Improving phase unwrapping efficiency and reliability of 3D fringe patterns profilometry for complex object measurement

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IMPROVING PHASE UNWRAPPING EFFICIENCY AND RELIABILITY OF 3D FRINGE PATTERNS PROFILOMETRY FOR COMPLEX OBJECT MEASUREMENT

A thesis submitted in fulfilment of the requirements for the award of the degree

Doctor of Philosophy

from

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by

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Statement of Originality

I, Ke Chen, declare that this thesis, submitted in fulfillment of the requirements for the award of Doctor of Philosophy, in the School of Electrical, Computer and Telecommunications Engineering, University of Wollongong, is wholly my own work unless otherwise referenced or acknowledged. The document has not been submitted for qualifications at any other academic institution.

Ke Chen

November 7, 2013
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ACRONYMS

FPP: Fringe Projection Profilometry
FTP: Fourier Transform Profilometry
PSP: Phase shifting Profilometry
LCD: Liquid Crystal Display
PSP: Phase Shifting Profilometry
QGFF: Quality-guided flood-fill
PDV: Phase Derivative Variance
PC: Pseudo-correlation
PM: Phase modulation
MPG: Maximum Phase Gradient
This thesis aims to improve the efficiency and accuracy of phase unwrapping for digital projection based three-dimensional fringe patterns profilometry (FPP). The proposed initiative is facilitated by the analysis of phase unwrapping principles and by the proposal of new phase unwrapping algorithms.

This thesis begins by tracing the history and development of FPP and then briefly introduces the principles, relevant conventional phase unwrapping methods, and typical applications of FPP. Focusing on phase unwrapping, the author theoretically analyses phase unwrapping errors in FPP and finds that these errors are triggered by disturbances on the phase map, which unavoidably makes conventional phase unwrapping methods unsuitable.

To solve the phase unwrapping problem, the author presents several approaches. The first approach is proposed to improve the efficiency of the quality-guided phase unwrapping method in terms of reducing the computational burden. This approach combines the conventional quality-guided method with a simple phase unwrapping algorithm. A quality threshold is used to classify pixels on the wrapped phase map into two types: high-quality (HQ) pixels corresponding to smooth phase variance and
low-quality (LQ) pixels corresponding to rough phase variance. These two types of pixels are then unwrapped using different approaches. The HQ pixels are unwrapped by a simple phase unwrapping algorithm and the LQ pixels are recovered by a conventional quality-guided method. It is shown by both theoretical analysis and experiment that the proposed approach is able to unwrap complex phase maps with similar accuracy to the conventional quality-guided method, but with a much lower computational burden, and therefore, much faster.

The second approach is proposed to improve both phase unwrapping reliability and efficiency when the object surface has large depth discontinuities and shadows. Although phase unwrapping errors caused by large depth discontinuities and shadows are unavoidable, the phase unwrapping accuracy can be improved if the propagation of errors is prevented. Based on this idea, this approach blocks large depth discontinuities and shadows from participating in phase unwrapping and, as a result, has better results than the conventional method. An image of the object is retrieved from the fringe patterns, and the shadow areas and the large depth discontinuities are identified. When carrying out phase unwrapping, the edges and shadow areas are isolated, and the remaining areas are then unwrapped by means of an efficient group merging technique proposed. The experimental results show that the proposed method is able to deal with
the phase unwrapping of objects with large depth discontinuities and shadows.

The third approach, a colour projection based method, is proposed to further improving phase unwrapping accuracy when an object has spatially isolated areas and distributed surface colour. Using this method, each fringe of the sinusoidal patterns is identified by a unique binary sequence. These sequences are encoded by a channel encoding scheme used in communications and the codewords are carried by binary coding stripes. With the colour projection technique, the sinusoidal fringes and the binary stripes can be projected simultaneously using red, green and blue (RGB) colours. The projected fringe patterns are captured and decomposed by an RGB 3-CCD camera into red, green and blue components. The wrapped phase map is obtained from the sinusoidal fringe patterns and the phase unwrapping is implemented based on the binary stripes. Compared with existing approaches, this approach provides reliable measurement of objects with spatially isolated areas and distributed surface colour.

Finally, the author concludes the thesis and proposes avenues for further research.
Chapter 1 Preliminaries

1.1 Principle of fringe patterns profilometry techniques

Fringe patterns profilometry (FPP) techniques play increasingly important roles in three-dimensional sensing, mechanical engineering, machine vision, intelligent robot control, biomedicine and other industrial applications [1-6]. A typical FPP system consists of a digital projector, a CCD camera and a computer. The digital projector generates fringe patterns those are first projected onto a reference plane, a flat white board with a uniformly reflective surface. Next, the same fringe patterns are projected onto the surface of the object to be measured. The deformed fringe patterns captured by the camera from the object surface differ from those directly obtained from the reference plane. The difference between the two provides information about the object surface and reflects the trend of the object’s profile relative to the reference plane. Therefore, the basic principle underlying FPP techniques is to retrieve the height distribution by retrieving the difference between fringe patterns from the reference plane and the object surface.

Assuming there is no nonlinear distortion introduced by the projector, the $n$th frame of the captured fringe patterns on the reference plane is:
1.1 Principle of fringe patterns profilometry techniques

\[
 s_n(x, y) = R_{ref} \left[ A + B \cos(2\pi f_0 x + \delta_n) \right] 
\]  

(1.1)

where, \( R_{ref} \) is a constant for the reflectivity of the reference plane. \( A \) is the background illumination intensity, which makes the fringe patterns visible for the camera, and \( B \) denotes the amplitude of the fringe patterns. Assuming each fringe covers \( m_0 \) mm on the reference plane, \( f_0 = 1/m_0 \) is the fundamental spatial frequency of the captured fringe patterns. \( \delta_n \) denotes the initial phase shift of sinusoidal fringe patterns at the \( n \)th frame.

The same fringe patterns are projected onto the object surface and captured by the camera. The following equation denotes the \( n \)th frame of the fringe patterns captured from the object surface:

\[
 d_n(x, y) = R_{obj}(x, y) \left[ a + b \cos(2\pi f_0 x + \phi(x, y) + \delta_n) \right] 
\]  

(1.2)

where, \( R_{obj}(x, y) \) is the reflectivity of the object surface and \( \phi(x, y) \) is the phase shift of fringe patterns caused by the object surface.

Figure 1.1 shows the geometric principle of a typical FPP system. A beam of light first reaches the reference plane at point C. The same point in the projected fringe patterns then reaches the object surface at point H, and through point D after the reference plane is removed. From the vision of the camera, the same point on the projected fringe
patterns is presented at point C and D without and with the object, respectively. Therefore, at point D, the phase shift between the fringe patterns on the reference plane and the fringe patterns on the object surface is equal to the phase difference between point C and D. Thus, the phase shift $\phi(x, y)$ at point H can be denoted by the phase difference from point C to D in the fringe patterns on the reference plane. According to Equation (1.2), the phase difference from C to D is $2\pi f_0 \overline{CD}$ and the phase shift $\phi(x, y)$ at point H can be denoted as follows:

$$\phi(x, y) = 2\pi f_0 \overline{CD}$$

(1.3)

Figure 1.1 Geometric principles of fringe patterns profilometry (FPP) system
Let \( h(x, y) \) denote the distance between the object surface and the reference plane, which contains the information about the object’s profile. As shown in Figure 1.1, the triangles \( \Delta CDH \) and \( \Delta E_E H \) are similar, and the following equation can therefore be obtained:

\[
\frac{CD}{h(x, y)} = \frac{d_0}{l_0 + h(x, y)} \tag{1.4}
\]

Substituting Equation (1.3) into Equation (1.4) gives:

\[
h(x, y) = \frac{l_0 \phi(x, y)}{\phi(x, y) + 2\pi f_0 d_0} \tag{1.5}
\]

As shown in Equation (1.5), once the phase shift \( \phi(x, y) \) is obtained, the three-dimensional profile of the object can be retrieved. Therefore, the main steps for a FPP system retrieving object surface information are as follows:

- System calibration, which retrieves the geometrical parameters of a FPP system.
- Fringe patterns projection and acquisition, which projects and captures the fringe patterns on both the object surface and the reference plane.
- Phase detection, which retrieves the wrapped phases from the fringe patterns.
1.1 Principle of fringe patterns profilometry techniques

- Phase unwrapping. The phases retrieved in the last step are wrapped into $(-\pi, \pi]$. To reconstruct the object surface shape, $\phi(x, y)$ needs to be recovered from its wrapped version.

- Phase-to-height conversion, which reconstructs the object surface shape according to the phase shift.

1.2 Introduction to phase unwrapping and its challenges

Many phase detection techniques used in the FPP end with an arctangent operation to retrieve the phase [10-12], but the arctangent operation can only return the principal value of the argument and the retrieved phase is wrapped into the interval $(-\pi, \pi]$. To correctly reconstruct the three-dimensional surface shape, the true phases are required and must be recovered from the wrapped version. The relationship between the true phase and its wrapped version is shown as follows:

$$\phi(x, y) = \phi'(x, y) + 2\pi \cdot k(x, y)$$  \hfill (1.6)

where, $k(x, y)$ is an integer, referred to as the fringe number index, which gives the location of $(x, y)$ in the patterns. For example, $k(x, y) = 1$ implies that $\phi(x, y)$ is on the second fringe of the patterns. The process to recover $\phi(x, y)$ from $\phi'(x, y)$ is called phase unwrapping and is the focus of this thesis.
1.2.1 Phase unwrapping operation

The phase unwrapping of a wrapped phase map is usually implemented from one pixel to another. At each pixel, an operation is carried out to recover the true phase by adding or subtracting $2\pi$ to the wrapped phase. Such an operation is called a phase unwrapping operation. To illustrate how it works, let us assume an arbitrary pixel $(x, y)$ on the phase map. Considering $(x', y')$ is a neighbour of $(x, y)$ with known wrapped and true phase. The phase unwrapping of pixel $(x, y)$ is based on $(x', y')$.

Let $\phi'(x', y')$ denote the wrapped phase and $\phi(x', y')$ denote the true phase. The relationship between $\phi'(x', y')$ and $\phi(x', y')$ is:

$$\phi'(x', y') = \left[\phi(x', y')\right]_{\text{mod}2\pi}$$  \hspace{1cm} (1.7)

where, $\left[\cdot\right]_{\text{mod}2\pi}$ denotes the module operation making the argument to be within $(-\pi, \pi]$ by adding or subtracting integral $2\pi$. $\phi'(x', y') \in (-\pi, \pi]$, $\phi(x', y') \in [-K\pi, K\pi]$, $K$ is the number of fringes, which is an integer.

When the object surface is absolutely flat and the phase map is absolutely noise free, the unwrapping is from pixel $(x', y')$ to $(x, y)$. If $\phi'(x, y)$ denotes the wrapped phase at $(x, y)$, and $\phi(x, y)$ denotes the corresponding true phase, the retrieval of $\phi(x, y)$ is implemented as follows:
1.2 Introduction to phase unwrapping and its challenges

\[
\phi(x, y) = \phi(x', y') + \Delta \phi(x, y) + \begin{cases} 
-2\pi, & \text{if } \Delta \phi(x, y) = 2\pi \\
0, & \text{if } \Delta \phi(x, y) = 0 \\
2\pi, & \text{if } \Delta \phi(x, y) = -2\pi
\end{cases}
\] (1.8)

where \( \Delta \phi'(x, y) = \phi'(x, y) - \phi'(x', y') \). Equation (1.8) provides a simple and straightforward way to recover the true phases. Equation (1.8) also shows that phase unwrapping depends on \( \Delta \phi'(x, y) \). \( \phi(x, y) \) can be recovered by adding or subtracting \( 2\pi \) to its wrapped value when a difference of \( 2\pi \) over adjacent pixels is detected.

