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Evaluating the volatility forecasting performance of best fitting GARCH models in emerging Asian stock markets

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Abstract

While modeling the volatility of returns is essential for many areas of finance, it is well known that financial return series exhibit many non-normal characteristics that cannot be captured by the standard GARCH model with a normal error distribution. But which GARCH model and which error distribution to use is still open to question, especially where the model that best fits the in-sample data may not give the most effective out-of-sample volatility forecasting ability. Approach: In this study, six simulated studies in GARCH(p,q) with six different error distributions are carried out. In each case, we determine the best fitting GARCH model based on the AIC criterion and then evaluate its outof- sample volatility forecasting performance against that of other models. The analysis is then carried out using the daily closing price data from Thailand (SET), Malaysia (KLCI) and Singapore (STI) stock exchanges. Results : Our simulations show that although the best fitting model does not always provide the best future volatility estimates the differences are so insignificant that the estimates of the best fitting model can be used with confidence. The empirical application to stock markets also indicates that a non normal error distribution tends to improve the volatility forecast of returns. Conclusion : The volatility forecast estimates of the best fitted model can be reliably used for volatility forecasting. Moreover, the empirical studies demonstrate that a skewed error distribution outperforms other error distributions in terms of out-of-sample volatility forecasting.

Keywords

models, garch, volatility, markets, forecasting, performance, best, stock, asian, evaluating, emerging, fitting

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Evaluating the Volatility Forecasting Performance of Best Fitting GARCH Models in Emerging Asian Stock Markets

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ABSTRACT

Problem statement : *While modeling the volatility of returns is essential for many areas of finance, it is well known that financial return series exhibit many non-normal characteristics that cannot be captured by the standard GARCH model with a normal error distribution. But which GARCH model and which error distribution to use is still open to question, especially where the model that best fits the in-sample data may not give the most effective out-of-sample volatility forecasting ability.* ***Approach:*** *In this study, six simulated studies in GARCH(p,q) with six different error distributions are carried out. In each case, we determine the best fitting GARCH model based on the AIC criterion and then evaluate its out-of-sample volatility forecasting performance against that of other models. The analysis is then carried out using the daily closing price data from Thailand (SET), Malaysia (KLCI) and Singapore (STI) stock exchanges.* ***Results :*** *Our simulations show that although the best fitting model does not always provide the best future volatility estimates the differences are so insignificant that the estimates of the best fitting model can be used with confidence. The empirical application to stock markets also indicates that a non normal error distribution tends to improve the volatility forecast of returns.* ***Conclusion :*** *The volatility forecast estimates of the best fitted model can be reliably used for volatility forecasting. Moreover, the empirical studies demonstrate that a skewed error distribution outperforms other error distributions in terms of out-of-sample volatility forecasting.*

Keywords: GARCH-models,stock market indices and volatility forecasting.

2000 Mathematics Subject Classification: 91G80,91G70.

1 Introduction

The general properties of financial time series that are called stylized characteristics become very important in applied economic analysis (Liu and Hung, 2010). Cont (2001) examined stylized statistical properties of asset returns, common to a wide set of financial assets, such as heavy tails, leptokurtic distribution, volatility clustering, absence of autocorrelations and leverage effect. The Autoregressive Conditional Heteroskedasticity (ARCH) model with normal innovations first introduced by Engle (1982) captured some of stylized characteristics of financial assets. Later generalized ARCH model (GARCH) by Bollerslev (1986) further improved the modeling process. But, traditionally, stock returns were modeled by time series with normal errors. Unfortunately, such models still failed to sufficiently capture the main stylized characteristics of financial time series, i.e. the heavy tails, leptokurtic and skewness.

A number of papers have investigated the performance of GARCH models with non-normal error distribution in mature stock markets. Hansen (1994) considered a GARCH model with skewed-student-t distribution to capture the skewness and the excess kurtosis. Liu and Hung (2010), and Bali (2007) proposed GARCH models with skewed generalized error distribution (SGED). Gokcan (2000) compared the performance on volatility forecasting of GARCH(1,1) model versus EGARCH(1,1) model using the the monthly stock market returns of seven emerging countries. It found that the GARCH(1,1) model outperforms the EGARCH model, even if the stock market return series exhibit skewed distributions. Chuang et al. (2007) investigated the volatility forecasting performance of GARCH (1,1) model with various distributional assumptions on stock market indices and exchange markets. Their results show that a GARCH(1,1) model combined with the logistic distribution, the scaled student's distribution or the Riskmetrics model is preferable both in stock markets and foreign exchange markets. Curto and Pinto (2009) considered ARMA-GARCH(1,1) models driven by Normal, Student's t and stable Paretian distributional assumptions. They found that a ARMA-GARCH(1,1) model with stable Paretian error fits returns better than normal distribution and slightly better than the Student's t distribution.

