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# Model-based optimization of ground source heat pump systems

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## **Keywords**

heat, pump, systems, model, ground, optimization, source

## **Disciplines**

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## Model-based optimization of ground source heat pump systems

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### Abstract

This paper presents the development of an optimization strategy for ground source heat pump (GSHP) systems equipped with variable speed pumps in the ground loop system. The optimization problem is formulated using a model-based approach, in which the component models are used to estimate the system performance under various trial settings and an exhaustive search method is used to identify the optimal settings under the search ranges defined. The variable optimized is the outlet water temperature from the ground heat exchangers, which can be used as a set-point to control the operation of the variable speed pumps in the ground loop system. The overall objective of the optimization is to minimize the system power consumption while providing required building heating and cooling demand. The performance of the proposed strategy is tested and evaluated through simulations. It is shown that, compared to a two-stage control strategy for variable speed pumps, the proposed strategy can save 4.2% of cooling power consumption of the GSHP system studied. The methodology used in the development of this proposed strategy can be potentially useful for control optimization of any types of GSHP systems. For complex systems, a performance map can be generated based on this method and then used to practically control the operation of GSHP systems.

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*Keywords:* Control optimization; Model-based approach; GSHP systems; Variable speed pumps

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### 1. Introduction

Ground source heat pump (GSHP) systems have attracted increasing attention due to their high energy performance and are being recognized as one of the most sustainable systems for heating and cooling of both residential and commercial buildings [1, 2]. The performance of a GSHP system is influenced by many variables such as the type of GSHP systems, ground heat exchanger (GHE) configuration, soil conditions, building load characteristics, equipment selection, system sizing, control optimization and local climate [3-5].

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Over the last several decades, many efforts have been made on appropriate design and optimal sizing of GSHP systems. Nagano et al. [6], for instance, presented the development of a design and performance prediction tool for GSHP systems. Sivasakthivel et al. [7] employed Taguchi and utility methods to optimize GHEs used for space heating applications. Li and Lai [8] applied the entropy generation minimization method to optimize the design of a vertical GHE. A multi-objective design optimization strategy for vertical GHEs was presented in [9] to minimize the system upfront cost and entropy generation number. The results from these studies showed that appropriate design of GSHP systems is essential to reduce their installation cost and lifetime operational cost.

Besides appropriate design, control and optimization of GSHP systems are also important to improve their operating efficiency while providing satisfied indoor thermal comfort. Through experimental investigation, Zhai et al. [10] concluded that optimization of indoor temperature settings can alleviate the thermal imbalance of the ground, which would be beneficial to the long-term operation of GSHP systems. Zhu et al. [11] reported the use of improved control strategies to select the operation modes of a ground water-source heat pump system. Gang et al. [12] investigated the performance of a GSHP system coupled with a supplemental heat rejecter by using a schedule based control, temperature differential based control and artificial neural network (ANN) based predictive control, respectively. The result showed that ANN-based predictive control is more energy efficient. The use of intermittent operation strategies for a hybrid GSHP system with cooling towers was studied by Yang et al. [13]. The simulation results showed that the proposed strategies can alleviate the soil heat accumulation and reduce the energy consumption of the hybrid GSHP system. Yavuzturk and Spitler [14] presented a comparative study of the control strategies for supplemental heat rejecters used in hybrid GSHP systems. It was shown that the best control strategy investigated was to operate the supplemental heat rejecters based on the difference between the fluid temperature exiting the heat pumps and ambient air wet-bulb temperature. An optimization methodology for frequency control of variable speed water pumps in GSHP systems was presented in [15]. The overall methodology consists of three steps, including frequency tests, system COP maps, and system performance factor maps. The results indicated that up to 32% energy savings can be achieved by using this optimization methodology. The above studies showed that proper control of GSHP systems is essential to substantially reduce the operational cost of GSHP systems. However, the research in this area is far from sufficient compared to the research on design optimization and optimal sizing of GSHP systems.

