3$ and the deviator stress $q = \sigma'_i - \sigma'_3$. $\sigma'_i$ and $\sigma'_3$ are the major and minor effective principal stresses, respectively.

The plastic volumetric strain $\varepsilon^p_v = \varepsilon^p_i + 2\varepsilon^p_3$ and the generalised plastic shear strain $\varepsilon^g_s = 2(\varepsilon^p_i - \varepsilon^p_3)/3$. $\varepsilon^p_i$ and $\varepsilon^p_3$ are the major and minor principal plastic strains, respectively.

$M = (6 \sin \phi_{cr} / (3 - \sin \phi_{cr}))$ is related to the critical state friction angle $\phi_{cr}$, $\dot{S}_s$ is the increment in surface area of particles within a sample, and $\Omega$ represents the surface energy.

$V_s$ is the soil volume. $\hat{R}$ is the ratio of energy dissipated by particle rearrangement to the energy dissipated by particle degradation. For simplicity of the analysis, $\hat{R}$ is assumed to be constant as also suggested by Russell (2011) and Nguyen and Einav (2009). Since this topic is rarely broached, a simple isotropic compression condition is considered here, which means that Equation (7.10) can be reduced as follows:

$$p' \dot{\varepsilon}^p_v = \frac{\Omega \dot{S}_s}{V_s(1 + e_o)} (1 + \hat{R})$$  \hspace{1cm} (7.11)

The fractal properties of a granular material under isotropic compression may now be combined with Equation (7.11) to derive a connection between the fractional order and the fractal dimension. It is recognised that soil particles cannot be truly distributed in a fractal way because of their different origin and types of mineral particles, but modelling the particle size distributions using fractals provides a reasonable fit to the experimental data (Coop et al., 2004; McDowell et al., 2013). The number of particles with a diameter $L$ that are larger than $l_s$ obey the following power law (Russell, 2011; McDowell et al., 2013):

$$N_s(L > l_s) = A l_s^{-D_s}$$  \hspace{1cm} (7.12)
where the subscript \( s \) denotes the solid, \( A \) is a constant of proportionality, and \( D_s \) is the fractal dimension of the aggregates. Therefore, the incremental particle surface area can be obtained by using Equation (7.12) as given by:

\[
\dot{S}_s = \pi l_s^2 \gamma_s \dot{N}_s = A D_s \gamma_s \pi l_s^{1-D_s} \dot{I}_s
\]  
(7.13)

where \( \gamma_s \) is the surface shape factor. Substituting Equation (7.13) into Equation (7.12) yields:

\[
p' \dot{\varepsilon}_v = \frac{\Omega(1 + \hat{R})}{V_s(1 + e_0)} AD_s \gamma_s \pi l_s^{1-D_s} \dot{I}_s
\]  
(7.14)

Equation (7.14) correlates the stress strain behaviour with the particle size \( l_s \) within granular soil, so the plastic deformation of granular soils is therefore attributed to a continuous degradation of particles within the sample. Suppose the minimum particle size decreases by \( k' \) times from its initial value for one loading cycle, then equation (7.14) can be further derived as:

\[
\dot{\varepsilon}_v = \mu k' \frac{\partial k'}{\partial N}
\]  
(7.15)

where \( \mu \) is expressed as:

\[
\mu = \frac{\Omega(1 + \hat{R})}{V_s(1 + e_0)p'} AD_s \gamma_s \pi l_s^{2-D_s}
\]  
(7.16)

and \( l_{s_0} \) denotes the initial minimum particle size. The decreasing rate \( k' \) changes with \( N \), so by recalling the relationship shown in Equation (7.9), and using Equation (7.2), \( k' \) can be expressed as:

\[
k' = \left[ (D_s - 2) N^\alpha / \alpha + 1 \right]^{(D_s - 2)}
\]  
(7.17)
A rapid increase followed by a stable variation of \( k' \) with an increasing number of load cycles can be expected. The physical relationship between \( \alpha \) and \( D_s \) can be obtained further via Equation (7.17) with \( N \) equal to 1.

\[
\alpha = (D_s - 2)/(k_1^{D_s - 2} - 1)
\]  

(7.18)

where parameter \( k'_1 \) corresponds to the degradation rate of the minimum particle size at the first loading cycle. The fractional derivative order decreases with an increase of the fractal dimension of a given granular soil (Figure 7.1), and approaches zero with a fractal dimension equal to 3, indicating that soils with higher fractal dimensions densify fast than those with lower fractal dimensions.

