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BELT CONVEYOR TRANSFERS: QUANTIFYING AND MODELLING MECHANISMS OF PARTICLE FLOW

A thesis submitted in fulfilment of the
requirements for the award of the degree

DOCTOR OF PHILOSOPHY

from

UNIVERSITY OF WOLLONGONG

by

DAVID BRYAN HASTIE, BE (Hons), ME (Hons)

SCHOOL OF MECHANICAL, MATERIALS AND
MECHATRONIC ENGINEERING

FACULTY OF ENGINEERING

2010

CERTIFICATION

I, David Bryan Hastie, declare that this thesis, submitted in fulfilment of the requirements for the award of Doctor of Philosophy, in the School of Mechanical, Materials and Mechatronic Engineering, Faculty of Engineering, University of Wollongong, is wholly my own work unless otherwise referenced or acknowledged. This document has not been submitted for qualifications at any other academic institution.

_____
(Signature)

David Bryan Hastie
May 2010

ABSTRACT

The purpose of this research was to determine if either analytical methods or numerical discrete element modelling could be used with accuracy to design conveyor transfers. This goal was achieved using two test materials, polyethylene pellets and corn, which were selected for their different particle and bulk properties and also for a third product, iron ore, but to a lesser extent due to test rig limitations.

The design of conveyor transfers has traditionally been based on either trial and error or previous experience and seen as a "black art" rather than a science, as such very few design guides are available.

The design of conveyor transfers can be based on experimental investigations, although this method can be costly to companies, taking vital resources away from the key goal of continuous production. The analytical models have existed for some time and have become widely accepted design tools; however, there is limited validation of these to determine their overall performance (both advantages and disadvantages). The analytical models are two dimensional in application and their accuracy with respect to the three dimensional nature of transfer chutes is not clear. This is an area which needs further investigation.

The design of transfer chutes has undergone an evolution since the advent of discrete element modelling (DEM) as well as increases in computer processing power. The potential to simulate and predict the behaviour of a transfer chute design before it is constructed can be highly desirable with the prospect of saving substantial time and money. This being said, there has been little validation published on the application of DEM in industrial applications, although in recent years this has started to increase with the realisation that companies need to be convinced this is a legitimate design tool. Additional DEM validation is warranted with respect to conveyor transfers.

An experimental test program was undertaken following the design and commissioning of a novel conveyor transfer research facility. This experimental work focussed on two main areas; investigation of particle flow of material through a conveyor transfer hood and spoon and the generation of conveyor trajectories. From these areas, 'real' data was obtained for a range of granular free-flowing products, using a combination of high-speed video capture and still photography, for the purposes of validation. The effect of belt speed, material feed rate and the positioning of the transfer hood and spoon were considered in these investigations.

Additional to this experimental work was the testing and collection of a wide range of particle and system characteristics for use in the analytical modelling and discrete element modelling components of this research.

Two analytical models were then used to predict the particle flow of the test materials through the conveyor transfer hood and spoon. Belt speed, material feed rate and positioning of the transfer hood and spoon were all considered as part of this analysis to provide direct comparisons with the data obtained from the experimental testing.

Prediction of the conveyor trajectories was performed using seven trajectory models available in the literature. These comparisons investigated the effect of belt speed and

mass flow rate on the trajectory profiles, again providing a direct link with the experimental data.

The discrete element method was used to generate three dimensional simulations of the material flow through the conveyor transfer hood and spoon and also conveyor trajectories, based on 3D CAD models of the conveyor transfer research facility. These simulation outputs were then compared to both the experimental data and data obtained from the analytical models. Two software packages were used, Chute Maven™ and EDEM. Chute Maven™ was used to produce the initial transfer chute and trajectory simulations using spherical particles. High material feed rates corresponding to those tested experimentally could not be simulated and so EDEM was employed to develop further simulations. The fact that EDEM has the ability to model both spherical and shaped (clustered) particles was utilised to investigate the effect of shape on simulation output.

A critical aspect of any discrete element modelling is whether the outputs are realistic. To minimise any potential issues, a wide range of bench-scale calibration experiments and simulations were also completed to validate both DEM packages used.

It can be concluded that the analytical models for conveyor transfers provided close approximations from a two dimensional perspective, however, there were some slight over-predictions evident in some situations. For conveyor trajectories, the models presented a substantial variation in prediction, however, one method stood out as being accurate under all conditions for the materials tested experimentally.

Findings from the discrete element modelling showed the dynamic behaviour mimics that of the experimental testing and there was a general agreement with both the experimental investigations and analytical models for the conveyor transfer comparisons. With respect to conveyor trajectories, the DEM results agreed with the results seen experimentally and also predicted the same trajectory path as the one “stand out” analytical trajectory method mentioned above.

The importance of DEM calibration and validation has also been documented and shown to be an absolute necessity in the successful simulation of industrial applications.