However, such an approach may not work in practice because \( \phi'(x, y) \) is often corrupted by phase discontinuities. In fact, the wrapped phase map may exhibit two types of discontinuities: Type 1 which is due to phase wrapping, and Type 2 which is due to disturbances caused by object surface changes or illumination noises. Considering both Type 1 and Type 2 discontinuities, the wrapped phase discontinuities may not exhibit \( 2\pi \) differences and phases may not flat within a phase period. Consequently, using Equation (1.8) may trigger incorrect unwrapping operations. To find the appropriate threshold for detecting Type 1 discontinuities, let us first investigate the relationship between the differences of wrapped phases and the true phases at \( (x, y) \) and its neighbour \( (x', y') \).

Let \( \Delta \phi'(x, y) = \phi'(x, y) - \phi'(x', y') \) and \( \Delta \phi(x, y) = \phi(x, y) - \phi(x', y') \) denote the respective differences of wrapped phases and true phases at \( (x, y) \) and \( (x', y') \). Denoting the true phases at \( (x', y') \) and \( (x, y) \) as:
\[
\phi(x', y') = 2\pi \cdot k_0 + \varphi 
\]  
\[
\phi(x, y) = 2\pi \cdot k_0 + \varphi + \delta 
\]

where, \( \varphi \in (-\pi, \pi] \) and \( k_0 \) is the fringe number index, which is an integer. \( \Delta\phi(x, y) = \delta \) is the true phase change, where \( |\delta| < \pi \). Substituting Equations (1.9) and (1.10) into (1.7):

\[
\Delta\phi'(x, y) = [2\pi \cdot k_0 + \varphi + \delta]_{\text{mod}2\pi} - [2\pi \cdot k_0 + \varphi]_{\text{mod}2\pi} \tag{1.11}
\]

Considering \( k_0 \) is an integer, \( [2\pi \cdot k_0]_{\text{mod}2\pi} = 0 \) and therefore Equation (1.11) can be further simplified as:

\[
\Delta\phi'(x, y) = [\varphi + \delta]_{\text{mod}2\pi} - [\varphi]_{\text{mod}2\pi} \tag{1.12}
\]

The value of \( \Delta\phi'(x, y) \) is obtained:

- If \( (\varphi + \delta) \in (-\pi, \pi] \), which implies that \((x, y)\) and \((x', y')\) are within the same phase period, \( \Delta\phi'(x, y) = \delta \), as illustrated in Figure 1.2(a).

- If \( (\varphi + \delta) > \pi \), which implies that \((x, y)\) is within one phase period in front of \((x', y')\), \( \Delta\phi'(x, y) = \delta - 2\pi \), as illustrated in Figure 1.2(b).
1.2 Introduction to phase unwrapping and its challenges

- If $(\varphi + \delta) \leq -\pi$, which implies that $(x, y)$ is within one phase period behind $(x', y')$, then $\Delta \varphi' (x, y) = \delta + 2\pi$, as illustrated in Figure 1.2(c).

![Diagram](image-url)
1.2 Introduction to phase unwrapping and its challenges

Figure 1.2 The possible values of \( \phi'(x', y') \) and \( \phi(x, y) \) with a difference \( |\delta| < \pi \), (a) if 

\( (\varphi + \delta) \in (-\pi, \pi] \), (b) if \( (\varphi + \delta) > \pi \), (c) if \( (\varphi + \delta) \leq \pi \).

Therefore, the operation to recover \( \phi(x, y) \) is summarised as follows:

\[
\phi(x, y) = \phi(x', y') + \Delta \phi'(x, y) + \begin{cases} 
-2\pi, & \text{if } -2\pi - \delta < \Delta \phi'(x, y) \leq 2\pi - \delta \\
0, & \text{if } -\delta < \Delta \phi'(x, y) \leq \delta \\
2\pi, & \text{if } -2\pi < \Delta \phi'(x, y) \leq -2\pi + \delta 
\end{cases} \tag{1.13}
\]

where, \( 0 < \delta \leq \pi \). Equation (1.13) implies that the phase unwrapping operation will only be triggered if \( \Delta \phi(x, y) \in (-2\pi, -2\pi + \delta] \cup (2\pi - \delta, 2\pi] \). In other words, only discontinuities with a height greater than \( 2\pi - \delta \) will be considered as Type 1 and will trigger a phase unwrapping operation.
1.2.2 The limitation to large depth discontinuities

In the last subsection, the Type 2 discontinuities are assumed to be smaller than \( \delta(0 < \delta \leq \pi) \); however, it is common for objects to exhibit large depth discontinuities. Consequently, wrapped phase maps may exhibit large Type 2 discontinuities. In this subsection, the limitation of Equation (1.13) for large depth discontinuities is discussed.

Let \( \phi(x, y) \) and \( \phi(x', y') \) be:

\[
\phi(x', y') = 2\pi \cdot k_0 + \varphi
\]

(1.14)

\[
\phi(x, y) = 2\pi \cdot k_0 + \varphi + (2\pi + \delta)
\]

(1.15)

where, \( \varphi \in (-\pi, \pi) \) \( |\delta| < \pi \) and the true phase change is \( \Delta \phi(x, y) = 2\pi + \delta \). Substituting the Equations (1.14) and (1.15) into (1.7):

\[
\Delta \phi'(x, y) = [2\pi \cdot k_0 + \varphi + 2\pi + \delta]_{\text{mod}2\pi} - [2\pi \cdot k_0 + \varphi]_{\text{mod}2\pi}
\]

(1.16)

Equation (1.16) can be written as:

\[
\Delta \phi'(x, y) = [2\pi \cdot (k_0 + 1)]_{\text{mod}2\pi} + [\varphi + \delta]_{\text{mod}2\pi} - [2\pi \cdot k_0]_{\text{mod}2\pi} - [\varphi]_{\text{mod}2\pi}
\]

(1.17)

Because \( k_0 \) is an integer, \([2\pi \cdot (k_0 + 1)]_{\text{mod}2\pi} = 0\) and \([2\pi \cdot k_0]_{\text{mod}2\pi} = 0\), and Equation (1.17) can be further simplified as:
\[
\Delta \phi'(x, y) = \left[ \phi + \delta \right]_{\text{mod}2\pi} - \left[ \phi \right]_{\text{mod}2\pi}
\] (1.18)

By calculating Equation (1.18), the values of \( \Delta \phi'(x, y) \) can be obtained and are summarised as follows:

- If \( (\phi + \delta) \in (-\pi, \pi] \), which implies that \((x, y)\) is within one phase period in front of \((x', y')\), \( \Delta \phi'(x, y) = \delta \), as illustrated in Figure 1.3(a).

- If \( (\phi + \delta) > \pi \), which implies that \((x, y)\) is within two phase periods in front of \((x', y')\), \( \Delta \phi'(x, y) = \delta - 2\pi \), as illustrated in Figure 1.3(b).

- If \( (\phi + \delta) \leq -\pi \), which implies that \((x, y)\) is within two phase periods behind \((x', y')\), \( \Delta \phi'(x, y) = \delta + 2\pi \), as illustrated in Figure 1.3(c).
1.2 Introduction to phase unwrapping and its challenges

(a)

(b)
1.2 Introduction to phase unwrapping and its challenges

Figure 1.3 The possible values of \( \phi'(x', y') \) and \( \phi'(x, y) \) with a disturbance of \( (2\pi + \delta) \), where

\[
|\delta| < \pi \quad (a) \text{ if } (\phi + \delta) \in (-\pi, \pi], (b) \text{ if } (\phi + \delta) > \pi, (c) \text{ if } (\phi + \delta) \leq \pi.
\]

The retrieved value of \( \phi(x, y) \) using Equation (1.13) is:

\[
\phi(x, y) = 2\pi \cdot k_0 + \varphi + \delta
\]

(1.19)

The retrieved value of \( \phi(x, y) \) shown in Equation (1.19) is not equal to the true value. Therefore, when \( |\Delta \phi(x, y)| \geq \pi \), an incorrect unwrapping decision will be unavoidably made.
1.2.3 The key issues in phase unwrapping

Phase unwrapping is rather simple in principle, but its implementation in practice is challenging. The first issue is to determine an appropriate $\delta$ in Equation (1.13). The determination of $\delta$ is a trade-off. If $\delta$ is too large, Equation (1.13) will be less sensitive to Type 1 discontinuities. On the other hand, if $\delta$ is too small, Equation (1.13) will have less tolerance to Type 2 discontinuities. The second issue is to appropriately select the unwrapping path. Because of the recursive nature of phase unwrapping, the error caused by an incorrect phase unwrapping operation may propagate to other pixels and cause global failure. Therefore, understanding how to effectively prevent these errors is another key issue in phase unwrapping.

1.3 Existing phase unwrapping algorithms

During the last three decades, many phase unwrapping algorithms, which vary in reliability and computational requirements, have been proposed to solve the phase unwrapping problems. In general, more reliability requires higher computational power and is therefore slow. Existing phase unwrapping techniques can be classified into two categories: the temporal algorithms and the spatial algorithms. For the temporal algorithms [19-31], a set of fringe patterns with different frequencies are projected,
1.3 Existing phase unwrapping algorithms

resulting in a set of wrapped phase maps. The true phase map is recovered from the relationships between these wrapped phase maps. For the spatial algorithms [32-86], phase unwrapping is completed using a single wrapped phase map. Phases are unwrapped by following a continuous path on the wrapped phase map.

In the following part of this section, some existing algorithms including the multi-frequency algorithm [24], the path-following algorithm [32], the branch-cut method [33-49] and the quality-guided method[50-70] will be reviewed.

1.3.1 Multi-frequency algorithm

The multi-frequency algorithm [24] is a temporal phase unwrapping algorithm. In this method, several groups of fringe patterns images with different spatial frequencies are projected. Each fringe patterns image can obtain a phase map by phase shifting profilometry (PSP). Assuming the projector has a resolution of $W \times H$, the fringe patterns are horizontal. Using $f_n$ to denote the spatial frequency of the $n$th group fringe patterns, the spatial frequency of each group satisfies the following equations:

$$f_1 = \frac{1}{H}$$  \hspace{1cm} (1.20)

$$f_n = 2f_{n-1}$$  \hspace{1cm} (1.21)
where, \( n \) is a positive integer. Because the fringe patterns in the first group cover the whole measurement area, the unwrapping step for this group is not required. The following equation can be obtained:

\[
\phi_1(x, y) = \phi'_1(x, y)
\]  

where, \( \phi_1(x, y) \) denotes the true phase map, and \( \phi'_1(x, y) \) denotes the wrapped phase map that was obtained from the first group fringe patterns.

The true phase and wrapped phase map from the second group fringe patterns has the relationship below:

\[
\phi_2(x, y) = 2\pi k_2(x, y) + \phi'_2(x, y)
\]  

where, \( k_2(x, y) \) is the fringe number index, which is an integer.