Figure 7.1. Relationship between the fractional order and the fractal dimension
7.5 Fractional Order Constitutive Model

The traditional bounding surface plasticity is modified in this section by using the fractional rate of strain accumulation. The total increments of strain $\dot{\varepsilon}$ consists of both the incremental plastic strain $\dot{\varepsilon}^p$ and the elastic strain $\dot{\varepsilon}^e$.

$$\dot{\varepsilon} = \dot{\varepsilon}^e + \dot{\varepsilon}^p$$  

(7.19)

The increments of the elastic strain can be defined as follows:

$$\dot{\varepsilon}^e = C^e \sigma$$  

(7.20)

where the elastic strain tensor $\varepsilon^e = \begin{bmatrix} \varepsilon^e_x, \varepsilon^e_y \end{bmatrix}$, and the stress tensor $\sigma = \begin{bmatrix} p', q' \end{bmatrix}$. $C^e$ denotes the elastic compliance matrix, defined by:

$$C^e = \begin{bmatrix} \frac{1}{K} & 0 \\ \frac{1}{3G} & \frac{1}{3G} \end{bmatrix}$$  

(7.21)

where the bulk modulus is $K = (1 + \nu) / \kappa p'$, and the shear modulus is $G = 3(1 - 2\nu) / 2 / (1 + \nu) K$; and $\nu$ and $\nu$ are the void ratio and Poisson ratio, respectively. As demonstrated before, the accumulation of strain can be described better using the fractional order derivative, so rather than use the traditional incremental definition of the accumulation rate (Indraratna et al., 2012b), the incremental plastic strain $\dot{\varepsilon}^p$ can be fractionally defined as:

$$D_{\chi}^{\alpha} \dot{\varepsilon}^p = \chi \frac{\partial g}{\partial \sigma}$$  

(7.22)

where $\chi$ is the plastic multiplier and $g$ is the plastic potential function. By recalling Equation (7.9), $\chi \frac{\partial g}{\partial \sigma}$ should remain as a constant in the fractional scale for a given loading state, so performing the derivation of $1-\alpha$ order on both sides of the Equation (7.21) means:
\[ \dot{e}^p = \chi \frac{\partial g}{\partial \sigma} N^{\alpha-1} \Gamma(\alpha) \]  
(7.23)

### 7.5.1 Loading Surface and Bounding Surface

Traditional bounding surface plasticity is a framework for constitutive modelling of Granular soils, including ballast (Chen et al., 2015) and rockfill (Xiao et al., 2014a). This theory consists of three import parts, i.e., loading surface, bounding surface, and hardening rule. For sake of simplicity, the following loading surface is used:

\[ g = q - Mp' \left[ \ln(p'_0 / p') \right] = 0 \]  
(7.24)

where \( p'_0 \) controls the size of the loading surface and represents the intercept with the abscissa. The bounding surface is usually assumed to have the same shape as the loading surface:

\[ f = q - M\overline{p}' \left[ \ln(p'_0 / \overline{p}') \right] = 0 \]  
(7.25)

where \( \overline{p}'_0 \) denotes the size of the current bounding surface. The image stress point on the bounding surface can be expressed by using a scalar \( \rho \) as (Xiao et al., 2014a):

\[ \overline{p}' = \rho \overline{p}'_0 \]  
(7.26a)

\[ \overline{q} = \rho \eta \overline{p}'_0 \]  
(7.26b)

in which the stress ratio \( \eta \) is defined by the radial mapping rule (Bardet, 1986) as follows:

\[ \eta = \frac{q}{p'} = \frac{\overline{q}}{\overline{p}'} \]  
(7.27)

Substituting Equations (7.26) and (7.27) into Equation (7.25), the scalar \( \rho \) which determines the image stress point on bounding surface can be given as follows:
\[
\rho = \exp\left[-\left(\frac{\eta}{M}\right)\right]
\]  

(7.28)

The position of the initial bounding surface \((p'_{oi})\) can be determined by intersecting the critical state line and the swell line at point \((e_0, p'_e)\):

\[
p'_{oi} = p_r \exp\left[\frac{e_f - e_0 - \kappa \ln p'_{e}}{\lambda - \kappa} + 1\right]
\]  

(7.29)

where \(e_0\) denotes the initial void ratio prior to shearing. \(p_r\) is the unit pressure. \(\lambda\) and \(\kappa\) are gradients of the critical state and swell lines, respectively. \(e_f\) is the critical state void ratio at \(p' = 1\).