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TABLE OF CONTENTS

BELT CONVEYOR TRANSFERS: QUANTIFYING AND MODELLING MECHANISMS OF PARTICLE FLOW

CERTIFICATION		ii
ABSTRACT		iv
ACKNOWLEDGEMENTS		vi
TABLE OF CONTENTS		vii
LIST OF FIGURES		xv
LIST OF TABLES		xxiii
NOMENCLATURE		xxv
CHAPTER 1 INTRODUCTION		1
1.1	BACKGROUND	2
1.2	OBJECTIVES AND SCOPE OF THE RESEARCH	3
1.3	ORGANISATION OF CHAPTER CONTENT	5
CHAPTER 2 LITERATURE REVIEW		6
2.1	INTRODUCTION	7
2.2	ISSUES RELATING TO CONVEYOR TRANSFER DESIGN	8
2.2.1	Conveyor Belt Wear and Damage	8
2.2.2	Spillage of Material	9
2.2.3	Degradation of Material	9
2.2.4	Material Hang-ups	9
2.2.5	Blockage of the Transfer Chute	10
2.2.6	Noise Emissions	10
2.2.7	High Maintenance Costs	10
2.3	DUST CONTROL	10
2.4	CONVEYOR DISCHARGE AND TRAJECTORY	11
2.4.1	C.E.M.A.	12
2.4.2	M.H.E.A. 1977	13
2.4.3	M.H.E.A. 1986	14
2.4.4	Korzen	14
2.4.5	Booth	16
2.4.6	Golka	17
2.4.7	Dunlop	20
2.4.8	Goodyear	20
2.4.9	Roberts	21
2.4.10	Trajectory Discussion	23
2.5	UPPER TRANSFER CHUTE DESIGN OPTIONS	24
2.5.1	Hood Design	24
2.5.2	Impact Plates	27
2.5.2.1	*Method of Lonie*	27
2.5.2.2	*Method of Korzen*	29

2.5.2.2.1 Initial Conditions 29
2.5.2.2.2 Non-cohesive Materials 30
2.5.2.2.3 Cohesive Materials 31
2.6 LOWER TRANSFER CHUTE DESIGN OPTIONS 33
2.6.1 Chute Angles 33
2.6.2 Rock Box 34
2.6.3 Spoon Design 34
2.6.3.1 Material Flow Through a Loading Chute 34
2.6.3.2 Material Flow in a Constant Radius Curved Chute 34
2.6.3.3 Material Flow in a Parabolic Curved Chute 35
2.7 MATERIAL FREEFALL 36
2.8 AIR SUPPORTED BELT CONVEYORS 41
2.9 DISCRETE ELEMENT MODELLING 43
2.10 APPLICATIONS OF DEM 43
2.11 CALIBRATION OF DEM 43
2.12 PARTICLE SHAPE IN DEM 46
2.12.1 Circular and Spherical Particles 47
2.12.2 Ellipsoid Particles 48
2.12.3 Multi-Sphere Particles 48
2.12.4 Spherocylinders 50
2.12.5 Superquadrics 50
2.12.6 Polygons 50
2.13 CONTACT FORCE MODELS 51
2.13.1 Linear Spring-Dashpot Model 51
2.13.2 Partially Latched Spring Model 53
2.13.3 Non-Linear Spring-Dashpot Model 54
2.13.4 Hertz-Mindlin Model 54
2.13.5 Improved Hertz-Mindlin Model 55
2.13.6 Hertz-Mindlin Model Without Slip 56
2.13.7 Hertz-Kuwabara-Kono Contact Model 56
2.13.8 Effect of Rolling Friction 56
2.14 DEM CODES AND LIMITATIONS 57
2.14.1 DEM Codes Developed by Research Groups 57
2.14.2 Commercially Available DEM Software Packages 58
2.14.2.1 PFC^{2D} and PFC^{3D} 58
2.14.2.2 Chute Maven™ 58
2.14.2.3 Chute Analyst 58
2.14.2.4 EDEM 59
2.14.2.5 Newton 59
2.14.3 Factors Influencing Simulation Time 59
2.15 COMPARISONS OF THE CONTACT FORCE MODELS 61

CHAPTER 3
THE CONVEYOR TRANSFER RESEARCH FACILITY 63

3.1 INTRODUCTION 64
3.2 KEY DESIGN CRITERIA 64
3.2.1 Conveyor Belt Speed 64

3.2.2 Conveyor Belt Width 64
3.2.3 Conveyor Belt Surface 64
3.2.4 Belt Conveyor Selection 65
3.3 FACILITY LAYOUT 65
3.3.1 Conveyor Support Frames 66
3.3.2 Transfer Chutes 66
3.3.2.1 Transfer Chute 1 67
3.3.2.2 Transfer Chute 2 67
3.3.2.3 Transfer Chute 3 68
3.3.3 Feed Bin 68
3.4 INSTALLATION OF THE CONVEYOR TRANSFER RESEARCH FACILITY 69
3.5 'DRY' COMMISSIONING 71
3.6 FINAL COMMISSIONING 71
3.7 FEED BIN AND CHUTE LINERS 72
3.8 DUST EXTRACTION 72
3.9 ADDITIONAL FRAMES 74

CHAPTER 4
CALIBRATION OF THE CONVEYOR TRANSFER RESEARCH FACILITY 75

4.1 INTRODUCTION 76
4.2 DATA ACQUISITION 76
4.3 CALIBRATION OF THE CONVEYOR BELT SPEED 76
4.3.1 Additional Conveyor Belt Speed Calibration 77
4.4 CALIBRATION OF THE FEED BIN LOAD CELLS 78
4.5 CALIBRATION OF THE HOGAN™ VALVE 79