Some researchers also applied GARCH models to daily closing price data from South East Asian emerging stock markets. Shamiri and Isa (2009) examined the relative efficiency of several different types of GARCH models in terms of their volatility forecasting performance. They compared the performance of symmetric GARCH, asymmetric EGARCH and non-linear asymmetric NAGARCH models with six error distributions (normal, skew normal, student-t, skew student-t, generalized error distribution and normal inverse Gaussian). Komain (2007) fitted stock market index in the stock exchange of Thailand (SET) by ARMA-GARCH(1,1) to examine the behaviour of stock prices. However, these previous investigations into the volatility forecasting performance of GARCH models in the emerging stock markets of South East Asia are reported mainly on the different types of GARCH model with order p=1 and q=1. Those investigation did not mention if higher order of model had been investigated or not. In this paper, we investigate whether it is more appropriate to use higher order GARCH model to fit some indices. Therefore, we are interested if the conclusion given by previous research is still holds for GARCH models without presetting their orders.

Several extensions of the traditional symmetric GARCH(p,q) model have been introduced to increase the flexibility of the original GARCH model such as asymmetric and non-linear asymmetric GARCH model which consist of various GARCH models, for example the exponential GARCH(EGARCH), GJR GARCH of Glosten Jagannathan and Runkle (1993), the quadratic GARCH of Sentana (1995) and the threshold GARCH (TGARCH) of Zakoian (1994). To sim-

plify the analysis, we restrict our study to GARCH(p,q) and compare the volatility forecasting performance of models with different error distributions, including Normal(N), Skewed Normal (SN), Student-t (STD), Skewed Student-t (SSTD), Generalized Error Distribution (GED) and Skewed Generalized Error Distribution (SGED). The main reason for choosing these six types of error distributions is to take into account the skewness, excess kurtosis and heavy-tails of return distributions. It is clear that Student-t and GED distributions exhibit heavy-tails. Moreover, Skewed Student-t and SGED distributions also allow various type of skewness and heavy-tails. The main objective is to investigate whether the best fitting model, in terms of the Akaike information criterion (AIC) also provides the best volatility forecasts of the underlying series in terms of the Mean Squared Error (MSE) criteria and the Mean Absolute Error (MAE).
Since emerging stock markets in South East Asia are of empirical interest to both of individual and institutional investors, three emerging stock markets in South East Asia, Thailand (SET), Kuala Lumpur Composite Index (KLCI) from Malaysia and Straits Time Index (STI) from Singapore are studied in this paper.
The paper is organized as follows. The next section describes the data used in this paper, methodology, the distributions of error in GARCH models and measurements used to evaluate forecast performance. The results of simulation to find the best fitted model and the best forecasting model are reported in Section 3. The studies on real data and the volatility forecasting performance of different models given by real data are presented in Section 4 . The last section gives our conclusions.

2 Data and Methodology

2.1 Data and GARCH models

The data employed in this study comprise 6,536 daily closing price on SET covering the period 4/01/1982 to 11/08/2008 ; 3,880 daily closing price on KLCI covering the period 3/12/1993 to 21/08/2009 and 5,407 daily closing price on STI covering the period 28/12/1987 to 21/08/2009. Each data set is divided into two subsets. The first subset is called in-sample data set used to build up a model for underlying data and the second subset is called out-sample data set used to investigate the performance of volatility forecasting.
The in-sample period for SET starts from 4/01/1982 to 14/05/1996 with 3,535 daily observations ; for KLCI starts from 3/12/1993 to 23/12/1999 with 1,500 daily observations and for STI starts from 28/12/1987 to 19/01/1998 with 2,500 daily observations. The out-sample period starts from 15/05/1996 to 11/08/2008 for SET; from 24/12/1999 to 21/08/2009 for KLCI and from 28/12/1987 to 21/08/2009 for STI.
Logarithm of daily return is defined as follows :

$$r_t = ln[p_t/p_{t-1}]$$

is considered in this study, where p_t denotes the closing price index at time t. The statistical package used in this study is R version 2.11.1. A GARCH (p,q) model for time series r_t is defined as follows :