According to Equation (1.5) :

\[
\phi_2(x, y) = 2\phi_1(x, y)
\]  

That is:

\[
k_2(x, y) = R\left[ \frac{\phi_1(x, y)}{\pi} - \frac{\phi'_1(x, y)}{2\pi} \right]
\]  

(1.25)
where, \( R[\cdot] \) indicates the closest integer of the argument. Therefore, \( \phi_2(x, y) \) can be unwrapped from its wrapped version by referring to \( \phi_1(x, y) \). Once \( \phi_2(x, y) \) is obtained, a similar process is used to retrieve \( \phi_3(x, y) \). This process continues until the wrapped phase map of the last group of fringe patterns is unwrapped.

\[
k_n(x, y) = R\left[\frac{\phi_{n-1}(x, y) - \phi_n'(x, y)}{2\pi}\right]
\]

(1.26)

\[
\phi_n(x, y) = 2\pi k_n(x, y) + \phi_n'(x, y)
\]

(1.27)

Equations (1.26) and (1.4) show that the true phase at each pixel is obtained without spatially accessing neighbouring pixels. Therefore, noise and disturbances will not interrupt the unwrapping results.

On the other hand, the multi-frequency method requires 6 groups of fringe patterns, each of which contains 18 frames of image patterns, to obtain satisfactory results. Although many attempts [25-30] have been made to improve the efficiency of the multi-frequency method in terms of reducing the number of fringe patterns frames, this method still needs at least 2 groups of fringe patterns to carry multi-frequency casting. Therefore, this intensive requirement for the number of image patterns makes it not suitable for most dynamic object measurements.
1.3 Existing phase unwrapping algorithms

1.3.2 Path-following algorithm

The path-following algorithm is the simplest local spatial phase unwrapping algorithm and is also known as the simple algorithm [32-33]. Using the path-following algorithm, phase unwrapping is performed using Equation (1.13) along vertical or horizontal directions. However, such an approach may not work when phase maps are corrupted by noise or disturbances as it is not able to change unwrapping direction. Due to the recursive nature of the path-following algorithm, phase unwrapping errors will propagate along the unwrapping path and cause a global unwrapping failure. Therefore, the path-following algorithm is not suitable for the measurement of objects with complex surfaces.

1.3.3 Branch-cut algorithms

The branch-cut algorithm [33-49] is also known as the residue-balancing method. Using the branch-cut method, noises and disturbances in a phase map are detected and marked by residues. Type 2 discontinuities are then identified by placing curves between positive and negative residues, referred to the branch-cuts. The branch-cuts behave as a blockage to prevent the path from getting through and the propagation of incorrect phase unwrapping operations can be prevented.
The residue is identified by calculating wrapped phase gradients in a $2 \times 2$ closed loop at each pixel on the wrapped phase map as shown in Equation (1.28) [48].

$$r(x, y) = R\left[\frac{\phi'(x, y) - \phi'(x+1, y)}{2\pi}\right] + R\left[\frac{\phi'(x+1, y) - \phi'(x+1, y+1)}{2\pi}\right]$$

$$+ R\left[\frac{\phi'(x+1, y+1) - \phi'(x, y+1)}{2\pi}\right] + R\left[\frac{\phi'(x, y+1) - \phi'(x, y)}{2\pi}\right]$$

(1.28)

where, $R[.]$ indicates an operation to retrieve the closest integer of the argument. $r(x, y)$ can only take three values: +1, -1 or 0. Pixels with $r(x, y) = +1$, and $r(x, y) = -1$ are considered to be positive residues and negative residues, respectively.

When all residues have been identified, branch-cuts are placed between every two opposite-sign residues, a process called residue-balancing.

After the branch-cuts have been placed, the true phase of each pixel is recovered along the path using Equation (1.13). The branch-cuts method is effective for detecting salt and pepper like illumination noises on the wrapped phase map but is sensitive to disturbances caused by an object surface and is therefore not suitable for the measurement of objects with complex surfaces.

### 1.3.4 Quality guided algorithm

Using the quality-guided method[50-70], every pixel on the phase map is associated with
1.3 Existing phase unwrapping algorithms

a quality parameter measuring the smoothness of the pixel compared with its surrounding pixels. Then, by employing a proper guiding strategy, the unwrapping process starts from the highest quality pixel and continues to the lower quality pixels. Because incorrect phase unwrapping operations are more likely to occur at low quality pixels, this method can effectively prevent the propagation of phase unwrapping errors.

1.3.4.1 Quality map generation

The quality map is a matrix of quality values that serves as a metric to evaluate the difficulties of unwrapping in terms of the smoothness of the wrapped phase map. Different methods to evaluate the quality include the phase derivative variance (PDV) method [59], the phase modulation (PM) method [55,61] and the maximum phase gradient (MPG) method [56,65]. All of these methods have been widely used in quality map generation for different phase unwrapping scenarios. The details of those methods are as follows.

The phase derivative variance quality map

The PDV measures the variance of the wrapped phase differences within a certain area to probe the difficulties of unwrapping in terms of the smoothness of the wrapped phase [59]. Because pixels of $2\pi$ discontinuities due to phase wrap also exist on the wrapped
phase map, these discontinuities must be removed when calculating the quality map. Assuming \( \phi' (x, y) \) is the wrapped phase map, a step referred to as removing intrinsic \( 2\pi \) discontinuities is implemented as follows:

\[
\phi(x, y) = \phi(x, y - 1) + \Delta \phi'(x, y) + \begin{cases} 
-2\pi, & \text{if } \pi < \Delta \phi'(x, y) \leq 2\pi \\
0, & \text{if } -\pi < \Delta \phi'(x, y) \leq \pi \\
2\pi, & \text{if } -2\pi < \Delta \phi'(x, y) \leq -\pi 
\end{cases}
\]  

(1.29)

where, assuming the fringe patterns are horizontal, \( \Delta \phi'(x, y) = \phi'(x, y) - \phi'(x, y - 1) \). The matrix of \( \phi(x, y) \) denotes the phase map without intrinsic \( 2\pi \) discontinuities.

The PDV at pixel \((x, y)\) is evaluated within a square window of size \( k \times k \) with \((x, y)\) being its centre, and \( k \) is an odd number usually at 3 or 5. That is:

\[
PDV(x, y) = \frac{1}{k^2} \left[ \sum_{i=-(k-1)/2}^{(k-1)/2} \sum_{j=-(k-1)/2}^{(k-1)/2} \left( \Delta^x \phi(x+i, y+j) - \overline{\Delta^x \phi(x, y)} \right)^2 ight]
\]

\[
+ \sum_{i=-(k-1)/2}^{(k-1)/2} \sum_{j=-(k-1)/2}^{(k-1)/2} \left( \Delta^y \phi(x+i, y+j) - \overline{\Delta^y \phi(x, y)} \right)^2
\]  

(1.30)

where, \((x, y)\) can be any pixel except borders of the wrapped phase map, and \( \Delta^x \phi(x, y) = \phi(x, y) - \phi(x-1, y) \) and \( \Delta^y \phi(x, y) = \phi(x, y) - \phi(x, y-1) \) are the phase differences in the \( x \) and \( y \) directions, respectively. \( \overline{\Delta^x \phi(x, y)} \) and \( \overline{\Delta^y \phi(x, y)} \) are the average phase differences in a \( k \times k \) window in the \( x \) and \( y \) directions, respectively, and are defined by
1.3 Existing phase unwrapping algorithms

following equations:

$$\Delta^x \phi(x, y) = \frac{1}{k^2} \left[ \sum_{i=-\frac{(k-1)}{2}}^{\frac{(k-1)}{2}} \sum_{j=-\frac{(k-1)}{2}}^{\frac{(k-1)}{2}} \Delta^x \phi(x + i, y + j) \right]$$  \hspace{1cm} (1.31)$$

$$\Delta^y \phi(x, y) = \frac{1}{k^2} \left[ \sum_{i=-\frac{(k-1)}{2}}^{\frac{(k-1)}{2}} \sum_{j=-\frac{(k-1)}{2}}^{\frac{(k-1)}{2}} \Delta^y \phi(x + i, y + j) \right]$$  \hspace{1cm} (1.32)$$

The quality map $Q(x, y)$ is generated according to $PDV(x, y)$, the larger $PDV(x, y)$, the lower the value of $Q(x, y)$.

The PDV is accurate in detecting phase discontinuities, but it requires extensive calculations and is not ideal for fast measurement.

**The phase modulation quality map**

In wrapped phase maps, pixels associated with lower reflectivity areas are more easily affected by illumination noise and are therefore more unreliable, causing phase unwrapping to also be unreliable. Therefore, the quality can be evaluated by probing the object surface reflectivity. The PM [55,61] is a metric associated with the reflectivity of the object surface, which can be obtained directly from the fringe patterns image. Assume the fringe patterns image is:

$$I(x, y) = I_0(x, y) + I_1(x, y) \cos(\phi(x, y))$$ \hspace{1cm} (1.33)
where, $I(x, y)$ is the fringe patterns image, $I_0(x, y)$ denotes the background light intensity at each pixel, and $I_1(x, y)$ denotes the fringe patterns intensity at each pixel. $\phi(x, y)$ denotes the phase map of the corresponding fringe patterns image. The PM is calculated by the following equation:

$$PM(x, y) = \frac{I_1(x, y)}{I_0(x, y)}$$ (1.34)

The larger $PM(x, y)$, the larger reflectivity of the corresponding pixel on the object surface. The quality map $Q(x, y)$ is therefore generated according to $PM(x, y)$: the larger $PM(x, y)$, the larger $Q(x, y)$.

The PM provides a simple and straightforward method to generate a quality map. It is effective in detecting low reflectivity areas, but it is not able to detect phase discontinuities and, therefore, has limited applications.

**The maximum phase gradient quality map**

The MPG [56,65] measures the smoothness of the phase map by evaluating the maximum phase gradient of each pixel. The calculation of MPG is shown in the following equation:
1.3 Existing phase unwrapping algorithms

\[ MPG(x, y) = \max \left\{ \left| \phi_d(x, y) - \phi_d(x-1, y) \right|, \left| \phi_d(x+1, y) - \phi_d(x, y) \right|, \left| \phi_d(x, y) - \phi_d(x, y-1) \right|, \left| \phi_d(x, y+1) - \phi_d(x, y) \right| \right\} \]  

(1.35)

where, the value range of MPG is \( 0 < MPG(x, y) \leq \pi \). The larger the MPG, the lower the quality \( Q(x, y) \).

1.3.4.2 Guiding strategies

Once a quality map has been generated, guiding strategies are used to implement phase unwrapping operations. The guiding strategy is so important to quality-guided phase unwrapping algorithm that an efficient guiding strategy can significantly improve the efficiency and reliability of quality-guided phase unwrapping algorithm. In this subsection, several existing guiding strategies will be reviewed, including the flood-fill [68], the group growing [66] and the fast scan-line [59].

The flood-fill guiding strategy

The flood-fill guiding strategy was proposed by Asundi and Wensen[68]. The details of the flood-fill guiding strategy are shown as follows.

To clearly describe the process for the flood-fill algorithm, assuming a cursor with its position indicating the pixel on which phase unwrapping is currently carried out, \( \phi'(x, y) \)
denotes the wrapped phase and \( \phi(x, y) \) denotes the unwrapped phase.

- Step 1. Identify the pixel with the highest quality value on the quality map \( Q(x, y) \), as shown in Figure 1.4(a). Assuming that the wrapped phase on this pixel is equal to its true phase, put the cursor on this point and start the unwrapping process as follows.