This paper presents an optimization strategy for optimal control of GSHP systems equipped with variable speed pumps in the ground loop system. The optimization strategy is developed using a model-based approach. The overall objective of the optimization is to minimize the total energy consumption of the heat pumps and the water pumps in the ground loop system without sacrificing the indoor thermal comfort and violating the relevant operating constraints.

## **2. Model-based optimization strategy**

### *2.1. Outline of the optimization strategy*

The proposed optimization strategy for GSHP systems is illustrated in Fig. 1. It is developed based on an assumption that a variable speed pump(s) is installed in the ground loop system to modulate the water flow rate. The overall optimization process consists of two steps. The first step is to use a rule-based sequence controller to determine the operating number of the heat pumps based on the building heating/cooling demand and the capacity of each heat pump as well as the operating constraints of practical applications. The second step is to determine the optimal combination of the outlet water temperature from the GHEs (i.e. the inlet water temperature to the source side heat exchangers of the heat pumps) and the water flow rate circulating through the GHEs to minimize the total power consumption of the water pumps in the ground loop and the water-to-water heat pumps. This is achieved by using a model-based approach, in which the simplified component models of heat pumps, water pumps and GHEs are used to predict the performance of the GSHP system under different trial settings of the optimization variables and an exhaustive search method is used to search for the optimal solutions of the optimization problem. For a given trial combination, the water-to-water heat pump model is first used to determine the power consumption of the heat pumps and the outlet water temperature from the source side heat exchangers of the heat pumps. This temperature is then used as the inlet water temperature of the GHEs. Using the GHE model and water pump models, the outlet

water temperature from the GHEs and the power consumption of the water pumps in the ground loop system can be determined. If the difference between the trial outlet water temperature from the GHEs and that of calculated through energy balance is less than a defined value, this trial combination will be considered as one of the candidate settings. Otherwise, it will be considered as an unfeasible combination and a penalty cost will be given to this trial combination. In practical applications, a supervisory strategy should be used to determine the final decision for the controller to avoid frequent ON/OFF of the heat pumps. The outputs from the optimization strategy are the on/off status of the heat pumps and the optimal setting of the outlet water temperature from the GHEs, which can be used as a set-point to control the operation of the variable speed pumps in the ground loop system.

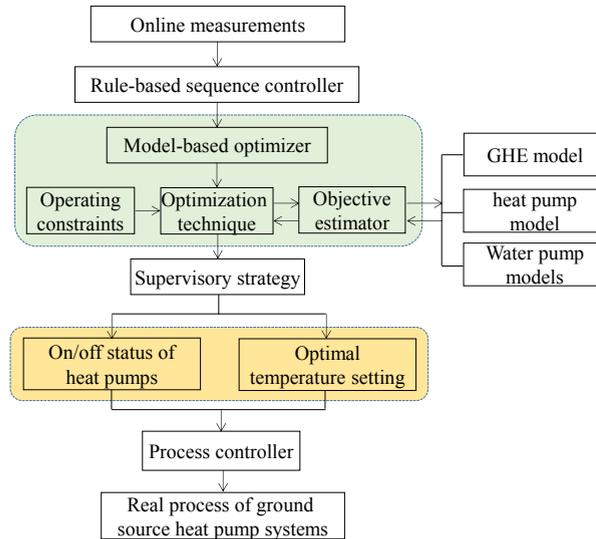


Fig. 1. Outline of the proposed optimization strategy.