### 7.5.2 Loading Direction and Flow Direction

The loading direction is normal to the bounding surface and can be expressed by:

\[
n = [n_{f_s}, n_{f_n}]^T
\]  

(7.30)

where the compression-related component is:

\[
n_{f_s} = \frac{\partial f}{\partial p} \left| \frac{\partial f}{\partial \sigma} \right|^{-1} = \frac{1}{p'} \left[1 - \left(\frac{\eta}{M}\right)\right] \left|\frac{\partial f}{\partial \sigma}\right|^{-1}  
\]  

(7.31a)

and the shear-related component is given by:

\[
n_{f_n} = \frac{\partial f}{\partial q} \left| \frac{\partial f}{\partial \sigma} \right|^{-1} = \frac{1}{p'} \left|\frac{\partial f}{\partial \sigma}\right|^{-1}
\]  

(7.31b)

where the gradient amplitude is:
\[ \left\| \frac{\partial f}{\partial \sigma} \right\| = \frac{1}{\rho} \sqrt{ \left[ 1 - \left( \frac{\eta}{M} \right) \right]^2 + 1 } \quad (7.32) \]

For railroad ballast, the non-associated flow rule is used. By recalling Equation (7.22), one has the following:

\[ \dot{\varepsilon} = \frac{\dot{\varepsilon}}{\varepsilon} = \frac{\partial g}{\partial p} = \frac{\partial g}{\partial p} \left[ \gamma \frac{\partial g}{\partial q} \right] = (M - \eta) \quad (7.33) \]

Note that parameter, \( \gamma \), is introduced to account for the difference of accumulation rate between the volumetric and shear strains. Accordingly, the plastic flow direction can be defined as:

\[ \mathbf{m} = \begin{bmatrix} m_{gr} \end{bmatrix} = \begin{bmatrix} \frac{d}{\sqrt{1 + d^2}} \end{bmatrix} \quad (7.34) \]

### 7.5.3 Hardening Rule

In bounding surface plasticity, the hardening modulus \( H \) is related to both the size of the bounding surface and the distance between the current loading and bounding surfaces. Therefore, it can be conveniently decomposed into two components:

\[ H = H_b + H_\beta \quad (7.35) \]

where \( H_b \) is the plastic modulus at \( \sigma \) on the bounding surface. \( H_\beta \) is related to the distance between the current loading surface and the bounding surface. Applying the consistency condition at the bounding surface (Equation (7.25)) and using isotropic hardening of the bounding surface with the plastic volumetric compression, the derivatives of the bounding surface can be obtained as:
\[
\frac{\partial f}{\partial p'} \bar{p}' + \frac{\partial f}{\partial q} \bar{q} + \frac{\partial f}{\partial \bar{p}'_0} \frac{\partial \bar{p}'_0}{\partial \varepsilon^p_v} \dot{\varepsilon}^p_v = 0
\]  
(7.36)

By recalling Equation (7.22), the plastic volumetric strain \( \dot{\varepsilon}^p_v \) can be rewritten as:

\[
\dot{\varepsilon}^p_v = \frac{\partial g}{\partial p'} \frac{\chi}{\Gamma(\alpha)} N^{\alpha-1}
\]  
(7.37)

Substituting Equation (7.37) into Equation (7.36) and using the definition of the unit vector normal to the bounding surface, the hardening modulus, \( H_b \), can be derived as:

\[
H_b = -\frac{\partial f}{\partial \bar{p}'_0} \frac{\partial \bar{p}'_0}{\partial \varepsilon^p_v} m_{ev} \frac{N^{\alpha-1}}{\Gamma(\alpha)}
\]  
(7.38)

where the relationship between \( p'_0 \) and \( \varepsilon^p_v \) under isotropic loading can be formulated by using the fractional rate for volumetric strain accumulation as:

\[
\frac{\partial \bar{p}'_0}{\partial \varepsilon^p_v} = \frac{(1 + e_0)}{\lambda - \kappa} \bar{p}'_0 N^{1-\alpha}
\]  
(7.39)

Note that Equation (7.39) decreases to the virgin loading condition with \( N = 1 \). The cumulative volumetric strain decreases with an increasing load cycle, as shown schematically in Figure 7.2, which indicates that ballast is in a state of cyclic densification. Equation (7.38) shows that the fractional derivative order has a distinct effect on the plastic modulus where it is \( H_b \), and then it decreases with a decreasing fractional derivative order, indicating that more plastic strain accumulated during the loading cycle.
Figure 7.2. Schematic show of the cyclic loading and unloading