- Step 2. Check the quality value of the four pixels surrounding the cursor (the pixels to the left, right, top and bottom). Then move the cursor to the pixel with the highest quality and use Equation (1.13) to carry out phase unwrapping of this pixel based on the start pixel, as shown in Figure 1.4(b).

- Step 3. Check the pixels surrounding the unwrapped pixels and move the cursor to the pixel with the highest quality value. Then carry out phase unwrapping, as shown in Figure 1.4(c). Repeat this step until all pixels have been unwrapped.
As the phase unwrapping path is always from high to low quality on $\phi_{xy}(x, y)$, incorrect phase unwrapping operations are less likely to occur and spread. The flood-fill algorithm requires excessive computation because it compares the quality of adjacent pixels and determines the direction of phase unwrapping for every pixel. If the phase map is large, the flood-fill algorithm is time consuming.
**The group growing guiding strategy**

The group growing guiding strategy was proposed by Herráez et al. [66]. This method aims to improve the guiding strategy efficiency in terms of reducing operations of comparison. In the group growing guiding strategy, the concept of a pixel pair is introduced. Any pixel can form a pixel pair with its left, right, upper or lower neighbouring pixel. The quality value of a pixel pair is defined as the sum of the quality value of those two pixels, as shown in Figure 1.5(a). Each block denotes a pixel on the quality map and the digit in each block indicates the quality value of the pixel, while the digit at the border of the blocks indicates the quality value of the pixel pair.

Figures 1.5(b-j) illustrate how the group growing guiding strategy works. Initially, all pixels are considered to not belong to any group. In Figure 1.5(b), the pair consisting of pixels $f$ and $g$ has the highest quality. Assuming the true phase at pixel $f$ is equal to its wrapped phase, recover the true phase at pixel $g$ with respect to $f$. Both pixels $f$ and $g$ belong to the first group. The same process is then carried out for the pair of pixels $a$ and $b$, which has the second highest quality. Pixel $a$ is unwrapped by $b$, and both belong to the second group. The unwrapping of pixels $i$ and $j$ follows, and they constitute the third group, as shown in Figure 1.5(c). The pair of pixels $a$ and $e$ has the fourth highest quality value. Pixel $a$ is already unwrapped with respect to $b$, and the
unwrapping of \(e\) is performed with respect to \(a\). Then, pixels \(a\), \(b\) and \(e\) are considered to be in the same group.

After several steps, the unwrapping process moves to the pixel pair \(e\) and \(i\), where, \(e\) and \(i\) belong to different groups. These two groups need to be merged, as shown in Figure 1.5(d). The group that contains \(e\) is larger than the group that contains \(i\). Thus, \(i\) is unwrapped by \(e\), the phase values of all the pixels in the smaller group are added or subtracted \(2\pi\) with respect the operation on \(i\), as shown in Figures 1.5(e-f). Then, the merger of those two groups is finished. The process of unwrapping continues following the rules above until no wrapped pixels remain, as shown in Figures 1.5(g-j).

The rules of this guiding strategy can be described as follows:

- **Step 1.** If both pixels of a selected pair have not been processed before, unwrap one of them with respect to the other. Mark them as a single group.

- **Step 2.** If only one of the pixels of a selected pair has been processed before, add the unprocessed pixel to the group the processed pixel belongs to, and unwrap the unprocessed pixel with respect to the processed one.

- **Step 3.** If both pixels of a selected pair have been processed before, and they belong to different groups, unwrap the pixel from the smaller group with respect to the pixel
from the larger group, then add or subtract $2\pi$ to all pixels in the smaller group corresponding to the unwrapping operation. Then, merge those two groups.
1.3 Existing phase unwrapping algorithms

Compared with the flood-fill guiding strategy, the group growing strategy requires no comparison of quality value over neighbouring pixels for each iteration and thus saves extensive computational expenses. On the other hand, the merging of different groups in this guiding strategy still requires lots of computations and is not ideal for fast measurements.

**Fast scan-line guiding strategy**

The fast scan-line guiding strategy was proposed by Zhang *et al.*[59] and aims to
improve phase unwrapping speed. In this strategy, the pixels on the quality map are classified into several levels according to their quality values. The unwrapping is performed from higher to lower levels. The details of fast scan-line guiding strategy are shown in the following. To clearly describe this strategy, assume a cursor with its position indicating the pixel on which phase unwrapping is currently carried out.

- Step 1. Classify pixels into levels by quality thresholds. For each level, the process of unwrapping starts from one high-quality pixel near the centre of the wrapped phase map. Assuming the starting pixel at the first level is $(x_0, y_0)$, as shown in Figure 1.6., put the cursor at $(x_0, y_0)$.

- Step 2. The cursor moves horizontally from the start pixel $(x_0, y_0)$ to the border and the true phases of those pixels are recovered sequentially. If the cursor encounters a pixel that does not belong to the current level, the cursor moves back to the first pixel of the row, and the row is considered scanned. For example, if the first row is being processed, the cursor will move back to the start pixel. The rest of the pixels in this row will be put into a stack. If the cursor does not encounter such a situation, the cursor keeps moving on until the entire row is scanned.

- Step 3. After the current row is scanned, the cursor moved vertically from the start
pixel to scan another row. This process continues until all pixels in the current level are scanned.

- Step 4. The pixels in the stack are then recovered based on their unwrapped neighbour pixel. The same process is then repeated on the next level of pixels and continues until all pixels are unwrapped.

![Figure 1.6 Schematic of the fast scan-line guiding strategy](image)

In contrast to the flood-fill algorithm that involves pixel-by-pixel comparison scheme, fast scan-line provides a level-by-level unwrapping scheme that can greatly reduce the computational burden of the phase unwrapping process. On the other hand, compared with the flood-fill algorithm, fast scan-line has less ability to prevent the spread of errors and may be more sensitive to disturbances on the object surface.
1.4 Outstanding research issues and summary of contributions

1.4.1 Summary of outstanding research issues

Much has been done in the literature on the topic of phase unwrapping, however, there are still a number of unresolved issues. Existing approaches including the path-following method [32] and the branch-cut method [33-49] are computationally efficient but they are sensitive to the phase discontinuities. On the other hand, existing approaches such as the multi-frequency method [24] and the quality-guided methods [50-70] are relatively reliable in phase unwrapping but they are computationally complex. A question is whether it is possible to improve the efficiency of quality-guided methods while remaining the same phase unwrapping reliability. Another question is whether it is possible to implement reliable and efficient phase unwrapping of the wrapped phase maps which are resulted from objects with large depth discontinuities and shadows.

Secondly, when object surface observes colour distribution and spatially isolated areas, the existing methods [32-70] are not able to correctly recover true phases based on a single phase map. A question is whether it is possible to implement reliable phase
unwrapping of such a wrapped phase map.

1.4.2 Summary of contributions

During this research, some outstanding issues for improving the phase unwrapping efficiency and reliability were addressed and discussed. Then, extensive work was done that included the performance evaluation of existing approaches, and the development of several new methods. The contributions of this research can be concluded as:

- An FPP 3D acquisition system is built up.

- The phase unwrapping efficiency of the quality-guided phase unwrapping algorithm is improved.

- The phase unwrapping reliability of the quality-guided phase unwrapping algorithm is improved.

- A colour projection based 3D profilometry for measuring objects with spatially isolated areas is proposed.

- The application of the colour projection based 3D profilometry is extended to colour
distribution objects.
Chapter 2 System configuration

In this chapter, a FPP-based 3D profile acquisition system is established, as shown in Figure 2.1(a). The wrapped phase maps provided by this system will be used later in this thesis as the basis for the research that focuses on phase unwrapping. The first section of this chapter describes the configuration of the FPP system. In the second section, the procedure for retrieving the wrapped phase map using the FPP system is described.

2.1 System setup

2.1.1 Optical device setup

The geometric configuration of the FPP system is shown in Figure 1.1 in the previous chapter. The optical devices and the object are placed as follows. The line from the projector to the camera $E_pE_c$ is parallel to the reference plane. The optical axis of the camera $E_cO$ is vertical to the reference plane and crosses at point $O$ on the reference plane. The optical axes of the projector and the camera cross the reference plane at point $O$. The axes $x$ and $y$ of the projected fringe patterns image are on the reference plane. Axis $y$ is along the line $E_pE_c$. 

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To obtain high-quality images, the camera and the projector need further fine adjustment. The rules for adjusting the optical devices are as follows:

- To make full use of the resolution of the optical devices, the image area of the projector should be slightly larger than the imaging area of the camera.

- To guarantee that the captured images are clear, the camera lens should be focused on the reference plane.

- Choose an appropriate aperture stop of the camera. A too large aperture stop introduces defocusing distortion to the camera, while a too small aperture stop results in lower quality of captured images.

For the experiments in this thesis, the projector was a HITACHI CP-X260 LCD projector and the camera was a Duncan Tech MS3100 3-CCD camera with a Nikon AF-S 16-35mm lens. The distance between the reference plane and the camera was 1300mm. The camera was placed above the projector so that the connecting line from the centre of their lenses was vertical to the reference plane, and the distance between the camera and projector was 330 mm. The positions of the projector, camera and reference plane were fixed by a stable frame. The lights in the lab were turned off to provide a relatively dark room.
2.1.2 System calibration

As explained in Chapter 1, the height of the object surface is related to the geometric configuration of the system. However, it is difficult to accurately obtain these parameters using traditional methods of measurement. Moreover, the optical lenses in the projector and the camera introduce distortions which need to be compensated for. To compensate for errors or distortions, the phase map coordinates are translated to real world 3D coordinates. To achieve this, a calibration step is introduced.

The captured fringe patterns image can be considered a two-dimensional matrix. The wrapped phase map is a matrix with the same size as the fringe patterns image. The height matrix, which is converted from the phase map, has the same size as the wrapped phase map. If \( h \) is considered to be the third coordinate, the height matrix can be denoted as \((x, y, h)\). If the real world coordinates are denoted as \((W_x, W_y, h)\), where, \( W_x \) and \( W_y \) are the coordinates in the \( x \) and \( y \) directions, respectively, then \( h \) is the real height value of the object surface. The calibration process aims to describe the relationship between \((W_x, W_y, h)\) and \((x, y, h)\) to obtain the real height value on the object surface \( h \).

The conversion is related to the optical setup parameters, including reference plane
location, lens focal distance, fringe spatial frequency, camera projection angle and geometric aberrations. The calibration in this thesis was based on the method proposed by Zhang [90] in which a specially designed calibration board marked with circles is used. As shown in Figure 2.1 (b), the calibration board has a grid of 9×11 circles, and the distances between the circles are known. The four larger circles are used to tag the direction of the board.

![Figure 2.1](image)

(a) The 3D FPP system. (b) The calibration board used in the FPP system

Zhang’s [90] method requires capturing images of the calibration board from several (at least 3) different positions. The distances between the circles on the board are known, and the value of \((W_x, W_y, h)\) on the board can therefore be retrieved. Because \((x, y, h)\) on the board can be retrieved from the captured images, the relationship between the real
world coordinates \((W_x, W_y, h)\) and \((x, y, h)\) can be found.