## 2.2. Cost function and operating constraints

The cost function used to formulate the model-based optimization strategy is expressed in Eq. (1), which is to minimize the total power consumption of the constant and variable speed water pumps in the ground loop system and the water-to-water heat pumps. The optimal combination of the outlet water temperature from the GHEs and the water flow rate in the ground loop system which gives the minimum total power consumption will be considered as the optimal control settings for the given working condition.

$$J = \min W_{tot} = \sum_{i=1}^{N_{HP}} W_{HP,i} + \sum_{j=1}^{N_{pu,con}} W_{pu,con,j} + \sum_{k=1}^{N_{pu,var}} W_{pu,var,k} \quad (1)$$

where  $J$  is the cost function,  $W$  is the power consumption,  $N$  is the number, and the subscripts  $tot$ ,  $HP$ ,  $pu$ ,  $con$  and  $var$  indicate total, heat pump, water pump, constant and variable, respectively.

In the optimization, the low limits of the outlet water temperature from the GHEs for cooling and heating mode operations were set as 20°C and 6.0°C, respectively. As a low water flow rate will lead to a laminar flow in the GHEs and a poor heat transfer to the ground [15], the minimum water flow rate in the ground loop is roughly estimated when the variable speed pump operates at 10 Hz by using pump affinity laws.

### 2.3. Description of the component models

#### Water-to-water heat pump model

As the performance of a water-to-water heat pump is mainly influenced by its load side and source side inlet water temperatures and water flow rates, the power consumption of the water-to-water heat pump is simulated using a slightly modified curve-fitting model presented in [16, 17], as expressed in Eq. (2). The coefficients of the model for the heating mode operation and cooling mode operation can be determined using the heat pump heating performance data and cooling performance data provided by the manufacturer, respectively.

$$\frac{W_{HP}}{W_{HP,ref}} = A_0 + A_1 \left[ \frac{T_{LHP,out}}{T_{LHP,out,ref}} \right] + A_2 \left[ \frac{T_{SHP,in}}{T_{SHP,in,ref}} \right] + A_3 \left[ \frac{m_{LHP}}{m_{LHP,ref}} \right] + A_4 \left[ \frac{m_{SHP}}{m_{SHP,ref}} \right] \quad (2)$$

where  $T$  is the temperature,  $m$  is the water flow rate,  $A_0$ - $A_4$  are the coefficients, and the subscripts *ref*, *LHP*, *SHP*, *in* and *out* represent the reference condition, the load side of the heat pump, the source side of the heat pump, inlet and outlet, respectively.

#### Water pump models

The performance of both constant and variable speed pumps was modeled using a series of polynomial approximations representing the head versus flow and speed, and the efficiency versus flow and speed, as expressed in Eqs. (3) and (4), respectively [18, 19]. The pump head and efficiency characteristics can be determined based on the manufacturing data at the rated operation and extended to the variable speed operation using pump affinity laws [19, 20]. The power input to a pump-motor-VFD set is computed using Eq. (5). In this study, the constant values were assumed for the motor efficiency and VFD efficiency.

$$H_{pu} = B_1 m_w^2 + B_2 \left( \frac{n}{n_0} \right) m_w + B_3 \left( \frac{n}{n_0} \right)^2 \quad (3)$$

$$\eta_{pu} = C_1 \left( \frac{n_0}{n} \right)^2 m_w^2 + C_2 \left( \frac{n_0}{n} \right) m_w + C_3 \quad (4)$$

$$W_{pu,in} = \frac{m_w \times H_{pu} \times SG}{102 \times \eta_v \times \eta_m \times \eta_{pu}} \quad (5)$$

where  $H$  is the pump head,  $\eta$  is the pump efficiency,  $SG$  is the specific gravity of the fluid being pumped,  $n$  is the pump operating speed, and  $n_0$  is the pump operating speed at the rated condition,  $B_1$ - $B_3$  and  $C_1$ - $C_3$  are the coefficients, and the subscripts *pu*, *m*, *v* and *in* represent pump, motor, VFD and input, respectively.