Similar to Russell and Khalili (2004), the arbitrary modulus $H_\delta$ can be given by:

$$H_\delta = h_0 p' \frac{1 + e_0}{\lambda - \kappa} \frac{\delta}{\delta_{in} - \delta}$$  \hspace{1cm} (7.40)

where $h_0$ is a material parameter and can be defined as a function of the initial conditions (Kan et al., 2014) or changing state (Bardet, 1986; Xiao et al., 2014a), for general application. For cyclic triaxial tests under different loading frequencies ($f$) or effective confining pressures ($\sigma'_3$), the following generalisation of $h_0$ can be given:
where $a$ and $c$ are material constants. $Ty = f(\sigma')$, indicating the tests under different load frequencies (or confining pressures, load amplitudes). $\delta'_m$ and $\delta$ are the distances from the stress origin and current stress point to the image stress point, respectively. Following the radial mapping rule (Bardet, 1986), it is easy to obtain:

$$\frac{\delta}{\delta'_m - \delta} = \frac{p'_b - p_b}{p'_b}$$

(7.41b)

Note that the hardening modulus $H$ approaches $H_b$ with increasing loading stress and finally turns out to be $H_b$ when the loading surface coincides with the bounding surface. $H_\delta$ approaches infinite if $\delta'_m - \delta \to 0$, implying a state of no plastic deformation of the ballast.

Combining Equations (7.22), (7.30), (7.34), and (7.35), a fractional order constitutive relationship between the incremental plastic strain and stress can be given by:

$$\dot{\varepsilon}^p = \frac{1}{H} \frac{\mathbf{mN}^{\alpha-1}}{\mathbf{n}^T} \left( \mathbf{C}^e + \frac{1}{H} \frac{\mathbf{mm}^T}{\Gamma(\alpha)} \right) \dot{\sigma}$$

(7.42)

Equation (7.42) is different from that of the traditional elasto-plastic model. It considers the number of load cycles by incorporating the concept of the fractional calculus. Substituting Equations (7.19) and (7.42) into Equation (7.20), the generalised constitutive equation can be obtained as:

$$\dot{\varepsilon} = \left( C^e + \frac{1}{H} \frac{N^{\alpha-1}}{\Gamma(\alpha) c} \right) \dot{\sigma}$$

(7.43)

It incorporates the concept of fractional calculus into the traditional plasticity theory. Semi-empirical elasto-plastic formulae with $\alpha = 1/3$ proposed by Liu and Carter (2000) and Indraratna et al. (2012b) are just special cases which can be theoretically derived from our
proposed approach. Equation (7.43) deduces to the traditional constitutive formulae for granular soils with $\alpha = 1$. As shown in Figure 7.3, the proposed model predicts equivalent stress strain behaviour for virgin loading and first unloading. However, the fractional derivative order greatly affects the subsequent densification of soils. The shear strain is observed to densify more quickly with a smaller value of $\alpha$. The rate of cyclic densification can be appropriately simulated by a proper value of $\alpha$.

Figure 7.3. Schematic representation of the effect of the fractional derivative order on soil densification: (a) stress strain response and (b) accumulated strain vs number of load cycles
7.5.4 Model Validation

7.5.4.1 Identification of Model Parameters

The proposed model requires 9 parameters to completely characterize the three dimensional behaviour of ballast. Two parameters, $\kappa$ and $\nu$, define the elastic behaviour. $\kappa$ can be determined by measuring the slope of the swell line of an isotropic test in $e$-$\ln p'$ space. The Poisson ratio, $\nu$, usually ranges from 0.05 to 0.35 for most granular soils, and can be determined from the following relationship (Xiao et al., 2014b):

$$\nu \approx \frac{9\epsilon_s - 2\epsilon_v}{18\epsilon_s + 2\epsilon_v}$$

(7.44)

However, for simplicity, a Poisson ratio of 0.3 for ballast is taken. There are three critical state parameters, $\phi_c$, $\lambda$, and $e_T$. The critical state friction angle, $\phi_c$, can be determined by fitting the critical state stress points in the $p' - q$ plane, as shown in Figure 7.4(a). Parameters, $\lambda$ and $e_T$, can be determined by fitting the critical state points in the $e - \ln p'$ space, as shown in Figure 7.4(b). $\lambda$ denotes the gradient of the critical state line and $e_T$ is the intersection of the critical state line with the $p' = 1$ line.
Figure 7.4(a). Determination of the critical state parameters in the $e - \ln p'$ space (data sourced from Salim and Indraratna, 2004)

\[ e = e_\Gamma - \lambda \ln p' \]
\[ e_\Gamma = 1.83, \lambda = 0.188 \]

Figure 7.4(b). Determination of the critical state parameters in the $p' - q$ space (data sourced from Salim and Indraratna, 2004)

\[ q = Mp' \]
\[ M = 1.87 \]
The hardening parameters, $a$ and $c$, determine the value of $h_0$ and can be obtained by fitting the stress strain curve in the $q - \varepsilon$ space, as shown in Figure 7.5.