2.2 Wrapped phase map acquisition

2.2.1 Fringe patterns projection and acquisition

Because of the software-driven nature of FPP, different patterns can be generated and implemented. Options for fringe patterns include triangular patterns [91], sawtooth patterns [92] and sinusoidal patterns [93-94]. In this thesis, sinusoidal patterns were used. The following equation shows an example of sinusoidal patterns:

\[
g(x, y) = a + b \cos(2\pi f_0 x + \delta)
\]  

(2.1)

If Fourier transform profilometry is employed, 2 fringe patterns images are required, one for the reference plane and the other for the object surface. If phase shifting profilometry (PSP) is employed, \(2N\) fringe pattern images are required, where \(N\) images are used for the reference plane and the other for the object surface, and where \(N\) is \(\geq 3\). The retrieved phase map from PSP has much higher quality than that from the Fourier transform profilometry. In this thesis, all phase maps were obtained using PSP. Considering a 5-step PSP with 10 fringe patterns images, each fringe patterns image has \(2\pi/5\) initial phase shift with its neighbours. The fringe patterns of a 5-step PSP at the
input of the projector can be denoted as follows:

\[ g_n(x, y) = a + b \cos \left( 2\pi f_{\text{proj}} x + \frac{2\pi (n-1)}{5} \right) \]  

(2.2)

where, \( n \) is the order of the frame of fringe patterns and is an integer from 1 to 5. \( a \) is the constant component that makes the projection intensity at each pixel bright enough for the camera. \( b \) is the amplitude of the sinusoidal component. These 5 frames of fringe patterns are first projected onto the reference plane and captured by the camera one at a time. Then, the same 5 frames of fringe patterns are projected onto the object surface and captured by the camera one at a time.

### 2.2.2 Non-linear distortion removal

Considering the non-linear distortion introduced by the projector, the \( n \)-th frame of a 5-step PSP from the reference plane \( s_n(x, y) \) and object surface \( d_n(x, y) \) can be denoted as follows:

\[ s_n(x, y) = R_{\text{ref}} \left[ a + \sum_{k=0}^{\infty} b_k \cos \left( 2\pi f_{\text{proj}} x + k \frac{2\pi (n-1)}{5} \right) \right] \]  

(2.3)

\[ d_n(x, y) = R_{\text{obj}} (x, y) \left[ a + \sum_{k=0}^{\infty} b_k \cos \left( 2\pi f_{\text{proj}} x + \phi_k (x, y) + k \frac{2\pi (n-1)}{5} \right) \right] \]  

(2.4)
where, $R_{\text{ref}}$ and $R_{\text{obj}}(x, y)$ are the reflectivity of the reference plane and the object surface, respectively. $b_k$ denotes the amplitude of the $k$-th order harmonic of the fringe patterns and $\phi_k(x, y)$ is the phase shifting caused by the object surface of the $k$-th order harmonic.

As shown in Equations (2.3) and (2.4), the fringe patterns captured by the camera are composed of harmonic components because of the non-linearity of the projector. Errors are introduced into measurements if the distorted patterns are used as sinusoidal patterns. Figure 2.2(a) shows the expected cross-section of the wrapped phase map, and Figure 2.2(b) shows the cross-section of the wrapped phase map extracted from the distorted patterns, in which we can see that obvious distortion has been introduced to the wrapped phase.
A look-up table [104] is an effective method to eliminate non-linear distortion. Using this method, the distorted fringe patterns are modelled as the $\gamma$ power of the original. For example, Equations (2.3) and (2.4) can be re-written as:

$$s_n(x, y) = R_{\text{ref}} \left[ a + b \cos \left( 2\pi f_0 x + \frac{2\pi(n-1)}{5} \right) \right]^\gamma \quad (2.5)$$

$$d_n(x, y) = R_{\text{obj}}(x, y) \left[ a + b \cos \left( 2\pi f_0 x + \phi(x, y) + \frac{2\pi(n-1)}{5} \right) \right]^\gamma \quad (2.6)$$

where, $\gamma$ is the non-linear distortion parameter. Once $\gamma$ is found, compensation can be
made to the fringe patterns, and the non-linear distortion can be eliminated.

For a certain digital projector, the non-linear distortion is constant [104] and the $\gamma$ can be retrieved by several tests of projector response. Equation (2.7) shows the relationship between the input of the projector, the output of the projector and $\gamma$:

$$O(x, y) = [I(x, y)]^\gamma$$  \hspace{1cm} (2.7)

where, $I(x, y)$ is the input of the projector which is known and $O(x, y)$ is the output intensity of the projector which can be observed. Figure 2.3 shows the response of the projector used in the FPP system, where the non-linear distortion parameter is $\gamma = \sqrt{6}$.

![Figure 2.3 Response of the projector in the FPP system](image)
Once $\gamma$ has been obtained, the fringe patterns for a 5-step PSP at the input of the projector is modified to be the following:

$$g_n(x, y) = \left[a + b \cos \left(2\pi f_0 x + \frac{2\pi (n-1)}{5}\right)\right]^\frac{1}{\gamma}$$

(2.8)

Therefore, the change of fringe intensity caused by non-linear distortion will be reduced to less than 0.5% compared to prior.

### 2.2.3 Wrapped phase map acquisition

In this step, the PSP is used to retrieve wrapped phase maps from the fringe patterns. Let $\phi_s(x, y) = 2\pi f_0 x$ and $\phi_d(x, y) = 2\pi f_0 x + \phi(x, y)$ be the true phases from the reference plane and the object surface, respectively. We use $\phi'_s(x, y)$ and $\phi'_d(x, y)$ to denote the wrapped version of $\phi_s(x, y)$ and $\phi_d(x, y)$, which can be retrieved as follows:

$$\phi'_s(x, y) = \arctan \left( \frac{\sum_{n=1}^{5} s_n(x, y) \cdot \sin \frac{2\pi (n-1)}{5}}{\sum_{n=1}^{5} s_n(x, y) \cdot \cos \frac{2\pi (n-1)}{5}} \right)$$

(2.9)

$$\phi'_d(x, y) = \arctan \left( \frac{\sum_{n=1}^{5} d_n(x, y) \cdot \sin \frac{2\pi (n-1)}{5}}{\sum_{n=1}^{5} d_n(x, y) \cdot \cos \frac{2\pi (n-1)}{5}} \right)$$

(2.10)
In the following chapter, these wrapped phase maps are used as the basis for the research on phase unwrapping.
Chapter 3 A composite quality-guided algorithm for improving phase unwrapping efficiency

This chapter discusses improving the phase unwrapping efficiency of the quality-guided method. In practice, it is very common for the object surface to contain both smooth areas and rough areas, implying that $\phi'(x, y)$ contains both high-quality (HQ) and low-quality (LQ) areas. If these two areas on $\phi'(x, y)$ can be identified, the simple algorithm can be applied to the HQ areas and the quality-guided flood-fill (QGFF) technique can be applied to the LQ areas; this will lead to an improvement in the efficiency of phase unwrapping. In this chapter, a technique that combines the simple algorithm and the QGFF algorithm is presented.

This chapter is organised as follows: Section 3.1 introduces the method for generating the quality map; Section 3.2 shows the scheme to identify HQ and LQ areas; Sections 3.3 and 3.4 propose a simple algorithm for HQ areas and a QGFF algorithm for LQ areas, respectively; Section 3.5 presents experimental results to verify the effectiveness of the proposed approach; and Section 3.6 concludes the chapter.
3.1 Quality map generation

For simplicity, we use the MPG [56,65] to evaluate quality. Figure 3.1 (a) illustrates an example of a wrapped phase map. Figure 3.1 (b) shows the corresponding quality map as a greyscale image where darker pixels correspond to lower quality values. Figure 3.1 (c) is the camera capture of the target.

Figure 3.1 (a) An example of a wrapped phase map, (b) the quality map and (c) the camera capture.
3.2 HQ and LQ area identification

A flow chart of this improved approach is shown in Figure 3.2. The first step is to divide \( \phi'(x, y) \) into HQ and LQ areas. To do that, the quality map is calculated using MPG and the quality values of every pixel are checked against a threshold \( \theta_H \), where \( \theta_H < \pi \). The pixel \((x, y)\) is considered to be HQ if \( MPG < \theta_H \) and LQ if \( MPG \geq \theta_H \).

![Figure 3.2 The flow chart of the approach](image)

It is important to choose a suitable value for \( \theta_H \). If \( \theta_H \) is too small, the resulting HQ areas may contain a lot of pixels with large Type 2 discontinuities, leading to errors in recovering \( \phi(x, y) \). On the other hand, if \( \theta_H \) is too large it will result in many LQ areas and low efficiency in recovering \( \phi(x, y) \). In practice, the value of \( \theta_H \) should be chosen...
together with \( \delta \), which is the margin used in Equation (1.13) to define the range of Type 1 discontinuities. In other words, all steps on \( \phi'(x, y) \) higher than \( \delta \) will be considered Type 1 and will be unwrapped. Hence the threshold should be set as \( \theta_y \leq \delta \) so that, in the HQ areas, no Type 2 discontinuities higher than \( \delta \) can be observed.

### 3.3 The simple algorithm for HQ areas

A simple algorithm [32] is proposed to unwrap the HQ areas. To track the status of the unwrapping operation, the unwrapped pixels are labelled ‘recovered’ and the others are labelled ‘unrecovered’. The simple algorithm used in this research was as follows:

- **Step 1.** Choose the pixel with the maximum quality (i.e., the lowest MPG) in the HQ area as the start pixel. Assuming that the phase value of this pixel on the wrapped phase map is the same as on the true phase, mark this pixel as ‘recovered’.

- **Step 2.** Unwrap all HQ pixels adjacent to ‘recovered’ pixels using Equation (1.13) and mark them as ‘recovered’.

- **Step 3.** Repeat Step 2 until there are no ‘unrecovered’ HQ pixels left.

An example of such an algorithm is shown in Figure 3.3. After the HQ area pixels have
been unwrapped, the flood-fill algorithm is applied to unwrap the LQ areas. The unwrapping process is complete when all pixels on the wrapped phase map have been recovered.

![Diagram](image)

Figure 3.3 An example of the simple algorithm for HQ areas: (a) step 1, (b) step 2 and (c) step 3.
3.4 The QGFF algorithm for LQ areas

To ensure that the unwrapping path is continuous, the start pixel for the QGFF algorithm must be on the border of the HQ and LQ areas. Assuming a cursor with its position indicating the pixel on which phase unwrapping is currently carried out, the QGFF algorithm used in this research was as follows:

- **Step 1.** Find the pixel with the highest quality value that is on the border of the current processing LQ area and unwrapped neighbour area. This pixel is already unwrapped. Put the cursor on this point.

- **Step 2.** Check the quality value of the pixels in the LQ area surrounding the cursor, then find the highest quality pixel and move the cursor to this pixel and unwrap it using Equation (1.13). Mark this pixel as ‘recovered’.

- **Step 3.** Check the unrecovered pixels around the pixel recovered in Step 2 and then select and move the cursor to the pixel with the highest quality value. Repeat Step 2 until all pixels in the current LQ area have been unwrapped.

An example of the QGFF algorithm is shown in Figure 3.4.
3.4 The QGFF algorithm for LQ areas

Figure 3.4 An example of the QGFF algorithm for unwrapping an LQ area: (a) step 1, (b) step 2

3.5 Experimental verification

To demonstrate the proposed algorithm and to compare it with the simple algorithm and the QGFF algorithm, each approach is applied to images acquired by a 3D shape FPP system. In the first experiment, a hand is used as the target and Figure 3.5(a) shows the camera snap of the target. Five-step PSP has been applied to acquire phase information and to calculate the surface height of the object. The phase shift between each step is $2\pi / 5$, each fringe covers 20 pixels in the reference image, $\delta = 0.25\pi$ and $\theta_o = 0.20\pi$.

