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A NEW METHOD OF FILTRATION DESIGN ON RAIL TRACK SUBBALLAST

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ABSTRACT

Apart from being a protection layer against subgrade attrition, the subballast of ballasted rail tracks is designed mainly to act as a stress dissipation layer or more commonly known as a capping layer. When considered as a filtration layer, subballast would prove to be inadequate due to the use of design criteria that are primarily based on steady seepage loading common in embankment dams. The seepage hydraulics through porous media would have to be influenced by the cyclic mechanical loading generated by the passing trains. Under the influence of cyclic train loading, subballast particles rearrange and attempt to attain a more stable configuration through the process of vertical settlement, lateral spreading, and particle degradation. The deformability of the pore medium itself would then affect the filter condition due to the changes in porosity and its subsequent impact on permeability. This paper presents a new design method that considers the effectiveness of the subballast as granular filter being dependent on the reduction of its porosity and permeability over time. The main factors that are found to contribute to the reduction in porosity within the subballast layer are (a) the plastic deformation generated by the cyclic load from the passing traffic, and (b) the accumulation of migrating fines trapped within the filter voids. Laboratory test results conducted on a novel cyclic loading permeameter apparatus were used to validate the proposed method. Two worked out examples are provided.

1 INTRODUCTION

The subgrade beneath the rail tracks is subjected to a series of cyclic loading transferred from the passing traffic. This cyclic loading enhances attrition by the overlying coarse layer (or subballast) on a previously dry subgrade creating fines at the interface (Selig and Waters, 1994). In addition, the fines resulting from degradation of upper ballast layer migrate into the subballast layer under heavy precipitation and cyclic train loads. The intrusion of fines changes the particle size distribution (PSD) of the subballast soils and affects its drainage characteristics.

In low lying coastal areas where the subgrade is generally saturated, water creates a slurry at the subballast-subgrade interface. Pore water pressure generated from cyclic rail loading applies a vertical seepage force, which reduces the effective stress of the subgrade and causes the slurry to be pumped into the overlying layer (Alobaidi and Hoare, 1999). This clay pumping phenomenon thereby clogs the ballast bed and promotes undrained shear failure (Indraratna *et al.*, 1992).

When granular filters are also expected to serve as drainage layers, as in the case of subballasts, changes in the hydraulic conductivity of the filters become important. As a result of the progressive entrapment of fine particles, subballasts undergo significant reductions in hydraulic conductivity. More fines accumulate within the subballast over time and slowly clog up the voids and reduce the drainage capacity of the layer. In poorly drained situations an increase in excess pore pressure adversely affects the strength and stiffness of the subgrade and track profile deformation behaviour, issues that may incur substantial maintenance costs.

Subballast clogging and its related problems are generally ignored in conventional rail track design. A number of empirical and analytical models of the filtration phenomenon in granular materials have been developed for embankment dams (e.g., Indraratna and Vafai, 1997, Locke *et al.*, 2001, Indraratna and Raut, 2006), but the loading system in a rail track environment is cyclic unlike the steady seepage force that usually occurs in them. There is a need to assess the impact of cyclic loading in order to improve our understanding of the mechanisms of filtration, interface behaviour, and time dependent changes to the filtration that occurs within subballast as a filter medium. These advances may potentially improve rail performance and safety, extend system life cycles and reduce maintenance costs.

2 THEORETICAL BACKGROUND OF THE PROPOSED METHOD

Cyclic loading condition in a wet environment presents a unique challenge to filtration mechanisms of the subballast layer. The deterioration of the hydraulic conductivity of the filter with time is related to the longevity of the drainage capacity of the filter. The reduction of porosity and the amount of fines accumulated through the profile of the filter, two of the key physical parameters that cause significant impact to hydraulic conductivity of a filter, are examined mathematically. The reduction of porosity has been observed to be a function of both the compressive effect of the cyclic loading and the amount of fines trapped within the filter voids. Moreover, the accumulated amount of foreign fine particles that are effectively captured within the filter voids alters the PSD of the original filter matrix. This new PSD of the combined filter-base soil matrix would result into a new effective diameter used in typical hydraulic conductivity prediction formulas.

Three steps of mathematical description and the physical basis of the filter mechanism under cyclic loading regime are discussed. The first step is to investigate the one dimensional cyclic compression behaviour of the subballast and its effect on the reduction of its controlling constriction size relative to the base soil representative diameter (Indraratna *et al.*, 2007). The coupling effect of the compression behaviour, which is developed in the framework of post shakedown plastic analysis, is then investigated with respect to base soil particle migration mechanism through the network of filter voids. Finally, a time based porosity reduction function is proposed and the Kozeny (1927)-Carman(1938) formula is extended to provide a practical tool in predicting the longevity of the drainage layer. Laboratory investigation results presented by Trani and Indraratna (2010) are used to validate the theoretical findings.