![Figure 7.5. Determination of the hardening parameters](image)

Parameter $\gamma$ control the plastic flow direction, therefore, can be determined by simulating the stress dilatancy relationship as comprehensively discussed by Fu et al. (2014). One additional parameter that is different from the traditional plasticity is the fractional order, $\alpha$, for strain accumulation. It could be determined by trial and error for a better description of the ballast densification, as shown in Figure 7.6.
Figure 7.6. Determination of the fractional order

Note that physical properties of the materials used in this study can be found in Table 7.1. The test ID symbol refers to the test condition, e.g., S100 indicates a static test under the confining pressure of 100 kPa, and C30f20 indicates a cyclic test under the confining pressure of 30 kPa and load frequency of 20 Hz. All corresponding parameters used in this study are listed in Table 7.2. A flow chart showing how to implement the proposed model in prediction of cumulative deformation of ballast can be found in Figure 7.7.
Start

Input model parameters, soil properties, and loading parameters

Calculate initial bounding pressure

Calculate the evolution of current bounding surface, radial mapping scalar, bounding pressures

Compute loading and flow directions in $p-q$ space

Stress transformation matrix

Calculate loading and flow directions in general stress space

Calculate the hardening modulus, elastoplastic stiffness matrix

Back-calculate the stress increments

Compute the strain increments

Update the stress and strain matrices, number of loading cycles with fractional order

Load stop?

Yes, output results

Stop

No, next load

Figure 7.7. Flow chart for model implementation
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7.5.4.1 Model Performance

Performance of the proposed model for predicting the cumulative deformations is now investigated by comparisons between the numerical and experimental results from current study and the other literatures (Salim and Indraratna, 2004; Anderson and Fair, 2008; Aursudkij et al., 2009; Indraratna et al., 2012b; Sun et al., 2015). The results of simulation are shown in Figures 7.8 – 7.18.

Figure 7.8. Model simulation for drained compression tests on ballast
Figures 7.8 and 7.9 show simulation results for monotonic and cyclic tests performed on ballast under different confining pressures. Results of the monotonic simulation (Figure 7.8) show that the model well captures the characteristic features, such as the strain hardening, stress dilation, and contraction, of ballast under different confining pressures. Some models can only simulate the accumulated strain for limited cycles and failed to predict the deformation for high cycles while others can only give reasonable prediction of the
deformation under high cycles but failed to address the deformation at low cycles. However, as shown in Figure 7.9, the proposed model can well capture the long-term axial and volumetric strains from the initiation of the load cycle up to 50000 cycles. The predicted axial and volumetric strains are observed to increase rapidly at the initial loading stage and then gradually approach stable with increasing number of load cycles, which are in good agreement with the experimental results. The fractional order $\alpha$ of 0.5 is found to properly describe the cyclic densification of ballast under different confining pressures.

Figure 7.10. Model simulation for drained compression tests on ballast (data sourced from Salim and Indraratna, 2004)
Figure 7.11. Model predictions of (a) shear strain and (b) volumetric strain of ballast (data sourced from Indraratna et al., 2012b) under different load frequencies.

Figures 7.10 and 7.11 show the simulation results for monotonic and cyclic tests conducted by Salim and Indraratna (2004) and Indraratna et al. (2012b) on ballast with different loading conditions. The monotonic tests were conducted on the specimens with 300 mm in diameter and 600 mm in height under the confining pressures equal to 100 kPa, 200 kPa, and 300 kPa. Detailed physical properties and test conditions of each sample can be found in Table 7.1. Figure 7.10 presents the comparisons between the test results and model simulations on stress
strain response of ballast under different confining pressures. It is observed that the proposed model can well capture the stress strain response of ballast, especially the stress dilation behaviour under relatively low confining pressure of 100 kPa. It can also characterize the strain hardening and volumetric contraction for ballast under relatively high pressure of 200 kPa and 300 kPa. The cyclic tests were performed at a constant confining pressure of 60 kPa with three different loading frequencies equal to 10 Hz, 20 Hz, and 40 Hz. A sinusoidal stress wave with minimum stress equal to 45 kPa and maximum stress equal to 230 kPa was applied during the cyclic loading period. A proper description of the cyclic densification is obtained by using $\alpha = 0.55$. As illustrated in Figure 7.11, with the increase of the number of loading cycles, the permanent deformation rapidly increases and then approaches stable, which can be well characterised by the fractional order plasticity model.

