Error exponent of amplify and forward relay networks in presence of I.I.D. interferers

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Abstract
In this paper, we derive the random coding error exponent of amplify-and-forward (AF) relay networks in presence of arbitrary number of independent and identically distributed (i.i.d.) interferers both at the relay and the destination. Multiuser networks are common examples of interference limited networks. We derive the ergodic capacity of the network and present simulation results on the performance of the network where we compare the capacity and error exponent performance of interference limited networks with noise limited networks. Numerical results show that noise limited networks outperform interference limited networks even when only a very few interferers exist in the network.

Keywords
i, presence, networks, relay, exponent, forward, error, amplify, interferers

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Error Exponent of Amplify and Forward Relay Networks in Presence of I.I.D. Interferers

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Abstract—In this paper, we derive the random coding error exponent of amplify-and-forward (AF) relay networks in presence of arbitrary number of independent and identically distributed (i.i.d.) interferers both at the relay and the destination. Multiuser networks are common examples of interference limited networks. We derive the ergodic capacity of the network and present simulation results on the performance of the network where we compare the capacity and error exponent performance of interference limited networks with noise limited networks. Numerical results show that noise limited networks outperform interference limited networks with only a very few interferers exist in the network.

Index Terms—Interference network, Random coding error exponent, amplify-and-forward, ergodic capacity

I. INTRODUCTION

Capacity analysis of wireless networks is crucial to determine the reliable data rate that a channel can provide. Capacity bounds of general relay channel has been studied in many papers using information theory [1]–[3]. However, most of these have characterized capacity bounds for fading and Gaussian relay networks. An approximate capacity bound has been derived for a Gaussian interference relay network in [3]. In reality, with the growing number of wireless devices interference is becoming unavoidable in practical networks. Performance analysis of interference in cooperative relay networks has been studied extensively in [4]–[7]. The authors in [4], [5] consider a scenario where the relay node is affected by interference in an interference limited network and the receiver node remains interference free. The relays considered can estimate the instantaneous channel state information (CSI) of interfering channels to use it in the gain. However, the assumption of AF relay gain parameter that includes the instantaneous or average channel information of interfering channels as considered in [4]–[6] requires additional computational capability at the relaying node, and in certain cases where the interfering signals are not known to the relay a priori, the technique is not applicable. A total interference limited cooperative network has been studied in [6], [7]. Outage performance of a dual hop network has been investigated using a fixed gain relay in [6] and hypothetical gain AF relay in [7] with arbitrary number of interferers.

C. E. Shannon defined a reliability function or error exponent to describe the probability of the error as a function of code rate $R$ and code length $W$ as,

$$E(R) \triangleq \lim_{W \to \infty} \sup_{R,W} \frac{-\ln P_{\text{opt}}(R,W)}{W}$$

(1)

where $P_{\text{opt}}(R,W)$ is the average block error probability for the optimal block code of length $W$ and rate $R$ [8]. In practice, derivation of exact error exponent (1) involves quite complex mathematical procedures, however, a lower bound on the error exponent known as random coding error exponent (RCEE), (defined in [9], [10]) exists. This RCEE measurement provides important information about the design requirements of a codeword to achieve a given target rate $R$ below the capacity $C$ of the channel. In [11] and [12], the authors have derived the RCEE of cooperative relay networks and two way relay networks respectively using CSI assisted ideal gain AF relays and obtained the ergodic capacity and cutoff rate for the network. Recently, the random coding error exponent and the capacity of a dual hop cooperative relay network using single antenna CSI assisted AF relay were derived in [13]. However, all the analysis regarding RCEE has been performed for noise limited relay networks only, and to the best of our knowledge RCEE and capacity analysis of cooperative relay networks in interference using RCEE has not been studied to date.

In this paper, we derive the closed form RCEE of cooperative relay network in presence of arbitrary number of i.i.d. interferers using an ideal gain AF relay. I.i.d. interferers can represent the worst case scenario of interference limited networks in a similar interference power constraints. In [14], the authors show that the performance of interference network does not depend on individual interferer’s power but on the aggregated power of the interferers. Thus, our i.i.d. assumption will provide a lower bound on the performance of the nonidentically distributed interference network, considering that the maximum power of the nonidentically distributed interferer allocated to the i.i.d interferers. Furthermore, we provide the ergodic capacity expression of the network.
where $x$ is the node interference sources for the relay and receiver nodes respectively, $G$ is the AF relay gain, $n_1 \sim \mathcal{CN}(0, \sigma_1^2)$ and $n_2 \sim \mathcal{CN}(0, \sigma_2^2)$ are AWGN at the relay and the destination respectively. Thus the SINR with arbitrary relay gain is given by

$$\gamma_{\text{SINR}} = \frac{G^2|h_1|^2|h_2|^2P_S}{G^2|h_2|^2|\Sigma_{1,1}h_{1,1}^T + h_{1,2}\Sigma_{1,2}|^2 + G^2|h_2|^2\sigma_1^2 + \sigma_2^2}$$

where $\Sigma_{1,1} = E\{x_{1,1}^*x_{1,1}\}$ and $\Sigma_{1,2} = E\{x_{1,1}^*x_{1,2}\}$ are diagonal matrices of the transmission powers of interfering signals at the relay node and the destination respectively and $P_S$ is the transmission power of the source node. Assuming that the network is interference limited, we set $\sigma_2^2 = 0$ in (3), where $i \in \{1, 2\}$. With an ideal/hypothetical AF relay gain the end-to-end signal to interference power ratio (SIR) is given by [7].