$$\begin{aligned} r_t &= \mu + \varepsilon_t, \\ \varepsilon_t &= \eta_t \sqrt{h_t}, \\ h_t &= \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j h_{t-j}, \end{aligned}$$

where μ is constant parameter; η_t are i.i.d with $E(\eta_t) = 0$ and $Var(\eta_t) = 1$; η_t is independent of h_t; $\omega >$ 0, $\alpha_i \geq$ 0 and $\beta_j \geq$ 0 are non-negative constants with $\sum_{i=1}^{p} \alpha_i + \sum_{j=1}^{q} \beta_j < 1$ to ensure the positive of conditional variance and stationarity as well. If $q = 0$ the model reduces to an Autoregressive Conditional heteroscedasticity (ARCH) model.

GARCH(p,q) models with normal error distribution often fail to capture leptokurtic (high kurtosis and heavy-tailed) of underlying time series but various non-normal error distributions have been suggested. Hansen (1994) used the skewed t distribution to capture the skewness and the excess kurtosis. Lee and Pai (2010) applied the student-t and SGED distributions to investigate the volatility prediction of the GARCH model. In order to capture leptokurtic of r_t, non-normal distribution for ε_t is suggested. For the purpose of this study, six types of error distributions are considered. The standard process is followed in this study to identify the order of a GARCH(p,q) model and AIC criterion is used to determine the best fitted GARCH model.

2.2 The distribution of error ε_t

Six different types of error distributions are considered in this paper.

1. Normal Distribution

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{\frac{-z^2}{2}}, -\infty < z < \infty,$$

2. Skewed Normal Distribution [1]

$$f(z) = \frac{1}{\omega\pi} \quad e^{\frac{-(z-\xi)^2}{2\omega^2}} \int_{-\infty}^{\alpha\frac{z-\xi}{\omega}} e^{\frac{-t^2}{2}} \quad dt, -\infty < z < \infty,$$

where ξ denotes the location ; ω denotes the scale and α denotes the shape of density.

3. Student-t Distribution [2]

$$f(z) = \frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\nu\pi}\Gamma(\frac{\nu}{2})} (1 + \frac{z^2}{\nu})^{-(\frac{\nu+1}{2})}, -\infty < z < \infty,$$

where ν denotes the number of degrees of freedom and Γ denotes the Gamma function.

4. Skewed Student-t Distribution [3]

$$f(z; \mu, \sigma, \nu, \lambda) = \begin{cases} bc(1 + \frac{1}{\nu-2}(\frac{b(\frac{z-\mu}{\sigma})+a}{1-\lambda})^2)^{-\frac{\nu+1}{2}}, & \text{if} z < -\frac{a}{b}, \\ bc(1 + \frac{1}{\nu-2}(\frac{b(\frac{z-\mu}{\sigma})+a}{1+\lambda})^2)^{-\frac{\nu+1}{2}}, & \text{if} z \geq -\frac{a}{b}, \end{cases}$$

[1]See Shamiri and Isa (2009)

[2]See Shamiri and Isa (2009)

[3]See Bali (2007)

where ν is a shape parameter with $2 < \nu < \infty$ and λ is a skewness parameter with $-1 < \lambda < 1$. The constants a,b and c are given below

$$a = 4\lambda c(\frac{\nu-2}{\nu-1}), b = 1 + 3\lambda^2 - a^2, c = \frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\pi(\nu-2)\Gamma(\frac{\nu}{2})}}.$$

μ and σ^2 are the mean and variance of the skewed student-t distribution.

5. Generalized Error Distribution (GED) [4]

$$f(z;\mu,\sigma,\nu) = \frac{\sigma^{-1}\nu e^{(-0.5|(\frac{z-\mu}{\sigma})/\lambda|^{\nu})}}{\lambda 2^{(1+(1/\nu))}\Gamma(1/\nu)}, 1 < z < \infty,$$

$\nu > 0$ is the degrees of freedom or tail -thickness parameter and

$$\lambda = \sqrt{2^{(-2/\nu)}\Gamma(1/\nu)/\Gamma(3/\nu)}.$$

If $\nu = 2$, the GED yields the normal distribution. If $\nu < 1$, the density function has thicker tails than the normal density function, whereas for $\nu > 2$ it has thinner tails.