2.1 TIME-BASED ONE-DIMENSIONAL GRANULAR FILTER COMPRESSION

The evolution of permanent granular filter deformation was studied over a large number of load cycles (N). When the amplitude of the cyclic loading was above the shakedown level, the internal material structure was altered during loading which caused the shakedown level to evolve (Melan, 1936). It is proposed that in this study a stress domain of Drucker-Prager potential applied in a viscoplastic model (Perzyna, 1966) in the form of a post shakedown cyclic densification regime would be used to describe the progressive plastic deformation of granular material under cyclic loading.

Suiker and de Borst (2003) proposed a detailed mathematical development of a cyclic densification model based on triaxial experiments that showed the plastic deformation of a ballast and subballast material subjected to cyclic loading. This model described two mechanisms which are essential parts of the granular material densification process, frictional sliding and volumetric compaction. Due to the existing one dimensional compression constraints of the present study, the function for irreversible plastic strain (ε_p) is set to correspond to the frictional shakedown evolution framework and is proposed as:

$$\varepsilon_p = \varepsilon_f \left(1 - e^{-t f / k_s}\right) \qquad \textbf{(1)}$$

where, ε_f = the shakedown plastic strain obtained from one dimensional cyclic consolidation test on a fully saturated specimen, t = time (sec), f = frequency (Hz), and k_s = scaling factor equal to $N_{max}/10$, where N_{max} is the maximum number of cycles used in the model. Figure 1(a) shows a comparison between the proposed plastic strain evolution model over a number of cycles and their corresponding experimental data (Trani, 2010).

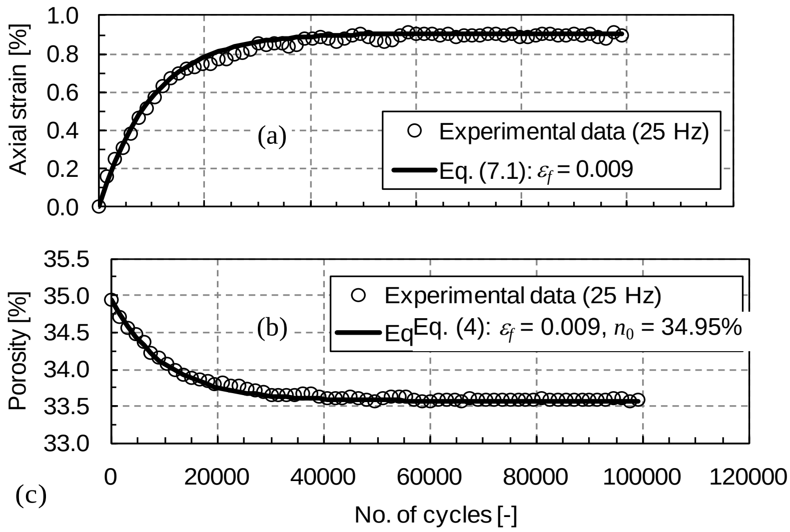

Figure 1: Comparison between the experimental data and (a) proposed plastic strain model and (b) proposed porosity predictive model at 25 Hz.

From the one dimensional compression principle, the plastic axial strain is given as:

$$\varepsilon_p = \frac{\Delta e}{1+e_0} \tag{2}$$

where Δe is the change in voids ratio and e_0 is the initial voids ratio of the filter matrix.

Using the voids ratio-porosity relationship ($n = e/(1+e)$), the change in porosity of a porous medium caused by axial compression during a single time step (Δn_c) is represented by:

$$\Delta n_c = \frac{\varepsilon_p}{1-n_0+\varepsilon_p} \tag{3}$$

where n_0 is the initial filter porosity. Substituting Eq. (1) into Eq. (3) results into:

$$\Delta n_c = \frac{\varepsilon_f\left(1-e^{-tf/k_s}\right)}{1-n_0+\varepsilon_f\left(1-e^{-tf/k_s}\right)} \tag{4}$$

The proposed function takes into account the reduction of porosity with time that depends on the densification energy (natural or imposed stress state) through the parameter ε_f. The prediction of actual material porosity is comparable with the experimental observations (Trani, 2010) as shown in Figure 1(b).