$$\gamma_{\text{SIR}} = \frac{\gamma_1 \gamma_2}{\gamma_1 \gamma_{\text{I,2}} + \gamma_2 \gamma_{\text{I,1}}}$$

where $\gamma_{\text{I,1}}$ and $\gamma_{\text{I,2}}$ are the total instantaneous interference power at the relay and the destination respectively. Considering i.i.d. interferers both at the relay and destination, the probability density function (PDF) of the end-to-end signal to interference ratio (SIR) $\gamma_{\text{SIR}}$ at the destination can be written as [7],

$$f_{\gamma_{\text{SIR}}} (\gamma) = \frac{L_1L_2}{\Gamma(L_1L_2)} \left( \frac{\lambda_{1,1} \gamma_1}{\lambda_1} + \frac{\lambda_{1,2} \gamma_2}{\lambda_2} \right)^{-L_1} \times \left( \frac{\lambda_1 \gamma_1 + \lambda_{1,2} \gamma_2}{\lambda_1} + \frac{\lambda_{1,1} \gamma_1 + \lambda_{1,2} \gamma_2}{\lambda_2} \right)_2 F_1 \left( L_1L_2; L_1L_2 + 1; \lambda_1 \gamma_1 + \lambda_{1,2} \gamma_2 \right)
\times \left( \frac{\lambda_1 \gamma_1 + \lambda_{1,2} \gamma_2}{\lambda_2} + \frac{\lambda_{1,1} \gamma_1 + \lambda_{1,2} \gamma_2}{\lambda_1} \right)_2 F_1 \left( L_1L_2 + 1; L_1L_2 + 2; \lambda_1 \gamma_1 + \lambda_{1,2} \gamma_2 \right)$$

where $\lambda_1 \gamma_1 + \lambda_{1,2} \gamma_2$ is the sum of the total instantaneous interference power at the relay and the destination respectively, and $k_1 (\gamma) = \lambda_{1,1} \gamma_1 + \lambda_{1,2} \gamma_2$. The probability density function of the end-to-end signal to interference ratio (SIR) $\gamma_{\text{SIR}}$ at the destination can be written as [7].

IV. ERROR EXPONENT: I.I.D. INTERFERENCE NETWORK

The random coding error exponent is defined as a function of input distribution function $Q(x)$, a factor $\rho \in [0, 1]$ and rate $R \leq C$ (for details please read ch. 5 of [9]), which is jointly optimized over $Q(x)$ and $\rho$ at a desired rate $R$. However, the Gaussian input distribution has often been used in many publications such as in [12], [17] to avoid the mathematical complexity involved in the joint optimization of the reliability function. This assumption provides near optimal result for the error exponent at a rate near the channel capacity. We analyze the error exponent for i.i.d. interference network where all the interfering channels to the relay and to the destination are

\footnote{Hypothetical relay gain proposed by Hasna et. al. in [15] simply inverses the instantaneous channel gain as $G^2 = \frac{P_R}{\gamma h_{1,2}^2}$.}
independent and identically distributed. The error exponent of the dual hop AF network with Gaussian input distribution can be written as [9],

$$E_r(R) = \max_{0 \leq \rho \leq 1} \{ E_0(\rho) - 2R \rho \}$$

with

$$E_0(\rho) = - \ln E_{\gamma_{SIR}} \left\{ \left( 1 + \frac{\gamma}{1 + \rho} \right)^{-\rho} \right\}$$

$E_{\gamma_{SIR}}$ denotes the statistical expectation operation over random variable $\gamma_{SIR}$. Using the series expression of Gauss hypergeometric function as in [16, eq. (15.1.1)] in eq. (5) and (7), random coding error exponent over i.i.d. interference channels can be written as,

$$E_0(\rho) = - \ln \left[ L_1 L_2 \sum_{n=0}^{\infty} \sum_{k=1}^{n} \binom{n}{k} \left( \frac{\lambda_I}{\lambda} \right)^{k+1} \right] \times \frac{\Gamma(L_1 + n) \Gamma(L_2 + n)}{\Gamma(\rho) \Gamma(L_1 + L_2 + n) \Gamma(L_1 + L_2 + 2n + 1)} \times \frac{\gamma^{k+1} G_{1,1}^{1,1} \left( \frac{\lambda_I}{\lambda} \right)}{\rho \Gamma(L_1 + n) L_2 + n) \Gamma(L_1 + L_2 + 2n + 1) \Gamma(L_1 + L_2 + n + 1)} \times \frac{\gamma^{k+1} G_{1,1}^{1,1} \left( \frac{\lambda_I}{\lambda} \right)}{\rho \Gamma(L_1 + n) L_2 + n) \Gamma(L_1 + L_2 + 2n + 1) \Gamma(L_1 + L_2 + n + 1)}$$

Using [18, eq. (8.24.1.1)] and after some manipulations the RCEE can be expressed as (9) shown at the bottom of the page 4, where $K$ represents the maximum number of sum terms required for convergence. From numerical calculations we found $K \geq 100000$ is sufficient for convergence. $G_{m,n}^a b(z)$ is the Meijer-G function defined as [18, eq. 8.2.1.1] and $\alpha = \frac{\lambda_I}{\lambda}$. 