6. Skewed Generalized Error Distribution [5]

$$f(z;\nu,\xi) = \nu[2\theta\Gamma(1/\nu)]^{-1}\exp(-\frac{|z-\delta|^{\nu}}{[1-sign(z-\delta)\xi]^{\nu}\theta^{\nu}})$$

where

$$\begin{aligned}
\theta &= \Gamma(1/\nu)^{0.5}\Gamma(3/\nu)^{-0.5}S(\xi)^{-1},\\
\delta &= 2\xi A S(\xi)^{-1},\\
S(\xi) &= \sqrt{1+3\xi^2-4A^2\xi^2},\\
A &= \Gamma(2/\nu)\Gamma(1/\nu)^{-0.5}\Gamma(3/\nu)^{-0.5},
\end{aligned}$$

where $\nu >$ 0 is the shape parameter controlling the height and heavy-tail of the density function while ξ is a skewness parameter of the density with $-1 < \xi < 1$.

In simulation studies in Section 3, all parameters in the above distribution are the default values in R package, location, scale and skewness parameter are equal to 0, 1 and 1.5 respectively. Shape parameter is equal to 5 for student-t and skewed student-t distribution and equal to 2 for GED and skewed GED distribution.

2.3 Evaluation of volatility forecasts

While there are several different measurements for evaluating volatility forecasting performances, the mean absolute error (MAE) and the mean square error (MSE) (See Lui et al, 2009) are used in this study. When the true underlying volatility process is unobservable, we adopt Awartani and Corradi's (2005) suggestion to use $(r_t - \bar{r})^2$ as a proxy for latent volatility in this scenario. The MAE and MSE for n step ahead forecast are defined as follows :

$$MAE(n) = \frac{1}{N}\sum_{t=1}^{N}|(r_{t+n}-\bar{r})^2 - \hat{h}_t(n)|, \quad (2.1)$$

$$MSE(n) = \frac{1}{N}\sum_{t=1}^{N}[(r_{t+n}-\bar{r})^2 - \hat{h}_t(n)]^2, \quad (2.2)$$

[4] See Bali (2007)

[5] See Lui et al (2009)

where
r_{t+n}: the return over horizon n steps ahead at current time t ,
$\bar{r}$: the mean of return ,
$\hat{h_t}(n)$: the forecasted conditional variance over horizon n steps ahead at current time t.
For simplicity, we will drop n from MAE(n) and MSE(n).

3 Simulation studies

Shamiri and Isa (2009) showed that the best fitted model based on AIC criterion is not necessarily a model that is able to provide the best forecast of volatility in terms of MSE and MAE. Their conclusion is made based on the study on KLCI fitted by GARCH(1,1), EGARCH(1,1) and NAGARCH(1,1). For some financial data sets, a higher order GARCH is more appropriate than a GARCH(1,1). Thus, we are interested to see if Shamiri and Isa's statement is still valid when the underlying best fitted model is a higher order GARCH model.
We use the data simulated from the following two models to carry out our study. The two models are defined as follows :

$$r_t = \mu + \varepsilon_t$$

GARCH(1,3) model :

$$h_t = 0.00007 + 0.02354\varepsilon_{t-1}^2 + 0.05387h_{t-1} + 0.00127h_{t-2} + 0.18574h_{t-3}; \tag{3.1}$$

GARCH(2,1) model :

$$h_t = 0.00008 + 0.05334\varepsilon_{t-1}^2 + 0.06147\varepsilon_{t-2}^2 + 0.08599h_{t-1}. \tag{3.2}$$

Both of models are higher order of GARCH models. The coefficients in (3.3) and (3.4) were borrowed from the fitted models with normal error distribution for SET and STI in Section 2.1 respectively.

We simulated 6,536 and 5,407 observations from (3.3) and (3.4) respectively. Six types of error distributions, Normal, Skewed normal, Student-t, Skewed student-e, GED and Skewed GED are considered for ε_t in this study. Each data set are divided into two parts. The first part is for in-sample observations which is used to estimate the coefficients in the fitted model, (3,535 and 2,500 observations for GARCH(1,3) and GARCH(2,1) model respectively). The second part is served as out-sample observations used for investigating the volatility forecasting performance.
We fit each data set by the same order of the GARCH model where the data were simulated from, with six different error distributions respectively. Then compare the values of AIC given by each fitting and determine which model is the best fitted model for the underlying data set. We also evaluate the out-of-sample 1 step ahead forecasting on conditional volatility and compare the performance of each GARCH model with different error distributions measured by MSE and MAE. The results are shown in Tables 1 and 2.