2.2 A MECHANISM OF ACCUMULATION OF FINES WITHIN FILTER VOIDS

A mechanism is proposed wherein fines are accumulated within the voids of adjacent filter particles based on the assumption that the dominant constriction size (D_{c35}) of the filter is smaller than or equal to the representative diameter (d_{85sa}) of the base soil (Raut and Indraratna, 2008). Depending on the manner by which the filter is prepared, the initial size of D_{c35} can be controlled by the level of compaction through the relative density (R_d) of the filter. Figure 2 shows a progressive reduction of the constriction size profile of an effective filter with time in comparison with the d_{85} and d_{50} of the base soil. Note that in this filtration test the slurry is introduced at the bottom of the filter with an upward flow (Trani 2010). Despite the initial filter constriction size exceeding the minimum requirement of d_{85sa}, the densification energy generated from cyclic loading led to granular filter permanent deformation over time. The eventual irrecoverable plastic strain affected the geometry of the constrictions in a way that the apparent D_{c35} was as close to the size where it satisfied the filtration criterion. The formation of the self-filtration layer at the bottom is also shown by the drastic reduction of constrictions within a few load cycles (represented by time axis). The gradual reduction of the constriction size profile over time indicates stability of the accumulated fines within the filter voids. This stability of the new base soil-filter formation consequently created finer constriction sizes much smaller than the d_{85sa} of the base soil.

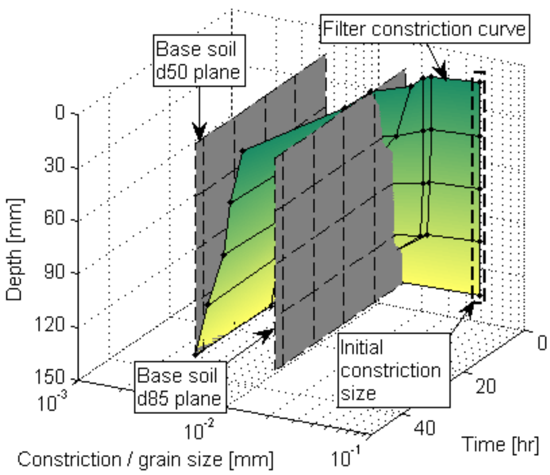

Figure 2: Reduction in filter constriction size due to accumulation of base particles.

The base soil particles with size d_{50}, which was still marginally smaller than the estimated filter constriction size, have the capacity to migrate upwards to the next filter layer. However, these particles only represent 50% of the total original soil mass. Compounded by the gradual formation of finer constrictions within the lower filter layers further limited the mass of the base particles from being transported into the next upper layer. This successive reduction of accumulation of fines along the profile of the filter is controlled by a dimensionless depth dependent accumulation factor (F_a) which can be described by a rate law relationship:

$$F_a = F_1 e^{F_2 z} \tag{5}$$

In the above equation, F_1 and F_2 are empirical indices related to slurry concentration and slurry loading rate, respectively. This proposed function creates an apparent threshold amount of fines that could occupy the voids in between the filter grains at a given depth. The apparent maximum mass of subgrade fines that corresponds to this threshold could occupy a part of the volume of voids that remained after compaction. The equivalent volume taken up by the trapped and accumulated fines is much less than the theoretical volume of voids (V_v) and it is given as:

$$m_{a\max} = F_a \frac{V_0 \rho_a}{1+\varepsilon_f} \tag{6}$$

where V_0 is the bulk volume of soil specimen [m^3] and ρ_a = solid density of accumulated fines [kg/m^3]. By substituting Eq. (5) into Eq. (6), the maximum amount of fines that can be accumulated with respect to the thickness profile of a filter could be predicted by:

$$m_{a\max} = F_1 e^{F_2 z} \frac{V_0 \rho_a}{1+\varepsilon_f} \tag{7}$$

2.3 POROSITY REDUCTION DUE TO ACCUMULATED FINES

In an ideal coarse packing, the filter matrix is assumed to be supported by the skeleton created by the contacts among filter grains. The porosity of the filter matrix is traditionally defined as the ratio of the volume of voids and the bulk volume of soil specimen ($n = V_v/V_0$). The plastic deformation due to compression ($V_0/(1+\varepsilon_p)$) impacts the filtering capacity of the filter by effectively reducing the size of its constrictions.