CHAPTER 5
PARTICLE AND BULK CHARACTERISTICS 81

5.1 PARTICLE AND BULK CHARACTERISATION AND MEASUREMENT 82
5.1.1 Loose-Poured Bulk Density 82
5.1.2 Particle Density 82
5.1.3 Equivalent Volume Diameter 83
5.1.4 Particle Sphericity 83
5.1.5 Particle Size Distribution 84
5.1.6 Coefficient of Restitution 85
5.1.7 Wall Friction Angle and the Coefficient of Wall Friction 86
5.1.8 Internal Friction Angle 88
5.1.9 Static and Kinetic Wall Friction 88
5.1.10 Particle-Particle Friction 90
5.1.11 Angle of Repose 90
5.1.12 Surcharge Angle 90
5.1.13 Terminal Velocity of Particles 91
5.1.14 Poisson's Ratio and Shear Modulus 91
5.2 TEST MATERIALS 92
5.2.1 Polyethylene Pellets 92

5.2.2 Yandicoogina Iron Ore 93
5.2.3 Corn 94
5.2.4 Determining the Shear Modulus and Poisson's Ratio 95

CHAPTER 6
DISCRETE ELEMENT MODELLING SOFTWARE 98

6.1 INTRODUCTION 99
6.2 CHUTE MAVEN™ 99
6.2.1 Three Dimensional CAD Models 100
6.2.2 Model Parameters 102
6.2.2.1 *Coefficient of Friction Between Particles* 103
6.2.3 Simulation Data 104
6.2.4 Performing a Simulation 105
6.2.4.1 Optimisation of Simulation Time 105
6.2.5 Interpreting Results 106
6.2.6 Calibration of DEM at the Bench-Scale Level 107
6.2.6.1 Slump Model 107
6.2.6.2 Hopper Model 108
6.2.7 Sensitivity Analysis 109
6.2.7.1 Trajectory Geometry 110
6.2.7.2 Effect of Particle Size Distribution 113
6.3 EDEM 116
6.3.1 Computing Power 116
6.3.2 Global Model Parameters 117
6.3.3 Particle Definition 117
6.3.4 Defining the Geometry 117
6.3.5 Defining the Domain 118
6.3.6 Particle Factory 118
6.3.7 Running a Simulation 118
6.3.8 Simulation Analysis 119
6.3.9 Sensitivity Analysis 119
6.3.9.1 Sensitivity Analysis for Polyethylene Pellets 120

CHAPTER 7
CONVEYOR DISCHARGE ANGLES 127

7.1 INTRODUCTION 128
7.2 CRITICAL BELT SPEEDS 128
7.3 CONVEYOR DISCHARGE ANGLE MODEL COMPARISONS 129
7.3.1 Effect of Belt Inclination Angle on Critical Belt Speed 129
7.3.2 Effect of Belt Speed and Pulley Diameter on Conveyor Discharge Angle 130
7.3.3 Effect of Static and Kinetic Friction 134
7.3.4 Effect of Adhesive Stress on Conveyor Discharge Angle 134
7.4 DETERMINING THE CONVEYOR DISCHARGE ANGLE 135
7.4.1 Experimental Determination 135
7.4.2 Determination by Analytical Method 136
7.4.3 Determination by Discrete Element Modelling 137

7.5 CONVEYOR DISCHARGE ANGLE FOR POLYETHYLENE PELLETS 137
7.5.1 Experimentally Determined Conveyor Discharge Angles 137
7.5.2 Analytically Determined Conveyor Discharge Angles 138
7.5.3 DEM Conveyor Discharge Angles 138
7.5.3.1 Chute Maven™ 138
7.5.3.2 EDEM 139
7.5.4 Comparison of Conveyor Discharge Angles for Polyethylene Pellets 140
7.6 CONVEYOR DISCHARGE ANGLE FOR IRON ORE 141
7.7 CONVEYOR DISCHARGE ANGLE FOR CORN 141
7.7.1 Experimental Conveyor Discharge Angles 141
7.7.2 Analytically Determined Conveyor Discharge Angles 142
7.7.3 DEM Conveyor Discharge Angles 142
7.7.3.1 Chute Maven™ 142
7.7.3.2 EDEM 142
7.7.4 Comparison of Conveyor Discharge Angles for Corn 143
7.8 DISCUSSION 144

CHAPTER 8
CONVEYOR TRAJECTORIES 146

8.1 INTRODUCTION 147
8.2 CONVEYOR TRAJECTORY MODEL COMPARISONS 148
8.2.1 Low-Speed Conveyor Trajectory Comparisons 149
8.2.2 High-Speed Conveyor Trajectory Comparisons 149
8.3 INFLUENCES ON CONVEYOR TRAJECTORY PROFILES 153
8.3.1 Effect of Belt Inclination Angle 153
8.3.2 Effect of Static and Kinetic Friction 153
8.3.3 Effect of Divergent Coefficients 155
8.3.4 Effect of Particle Shape and Size 156
8.3.5 Effect of Adhesive Stress 157
8.3.6 Effect of Bulk Density 157
8.4 CONVEYOR TRAJECTORIES OF POLYETHYLENE PELLETS 158
8.4.1 Experimental Conveyor Trajectories 158
8.4.1.1 Preliminary Setup 158
8.4.1.2 Laser Scanning 159
8.4.1.3 Final Setup 160
8.4.2 Analytically Determined Conveyor Trajectories 166
8.4.3 DEM Conveyor Trajectories 170
8.4.3.1 Scope of Simulations 171
8.4.3.2 Particle Geometry 171
8.4.3.3 Calibration of the Mass Flow Rate 171
8.4.3.4 Conveyor Geometry 173
8.4.3.5 Particle Parameters 174
8.4.3.6 Low Mass Flow Rate EDEM Trajectory Simulations 175
8.4.3.7 High Mass Flow Rate EDEM Trajectory Simulations 179
8.4.3.8 Further Investigation of Rolling Friction 180
8.4.3.9 Re-Visiting Low Mass Flow Rate EDEM Trajectory Simulations 183