From Tables 1 and 2, we can see that the true model is always the best fitted model in terms of the AIC criterion but the true model does not necessarily provide the minimum values of MSE and MAE and might not produce the best performance of forecasting volatility. For this particular sample, our simulation study shows that the statement "the best fitted model does not necessarily provide the best forecast on volatility" also holds for higher order of GARCH models.

Shamiri and Isa (2009) argue that there are several plausible models that we can select to use for our forecast and we should not be fooled into thinking that the one with the best fit is the one that will forecast the best. However, how much difference between the best forecast and the forecast given by the best fitted model? To investigate this question, for each of the six different distribution of ε_t we independently simulated 100 samples from (3.3). Each sample has size 6,536. The first 3,535 observations were considered as in-sample data and the remains were considered as out-sample data. For each set of simulated data, we fit the data by models with the six different error distributions respectively, and then calculate the value of MSE and MAE. We carry out pair t-test on the following hypothesis :

$$\begin{aligned} H_0 &: \mu_a - \mu_b = 0 \\ H_1 &: \mu_a - \mu_b > 0 \end{aligned}$$

where

μ_a denotes the mean of MSE (MAE) given by the best fitted model

μ_b denotes the mean of MSE (MAE) given by the best performance model

The reason to carry out this test is to check if the mean of MSE and MAE from the best fitted model are statistically significantly larger than the mean of MSE and MAE from the best performance model. If the null hypothesis cannot be rejected, it will mean that statistically the best performance model will not provide better volatility forecast than the best fitted model in terms of MSE(MAE) value. The P-values of the positive one tail paired t-test for 1-step and 10-step ahead forecast are shown in Table 3.

Table 3 shows that all the tests are not significant. It indicates that, although based on the outcomes in Tables 1 and 2, the best fitted GARCH model does not provide the best volatility forecast. Therefore, the best fitted model is still able to make the reasonable forecast on volatility.

4 Empirical studies

Our simulation studies in previous section indicated that sometimes the best fitted model might not necessarily provide the minimum values of MSE and MAE and might not necessarily produce the best performance of forecasting volatility. However, the best fitted model is still able to provide a reasonable forecast on volatility. In this section, we want to further find out whether the best fitted is appropriate to be used for forecast on volatility based on the real data.

Table 1: AIC, MSE and MAE for Simulated Data from GARCH(1,3)

	The distribution used in the fitted model					
	Normal	Skewed Normal	Student-t	Skewed Student-t	GED	Skewed GED
Normal						
AIC	**-6.39728**	-6.39673	-6.38562	-6.38507	-6.39678	-6.39622
MSE	33.27150	33.19940	39.41210	39.40620	32.80390	**32.73420**
MAE	5.76153	5.75527	6.27168	6.27122	5.72081	**5.71471**
Skewed Normal						
AIC	-6.46464	**-6.52380**	-6.45955	-6.51125	-6.46409	-6.52332
MSE	1.69244	1.96543	2.58870	**0.74882**	1.69292	1.92887
MAE	1.24786	1.36565	1.56406	**0.80091**	1.24806	1.35239
Student-t						
AIC	-6.35970	-6.36029	**-6.47920**	-6.47864	-6.46384	-6.46331
MSE	**0.06376**	0.07311	0.07454	4.53249	6.78319	6.73794
MAE	**0.13589**	0.21205	0.21138	2.11192	2.59203	2.58324
Skewed Student-t						
AIC	-6.37249	-6.49380	-6.53352	**-6.60656**	-6.57894	-6.57441
MSE	0.18411	22.03428	1.32608	2.91688	**0.17645**	0.17655
MAE	0.28982	4.68293	1.09884	1.67549	**0.28451**	0.28541
GED						
AIC	-6.38848	-6.38847	-6.37966	-6.37958	**-6.38870**	-6.38831
MSE	50.33190	50.40620	**6.83721**	47.55250	50.10220	50.18850
MAE	7.08312	7.08836	**2.58594**	6.88431	7.06691	7.07302
Skewed GED						
AIC	-6.44193	-6.50376	-6.43582	-6.49304	-6.44138	**-6.50432**
MSE	**7.06841**	12.10115	7.69763	49.95989	7.13673	12.16423
MAE	**2.63916**	3.46648	2.75687	7.06309	2.65211	3.47556

Notes: MSE $(\times 10^{-10})$ and MAE $(\times 10^{-5})$. Bold value in each row is the minimum value

4.1 Descriptive statistics

The summary statistics of return series r_t for SET, KLCI and STI are presented in Table 4. It shows that the mean of the returns for SET is slightly larger than the means of the returns for KLCI and STI markets. Both of the return series for SET and KLCI display negative skewness while the STI shows the positive skewness. All return series are leptokurtic and normality test for all return series are firmly rejected by the Jarque-Bera statistics. All return series have non-normal distributions.