Due to the cyclic loading action of the passing train, base soil particles are pumped upwards into the subballast filter from the fully saturated subgrade. Fines are trapped by the filter constrictions and are deposited within the pore network. With the simultaneous action of one dimensional compression and pumping of subgrade fines, the volume of voids is reduced by the volume of accumulated fines (V_a) trapped in the original voids while the bulk volume of the filter reduces with time:

$$\Delta n_a = m_a \frac{1+\varepsilon_p}{V_0 \rho_a} \tag{8}$$

where, m_a = mass of accumulated fines within the filter voids [kg]. The amount of fines trapped by the constrictions of an effective filter is proposed to follow the given relationship:

$$m_a = m_{a\max}\left(1-e^{-t f/k_s}\right) \tag{9}$$

Subsequent back substitution of Eq. (9) into Eq. (8) yields:

$$\Delta n_a = \frac{m_{a\max}}{V_0 \rho_a}\left(1+\varepsilon_p\right)\left(1-e^{-t f/k_s}\right) \tag{10}$$

Combining Eqs. (7) and (10) simplifies into:

$$\Delta n_a = F_1 e^{F_2 z} \frac{1+\varepsilon_p}{1+\varepsilon_f}\left(1-e^{-t f/k_s}\right) \tag{11}$$

By using Eq. (1) and the more compact Eq. (5) into Eq. (11), a time dependent porosity reduction function due to accumulated base soil fines is derived Eq. (12). The sum of Eqs. (4) and (12) is the time dependent total porosity reduction of the filter matrix as a collective effect of one dimensional compression and the accumulation of fines within the filter voids (Eq. (13)).

$$\Delta n_a = F_a \frac{1+\varepsilon_f\left(1-e^{-t f/k_s}\right)}{1+\varepsilon_f}\left(1-e^{-t f/k_s}\right) \tag{12}$$

$$\Delta n_T = \frac{\varepsilon_f\left(1-e^{-t f/k_s}\right)}{1-n_0+\varepsilon_f\left(1-e^{-t f/k_s}\right)} + F_a \frac{1+\varepsilon_f\left(1-e^{-t f/k_s}\right)}{1+\varepsilon_f}\left(1-e^{-t f/k_s}\right) \tag{13}$$

Figure 3 shows a comparison of measured porosity of effective granular filter between a non-slurry (control) and slurry tests.

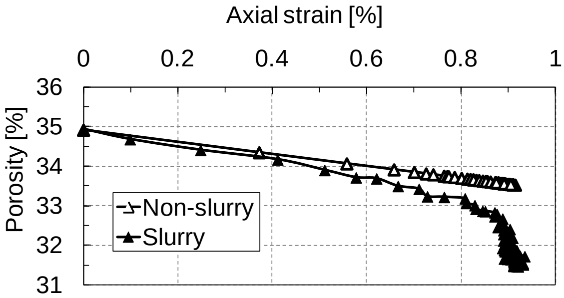

Figure 3: Reduction of filter constriction size due to accumulation of base particles.

2.4 TIME BASED HYDRAULIC CONDUCTIVITY MODEL

The Kozeny-Carman equation forms the basis of the derivation of the formulation employed to predict the deterioration in hydraulic conductivity of the granular filter specimens. The initial hydraulic conductivity of the granular filters (k_0) can be estimated as follows:

$$k_0\left[ms^{-1}\right]=\frac{1}{72\tau}\frac{\gamma}{\mu}\frac{d_{e.0}^2}{\alpha}\frac{n_0^3}{(1-n_0)^2} \qquad \textbf{(14)}$$

where τ = tortuosity, γ = unit weight of the permeant [N/m^3], μ = dynamic viscosity of the permeant [Pa-s], $d_{e.0}$ = initial effective diameter of the granular filter, and α = shape coefficient. Considering that the filter porosity decreases as the filter layer is being compressed and the clogging material accumulates with time (Eq. (13)), the reduced hydraulic conductivity with time (k_t) can be obtained from:

$$k_t=\frac{1}{72\tau}\frac{\gamma}{\mu}\frac{d_{e.t}^2}{\alpha}\frac{(n_0-\Delta n_T)^3}{\left[1-(n_0-\Delta n_T)\right]^2} \qquad \textbf{(15)}$$

In the above, $d_{e.t}$ = effective diameter of the granular filter at any time t. Rearranging Eq.(14), the constants can be expressed as:

$$\frac{1}{72\tau\alpha}\frac{\gamma}{\mu}=\frac{(1-n_0)^2}{n_0^3}\frac{k_0}{d_{e.0}^2} \qquad \textbf{(16)}$$

Substituting Eq. (16) into Eq. (15), a decreased hydraulic conductivity as a result of time based compression and clogging can be represented as a function of the initial hydraulic conductivity, the change in porosity, and the change in effective matrix diameter. The resulting equation is as follows:

$$k_t=k_0\frac{(1-n_0)^2}{n_0^3}\left[\frac{d_{e.t}^2}{d_{e.0}^2}\frac{(n_0-\Delta n_T)^3}{(1-(n_0-\Delta n_T))^2}\right] \qquad \textbf{(17)}$$

In this expression, the effective diameters $d_{e.0}$ and $d_{e.t}$ are the geometric-weighted harmonic mean of their respective PSDs.