8.4.3.10 Re-Visiting High Mass Flow Rate EDEM Trajectory Simulations 183
8.4.3.11 Trajectory Simulation Comparison of 1% and 0.3 Coefficient of Rolling Friction 184
8.4.4 Conveyor Trajectory Comparisons for Polyethylene Pellets 185
8.5 CONVEYOR TRAJECTORIES OF IRON ORE 189
8.6 CONVEYOR TRAJECTORIES OF CORN 189
8.6.1 Experimental Conveyor Trajectories 189
8.6.2 Analytically Determined Conveyor Trajectories 193
8.6.3 DEM Conveyor Trajectories 197
8.6.3.1 Particle Geometry 198
8.6.3.2 Calibration of the Mass flow Rate 199
8.6.3.3 Conveyor Geometry 200
8.6.3.4 Calibration of Rolling Friction 200
8.6.3.5 High Mass Flow Rate EDEM Trajectory Simulations 201
8.6.4 Conveyor Trajectory Comparisons for Corn 202
8.7 DISCUSSION 205

CHAPTER 9
CONVEYOR TRANSFER HOOD ANALYSIS 209

9.1 INTRODUCTION 210
9.2 THE FLOW OF POLYETHYLENE PELLETS THROUGH A TRANSFER HOOD 210
9.2.1 Experimental Particle Flow Investigation 210
9.2.2 Analytical Method Analysis 214
9.2.2.1 Analytical Method of Roberts 215
9.2.2.2 Analytical Method of Korzen 217
9.2.3 Discrete Element Modelling of Particle Flow 219
9.2.3.1 Chute Maven™ Simulations 219
9.2.3.2 EDEM Simulations 224
9.2.4 Method Comparisons for Polyethylene Pellets 225
9.3 FLOW OF IRON ORE THROUGH A CONVEYOR TRANSFER HOOD 228
9.3.1 Experimental Particle Flow Investigation 228
9.3.2 Analytical Method Analysis 231
9.3.2.1 Analytical Method of Roberts 232
9.3.2.2 Analytical Method of Korzen 232
9.3.3 Discrete Element Modelling of Particle Flow 233
9.3.3.1 Chute Maven™ Simulations 234
9.3.3.2 EDEM Simulations 235
9.3.4 Method Comparisons for Iron Ore 238
9.4 DISCUSSION 239

CHAPTER 10
PARTICLE FREEFALL 242

10.1 INTRODUCTION 243
10.2 PARTICLE FREEFALL VELOCITY OF POLYETHYLENE PELLETS 243
10.2.1 Experimental Measurement of Freefall and Terminal Velocity 243

10.2.2 Analytically Determining Terminal Velocity 246
10.2.3 Chute Maven™ DEM Simulation of Terminal Velocity 247
10.3 PARTICLE FREEFALL VELOCITY OF IRON ORE 250
10.3.1 Experimental Measurement of Freefall and Terminal Velocity 250
10.3.2 Analytically Determining Terminal Velocity 251
10.3.3 Chute Maven™ DEM Simulation of Terminal Velocity 252
10.4 PARTICLE FREEFALL VELOCITY OF CORN 252
10.4.1 Experimental Measurement of Freefall and Terminal Velocity 252
10.4.2 Analytically Determining Terminal Velocity 253
10.4.3 Chute Maven™ DEM Simulation of Terminal Velocity 254
10.5 DISCUSSION 254

CHAPTER 11
CONVEYOR TRANSFER SPOON ANALYSIS 255

11.1 INTRODUCTION 256
11.2 FLOW OF POLYETHYLENE PELLETS THROUGH A TRANSFER SPOON 256
11.2.1 Experimental Particle Flow Investigation 256
11.2.2 Analytical Method Analysis of Roberts 260
11.2.3 Discrete Element Modelling of Particle Flow 263
11.2.3.1 Chute Maven™ Simulations 263
11.2.3.2 EDEM Simulations 266
11.2.4 Method Comparisons 268
11.3 FLOW OF IRON ORE THROUGH A CONVEYOR TRANSFER SPOON 271
11.3.1 Experimental Particle Flow Investigation 271
11.3.2 Analytical Method Analysis of Roberts 273
11.3.3 Discrete Element Modelling of Particle Flow 273
11.3.3.1 Chute Maven™ Simulations 273
11.3.3.2 EDEM Simulations 273
11.3.4 Method Comparisons 281
11.4 DISCUSSION 283