Figure 7.12 presents the model simulations of the cumulative axial and volumetric strains under a loading frequency of 10 Hz and two different confining pressures equal to 30 kPa and 60 kPa. The ballast used had a coefficient of uniformity equal to 1.5 and was compacted to have a $e_0$ around 0.72 (Sun et al., 2015). As can be observed, with the increase of the confining pressure, axial strain increased while volumetric strain decreased due to the higher extent of radial expansion by lower confinement. These can be well captured by the fractional order model with $\alpha = 0.53$. 
Figure 7.12. Model predictions of (a) axial strain and (b) volumetric strain of ballast (data sourced from Sun et al., 2015) under different confining pressures.

Figures 7.13 and 7.14 show the simulation results for monotonic and cyclic tests conducted by Anderson and Fair (2008). The monotonic tests were conducted on the cylindrical specimens with a nominal height of 455 mm and a nominal diameter of 236 mm under the confining pressures equal to 40 kPa and 90 kPa. Figure 7.13 shows the comparison between the monotonic test results and the corresponding model predictions where a good agreement can be observed. The cyclic tests were conducted on samples with two different confining pressures.
pressures and a fixed loading frequency of 0.5 Hz as indicated in Table 7.1. Figure 7.14 shows the simulation results of the corresponding experimental data. As can be expected, with the increase of the confining pressure, the cumulative axial strain decreases while the cumulative volumetric strain increases. The proposed model can accurately simulate such evolution trend by using $\alpha = 0.48$.

Figure 7.13. Model predictions of (a) axial strain vs deviator stress and (b) axial strain vs volumetric strain of ballast (data sourced from Anderson and Fair, 2008)
Figure 7.14. Model predictions of (a) axial strain and (b) volumetric strain of ballast under different confining pressures (data sourced from Anderson and Fair, 2008)

Figures 7.15 and 7.16 show the simulation results for monotonic and cyclic tests conducted by Aursudkij et al. (2009). The monotonic tests were conducted on the cylindrical specimens with a height of 450mm and a diameter of 300 mm under the confining pressures equal to 10 kPa, 30 kPa, and 60 kPa. Figure 7.15 presents the model predictions of the corresponding test results. It is observed that the proposed model can well capture the stress strain response of ballast, especially the stress dilation behaviour at low confining pressures. The cyclic tests
were performed at the confining pressures equal to 30 kPa and 60 kPa with the loading frequency equal to 4 Hz. Thousands cycles of sinusoidal stress wave with minimum stress equal to 15 kPa and maximum stress equal to 180 kPa was applied during the cyclic loading period. Figure 7.16 presents the simulation results of the corresponding experimental results. The decrease of the cumulative axial strain with increasing confining pressure can be well captured by using $\alpha = 0.6$.

Figure 7.15. Model predictions of (a) axial strain vs deviator stress and (b) axial strain vs volumetric strain of ballast (data sourced from Aursudkij et al., 2009)
Figure 7.16. Model predictions of (a) axial strain and (b) volumetric strain of ballast under different confining pressures (data sourced from Aursudkij et al., 2009)

Figures 7.17 and 7.18 show the simulation results for monotonic and cyclic tests conducted by Fu et al. (2014). The monotonic tests were conducted on the cylindrical specimens with a height of 700 mm and a diameter of 300 mm. Figure 7.17 presents the comparison between the monotonic test results and the corresponding model predictions where a good agreement can be observed. The cyclic tests were performed at two different load amplitudes equal to 1 MPa and 2 MPa with the constant loading frequency of 0.1 Hz. Figure 7.18 shows the
prediction results of the corresponding experimental data. The increase of the cumulative axial and volumetric strains with the increasing load amplitude can be well captured by using $\alpha = 0.4$.

Figure 7.17. Model predictions of (a) axial strain vs deviator stress and (b) axial strain vs volumetric strain (data sourced from Fu et al., 2014)
Figure 7.18. Model predictions of (a) axial strain and (b) volumetric strain under different loading amplitudes (data sourced from Fu et al., 2014)

7.6 Conclusions

Fractional calculus was found to be a powerful instrument in characterising the memory-intensive phenomena, such as the soil deformation and diffusion, etc. Previous studies have demonstrated the flexible ability of the fractional calculus in characterising the monotonic behaviour of different geomaterials. However, the use of the fractional calculus in modelling the cyclic behaviour of ballast has not yet been explored. In this chapter, the novel concept of
fractional calculus was employed to model the cumulative behaviour of ballast. Several main findings are summarised as below:

(1) Strain accumulation rate of ballast obeyed a power law in relation to the number of load cycles, which can be well described by using the concept of fractional calculus. By adjusting the value of the fractional derivative order, good predictions of the strain accumulation rate with increasing number of loading cycles could be obtained.