Figure 3.5(b) shows the HQ and LQ pixels on the quality map.
3.5 Experimental verification

Figure 3.5 (a) The camera capture of target and (b) HQ area pixels (white) and LQ area pixels (black) on the quality map at $\theta_H = 0.20\pi$

The phase maps recovered by the simple algorithm, the proposed algorithm and the QGFF algorithm were compared and the results are shown in Figure 3.6(a), (b) and (c). To compare the robustness, it was assumed that the QGFF algorithm is the most robust algorithm and can successfully recover the true phase. Differences between the recovered phase maps of the QGFF algorithm and the phase maps of the simple algorithm and the proposed algorithm are shown in Figure 3.7(a) and (b). These show that the proposed algorithm produced a similar result to that of QGFF algorithm and is much better than the simple algorithm.
3.5 Experimental verification

Figure 3.6 The unwrapped phase maps obtained by (a) the simple algorithm, (b) the proposed algorithm and (c) the QGFF algorithm.

The corresponding 3D results are compared in Figure 3.8 which shows that the proposed algorithm is able to successfully reconstruct the 3D shape of objects.
Figure 3.7 (a) The difference map between Figure 3.6(a) and Figure 3.6(c) with greyscale black denoting the difference and (b) the difference map between Figure 3.6(b) and Figure 3.6(c) with greyscale black denoting the difference.
3.5 Experimental verification

Figure 3.8 The 3D results using phase maps recovered by (a) the simple algorithm, (b) the proposed algorithm and (c) the QGFF algorithm

In the second experiment, the proposed algorithm was used to measure a human face, which is more complicated than the human hand. Figure 3.9(a) is the camera capture of the target, Figure 3.9 (b) shows the true phase map recovered by the proposed algorithm, and Figure 3.10 shows the 3D result.

Figure 3.9 (a) The camera capture of the target and (b) the unwrapped phase map of the proposed algorithm
In the third experiment, the proposed algorithm was tested using a horse statue as the target. Figure 3.11(a) is the camera capture of the target, Figure 3.11(b) shows the true phase map recovered by the proposed algorithm and Figure 3.12 shows the 3D result.
The comparison of the phase unwrapping times for the three algorithms is shown in Table 3.1, which shows that the proposed method can successfully reduce the time used for phase unwrapping compared with the QGFF algorithm.

Table 3.1 Comparison of phase unwrapping time of different algorithms

<table>
<thead>
<tr>
<th></th>
<th>Path-following</th>
<th>Quality-guided flood-fill</th>
<th>Proposed method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unwrapping time for hand</td>
<td>0.029s</td>
<td>37.57s</td>
<td>1.054s</td>
</tr>
<tr>
<td>Unwrapping time for face</td>
<td>0.034s</td>
<td>40.65s</td>
<td>1.893s</td>
</tr>
<tr>
<td>Unwrapping time for horse statue</td>
<td>0.028s</td>
<td>39.48s</td>
<td>0.983s</td>
</tr>
</tbody>
</table>

The computation time was acquired by a Deltacom desktop (Intel Q9400@2.66GHz, 4 G memory in Matlab; the image size is 1026 x 1026 pixels)

Based on these experiments, it is clear that the proposed algorithm is able to achieve unwrapping reliability close to that of the QGFF algorithm with much less computation, and will therefore be much faster. For both algorithms, the computational burden is
measured by counting the number of pixels unwrapped by the QGFF algorithm. The details of the number of operations used by different algorithms are shown in Table 3.2.

Table 3.2 Comparison of the number of pixels unwrapped by QGFF algorithm

<table>
<thead>
<tr>
<th></th>
<th>Proposed algorithm</th>
<th>Quality-guided flood-fill</th>
</tr>
</thead>
<tbody>
<tr>
<td>Human hand</td>
<td>0.83×10^3</td>
<td>1.07×10^6</td>
</tr>
<tr>
<td>Human face</td>
<td>1.62×10^3</td>
<td>9.87×10^5</td>
</tr>
<tr>
<td>Horse statue</td>
<td>0.72×10^3</td>
<td>1.05×10^6</td>
</tr>
</tbody>
</table>

The image size is 1026×1026 pixels

To understand the reason for the efficiency improvement, the number of pixels identified as LQ using the proposed method for the three object models is counted. The LQ pixels are unwrapped by the QGFF algorithms, which account for most of the computational burden. As shown in Table 3.2, for all three objects, only 0.2% or less of the pixels are identified as LQ. This means that the number of pixels that need to be unwrapped by the QGFF algorithm is greatly reduced. Thus, the computational burden associated with the proposed algorithm is significantly lower than that of the QGFF algorithm.

Table 3.3 shows the percentage of pixels in the proposed algorithm and the simple algorithm that are different from those obtained by the QGFF algorithm. While there is little difference between the proposed algorithm and the QGFF algorithm, the difference
between the simple algorithm and the QGFF algorithm can be as high as 40%, implying that there are significant errors.

Table 3.3 Percentages of difference pixels in different algorithms compare to the total

<table>
<thead>
<tr>
<th>Percentage of different pixels over total</th>
<th>Proposed algorithm</th>
<th>Simple path-following algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Human hands</td>
<td>0.005%</td>
<td>37.339%</td>
</tr>
<tr>
<td>Human faces</td>
<td>0.008%</td>
<td>49.925%</td>
</tr>
</tbody>
</table>

The image size is 1026×1026 pixels

3.6 Summary

In this chapter, a fast quality-guided phase unwrapping algorithm is proposed. The basic idea of this method is to combine a time-efficient simple algorithm and a reliable QGFF algorithm, and take advantage of the benefits of both. In the proposed algorithm, the quality map was generated according to the MPG of pixels on the phase map and pixels were manually or automatically classified as HQ or LQ. The simple algorithm was then applied to unwrap the HQ pixels and the QGFF algorithm was applied to unwrap the LQ pixels. As demonstrated by the experimental results, the proposed algorithm used <5% of the computational burden of the QGFF algorithm at image size 1026×1026 pixels but was able to achieve similar results to those of the QGFF algorithm.
Chapter 4 A group merging approach for unwrapping phase maps with large depth discontinuities and shadows

This chapter considers improving phase unwrapping reliability and efficiency while the object surface has large depth discontinuities and shadows. Large depth discontinuities or shadows on the object surface will unavoidably trigger incorrect phase unwrapping operations. An idea to solve this problem is to identify large depth discontinuities and shadows and then remove them from phase unwrapping. In this chapter, a method is introduced whereby an image of the object is acquired by averaging the fringe patterns reflected from the object surface. Based on this image, shadow areas and phase discontinuities are identified. When carrying out phase unwrapping, the edges and shadow areas will be isolated first and the remaining areas are unwrapped using a computationally efficient technique. Experiments show that the proposed approach is able to quickly and reliably recover complex phase maps with large depth discontinuities.

This chapter is organised as follows: the schematics of shadows and large depth discontinuities identification are discussed in Section 4.1 and Section 4.2; Section 4.3
proposes a new group merging approach to commence the unwrapping; Section 4.4 presents experimental results to verify the effectiveness of the proposed approach; and Section 4.5 concludes the chapter.

**4.1 Shadow areas identification**

Assuming N-step PSP is used, a series of $N$ sinusoidal fringe patterns are projected onto the object surface, and the reflected images $d_n(x, y)$ are:

$$d_n(x, y) = r_d(x, y) \left[ I_1 + I_2 \cos \left( \phi_d(x, y) + \frac{2\pi(n - 1)}{N} \right) \right]$$

(4.1)

where, $n = 1, 2, ..., N$, and $\phi_d(x, y)$ is the phase map that contains the object surface 3D information. $r_d(x, y)$ is the reflectivity of the object surface. $I_1$ and $I_2$ are the intensity of the background and fringe components, respectively, and are assumed to be constant. Each fringe patterns consists of two components: the background component $I_1 \cdot r_d(x, y)$ and the fringe component $I_2 \cdot r_d(x, y) \cos \left( \phi(x, y) + \frac{2\pi(n - 1)}{N} \right)$. To acquire an image of the object, it is necessary to keep the background component $I_1 \cdot r_d(x, y)$. Considering the phase-shift property of the fringe component, the background component can be estimated by averaging the $N$ fringe patterns as follows:
4.1 Shadow areas identification

\[ I_d(x, y) = \frac{1}{N} \sum_{n=1}^{N} d_n(x, y) \]  

(4.2)

With \( I_d(x, y) \) available, the shadow areas can be identified. Because reflection from the object surface is stronger than that from the shadow areas, a threshold \( I_{th} \) is used to identify the two types of pixels. The pixels with a lower intensity than the threshold can be considered shadows. With all pixels checked against the threshold, a binary mask \( B(x, y) \) can be obtained:

\[
B(x, y) = \begin{cases} 
1 & \text{if } I_d(x, y) \geq I_{th} \\
0 & \text{if } I_d(x, y) < I_{th} 
\end{cases}
\]  

(4.3)

4.2 Large depth discontinuities identification

The determination of \( B(x, y) \) is able to block all the shadow pixels from participating the phase unwrapping process. However, phase discontinuities can still be observed in the areas corresponding to the object surface, and these are often caused by large depth discontinuities on the object surface shape. These large depth discontinuities usually result in the edges of the object image and can be identified by using existing technology for edge detection in the area of computer vision. In this approach, the Laplacian edge
detector is used [108-110]. Using the operational principle of the Laplacian edge detector, if there is a sharp change in greyscale in the image of interest, a second-order derivative of the intensity will exhibit a zero crossing. Following this concept, the edge of the object image \( I_d(x, y) \) can be detected as follows:

- **Step 1.** Calculate the following second-order derivative of \( I_d(x, y) \):

\[
I_d''(x, y) = \frac{\partial^2 I_d(x, y)}{\partial x^2} + \frac{\partial^2 I_d(x, y)}{\partial y^2}
\]  

(4.4)

where, \( \frac{\partial^2 I_d(x, y)}{\partial x^2} \) and \( \frac{\partial^2 I_d(x, y)}{\partial y^2} \) denote the second order derivative of \( I_d(x, y) \) in the \( x \) and \( y \) direction.

- **Step 2.** Check the first-order derivative \( I_d'(x, y) = \frac{\partial I_d(x, y)}{\partial x} + \frac{\partial I_d(x, y)}{\partial y} \) in the \( x \) and \( y \) direction for every pixel and mark the corresponding pixel as a potential edge pixel if \( I_d'(x, y) > \sigma \). \( \sigma \) is a pre-set threshold, and selection of its value is a trade-off. With a small \( \sigma \), tiny changes on an object may be detected as edges, and when a large \( \sigma \) is used, real edges may be missed.

- **Step 3.** Check the second-order derivative \( I_d''(x, y) \) of every potential edge pixel and look for significant change. Mark the pixel as an edge pixel if a sign change is observed.
4.2 Large depth discontinuities identification

With all pixels checked, a binary mask $E(x,y)$ can be obtained. $E(x,y) = 1$ if the corresponding pixel is an edge pixel, otherwise $E(x,y) = 0$. Pixels with $E(x,y) = 1$ on the wrapped phase map will be then isolated from phase unwrapping.

The proposed method employs a new algorithm, described in the following section, to unwrap the remaining pixels. Figure 4.1 shows the flow chart of the proposed approach. Note that the operator $\bigotimes$ denotes the element-by-element product of the two corresponding arrays.