CHAPTER 12
BRING IT ALL TOGETHER 285

12.1 INTRODUCTION 286
12.2 POLYETHYLENE PELLET COMPARISONS 286
12.3 IRON ORE COMPARISONS 292
12.4 DISCUSSION 293

CHAPTER 13
CONCLUSIONS AND FURTHER WORK 294

13.1 OVERVIEW 295
13.2 CONCLUSIONS 295
13.2.1 Conveyor Discharge Angle 295
13.2.2 Conveyor Trajectories 296

13.2.3 Conveyor Transfer Hoods 297
13.2.4 Particle Freefall 298
13.2.5 Conveyor Transfer Spoons 299
13.2.6 Discrete Element Modelling 299
13.3 FURTHER WORK 300
13.3.1 Experimental 300
13.3.1.1 Transfer Hood 300
13.3.1.2 Transfer Spoon 301
13.3.1.3 Conveyor Trajectories 302
13.3.1.4 Test Materials 302
13.3.1.5 Dust Generation 302
13.3.2 Analytical Modelling 303
13.3.3 Discrete Element Modelling 303

REFERENCES 304

BIBLIOGRAPHY 322

APPENDIX A
LIST OF PUBLICATIONS 329

A1 CONFERENCE PAPERS 330
A2 JOURNAL PAPERS 331
A3 OTHER PUBLICATIONS 331
A4 POSTER PRESENTATIONS 331

APPENDIX B
DETAILED CAD DRAWINGS OF THE TRANSFER HOOD AND SPOON 332

APPENDIX C
MATLAB TRANSFER HOOD M-file 339

APPENDIX D
MATLAB TRANSFER SPOON M-file 343

LIST OF FIGURES

Figure 2.1	Transfer chute mechanisms	8
Figure 2.2	Element of material travelling around head pulley (Roberts, 2001)	21
Figure 2.3	Material discharge when belt and head pulley first come in contact (Roberts, 2001)	22
Figure 2.4	Material discharge incorporating transition angle (Roberts et al., 2004)	23
Figure 2.5	Hood design (McBride, 1997)	25
Figure 2.6	Inverted curved chute	25
Figure 2.7	(a) Rectangular cross-section, (b) circular cross-section and (c) varying widths (Roberts, 2003)	26
Figure 2.8	Sliding on a straight chute	33
Figure 2.9	Sliding on a curved chute	33
Figure 2.10	Constant radius curved chute	35
Figure 2.11	The effect of particle shape on terminal velocity (Marcus et al., 1990)	37
Figure 2.12	Sectional view of an Aerobelt^{TM} (Read, 1985)	41
Figure 2.13	Spherodisc representations of a tablet, (a) tablet shaped particle, (b) 10 sphere representation and (c) 178 sphere representation (Song et al., 2006b)	48
Figure 2.14	Examples of axisymmetrical particles	49
Figure 2.15	Bullet head nail composite particle (Nolan and Kavanagh, 1995)	49
Figure 2.16	(a) arbitrary particle shape, (b) represented by 1 sphere and (c) represented by multiple spheres (Jensen et al., 1999)	49
Figure 2.17	Representation of a spherocylinder (Pournin et al., 2005)	50
Figure 2.18	Representation of the linear spring-dashpot model (Asmar et al., 2002)	52
Figure 2.19	Representation of the partially latched spring model (Walton and Braun, 1986)	53
Figure 3.1	Final conveyor transfer research facility design	66
Figure 3.2	Hood and spoon geometry of the first design - (a) CAD design, (b) conveyor transfer research facility	67
Figure 3.3	Square feed bin assembly	70
Figure 3.4	Conveyor in position	70
Figure 3.5	Splicing the cleated belt	70
Figure 3.6a	Control panel front	70
Figure 3.6b	Control panel internals	70
Figure 3.7	Impact of material inside feed bin	73
Figure 3.8	Donaldson shaker unit	73
Figure 3.9	Extraction after second transfer	73
Figure 3.10	Extraction at the transfer zone	73
Figure 3.11	Extraction at feed point	73
Figure 3.12	The conveyor transfer research facility installed at new location set for trajectory investigations	74

Figure 4.1	DT80 data acquisition unit	76
Figure 4.2	Millivolt reading vs. time	78
Figure 4.3	Calibrated mass vs. time	78
Figure 4.4	Angular scale for the Hogan™ valve	80
Figure 4.5	Hogan™ valve calibration graph for test materials	80
Figure 5.1	Stereopycnometer	82
Figure 5.2	Sample particle and circumscribing circle	84
Figure 5.3	(a) Sample of sieves, (b) sieves in mechanical sieve shaker	85
Figure 5.4	Jenike shear tester	87
Figure 5.5	(a) Melting of polyethylene pellets to create a test sheet, (b) partially melted polyethylene pellets and (c) the final polyethylene pellet wall sample	87
Figure 5.6	Jenike shear tester configuration for IYL test	88
Figure 5.7	Inclination tester	89
Figure 5.8	Preparation of test material for the friction test with (a) polyethylene pellets, (b) iron ore and (c) corn	89
Figure 5.9a	Angle of repose	91
Figure 5.9b	Surcharge angle	91
Figure 5.10	(a) Polyethylene pellets, (b) corn, and (c) iron ore, size range 2.36 – 6.3 mm	94
Figure 6.1	Three dimensional CAD model of a conveyor transfer	101
Figure 6.2	Chute Maven™ model parameters	103
Figure 6.3	Chute Maven™ simulation data	104
Figure 6.4	The results of an experimental slump test (Kamaras, 2007)	107
Figure 6.5	Comparison of (a) experimental slump test and (b) DEM slump test with restrain = 63% (Kamaras, 2007)	108
Figure 6.6	The results of an experimental hopper test (Kamaras, 2007)	108
Figure 6.7	Comparison of (a) experimental hopper test and (b) DEM hopper test with restrain = 88% (Kamaras, 2007)	109
Figure 6.8	Set 4 trajectory curve for mass flow rate, m_s = 0.5 t/h	112
Figure 6.9	Set 4 trajectory curve for mass flow rate, m_s = 5 t/h	112
Figure 6.10	Set 8 trajectory curve with coefficient of friction between particles, μ_p = 0.222	112
Figure 6.11	Set 8 trajectory curve with coefficient of friction between particles, μ_p = 0.966	112
Figure 6.12	System geometry used for sensitivity analysis	120
Figure 6.13	Steady-state EDEM outputs from the 15 sensitivity analysis tests for spherical particles	123
Figure 6.14	Steady-state EDEM outputs from the 15 sensitivity analysis tests for shaped particles	125
Figure 7.1	Critical belt speed for (a) D_p = 0.5 m, (b) D_p = 1.0 m, (c) D_p = 1.5 m	131
Figure 7.2	Variation in discharge angle based on belt speed for a pulley diameter of D_p = 0.5 m	131