(2) Accordingly, a simplified cyclic constitutive model for predicting the cumulative deformation of ballast was developed by incorporating the fractional strain accumulation rate into a traditional plasticity model. The proposed model deduces to the traditional plasticity model when the fractional order approaches unit. The proposed model was calibrated and validated against a series of independent laboratory test results for railroad ballast. Unlike other constitutive models, the model can efficiently simulate the cumulative deformation of ballast from the load onset to a high number of load cycles.

(3) The fractional order was found to decrease with the increasing fractal dimension of a given material. With the decrease of the fractional order, the model was observed to exhibit an increasing rate for cyclic densification. The fractional order for ballast was found to be around 0.4 – 0.6.
CHAPTER 8 CONCLUSIONS AND RECOMMENDATIONS

8.1 Introduction

While faster and heavier trains have recently been introduced into many countries, the ballast layer has not been modified much to compensate for the increasing load. Laboratory assessment of the deformation and degradation behaviour of railroad ballast with different particle size distributions under long-term cyclic loading has been conducted. The following sections summarise the major findings obtained regarding the variations in shape of ballast aggregates, the effect of particle size distribution on permanent deformation, resilient deformation, and degradation, as well as the use of fractional order calculus to model the cumulative deformation of railroad ballast. Recommendations for further studies are also provided.

8.2 Shape Characterisation of Ballast Aggregates

(1) The elongation and flatness ratios of ballast particles changed as the particle diameters changed. With the increase of particle size, ballast becomes more rounded and less columnar.

(2) Particles become more spherical as their size increases, which indicates the representative lengths of $L$, $I$, and $S$ in the larger particles were similar. Moreover, a comparison between the true sphericity index ($\psi$) and the two alternative 2D sphericity indices ($\psi_1$ and $\psi_2$) indicated that the 2D sphericity index would underestimate the actual particle sphericity.
The roundness of railroad ballast varied slightly as the particles changed in size; for instance, there was a slight increase in angularity in larger ballast particles and ellipsoidness decreased slightly with increasing particle size; this means that coarser particles of ballast would be relatively more irregular.

A 3D laser scan system can reliably determine the size and shape of ballast. Based on the measured values of volume and mass, the density of ballast particles was 2.66 g/cm³.

8.3 Permanent Deformation Behaviour

Particle size distribution (PSD) significantly influenced the permanent deformation of railroad ballast. Larger aggregates experienced smaller axial and radial strains, while smaller particles underwent dilation within the first few hundred loading cycles, followed by compression as the number of load cycles increased. The transition from dilation to compression was influenced by the loading frequency.

The permanent axial and volumetric stains of ballast decreased initially and then increased as the coefficient of uniformity increased. Ballast with a larger coefficient of uniformity deformed rapidly during the first few hundred loading cycles and then stabilised.

The rates of permanent axial (shear) and volumetric strain accumulation under cyclic loading were scale invariant and obeyed a power law in relation to the number of load cycles.

Resistance to plastic deformation under cyclic loading can be improved significantly by increasing the relative density of ballast. Based on these test results, an optimum
PSD with $1.8 \leq C_u \leq 2.0$ for ballast was proposed to minimise deformation under high frequency cyclic loading.

### 8.4 Resilient Deformation Behaviour

1. The particle size distribution (PSD) of railroad ballast influences its resilient behaviour. For ballast with the same coefficient of uniformity, the resilient modulus decreased as the particle size increased, but it increased as the coefficient of uniformity of ballast with fixed minimum and maximum particle sizes decreased.

2. The combined effects of deviation between the maximum and minimum particle sizes and the coefficient of uniformity influenced the resilient modulus of granular aggregates. The highest resilient modulus was expected to be in samples with almost the same minimum and maximum particle sizes. Any decrease in the minimum particle size or increase in the maximum particle size would reduce the resilient modulus. A constant resilient modulus was expected for granular aggregates with any type of uniform PSD, given the same testing conditions.

3. Apart from the PSD, particle breakage also influenced the resiliency of granular aggregates, with more serious breakage occurring in samples tested under a higher loading frequency where a low resilient modulus was observed. The extent of particle breakage under a higher loading frequency increased, which shifted the initial PSD towards a broader distribution; this may be regarded as one possible reason for the overall decrease in the resilient modulus with an increasing loading frequency.
8.5 Degradation Behaviour

(1) The degradation of ballast was influenced by its PSD and density such that the extent of breakage decreased when the sample density increased. All the breakage indices were able to capture the breakage of ballast with a fixed initial PSD, but the extent of breakage as defined by the breakage potential and grading entropy could not evaluate the breakage of ballast with multi-initial PSDs.