V. ERGODIC CAPACITY

Ergodic capacity $\langle C \rangle$ of this dual hop network is given by,

$$\langle C \rangle = \frac{1}{2} \left[ \frac{\partial E_0(\rho)}{\partial \rho} \right]_{\rho=0}$$

$$= \frac{1}{2} \int_0^\infty \ln (1 + \gamma) f_{\gamma_{SIR}}(\gamma) d\gamma$$

Let

$$J(\gamma, \alpha, k) = \int_0^\infty \gamma^k \ln (1 + \gamma) \left( 1 + \frac{\gamma}{\alpha} \right)^{-n} d\gamma$$

Using [19, eq. 2.6.10.60], for $\alpha > 1$ we have,

$$J(\gamma, \alpha, k) = \alpha^{k+1} B(k + 1, \alpha, k - 1) \left[ \frac{1}{\alpha} \sum_{l=0}^{\infty} \frac{(k + 1)l}{l!} \right] \psi(n + l) - \psi(n - k - 1)$$

VI. NUMERICAL RESULTS

For numerical evaluation we assume the average gain of the main channels and the interfering channels are unity. Furthermore, in noise limited networks we consider the noise variances at the relay and the destination are equal to $\sigma^2$. RCEE and the ergodic capacity expressions (9) and (13) contain functions of infinite sums. We observed that for $n \geq 100000$ sum terms, the expressions converge to the simulation results. In all figures, parameters average SIR per hop in interference limited network and average SNR per hop when the network is noise limited have been used.

Fig.2 compares the probability density function in equation
Fig. 3 plots the random coding error exponent as a function of data rate in nats/Hz for 10, 20, and 30 dB average SIR per hop. The figure compares the RCEE of interference limited network consisting of 2 and 4 interferers at the relay and destination nodes with the noise limited network. It shows that, noise limited networks outperform the interference limited networks even when there are only 2 interferers both at the relay and destination. At 20 dB per hop average SIR (SNR in noise limited network), for example, the RCEE of noise limited network is almost 1.6 times higher than the interference limited networks with 2 interferers both at the relay and the destination.

VII. CONCLUSION

In this paper, we have derived the RCEE of cooperative relay network in presence of arbitrary number of i.i.d. interferers using an ideal gain AF relay. The expression of ergodic capacity of the network is also derived. Numerical results on RCEE and ergodic capacity show that interference limited networks perform worse than noise limited networks even when only a very few interferers exist in the network.

VIII. ACKNOWLEDGMENTS

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\[
E_0(\rho) = -\ln \left[ L_1 L_2 \sum_{n=0}^{K} \sum_{k=0}^{n} \binom{n}{k} \frac{\Gamma(L_1 + n) \Gamma(L_2 + n) 2^k}{\Gamma(\rho) \Gamma(L_1 + L_2 + n + 1) n!} \left\{ \begin{array}{c} (L_1 + L_2) \\ G_{2,2}^2 \left( \frac{\alpha}{1 + \rho} \right) 0, L_1 + L_2 + 2n - k - 1 \\ \Gamma(L_1 + L_2 + 2n + 1) \end{array} \right. \right] 
\]

\[
(C) = \sum_{n=0}^{K} \sum_{k=0}^{n} \binom{n}{k} \frac{L_1 L_2 \Gamma(L_1 + n) \Gamma(L_2 + n) 2^{k-1}}{\Gamma(L_1 + L_2 + n + 1) n!} \left( \begin{array}{c} (L_1 + L_2) \end{array} \right) \left( \begin{array}{c} k + 1, L_1 + L_2 + 2n - k \end{array} \right) 
\]

\[
\left\{ \begin{array}{c} \sum_{l=0}^{\infty} \frac{(k + 1)_l}{\alpha^{k+1} l!} \left( 1 - \frac{1}{\alpha} \right)^l \psi(L_1 + L_2 + 2n + l + 1) - \psi(L_1 + L_2 + 2n - k) \\ 2(L_1 + n)(L_2 + n) \left( L_1 + L_2 + n + 1 \right) \left( \begin{array}{c} k + 2, L_1 + L_2 + 2n - k + 1 \end{array} \right) \sum_{l=0}^{\infty} \frac{(k + 2)_l}{\alpha^{k+2} l!} \left( 1 - \frac{1}{\alpha} \right)^l \right. \\
\times \psi(L_1 + L_2 + 2n + l + 3) - \psi(L_1 + L_2 + 2n - k + 1) \end{array} \right. \right\}, \quad \alpha > 1
\]
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