Figure 7.3 Variation in discharge angle based on pulley diameter for a belt speed of V_b = 1.00 m/s 133
Figure 7.4 Variation in discharge angle based on pulley diameter for a belt speed of V_b = 2.00 m/s 133
Figure 7.5 Variation in discharge angle based on pulley diameter for a belt speed of V_b = 3.00 m/s 133
Figure 7.6 Effect of adhesive stress on discharge angle 134
Figure 7.7 Setup for determining material discharge angle and trajectory 136
Figure 7.8 Low-speed discharge of polyethylene pellets at V_b = 1.0 m/s 136
Figure 7.9 Example of the discharge of polyethylene pellets using EDEM with spherical particles 139
Figure 7.10 Comparison of polyethylene pellet conveyor discharge angles from experiments, trajectory models and DEM 140
Figure 7.11 Comparison of corn conveyor discharge angles from experiments, trajectory models and DEM 144

Figure 8.1 Low-speed, horizontal conveyor, lower path, pulley diameter, D_p = 0.5 m, belt velocity, V_b = 1.25 m/s 150
Figure 8.2 Low-speed, horizontal conveyor, upper path, pulley diameter, D_p = 0.5 m, belt velocity, V_b = 1.25 m/s 150
Figure 8.3 Low-speed, horizontal conveyor, pulley diameter, D_p = 1.0 m, belt velocity, V_b = 1.25 m/s 150
Figure 8.4 Low-speed, inclined conveyor, belt inclination angle, α_b = 10°, pulley diameter, D_p = 1.0 m, belt velocity, V_b = 1.25 m/s 150
Figure 8.5 High-speed, horizontal conveyor, pulley diameter, D_p = 1.0 m, belt velocity, V_b = 3.00 m/s 151
Figure 8.6 High-speed, inclined conveyor, belt inclination angle, α_b = 10°, pulley diameter, D_p = 1.0 m, belt velocity, V_b = 3.00 m/s 151
Figure 8.7 High-speed, inclined conveyor, belt inclination angle, α_b = 10°, pulley diameter, D_p = 1.5 m, belt velocity, V_b = 6.00 m/s 153
Figure 8.8 Booth V_b = 1.5 m/s 154
Figure 8.9 Booth V_b = 3.0 m/s 154
Figure 8.10 Korzen V_b = 1.5 m/s 154
Figure 8.11 Korzen V_b = 3.0 m/s 154
Figure 8.12 Variation in trajectories based on different divergent coefficients 156
Figure 8.13 Effect of particle size distribution on Korzen (1989) method 157
Figure 8.14 Effect of bulk density on trajectory profile 158
Figure 8.15 Trajectory, V_b = 1.5 m/s, m_s = 24 tph 159
Figure 8.16 Laser scanned upper trajectory profile (Andrews, 2008) 160
Figure 8.17 Final conveyor trajectory test arrangement 161
Figure 8.18 Example grid referencing, V_b = 2 m/s, m_s = 2.6 tph 162
Figure 8.19 Flat underside of the trajectory stream at the point of discharge for a belt speed of V_b = 4 m/s and mass flow rate of m_s = 37.8 tph 163
Figure 8.20 Trajectory 'wings' for a belt speed of V_b = 4 m/s and mass flow rate of m_s = 37.8 tph 163