Figure 1 shows the time series plots of the daily returns for SET, KLCI and STI respectively. Volatility clustering phenomenon are clearly observed from the plots. It indicates that GARCH models may be appropriate models for explaining these data.

Table 2: AIC, MSE and MAE for Simulated Data from GARCH(2,1)

	The distribution used in the fitted model					
	Normal	Skewed Normal	Student-t	Skewed Student-t	GED	Skewed GED
Normal						
AIC	**-6.34650**	-6.34588	-6.33400	-6.33349	-6.34572	-6.34509
MSE	3.31193	3.30401	5.85204	5.84641	3.31212	**3.30389**
MAE	1.76076	1.75851	2.37478	2.37358	1.76081	**1.75847**
Skewed Normal						
AIC	-6.34699	**-6.42474**	-6.34473	-6.41400	-6.34648	-6.42411
MSE	17.09890	14.46090	19.47580	17.37420	17.42360	**14.22260**
MAE	7.03734	3.69947	4.32193	4.07312	4.07724	**3.66710**
Student-t						
AIC	-6.32700	-6.32730	**-6.42443**	-6.42372	-6.41384	-6.41317
MSE	**1.10375**	1.20011	3.20177	3.24368	1.57030	1.59460
MAE	0.93421	0.99052	1.70281	**0.17149**	1.16479	1.17546
Skewed Student-t						
AIC	-6.33295	-6.45439	-6.50478	**-6.58470**	-6.56589	-6.55410
MSE	4.34869	1.50253	3.37669	3.48624	**1.50023**	1.51042
MAE	2.05233	1.16904	1.79784	1.82853	**1.69584**	1.67588
GED						
AIC	-6.33936	-6.33926	-6.33103	-6.33097	**-6.33913**	-6.33784
MSE	3.75521	**3.75199**	5.76655	5.77420	3.75648	72.33824
MAE	1.84973	**1.84889**	2.31291	2.31451	1.85006	8.47768
Skewed GED						
AIC	-6.38937	-6.44594	-6.38137	-6.43504	-6.38863	**-6.44673**
MSE	28.74100	23.95710	**3.35241**	21.70430	28.52270	23.92570
MAE	5.30064	4.80787	**1.70891**	4.56184	5.28007	4.80455

Notes: MSE $(\times 10^{-10})$ and MAE $(\times 10^{-5})$, Bold value in each row is the minimum value

4.2 In-sample parameter estimation and model diagnostics

R package "Garch" is used to find the best fitted GARCH model among the models with different error distributions for in sample data and the estimation of parameters in the model. According to the results of sensitivity analysis, we found that changing error distributions does not change the order of the best fitted GARCH model (the reports are missing from this paper). Therefore, to save time in analysis processes, the orders of the best fitted model with different error distributions are chosen as the best fitted model with normal distribution. It was found that GARCH (1,3), GARCH(1,1) and GARCH(2,1) are appropriate for SET, KLCI and STI respectively. Then we apply GARCH(1,3), GARCH(1,1) and GARCH(2,1) to SET, KLCI and STI respectively, by assigning the six different distributions to the error in the models. The AIC

Table 3: The P-values of Paired Test result between the best fitted model and the best performance model given by samples from GARCH(1,3)

	1-step		10-step	
The best fitted model	MSE	MAE	MSE	MAE
N	0.3254	0.2517	0.3452	0.3543
SN	0.5871	0.9945	0.9845	0.9541
STD	0.8124	0.2641	0.9802	0.9678
SSTD	0.1934	0.3454	0.3546	0.2276
GED	0.0842	0.1104	0.1489	0.2978
SGED	0.3891	0.4512	0.3845	0.3312

Table 4: Summary Statistics for returns

	Sample	Mean $(\times 10^{-3})$	Standard Deviation	Skewness	Excess Kurtosis	Jarque-Bera
SET	6,536	0.289	0.01562	-0.06808	7.9577	17220^{***}
KLCI	3,880	-0.030	0.01554	-0.43354	40.7329	267574^{***}
STI	5,407	-0.200	0.01331	0.11628	8.3077	15526^{***}

values given by different models with different error distributions are reported in Table 5. Based on AIC criterion, GARCH(1,3) with error distribution SSTD is the best fitted GARCH model for SET, GARCH(1,1) with GED for KLCI and GARCH(2,1) with STD for STI.