3 WORKED OUT EXAMPLES

3.1 EXAMPLE 1

An illustration of the practical application of the proposed method is discussed in this section. Due to the limitation of the availability of relevant data from the literature, the discussions and validations that is paramount to the credibility of the procedure is restricted to the laboratory results described by Trani, 2010. The sequence by which the presented set of equations could be used follows the steps enumerated below.

1. The worked out example presented here is based on the recent findings by Trani and Indraratna (2010). From the given PSD of the base soil, the suitable PSD of the filter could be calculated according to the recommended D_{c35} model (Indraratna *et al*., 2007). Considering the compressive effect of the cyclic loads on porous media the value of the D_{c35}/d_{85sa} ratio could be relaxed from 2.0 to 3.33. The tested effective filter coefficient uniformity (C_u) ranges from 3 to 6.

2. Once the PSD of the filter is obtained, it is necessary to determine the shakedown plastic strain (ε_f) of a fully saturated specimen. This parameter can be deduced from a simple one dimensional compression test. Typical values with respect to material type are available in the literature (e.g. Werkmeister, 2003). The maximum load used in the compression test should not be less than the anticipated *in situ* stress state. The initial and final state voids ratio (or porosity, n) of the specimen should also be noted.
3. Determine the depth dependent accumulation factor (F_a) that controls the successive reduction of accumulation of fines along the profile of the filter. The specimen used in step 2 is then subsequently subjected to a filtration test. During the filtration test, two assumptions are made: (a) slurry concentration (*i.e.* mixture of 1500 g with 8 litres of water in this case), and (b) slurry loading rate (*i.e.* 15 kPa in this case).
4. The parameters obtained in steps 1 and 2, together with the solid density of the base soil and the original volume of the specimen, are then used in Eq. (7). In this equation, the profile of the apparent threshold amount of fines that could be deposited along the depth of the filter is determined. For this example, the values shown in Figure 4 are used.
5. Provided with a cyclic loading frequency (f) and a scaling factor (k_s), the amount of fines that could be accumulated at a specified depth and at given time can then be predicted (Figure 4a). This is done by performing Eq. (9).
6. Any amount of accumulated fines alters the PSD of the original porous medium. By factoring in the accumulated fines at a given layer and at a given time, the modified PSD of the base soil-filter mix is then calculated. The effective diameter d_e of the new PSD by mass of the filter and accumulated fines is its geometric – weighted harmonic mean.
7. The new d_e is then used in Eq. (17). This equation can be completed by knowing the initial hydraulic conductivity (k_0) of the filter and by performing Eq. (13).
8. The set of equations that are an integral part of the proposed mathematical procedure is summarised in a family of curves shown in Figure 4. Where applicable, the predictive equations are compared and validated against the experimental results obtained from the novel laboratory apparatus.