Figure 8.21	Experimental polyethylene pellet trajectories for low mass flow rates	164
Figure 8.22	Experimental polyethylene pellet trajectories for high mass flow rates	164
Figure 8.23	Comparison of belt speed to material discharge velocity (the yellow line represents the distinction between the lower and upper halves of the particle stream)	165
Figure 8.24	Comparison of belt speed to material discharge velocity	166
Figure 8.25a	Analytically determined conveyor trajectories for V_b = 1 m/s	168
Figure 8.25b	Analytically determined conveyor trajectories for V_b = 2 m/s	168
Figure 8.25c	Analytically determined conveyor trajectories for V_b = 3 m/s	169
Figure 8.25d	Analytically determined conveyor trajectories for V_b = 4 m/s	169
Figure 8.25e	Analytically determined conveyor trajectories for V_b = 5 m/s	170
Figure 8.26	Particle representations of polyethylene pellets used in EDEM	171
Figure 8.27	Mass flow rate calibration with EDEM simulating 100,000 particles	172
Figure 8.28	Calibration curves for mass flow rate of polyethylene pellets	173
Figure 8.29	Conveyor geometry imported into EDEM	174
Figure 8.30	Bins used for data extraction for (a) low mass flow rate simulations and (b) high mass flow rate simulations	175
Figure 8.31	Graph of discharge velocities versus width of belt for spherical particles	177
Figure 8.32	Graph of discharge velocities versus width of belt for shaped particles	177
Figure 8.33	Graph of discharge velocities versus width of belt for spherical particles using a 0.3 coefficient of rolling friction	182
Figure 8.34	Graph of discharge velocities versus width of belt for shaped particles using a 0.3 coefficient of rolling friction	182
Figure 8.35	Low mass flow rate EDEM simulations for spherical and shaped particles using 0.3 coefficient of rolling friction	183
Figure 8.36	High mass flow rate EDEM simulations for spherical and shaped particles using 0.3 coefficient of rolling friction	184
Figure 8.37	Comparison of the low mass flow rate EDEM simulations of spherical particles using 1% coefficient of rolling friction and 0.30 coefficient of rolling friction	185
Figure 8.38	Comparison of the high mass flow rate EDEM simulations of spherical particles using 1% coefficient of rolling friction and 0.30 coefficient of rolling friction	185
Figure 8.39	Upper trajectory boundary for the high mass flow rates for the experimental data and trajectory models	187
Figure 8.40	Low experimental trajectories super-imposed over the low mass flow rate EDEM trajectories for spherical and shaped particles with 0.3 coefficient of rolling friction	188
Figure 8.41	High experimental trajectories super-imposed over the high mass flow rate EDEM trajectories for spherical and shaped particles with 0.3 coefficient of rolling friction	188
Figure 8.42	High mass flow rate trajectory streams for the trajectory models and EDEM simulations	190

Figure 8.43	Experimental corn trajectories for high mass flow rates	191
Figure 8.44	(a) Vertical positioning of the Redlake X3 MotionPro high-speed digital video camera for analysis of the particle discharge velocity and (b) an example of corn for V_b = 1 m/s	192
Figure 8.45	Comparison of the particle speed of corn and the belt speed at the discharge point of the conveyor	192
Figure 8.46a	Analytically determined conveyor trajectories for V_b = 1 m/s	195
Figure 8.46b	Analytically determined conveyor trajectories for V_b = 2 m/s	195
Figure 8.46c	Analytically determined conveyor trajectories for V_b = 3 m/s	196
Figure 8.46d	Analytically determined conveyor trajectories for V_b = 4 m/s	196
Figure 8.46e	Analytically determined conveyor trajectories for V_b = 5 m/s	197
Figure 8.47	Particle representations of corn used in EDEM	198
Figure 8.48	High mass flow rate EDEM simulations for spherical and shaped particles	202
Figure 8.49	Upper trajectory boundary for the high mass flow rates for the experimental data and trajectory models	204
Figure 8.50	High experimental trajectories super-imposed over the high mass flow rate EDEM trajectories for spherical and shaped particles	205
Figure 8.51	High mass flow rate trajectory streams for the trajectory models and EDEM simulations	206
Figure 9.1	Detail of the conveyor transfer hood	210
Figure 9.2	Material flow through the conveyor hood (a) V_b = 2 m/s and m_s = 2 tph, (b) V_b = 2 m/s and m_s = 31 tph, (c) V_b = 3 m/s Pos A m_s = 2 tph, (d) V_b = 3 m/s Pos A m_s = 38 tph, (e) V_b = 3 m/s Pos B m_s = 10 tph, (f) V_b = 3 m/s Pos B m_s = 38 tph	212
Figure 9.3	Particle tracking using Image Pro Plus	214
Figure 9.4	Average particle velocity at each angular position around transfer hood	214
Figure 9.5a	Material stream height through the hood	215
Figure 9.5b	Material stream width through the hood	215
Figure 9.6	Force diagram for the inverted curved chute	215
Figure 9.7	Predicted stream velocity of polyethylene pellets through the transfer hood using the Roberts method	217
Figure 9.8	Flow representation for analysis by Korzen	217
Figure 9.9	Predicted stream velocity of polyethylene pellets through the transfer hood using the Korzen method	219
Figure 9.10	Example output from a Chute Maven™ simulation	222
Figure 9.11	Extracted Chute Maven™ simulation outputs of the various product feed rates for the different belt speeds and transfer hood positions	222
Figure 9.12	Chute Maven™ DEM simulation results for all mass flow rates	223
Figure 9.13	Conveyor transfer geometry for V_b = 2 m/s imported into EDEM	224
Figure 9.14	EDEM simulation results for transfer hood geometries	225
Figure 9.15	Comparison of methods for a belt speed of 2 m/s	226