![Figure 4.1 Flow chart of the proposed method](image-url)
4.3 Group merging phase unwrapping algorithm

Once the shadow areas and edge pixels have been removed, the phase unwrapping of the remaining areas is implemented using a proposed group merging phase unwrapping algorithm.

The basic idea of this method is that an MPG quality map is generated and the quality value of each pixel in the area of interest is checked against $n-1$ thresholds, and hence sorts the whole pixels into $n$ levels. Assuming the first threshold is $\theta_{\text{H}1}$, the pixels with $MPG < \theta_{\text{H}1}$ are marked as the first level pixels, which gives them the highest priority to be processed in the unwrapping process. Then, the quality values of the remaining pixels are checked against a threshold $\theta_{\text{H}2}$, where $\theta_{\text{H}2} > \theta_{\text{H}1}$, pixels with $\theta_{\text{H}1} \leq MPG < \theta_{\text{H}2}$ are marked as the second level pixels, giving them the second highest priority to be processed. After that, the quality values of the remaining pixels are checked against a threshold $\theta_{\text{H}3}$, where $\theta_{\text{H}3} > \theta_{\text{H}2}$, pixels with $\theta_{\text{H}2} \leq MPG < \theta_{\text{H}3}$ are marked as the third level pixels. This process continues until the last level pixels with $\theta_{\text{H}(n-1)} \leq MPG < 2\pi$ are identified.

Once the identification process is complete, phase unwrapping is implemented level by
4.3 Group merging phase unwrapping algorithm

level. To track the status of the phase unwrapping, pixels within the current processing
level are considered ‘active’, and pixels within the following processing levels are
considered ‘inactive’. Pixels that have been unwrapped based on the start pixel are
considered ‘recovered’, and those that have been unwrapped based on a pixel other than
the start pixel are considered ‘processed’. To clearly show the unwrapping process in
each level, a cursor is positioned to indicate the pixel on which phase unwrapping is
currently carried out. The following steps describe the unwrapping of the pixels within
the first level:

◆ Step 1. Check the quality value of each ‘active’ pixel and choose the pixel with
the highest quality to be the start pixel. Assuming this pixel is \((x_i, y_j)\), as
shown in Figure 4.2(a), let the true phase value of this pixel be equal to its
wrapped phase value. Mark this pixel as ‘recovered’.

◆ Step 2. The cursor moves horizontally from the start pixel \((x_i, y_j)\) to the left
border and the true phases of those pixels are unwrapped sequentially. Mark
the unwrapped pixels as ‘recovered’. If the cursor encounters an ‘inactive’
pixel, jump to the next ‘active’ pixel in this row. In Figure 4.2(b), this pixel is
\((x_{i-3}, y_j)\). Move the cursor to the following ‘active’ pixels and unwrap them
based on \((x_{i-3}, y_j)\) until the cursor encounters the next ‘inactive’ pixel. The
4.3 Group merging phase unwrapping algorithm

Pixels unwrapped based on \((x_{i-3}, y_j)\) are marked as ‘processed’ and constitute group 1, which is denoted as blue in Figure 4.2(b). The same operation is used on the rest of the pixels in this row. As shown in Figure 4.2(c), the next ‘inactive’ pixel is \((x_{i-7}, y_j)\). The following pixels are unwrapped based on \((x_{i-8}, y_j)\) and constitute group 2, which is denoted as yellow.

- **Step 3.** When the cursor reaches the left border, move it back to the start pixel and repeat Step 2 from \((x_i, y_j)\) to the right border, as illustrated in Figure 4.2(d).

- **Step 4.** Repeat Steps 2 and 3 to unwrap the rows above and below the start pixel \((x_i, y_j)\) until all rows have been unwrapped, as shown in Figure 4.2(e) and Figure 4.2(f).

- **Step 5.** Check whether the ‘processed’ pixels have a ‘recovered’ neighbour. If yes, add or subtract \(2\pi\) to remove wrap discontinuities between the ‘processed’ pixel and its ‘recovered’ neighbour. Then, do the same addition or subtraction operation to the pixels within the same group. Mark these pixels as ‘recovered’, as shown in Figure 4.2(g).

- **Step 6.** Repeat Step 5 until all ‘processed’ pixels have been ‘recovered’, as
shown in Figure 4.2(h–j).

(a)  

(b)  

(c)  

(d)
4.3 Group merging phase unwrapping algorithm
Figure 4.2 (a) Selection of start pixel, (b) ‘processed’ pixels in group 1, (c) ‘processed’ pixels in group 2, (d) unwrapping of the rest of the row, (e) unwrapping of the rows above \((x_0,y_0)\), (f) unwrapping of the rows below \((x_0,y_0)\), (g-i) merging of ‘processed’ pixels and ‘recovered’ pixels, and (j) merging of ‘processed’ pixels and ‘recovered’ pixels is completed.

After all pixels at level 1 are ‘recovered’, mark pixels at level 2 as ‘active’. Then select a level 1 pixel that has a level 2 neighbour as the start point, and carry out the unwrapping process within level 2. This process continues to the next levels until all pixels have been unwrapped.

This scheme requires no computationally expensive comparisons to select the unwrapping path at each pixel. Compared with the QGFF algorithm, which normally
4.3 Group merging phase unwrapping algorithm

requires operations proportional to $N^3$ for a size of $N \times N$ phase map, the proposed is much more efficient.

4.4 Experimental verification

In the experimental verification, a plaster hand model is used as the object and Figure 4.3 (a) shows the object image obtained from the fringe patterns images using Equation (4.2). The resolution of the image is 925×925 pixels. Figure 4.3 (b) shows the mask of shadow pixels when $I_{th} = 50$, and the maximum greyscale value on the object image is 255. $I_{th}$ was experientially selected to be $I_{th} = 50$, which can produce satisfactory results. Figure 4.3(c) shows the edge detection result on the object image.
4.4 Experiment verifications

Figure 4.3 (a) The object image, (b) the corresponding validation mask $B(x, y)$ when $I_m = 50$, black indicates shadow pixels and (c) the identification of edge pixels (black).

After the shadows and edges had been removed from the phase map, the remaining pixels were then classified into 3 levels with $\theta_{H1} = 0.10\pi$ and $\theta_{H2} = 0.20\pi$. Figure 4.4(a) shows the pixels in the first level, Figure 4.4 (b) shows the pixels in the second level, and Figure 4.4 (c) shows the pixels in the third level.
Figure 4.4 Pixels in different levels, (a) the first level, (b) the second level and (c) the third level, where black indicates the pixels in corresponding level.

The true phase maps recovered using the proposed algorithm were compared with the results obtained from the multi-frequency method [24] and the QGFF algorithm [68]. The true phase maps recovered using the proposed algorithm, the multi-frequency method and the QGFF algorithm are shown in Figure 4.5(a), (b) and (c). Note that the multi-frequency method is assumed to be the most reliable algorithm for recovering the true phase. Figure 4.5 (d) shows the difference between the recovered phase maps for the proposed method and the multi-frequency method. Figure 4.5 (e) shows the difference of between the recovered phase maps for the QGFF algorithm and the multi-frequency method.
Figure 4.5 (a) Recovered phase map by the proposed method (b) recovered phase map by the
multi-frequency algorithm, (c) recovered phase map by the QGFF algorithm, (d) the difference map between (a) and (b) where black denotes the difference, and (e) the difference map between (b) and (c) where black denotes the difference.

Table 4.1 Percentages of different pixels in using the proposed approach and the QGFF algorithm compared with the total

<table>
<thead>
<tr>
<th>Percentage of difference pixels compares to the total</th>
<th>The proposed approach</th>
<th>The QGFF algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plaster hand</td>
<td>0.006%</td>
<td>29.437%</td>
</tr>
<tr>
<td>Plaster head</td>
<td>0.004%</td>
<td>42.139%</td>
</tr>
<tr>
<td>Plastic bottle</td>
<td>0.008%</td>
<td>37.852%</td>
</tr>
</tbody>
</table>

The image size is $925 \times 925$ pixels

Table 4.1 shows the percentage of pixels that differ from the results obtained by the multi-frequency method. There is little difference between the proposed method and the multi-frequency method, but the difference between the QGFF algorithm and the multi-frequency method can be as high as 40%, implying that there are significant errors. Figure 4.6 shows the 3D reconstruction results using the proposed approach, the multi-frequency method and the QGFF algorithm. Both the proposed approach and the multi-frequency method are much better than the QGFF algorithm in terms of their ability to reconstruct the 3D surface. The proposed approach was also tested on other objects such as a plaster head and a milk bottle, and similar results were obtained.
4.4 Experiment verifications

Figure 4.6 (a) 3D results by the proposed method, (b) 3D results by the multi-frequency method and (c) 3D results by the QGFF algorithm

4.5 Summary

In this chapter, a new quality-guided phase unwrapping algorithm was presented. Using the proposed approach, an image of the object surface was first constructed by averaging the fringe patterns. Using the object image, the shadow areas were identified by an
4.5 Summary

Intensity threshold, and the large depth discontinuities were identified by detecting the edges of the object image. When carrying out phase unwrapping, the large depth discontinuities and shadow areas were isolated first, and then the remaining areas were unwrapped using the proposed group merging phase unwrapping algorithm. As demonstrated by the experimental results, the proposed can achieve a higher level of phase unwrapping reliability and requires less unwrapping time than the conventional QGFF method when objects have large depth discontinuities and shadow areas.
Chapter 5 A colour projection based profilometry for recovering phase maps with isolated phase areas

This chapter concerns implementing phase unwrapping of objects that have both surface colour distribution and large depth discontinuities. In this chapter, a new colour projection based 3D FPP approach is proposed, where each sinusoidal fringe is identified by a unique binary sequence. These sequences are designed to have the maximal difference using the channel encoding theory in digital communication systems. With the colour projection technique, the sinusoidal fringes and the binary stripes are projected simultaneously using red, green and blue colours. The reflected images are then captured by 3-CCD camera and decomposed into red, green and blue components. The wrapped phase map is obtained from the sinusoidal fringe patterns and phase unwrapping is implemented based on the binary sequences retrieved from the binary coding stripes. The experimental results show that this approach is able to be applied to the measurement of objects with both surface colour distribution and large depth discontinuities.

This chapter is organised as follows: Section 5.1 presents the approach; Section 5.2 demonstrates experimental results to verify the effectiveness of the proposed approach;
and conclusions are drawn in Section 5.3.

5.1 Description of the algorithm

5.1.1 Composite fringe patterns projection

In this method, each sinusoidal fringe is associated with a digital binary sequence, which is then employed to retrieve the fringe number index $k_d$. The primary colours (red, green and blue) are used to generate three images. One image is sinusoidal fringe patterns for PSP, and the other two images are binary stripes that are used to encode the binary sequences to identify $k_d$ for phase unwrapping.

The following is an example to demonstrate the implementation of the method described above. Without loss of generality, we assume that the sinusoidal patterns are projected in red and the two binary stripe patterns are in green and blue, respectively. To implement an $M$-step PSP, $M$ frames of colour images are projected, which is the combination of $M$ sinusoidal fringe patterns in red and $2M$ green and blue binary strip patterns. The sinusoidal patterns are equally shifted in their initial phase as:

$$g_m(x, y) = I_1 + I_2 \cos \left(2\pi f_0 x + \frac{2\pi(m-1)}{M}\right)$$

(5.1)
where, \( m = 1, 2, \ldots, M \). \( g_m(x, y) \) is a periodic function of \( x \), with each period corresponding to a fringe. Assuming that there are \( N \) fringes on \( g_m(x, y) \), \( N \) different binary sequences \( q_n \) (\( n = 1, 2, \ldots, N \)) are required to identify the fringe number index \( k_d \), implying that the length of each \( q_n \) must be longer than \( \left\lceil \log_2 N \right\rceil \) bits.