Table 5: The AIC values given by models with different error distributions

AIC	Normal	Skewd normal	Student-t	Skewed Student-t	GED	Skewed GED
SET	-6.4703	-6.4749	-6.5421	**-6.5432**	-6.5376	-6.5409
KLCI	-6.9142	-6.9137	-6.9583	-6.9575	**-6.9608**	-6.9596
STI	-6.0739	-6.0735	**-6.1104**	-6.1100	-6.1061	-6.1058

Tables 6-8 show the estimates of parameters in each of the best fitted GARCH models, as well as the test statistics given by Lagrange Multiplier Test (LM test) and Ljung-Box Q statistic.

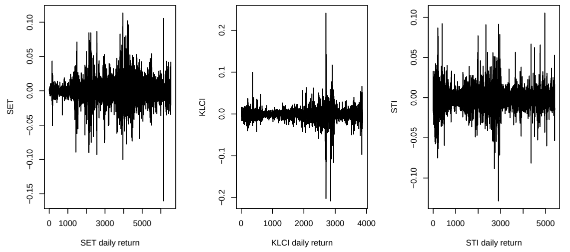

Figure 1: The daily return of SET, KLCI and STI

Table 6: Estimated parameters and diagnostic of GARCH(1,3)-SSTD model for SET

SET GARCH(1,3)	Skewed Student-t	P-value
μ	1.193$*10^{-4}$	0.2798
ω	7.128$*10^{-7}$	0.0014**
α_1	2.728$*10^{-1}$	1.11$*10^{-15***}$
β_1	5.839$*10^{-1}$	0.0007***
β_2	1.000$*10^{-3}$	0.0752*
β_3	1.897$*10^{-1}$	0.00812**
λ	9.469$*10^{-1}$	$< 2*10^{-16***}$
ν	5.0330	$< 2*10^{-16***}$
LM Test	14.6032	0.2638
Q(15)	17.1584	0.3094

Table 7: Estimated parameters and diagnostic of GARCH(1,1)-GED model for KLCI

KLCI GARCH(1,1)	GED	P-value
μ	-5.005$*10^{-4}$	0.0004***
ω	8.607$*10^{-7}$	0.0259*
α_1	1.136$*10^{-1}$	1.11$*10^{-7***}$
β_1	8.809$*10^{-1}$	$< 2*10^{-16***}$
ν	1.3110	$< 2*10^{-16***}$
LM Test	15.2401	0.2286
Q(15)	15.3440	0.4269

All parameters in GARCH(1,3), GARCH(1,1) and GARCH (2,1) for SET, KLCI and STI respectively are significant at 5% level. For each index data set, LM test supports the absence of ARCH effect in the residuals and the value of Q statistic is not significant. It shows that all the best fitted GARCH models are sufficient to correct the serial correlation of the returns series in the conditional variance equation (Liu and Hung 2010).

Table 8: Estimated parameters and diagnostic of GARCH(2,1)-STD model for STI

STI GARCH(2,1)	Student-t	P-value
μ	-5.850$*10^{-4}$	0.0015**
ω	1.962$*10^{-6}$	0.0017**
α_1	5.339$*10^{-2}$	0.0464*
α_2	6.893$*10^{-2}$	0.0262*
β_1	8.733$*10^{-1}$	$< 2*10^{-16***}$
ν	7.6690	2.27$*10^{-12***}$
LM Test	8.4077	0.7525
Q(15)	9.4368	0.8535

4.3 The performance of volatility forecasting

In this section, we adopt the best fitted models for SET, KLCI and STI in Section 4.2. By taking the same order of model and replacing the error distribution by other distributions mentioned in Section 2.2, we obtain six different fitted models for each index. Then we use each of these models to make out-sample forecasts. The performances of 1,2,10 and 15 steps ahead forecasts are evaluated and reported in Tables 9 and 10.