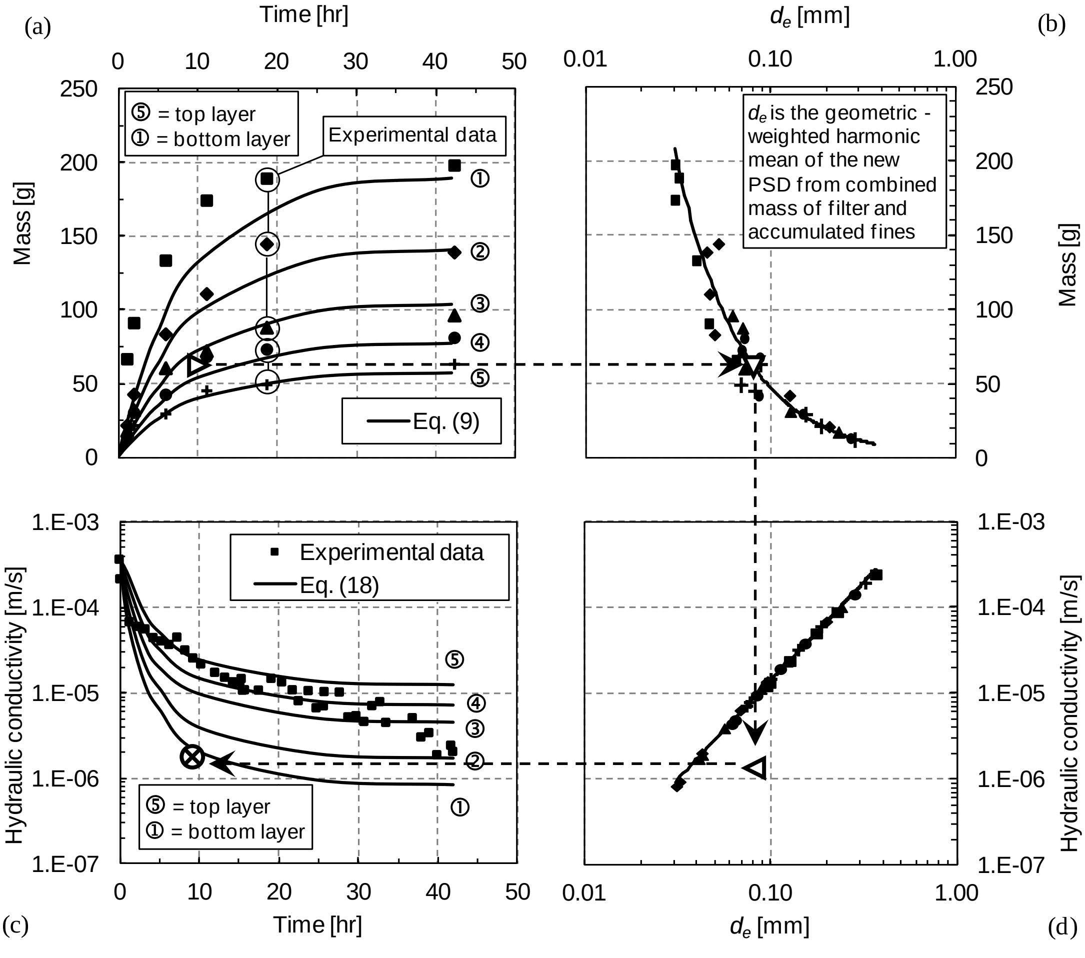

Figure 4: Porosity based family of curves generated from Eqs. (1) to (17); circled numbers represent the filter layers; n_0 = 34.96%, k_0 = 3.66x10^{-4} m/s, ε_f = 0.009, V_0 = 1.44x10^{-3} m^3, ρ_a = 2,700 kg/m^3, N$_{max}$ = 763,500 cycles, f = 5 Hz.

This illustration of the proposed mass transport mechanism is compared against the findings of Trani (2010) (Figure 4a). Apart from the slight underestimation in the first 2 layers during the first 20 hours of cyclic loading, a good agreement between the experimental and theoretical values is observed. Plotting the mathematically derived mass of accumulated fines against d_e generates a single trend line in semi-logarithmic space that conforms to the pattern of the actual measurements **(Figure 4**b**).** Note also that F_a can be used as a predictive tool of filter porosity deterioration. The value of F_a of 0.049 at layer 1 is comparable to the amount of porosity reduction during filtration tests by Locke *et al.* (2001).

In Figure 4(c), the predicted hydraulic conductivity of the filter in each of the five layers is compared with the measured steady state hydraulic conductivity values. The rapid formation of the self-filtration layer is reflected in the abrupt reduction of the measured hydraulic conductivity in the first 2 hours of testing. The gradual accumulation of fines in the voids along the profile of the filter, while maintaining internal stability condition, explains the deterioration of the measured hydraulic conductivity with time. Similar to Figure 4(b), the plot of predictive hydraulic conductivity values against d_e follows a single rate law curve in a fully logarithmic space Figure 4(d).

3.2 EXAMPLE 2

An alternative to using the estimated accumulated fines is the calculation of changes of the filter porosity through Eq. (13). As shown in Figure 5(a), the prediction of porosity reduction behaviour against time is compared against the laboratory measured porosity readings taken at layers 2 and 4. Plotting the porosity predictions with d_e generates curves that follow a rate law trend in semi-logarithmic space (Figure 5b).

The filtration criterion based on filter constrictions implicitly satisfies the requirement of internal stability during seepage. By using this criterion as a key condition in developing the proposed model, the assumption that the self-filtration layer is formed and the clogging through the profile of the filter would eventually occur is addressed.

Both Figure 4 and Figure 5 demonstrate the capability of the mathematical model to predict the extent of filter clogging and its effect on the drainage capacity at a particular time. If the accumulated fines (Figure 4a) or porosity (Figure 5a) of a particular layer is known without the information about its time of occurrence, the point in time when the corresponding hydraulic conductivity of the said layer in question can still be estimated by following three steps:

1. Draw a horizontal line towards d_e-mass space (Figure 4b), if mass is known) or d_e-porosity plot Figure 5(b), if porosity is determined).
2. A vertical isodiametric line is then drawn towards the d_e-k_t line in Figure 4(d) or Figure 5(d), if the mass or porosity is known, respectively.
3. A horizontal line is drawn to the hydraulic conductivity prediction lines against time in Figure 4(c) or Figure 5(c), for mass or porosity, respectively.