(2) An increase in the extent of breakage with a decreasing particle size was reported by using the breakage extent defined by the breakage potential, which indicated that smaller particles of ballast broke more. However, an increase in Marsal’s breakage ratio and the grading entropy occurred as the particle size increased, implying a higher extent of breakage in larger ballast.

(3) Breakage indices defined by breakage potential and grading entropy failed to characterise the extent of ballast breakage with different initial coefficients of uniformity. However, Marsal’s breakage ratio generally decreased as the coefficient of uniformity for ballast with similar initial relative densities increased. The degradation of ballast was thus divided into two zones depending on the range of the coefficient of uniformity, i.e., a high breakage zone (zone A) and a reduced breakage zone (zone B).

(4) Ballast samples tested under similar initial void ratios experienced an initial decrease followed by a slight increase of Marsal’s breakage ratio. This was attributed to the initially unstable packing which induced a high concentration of stress concentration and a greater extent of breakage among aggregates with an increasing coefficient of uniformity.

(5) The extent of breakage increased with an increasing load frequency. Higher increasing extents of the elongation ratio, flatness ratio, aspect ratio, and ellipsoidness were observed in samples tested under a higher load frequency. Moreover, particle breakage
increasingly rounded the parent ballast aggregates, whereas smaller aggregates created by particle breakage seemed to have a lower ellipsoidness and elongation ratio.

(6) The original Australian standard (AS 1141.15) PSD had a low irregularity ($E_e \approx 0.372$) and coefficient of uniformity ($C_u \approx 1.5$), which fell within the high breakage zone (zone A) that resulted in severe ballast breakage and deformation under high frequency cyclic loading. However, based on the above test results, a broader PSD was suggested to minimise deformation and degradation of ballast under long-term cyclic loading. Unlike the Australian standard (AS 1141.15), the suggested PSD had a moderate ellipsoidness ($E_e \approx 0.376$) and a better coefficient of uniformity ($C_u \approx 2.0$), which belonged to the reduced breakage zone (Zone B) that provided denser packing, better interlocking, and hence less settlement and reduced degradation.

**8.6 Fractional Order Constitutive Model**

(1) The strain accumulation rate of ballast obeys a power law in relation to the number of load cycles, and this can be described very well using fractional calculus. By adjusting the value of the fractional derivative order, good predictions of the strain accumulation rate with increasing number of loading cycles could be obtained.

(2) Accordingly, a simplified cyclic constitutive model for predicting the cumulative deformation of ballast was proposed by incorporating the fractional strain accumulation rate into a traditional plasticity model. The proposed model deduces to the traditional plasticity model when the fractional order approaches unit. The proposed model was calibrated and validated against a series of independent laboratory test results for railroad ballast. Unlike other constitutive models, the model
can efficiently simulate the cumulative deformation of ballast from the load onset to a high number of load cycles.

(3) The fractional order was found to decrease with the increasing fractal dimension of a given material. With the decrease of the fractional order, the model was observed to exhibit an increasing rate for cyclic densification. The fractional order for ballast was found to be around 0.4 – 0.6.

8.7 Recommendations for Further Study

The research in this study mainly investigated the effect of particle size distribution of ballast under cyclic loading without any rest period. However, field ballast usually experiences traffic loads that have rest periods and rotation of the principal stress axes. Therefore, for a comprehensive understanding of how ballast behaves under real track loads and different train speeds, the following recommendations are suggested:

(1) The experiments reported herein were limited to only one type of loading amplitude and confining pressure under triaxial loading condition. Further tests should be carried out using different loading amplitudes and confining pressures with rest periods or rotation of the principal stress axes to ensure the experimental results are more representative of the in-service loading conditions.

(2) The loading frequencies should also be varied with an increasing number of load cycles to evaluate the effect of train acceleration on ballast deformation. Substructures layers underlying the ballast should be also simulated in the triaxial apparatus to examine how ballast performs with different particle size distributions over different types of subgrade.
(3) Monotonic triaxial tests on ballast with different initial particle size distributions can be carried out to study the mechanical characteristics of ballast such as variations of its critical state and dilation with varying particle size distributions.

(4) A new particle breakage index that does not rely on the measurement of breakage potentials may be proposed to capture the breakage of ballast with different particle size distributions, under varied subgrade conditions.

(5) The physical origin of the fractional order should be validated with an experimental study because the current model did not consider plastic deformation during unloading. Further study can be extended to consider plastic deformation during unloading. Finally, short-term stress strain loops should be addressed properly by using the fractional order plasticity model.
REFERENCES


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