Table 9: Out-of-sample volatility forecasting evaluated by MSE

Step-Ahead	Normal	Skewed Normal	Student-t	Skewed Student-t	GED	Skewed GED
SET:GARCH(1,3)						
1	2.561	**2.551**	2.560	2.553	2.557	2.543
2	2.534	**2.524**	2.546	2.539	2.539	2.525
10	2.995	**2.992**	3.139	3.011	3.033	3.058
15	2.985	**2.982**	3.207	3.004	3.028	3.084
KLCI:GARCH(1,1)						
1	6.847	7.105	4.390	4.096	3.471	**3.468**
2	1.050	1.051	1.032	1.032	1.035	**1.027**
10	3.614	3.771	2.126	1.942	1.561	**1.544**
15	9.586	5.001	5.639	5.153	4.122	**4.096**
STI:GARCH(2,1)						
1	1.929	**1.922**	1.926	1.934	1.933	1.929
2	1.927	**1.918**	1.933	1.929	1.928	1.924
10	2.694	**2.688**	2.710	2.708	2.703	2.701
15	2.629	**2.624**	2.644	2.643	2.637	2.636

Notes: The reported value is multiplied by $(\times 10^{-7})$. The minimum value of MSE in the same raw is in bold.

Table 10: Out-of-sample volatility forecasting evaluated by MAE

Step-Ahead	Normal	Skewed Normal	Student-t	Skewed Student-t	GED	Skewed GED
SET:GARCH(1,3)						
1	2.244	**2.241**	2.325	2.453	2.281	2.287
2	2.312	**2.311**	2.419	2.334	2.361	2.372
10	2.614	**2.601**	2.857	2.622	2.688	2.745
15	2.771	**2.763**	3.018	2.792	2.831	2.903
KLCI:GARCH(1,1)						
1	2.528	2.578	1.995	1.921	1.764	**1.747**
2	2.440	2.448	2.311	2.323	2.325	**2.305**
10	1.898	1.939	1.454	1.390	1.252	**1.238**
15	3.093	3.160	2.371	2.266	2.044	**2.020**
STI:GARCH(2,1)						
1	2.207	**2.203**	2.218	2.217	2.214	2.213
2	2.185	**2.182**	2.199	2.198	2.195	2.193
10	2.538	**2.535**	2.552	2.549	2.545	2.543
15	2.509	**2.507**	2.521	2.518	2.515	2.513

Notes: The reported value is multiplied by $(\times 10^{-7})$. The minimum value of MAE in the same raw is in bold.

Table 11: The percent error of MSE and MAE given by the best fitted model and the best performance model

	The percent error of MSE error					
	SET		KLCI		STI	
Step-Ahead	Difference	PE(%)	Difference	PE(%)	Difference	PE(%)
1	0.002	0.078	0.003	0.086	0.004	0.207
2	0.015	0.591	0.008	0.773	0.015	0.776
10	0.019	0.631	0.017	1.089	0.022	0.812
15	0.022	0.732	0.026	0.631	0.020	0.756
	The percent error of MAE error					
	SET		KLCI		STI	
Step-Ahead	Difference	PE(%)	Difference	PE(%)	Difference	PE(%)
1	0.012	0.489	0.017	0.964	0.015	0.676
2	0.023	0.985	0.002	0.860	0.017	0.773
10	0.021	0.801	0.0014	1.118	0.017	0.666
15	0.029	1.039	0.024	1.174	0.014	0.555

Results in Tables 9 and 10 show that the best fitted models in each stock market does not provide the best volatility forecasts in terms of the values of MSE and MAE. To investigate the different of the values of MSE(MAE) given by the best fitted model and the best performance model, we evaluate the Percent Error (PE) of MSE(MAE) for each underlying cases where PE is defined as follows :

$$PE = \frac{A - B}{A} \times 100\%,$$

where

A denotes MSE (MAE) given by the best fitted model

B denotes MSE (MAE) given by best performance model

The PE values are reported in Table 11. It shows that the majority of PE values are small and less than 1.2%. It indicates that MSE(MAE) given by the best fitted model is not statistically different from that given by the best performance model. In practical situations, we still can use the best fitted model for volatility forecasting.

5 Conclusion

This paper investigates the volatility forecasting capability of GARCH(p,q) models with six different type of error distributions and apply them to three South East Asian emerging stock markets. Our results show that a GARCH(p,q) model with non-normal error distributions tends to provide better out-of-sample forecast performance than a GARCH(p,q) model with normal error distribution.

Simulation and empirical studies show that MSE(MAE) given by the best fitted model is insignificantly different from that given by the best forecast performance model. Since it is not practicable to identify the best performance model in practice, this study clearly demonstrates that it is reliable to use the best fitted model for volatility forecasting.

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