#### Ground heat exchanger models

Vertical ground heat exchangers have been commonly used in GSHP systems. In this study, the borehole wall temperature ( $T_b$ ) at the end of the  $n^{th}$  time step is determined by Eq. (6) [21]. The  $g$  function in Eq. (6) is calculated by Eq. (7), in which the time-dependent borehole wall temperature for a single step pulse of a given ground thermal conductivity was simulated using the numerical model of vertical GHEs available in TRNSYS. The thermal resistance per borehole length ( $R_b$ ) is determined by Eq. (8), which includes the thermal resistances of the pipe wall and the inside fluid and the thermal resistance of the grout material [1].

Based on the thermal resistance per borehole length, the outlet water temperature from GHEs can be easily determined based on the relationship described in Eq. (9).

$$T_b = T_{s,0} + \sum_{i=1}^n \frac{(Q_i - Q_{i-1})}{2N\pi k_s H} g\left(\frac{t_n - t_{i-1}}{t_s}, \frac{r_b}{H}\right) \quad (6)$$

$$g\left(\frac{t_i}{t_s}, \frac{r_b}{H}\right) = \frac{2\pi k_s (T_b - R_b \times Q / NH - T_{s,0})}{Q / NH} \quad (7)$$

$$R_b = \frac{1}{4\pi k_g} \left[ \ln \frac{r_b}{r_{p,o}} + \ln \frac{r_b}{2x_c} + \frac{k_g - k_s}{k_g + k_s} \ln \left( \frac{r_b^4}{r_b^4 - x_c^4} \right) \right] + \frac{\ln(r_{p,o} / r_{p,i})}{4\pi k_p} + \frac{1}{4\pi r_{p,i} h_f} \quad (8)$$

$$R_b = \frac{(T_{w,in} + T_{w,out}) / 2 - T_b}{Q / NH} \quad (9)$$

where  $T$  is the temperature,  $r$  is the radius,  $k$  is the thermal conductivity,  $t$  is the time,  $t_s$  is the time scale,  $H$  is the borehole depth,  $Q$  is the total heat rejection,  $N$  is the number of boreholes,  $R$  is the thermal resistance,  $x_c$  is the shank spacing and defined as the center-to-center distance between the two legs of the U-tube,  $h_f$  is the convective heat transfer coefficient, and the subscripts  $b$ ,  $g$ ,  $i$ ,  $o$ ,  $p$ ,  $s$  and  $\theta$  represent the borehole, grout material, inner, outer, pipe, soil and initial, respectively.

### 3. Test results and discussions

#### 3.1. Case study GSHP system

The ground source-air source combined heat pump system implemented in the Sustainable Buildings Research Centre at the University of Wollongong is used as a case study. In this system, an air-to-water heat pump and two identical water-to-water heat pumps are used to provide the required heating and cooling demand of the building. The two water-to-water heat pumps connected with three vertical borehole heat exchangers and a total of twelve horizontal linear heat exchangers. The design capacity of the two water-to-water heat pumps in the cooling mode operation is 32.8 kW and the heating mode operation is 40.8 kW. In the source side of the GSHP system, a constant speed water pump is dedicated to each water-to-water heat pump and a variable speed water pump is installed in the ground loop to provide sufficient force to circulate water flowing through the GHEs. In order to simplify the optimization process, it is assumed that six vertical borehole heat exchangers are connected with two water-to-water heat pumps. The diameter and depth of each borehole are 150 mm and 91 m, respectively. The soil thermal conductivity is 2.0 W/mK.

#### 3.2. Setup of the test

The performance of the proposed strategy was tested and evaluated based on the cooling mode operation. The test was carried out in three consecutive days under Sydney weather conditions, during which the building has a relatively low cooling demand in order to examine the response of the optimization strategy to the change of the working condition (i.e. load ratio) of the water-to-water heat pumps. Fig. 2 illustrates the simulated cooling load profile of the building in these three test days using DesignBuilder.

In the test, only one water-to-water heat pump with three vertical boreholes was used when the cooling demand of the building is less than the design cooling capacity of the single heat pump and the second water-to-water heat pump was put into operation when the cooling demand of the building exceeds the design cooling capacity of the

single water-to-water heat pump. The air source heat pump was switched on when the building cooling demand is greater than the design capacity of the two water-to-water heat pumps.