As a general guideline, the overall behaviour of the deterioration of the hydraulic conductivity of the filter with time can be estimated by using a multi-layer approach. Layer 1 corresponds to the first 2 hours of testing, layer 5 from the 2nd up to 12th hr before it deteriorates to level 4 predictions up to the 30th hr. Eventually, the gradual deterioration of the hydraulic conductivity can then be predicted using level 3 and finally to level 2. In the context of the number of cycles, assuming that the field conditions are primed for clay pumping to occur, the estimated time can be converted into the actual length of time subballast exceeds its prescribed drainage capacity by considering the frequency by which the trains are passing through the rail tracks.

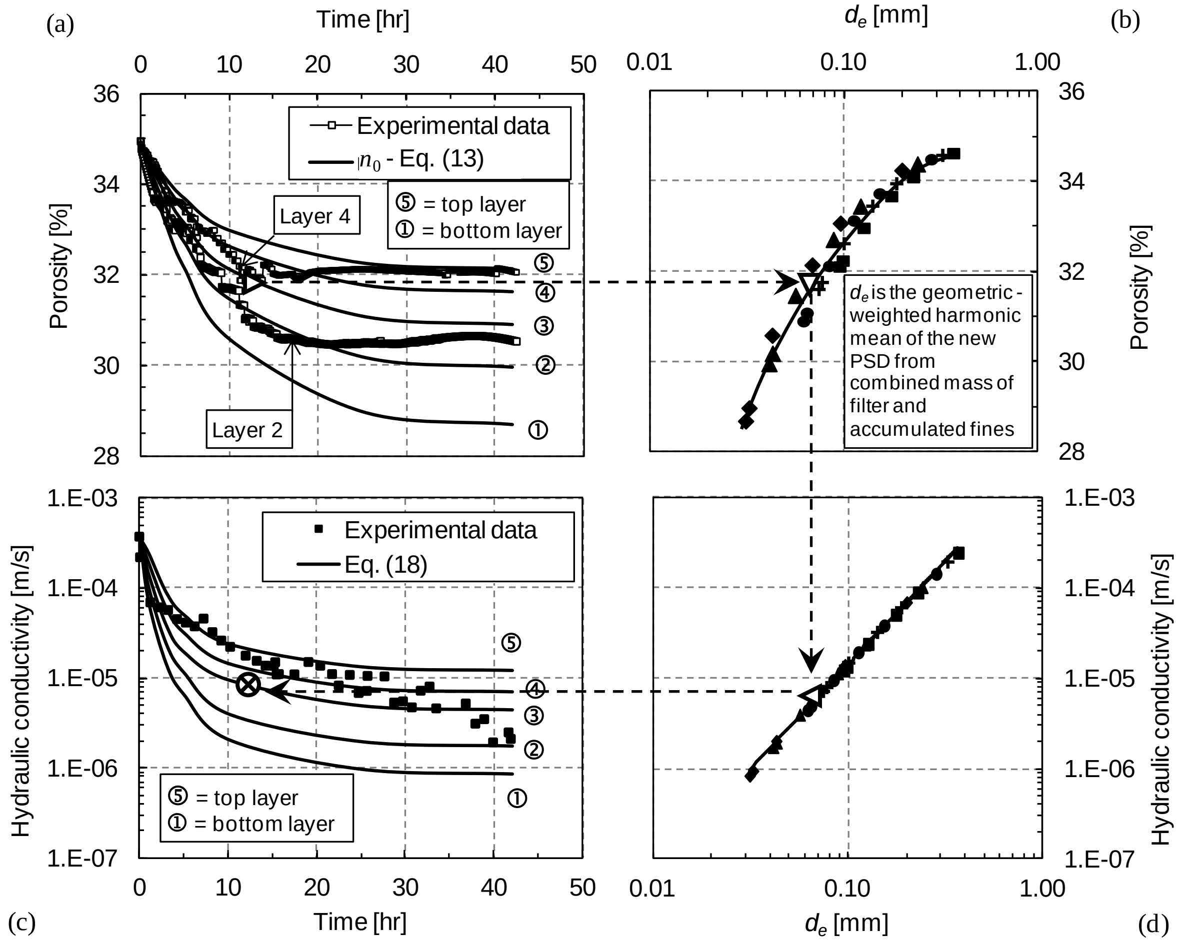

Figure 5: Porosity based family of curves generated from Eqs. (1) to (17).