As mentioned above, the \( 2M \) binary stripe patterns are used to carry the digital sequences, and are projected together with the sinusoidal patterns with the aim of identifying the fringe number indices. However, the binary stripes are often disturbed by object surface colour or environmental illumination. If the binary stripes are considered as signals, these disturbances can be modelled as interferences and noises to the signal. To cope with disturbances, the channel coding technique [148] can be used to encode a length \( k \) message sequence by adding \( (n-k) \) redundant bits in a controlled manner to produce an \( n \) bit sequence, called a codeword, which yields extra resistance to interference and noise. The set of all codewords is called an \((n,k)\) code. In the proposed approach, each \( q_n \) is encoded with length \( k \) into a length \( 2M \) codeword \( C_n \). The codeword \( C_n \) is then transmitted as binary stripes, which will be corrupted by the disturbances. When implementing phase unwrapping at the receiver side, this corrupted binary stripe is then decoded according to the structure introduced by the channel code and an estimation of \( q_n \) is generated. It is well known that the correction capability of a channel code (i.e., the
number of error bits the code can correct) is determined by its minimum Hamming distance $d_H$ between codewords. A channel code with a minimum Hamming distance of $d_H$ can correct $\left\lfloor (d_H - 1)/2 \right\rfloor$ bit errors. Consider an example where the number of fringe patterns is 20, the shortest message length will be $k=5$. Because the length of $C_n$ is even, $(6,5)$, $(8,5)$, $(10,5)$ and $(12,5)$ binary linear block codes were considered. The corresponding upper bounds on the minimum Hamming distances are 2, 2, 4, and 4. In consideration of both the minimum distance and code length, a $(10,5)$ code with a minimum distance of 4 is attempted to be constructed. In fact, a code can be built by fixing the first 6 message bits of the $(16,11)$ extended Hamming code to be 0, which leads to a $(10,5)$ shortened extended Hamming code with a minimum distance of 4.

Once the $C_n$ are obtained, the binary stripes in blue and green are generated based on $C_n$. If $C_n$ is denoted as $C_n = \{a_{1n}, a_{2n}, ..., a_{Mn}, b_{1n}, b_{2n}, ..., b_{Mn}\}$, each element in $C_n$ is a binary codeword ‘1’ or ‘0’. The first half of $C_n$, which is $\{a_{1n}, a_{2n}, ..., a_{Mn}\}$, determines the intensity of the $n$th binary stripe in green from the first frame to the $M$th frame. The second half of $C_n$, which is $\{b_{1n}, b_{2n}, ..., b_{Mn}\}$, determines the intensity of the $n$th binary stripe in blue from the first frame to the $M$th frame. The intensity of a binary stripe has two levels: bright or dark, corresponding to ‘1’ or ‘0’ in $C_n$. Using two matrices, $A$ and $B$, to denote the intensity of the binary stripes to be projected:
5.1 Description of the algorithm

\[
A = \begin{pmatrix}
a_{11} & a_{12} & \ldots & a_{1N} \\
a_{21} & a_{22} & \ldots & a_{2N} \\
\vdots \\
a_{M1} & a_{M2} & \ldots & a_{MN}
\end{pmatrix}
\]

and

\[
B = \begin{pmatrix}
b_{11} & b_{12} & \ldots & b_{1N} \\
b_{21} & b_{22} & \ldots & b_{2N} \\
\vdots \\
b_{M1} & b_{M2} & \ldots & b_{MN}
\end{pmatrix}
\]

where, the transpose of the \( n \)th column in \( A \) is the first half of \( C_n \) and the transpose of the \( n \)th column in \( B \) is the second half of \( C_n \). Codeword \( a_{mn} \) in matrix \( A \) determines the intensity of the \( n \)th binary stripe in green at the \( m \)th frame. Similarly, codeword \( b_{mn} \) in matrix \( B \) determines the intensity of the \( n \)th binary stripe in blue at the \( m \)th frame.

Figure 5.1 shows an example of the composite fringe patterns at the \( m \)th frame and the red, green and blue components, where, \( N=20 \) and \( M=5 \). The binary stripes in green start from the top to the bottom and are determined by the codewords \( \{a_{m1}, a_{m2}, \ldots, a_{m20}\} \), which is the \( m \)th row of matrix \( A \), where \( \{a_{m1}, a_{m2}, \ldots, a_{m20}\} = \{1,0,1,1,1,0,0,1,1,0,0,1,1,1,0,0,1,1\} \).

The binary stripes in blue start from the top to the bottom and are determined by \( \{b_{m1}, b_{m2}, \ldots, b_{m20}\} \), where \( \{b_{m1}, b_{m2}, \ldots, b_{m20}\} = \{1,1,0,1,0,1,1,0,0,1,1,0,0,0,1,1,1,0,1,0\} \).

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which is the $m$th row of matrix $B$.

Figure 5.1 (a) A colour coded fringe patterns at the $m$th frame where the order of fringes is from the top to the bottom, (b) sinusoidal fringes in red, (c) binary coding stripes in green where $N=20$ and (d) binary coding stripes in blue where $N=20$
5.1 Description of the algorithm

5.1.2 Phase unwrapping

Once the composite fringe patterns are captured by the 3-CCD camera, they are decomposed into red, green and blue components. The codes from the binary stripes in green and blue at each pixel are retrieved by a suitable intensity threshold. The wrapped phase map is retrieved from the sinusoidal fringes and the encoded sequences are retrieved from the binary stripes. Using $C(x, y)$ to denote the retrieved encoded sequence at pixel $(x, y)$, the fringe identification sequence at each pixel can be decoded from $C(x, y)$. $q(x, y)$ denotes the retrieved fringe identification sequence and is compared with $q_n$ to determine the fringe number index $k_n$ at pixel $(x, y)$. Figure 5.2 shows the flowchart of the proposed approach.
5.1 Description of the algorithm

5.2 Experimental verification

To demonstrate the proposed approach, a colour plaster doll was used as the object. A 5-step PSP was used so that the number of frames was 5 and the number of fringes was 32. The fringe identification sequences $q_n$ are 32 5-bit binary digits from 0 to 31, where $q_1 = [0, 0, 0, 0, 0]$ and $q_{32} = [1, 1, 1, 1, 1]$. Then, $C_n$ was obtained by encoding each corresponding $q_n$ through the (10,5) encoder. Figure 5.3 shows the 5 frames of composite fringe patterns. The sinusoidal fringe patterns and binary stripes images were then decomposed as shown in Figure 5.4.

Figure 5.2 The flowchart of the proposed approach
Figure 5.3 (a) The first frame, (b) the second frame, (c) the third frame, (d) the fourth frame and (e)
5.2 Experimental verification

the fifth frame of the composite fringe patterns projected on the object surface

Figure 5.4 (a) The sinusoidal fringe patterns in red (b) the binary coding stripes in green and (c) the binary coding stripes in blue

The wrapped phase map was then obtained by taking phase detection on the sinusoidal fringe patterns. The codeword \( C(x, y) \) at each pixel was acquired from the binary stripes and the fringe identification sequence \( q(x, y) \) was decoded from \( C(x, y) \). In some areas, the retrieved \( C(x, y) \) observed 1 digit error. These errors were then corrected when decoding \( C(x, y) \). Figure 5.5 shows the unwrapped phase map from which the proposed
method was able to recover the phase even 1 digit error at \( C(x, y) \).

Figure 5.5 (a) Wrapped phase map and (b) unwrapped phase map

Figure 5.6 The 3D result

The 3D result obtained by the proposed approach is shown in Figure 5.6.

To prove the feasibility of recovering a phase map that contains spatially isolated areas, the proposed approach was tested on two spatially independent objects. Figure 5.7(a)
5.2 Experimental verification

shows the appearance of two spatially independent objects. Figure 5.7(b) shows the recovered true phase map, which implies that the proposed approach is able to recover a phase map that contains spatially isolated areas.

Figure 5.7 (a) The appearance of two spatially independent objects, (b) the recovered true phase map and (c) The 3D result
5.3 Summary

In this chapter, a new colour projection based 3D FPP method was presented to recover the 3D shape of objects with both spatially isolated areas and surface colour. In the proposed approach, $M$ frames of sinusoidal fringe patterns images were used to recover the phases. Each fringe of the sinusoidal patterns was identified by a binary sequence $q_n$. A (10,5) shortened extended Hamming code was constructed with a minimum Hamming distance $d_H$ at 4, with which the $q_n$ were embedded into $2M$ frames of binary stripes. With the colour projection technique, the sinusoidal fringe patterns and binary stripes were projected simultaneously using red, green and blue colours. The composite images were then decomposed into red, green and blue components. The wrapped phase map was obtained from the sinusoidal fringes and the fringe identification sequences for unwrapping were obtained from the binary stripes. As proved by the experiments, the proposed method is able to measure objects with both surface colour and large depth discontinuities.
Chapter 6  Conclusion

6.1 Thesis Summary

The main aim of this thesis was to improve the efficiency and reliability of phase unwrapping approaches for the 3D FPP technique. During the study of the existing phase unwrapping methods for the 3D FPP technique, some outstanding issues for improving the phase unwrapping efficiency and reliability were addressed and discussed. Then, extensive work was done that included the performance evaluation of existing approaches, and the development of several new methods.

This thesis first theoretically derived the phase unwrapping operation and its limitation for discontinuities on the object surface. Then, it improved phase unwrapping efficiency of the QGFF algorithm. This thesis also enabled reliable phase unwrapping of phase maps from objects with large depth discontinuities and shadows. Finally, this thesis developed a new colour projection based 3D FPP, which can be applied to measure objects with large depth discontinuities and distributed surface colour.

Experiments were conducted in each chapter to demonstrate the feasibility of the proposed algorithms. Results showed that the new algorithms can reliably unwrap phase
maps in different conditions with significant efficiency improvements compared with existing phase unwrapping approaches.

6.2 Future Work

While this thesis has demonstrated that fast and reliable phase unwrapping for FPP systems has been successfully achieved, there are still many other research issues for FPP techniques.

◆ Developing efficient and reliable phase detection method for real-time applications.

In real-time applications, people are always in pursuit of faster phase unwrapping and detection methods. Conventional reliable phase detection methods such as PSP always require three or more fringe images, and this makes them too slow for real-time applications. Thus, the development of efficient and reliable phase detection methods is important.

◆ Panoramic 3D object reconstruction.

Current FPP techniques are only able to provide 3D profile imaging. To obtain a panoramic view of a real object, the object has to be turned with particular
stepping angles several times during the measurement. Consequently, the turning angles must be precisely adjusted, in which case there will be no problem in assembling the different views. However, if the turning is implemented by a mechanical device, the turning may not be precise enough. Therefore, fringe patterns images need to overlap and image stitching and merging are required to reconstruct panoramic 3D views.

**Colour cross talk elimination**

Colour digital projection enables the FPP technique to project fringe patterns images simultaneously, and reliable phase unwrapping or faster fringe patterns projection can be achieved. However, the sensing domains of different colour channels in a camera are always designed overlapped. In other words, light in one colour may ‘leak’ to other sensors and lead to significant measurement errors. These errors must be compensated for to improve the measurement accuracy of colour digital projection based FPP.
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