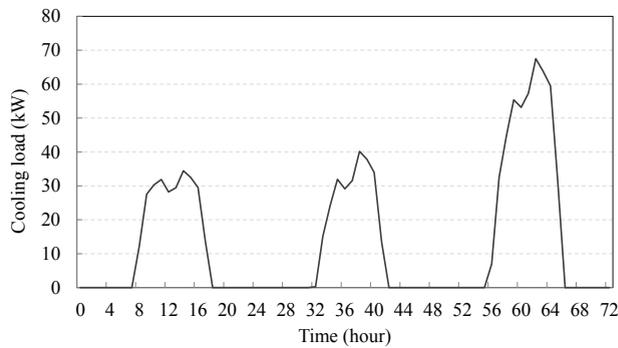


Fig. 2. Building cooling load profile used in the simulation test.

### 3.3. Test results

The identified optimal outlet water temperatures from the GHEs and the corresponding water flow rates required achieving these temperatures are shown in Fig. 3. It can be seen that both varied with the variation of the operating conditions. The optimal values were identified based on the tradeoffs between the power consumption of the water-to-water heat pumps and the power consumption of the source-side constant/variable speed water pumps.

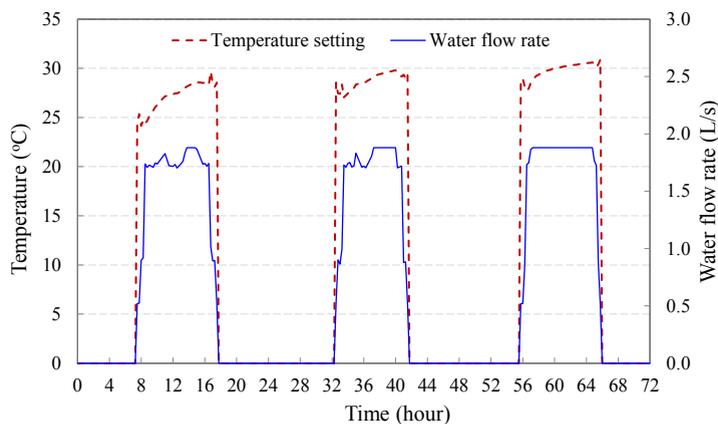


Fig. 3. Identified optimal temperatures and corresponding water flow rates.

As the building cooling loads in the second afternoon and the most daytime in the third test day were higher than the cooling capacity of the two water-to-water heat pumps, the air-source heat pump was therefore switched into operation during these time periods. As a result, the almost same water flow rates were identified for these time periods. However, due to the continuous heat rejection to the ground, the identified temperature settings were slightly increased.

The power consumption of the two water-to-water heat pumps, the source-side constant speed pumps and the variable speed pump in the ground loop system under the optimal settings identified as well as the operating number of the water-to-water heat pumps are shown in Fig. 4. It can be observed that the two water-to-water heat pumps were used in the most daytime test periods. The power consumption of the constant speed water pumps was relatively stable while the power consumption of the variable speed pump varied with the change of the working conditions. The instantaneous power consumption of the two water-to-water heat pumps during the most daytime test periods was around 8.0-9.0 kW, which was much higher than that of the power consumption of the water pumps.