4 CONCLUSIONS

A semi-empirical mathematical model is presented to predict the filtration and drainage of saturated subballast under cyclic train loading. The key assumptions of the model included: (a) a stress domain of Drucker-Prager potential applied in a viscoplastic model in the framework of a post shakedown cyclic densification regime, (b) plastic strain evolution under one dimensional consolidation principle, (c) the dominant constriction size (D_{c35}) of the filter is smaller than or equal to the representative diameter (d_{85sa}) of the base soil, and (d) steady state conditions under laminar flow to ensure the applicability of Darcy's law and the phenomenological Kozeny-Carman equation.

A suitable granular filter for a given base soil must satisfy the constriction-based filtration criterion. When applied to the transport sector where a degree of plastic deformation to an anticipated cyclic load generated from passing traffic is expected, a relatively relaxed criterion is proposed whereby the original size of D_{c35} can be 2 to 3.3 times the size of d_{85sa}. It is also suggested that the coefficient of uniformity of the filter is kept within the range of 3 to 6.

Although the theoretical background of proposed method complies with the basic principles of soil mechanics, it should be emphasised that the development of this mathematical tool was also partly based on laboratory conditions. This paper only illustrates an effective filter for a single type base soil. Furthermore, the aspects of a triaxial stress system, the presence of lateral hydraulic gradient, and the physicochemical aspects of filtration and clogging were not considered. Nevertheless, this proposed multi-layer approach of a time based prediction of the overall hydraulic conductivity of a filter, is a framework wherein future advances to develop more comprehensive design guidelines, can be incorporated.

5 REFERENCES

Alobaidi, I. M. and Hoare, D. J., *Mechanism of Pumping at the Subgrade-Subbase Interface of Highway Pavements*, Geosynthetics International, 1999, **6**(4): p. 241-259.

Carman, P. C., *Determination of the specific surface of powders I*, Transactions on Journal of the Society of Chemical Industries, 1938, **57**: p. 225-234.

Indraratna, B. and Raut, A. K., *Enhanced Criterion for Base Soil Retention in Embankment Dam Filters*, Journal of Geotechnical and Geoenvironmental Engineering, ASCE, 2006, **132**(12): p. 1621-1627.

Indraratna, B. and Vafai, F., *Analytical Model for Particle Migration Within Base Soil - Filter System*, Journal of Geotechnical and Geoenvironmental Engineering, ASCE, 1997, **123**(2): p. 100-109.

Indraratna, B., Balasubramaniam, A. and Balachandran, S., *Performance of Test Embankment Constructed to Failure on Soft Marine Clay*, Journal of Geotechnical Engineering, 1992, **1181**: p. 12-33.

Indraratna, B., Raut, A. K. and Khabbaz, H., *Constriction-Based Retention Criterion for Granular Filter Design*, Journal of Geotechnical and Geoenvironmental Engineering, ASCE, 2007, **133**(3): p. 266-276.

Kozeny, J., *Ueber Kapillare Leitung Des Wassers Im Boden*, Sitzungsberichte der Akademie der Wissenschaften, Wien, 1927, **136**(2a): p. 271-306.

Locke, M., Indraratna, B. and Adikari, G., *Time-Dependent Particle Transport Through Granular Filters*, Journal of Geotechnical and Geoenvironmental Engineering, ASCE, 2001, **127**(6): p. 521-529.

Melan, E., *Theorie Statisch Unbestimmer Systeme Aus Ideal-Plastischen Baustoff*, Sitzungsberichte der Akademie der Wissenschaften, Wien, 1936, p. 145-195.

Perzyna, P., *Fundamental Problems in Viscoplasticity*, Advanced Applied Mechanics, 1966, **9**: p. 243-377.

Raut, A. K. and Indraratna, B., *Further Advancement in Filtration Criteria Through Constriction-Based Techniques*, Journal of Geotechnical and Geoenvironmental Engineering, ASCE, 2008, **134**(6): p. 883-887.

Selig, E. T. and Waters, J. M., *Track Technology and Substructure Management*, Thomas Telford, 1994, London.

Suiker, A. S. J. and de Borst, R., *A Numerical Model for The Cyclic Deterioration of Railway Tracks*, International Journal of Numerical Methods in Engineering, 2003, **57**: p. 441-470.

Trani, L. D. and Indraratna, B., *An Assessment of Subballast Filtration Under Cyclic Loading*, Journal of Geotechnical and Geoenvironmental Engineering, ASCE, 2010, **136**(11): p. 1519-1528.

Trani, L. D., *Application of Constriction Size Based Filtration Criteria for Railway Subballast Under Cyclic Conditions*, PhD Thesis, University of Wollongong, 2010, Australia.

Werkmeister, S., *Permanent Deformation Behaviour of Unbound Granular Materials in Pavement Constructions*, PhD Thesis, Dresden University of Technology, 2003, Germany.