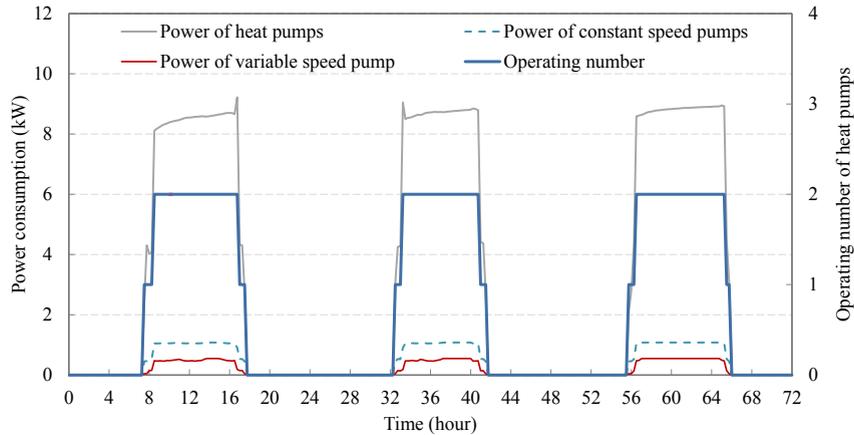


Fig. 4. Power consumption and operating number of water-to-water heat pumps and power consumption of water pumps.

Table 1 summarizes the power consumption of the water-to-water heat pumps, and constant/variable speed pumps in the ground loop system when using the proposed optimization strategy and a rule-based control strategy under the same three test days. In the rule-based control strategy, a two-stage control was used to operate the variable speed pump in the ground loop system. When one water-to-water heat pump is in operation, the pump operating frequency was set as 25 Hz. Otherwise, the pump operating frequency was set as 50 Hz. It can be seen that the total power consumption of the GSHP system in the three test days using the rule-based control strategy and the proposed optimization strategy were 290.6 and 278.5 kWh, respectively. Compared to the rule-based control strategy, 4.2% power consumption can be saved due to the use of the proposed optimization strategy.

Table 1. Power consumption of the GSHP system under three test days when using different control strategies

Control strategy	Power consumption of water-to-water heat pumps (kWh)	Power consumption of constant/variable speed pumps (kWh)	Total power (kWh)	Total power difference (%)
Rule-based control	230.0	60.6	290.6	-
Proposed strategy	236.2	42.3	278.5	4.2%

### 3.4. Discussions

Although the power savings due to the use of the proposed optimization strategy in the case study building are not significant, this part of savings was achieved by using optimal control only and without adding any additional cost. It is also worthwhile to mention that the power savings are highly dependent on the baseline control strategy used. The rule-based control strategy used as the baseline in this study also provided near optimal operation of the variable speed water pump.

This study attempted to use a model-based approach to developing optimization strategies for GSHP systems. Unlike most existing studies for control optimization of GSHP systems, simplified physical models of major components were used in this study to design the optimization strategy. As a priori knowledge of the component is

generally incorporated in the model parameters as constraints, simplified physical models have certain physical significance, and their model complexity and computational cost are manageable. The type of models can also be easily trained using the manufacturing data or the short-term operation data.

Most GHE models available in the public domain were primarily developed for design purposes. For control optimization, the models with controllable computational costs and acceptable prediction accuracies are highly needed. As the borehole wall temperature directly influences the selection of optimal control settings, the models used should accurately predict the time-varying borehole wall temperature under dynamic working conditions.

The exhaustive search method used in this study is not a good search method for real-time control applications due to the nature of its exhaustive search, thereby relatively high computational cost requirement. However, the results obtained using this method can be potentially useful to generate a performance map, which can then be used for real-time control applications.

#### 4. Conclusions

A model-based optimization strategy for GSHP systems was developed in this study in order to minimize the total power consumption of the water-to-water heat pumps and the water pumps in the ground loop system. The simplified component models were used as the performance predictors and the exhaustive search method was used as the optimization technique. The performance of the proposed strategy was tested in three consecutive days in Sydney weather conditions under the cooling mode operation. Compared to a rule-based control strategy with two-stage control for the variable speed pump in the ground loop, the proposed strategy can save 4.2% of the total power consumption of the water-to-water heat pumps, the source-side constant speed pumps and the variable speed pump in the ground loop system. The methodology used in the development of the proposed optimization strategy can be easily adapted to control and optimize the operation of any types of GSHP systems.

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