Figure 9.16	Comparison of methods for a belt speed of 3 m/s with hood position A	226
Figure 9.17	Comparison of methods for a belt speed of 3 m/s with hood position B	227
Figure 9.18	Iron ore dust build upon the transfer hood wings	230
Figure 9.19	Iron ore particle flow through the transfer hood	230
Figure 9.20	Transfer hood setup for V_b = 3 m/s showing excessive dust	231
Figure 9.21	Average particle velocity in the transfer hood	231
Figure 9.22	Predicted stream velocity of iron ore through the transfer hood using the Roberts method	232
Figure 9.23	Predicted stream velocity of iron ore through the transfer hood using the Korzen method	233
Figure 9.24	Chute Maven™ DEM simulation results for all mass flow rates	234
Figure 9.25	Shaped representations of iron ore particles	235
Figure 9.26	EDEM simulation results for low and high mass flow rates	237
Figure 9.27	Comparison of EDEM simulation outputs for varying coefficient of rolling friction and shear modulus	237
Figure 9.28	Comparison of methods for a belt speed of 2 m/s	239
Figure 10.1	Experimental particle freefall setup	244
Figure 10.2	Experimental and theoretical freefall results for polyethylene pellets, (a) V_b = 2 m/s, m_s = 1 tph, (b) V_b = 2 m/s, m_s = 9 tph, (c) V_b = 3 m/s, m_s = 1 tph, (d) V_b = 3 m/s, m_s = 9 tph, (e) V_b = 4 m/s, m_s = 9 tph	245
Figure 10.3	Comparison of experimental freefall velocity results	246
Figure 10.4	Particle freefall velocity obtained from Chute Maven™ simulations	248
Figure 10.5	Particle freefall velocity obtained from Chute Maven™ simulations	248
Figure 10.6	Chute Maven™ DEM freefall data from conveyor transfer simulations	249
Figure 10.7	Experimental setup to measure the freefall velocity of iron ore	251
Figure 10.8	Experimental and theoretical freefall results for iron ore	251
Figure 10.9	Experimental and theoretical freefall results for corn	253
Figure 11.1	Detail of the transfer spoon	256
Figure 11.2	Material flow through the conveyor spoon (a) V_b = 1 m/s and m_s = 2 tph, (b) V_b = 2 m/s and m_s = 2 tph, (c) V_b = 3 m/s position A m_s = 2 tph, (d) V_b = 3 m/s position B m_s = 10 tph, (e) V_b = 3 m/s position C m_s = 10 tph	259
Figure 11.3	(a) Average experimental particle velocities for low mass flow rates	260
Figure 11.3	(b) Average experimental particle velocities for high mass flow rates	260
Figure 11.4	Force diagram for the curved chute	261
Figure 11.5	(a) Predicted average stream velocity through the spoon by Roberts method for low mass flow rates	262

Figure 11.5 (b) Predicted average stream velocity through the spoon by Roberts method for high mass flow rates 263
Figure 11.6 Two examples of the DEM simulation outputs showing both good and bad flow trends 264
Figure 11.7 Simulation results for all experimental belt speeds and spoon geometries from the Chute Maven™ transfer hood and spoon simulations 266
Figure 11.8 Simulation results for all experimental belt speeds and spoon geometries from the Chute Maven™ transfer spoon simulations 266
Figure 11.9 EDEM simulation results for transfer spoon geometries 267
Figure 11.10 Comparison of the three analysis methods for each belt speed and spoon geometry 270
Figure 11.11 Experimental testing of iron ore showing dust generation 272
Figure 11.12 Average particle velocity in the transfer spoon 272
Figure 11.13 Predicted stream velocity through the spoon by Roberts method 274
Figure 11.14 Simulation results from the Chute Maven™ transfer hood and spoon simulations and the spoon only simulations 274
Figure 11.15 EDEM simulation results for low and high mass flow rates 275
Figure 11.16 Ten EDEM simulations for the high mass flow rate looking at variations of coefficient of rolling friction and shear modulus for spherical and shaped particles 276
Figure 11.17 The angular velocity (rpm) of particles with respect to the vertical displacement of the particles through the transfer spoon 278
Figure 11.18 The horizontal displacement of particles across the transfer spoon with respect to the angular velocity (rpm) of particles through the transfer spoon 279
Figure 11.19 Front view of the particle velocity through the spoon for all ten simulations used in these comparisons 280
Figure 11.20 Adjusted EDEM simulation results for low and high mass flow rates 282
Figure 11.21 Average particle velocities through the transfer spoon for all methods 282

Figure 12.1 Representation of how to estimate the impact velocity on the spoon 287
Figure 12.2 Polyethylene pellets, V_b = 1 m/s, m_s = 2 tph 288
Figure 12.3 Polyethylene pellets, V_b = 1 m/s, m_s = 19 tph 289
Figure 12.4 Polyethylene pellets, V_b = 2 m/s, m_s = 2 tph 289
Figure 12.5 Polyethylene pellets, V_b = 2 m/s, m_s = 31 tph 289
Figure 12.6 Polyethylene pellets, V_b = 3 m/s, Position A, m_s = 2 tph 290
Figure 12.7 Polyethylene pellets, V_b = 3 m/s, Position A, m_s = 37.8 tph 290
Figure 12.8 Polyethylene pellets, V_b = 3 m/s, Position B, m_s = 10 tph 290
Figure 12.9 Polyethylene pellets, V_b = 3 m/s, Position B, m_s = 37.8 tph 291
Figure 12.10 Polyethylene pellets, V_b = 3 m/s, Position C, m_s = 10 tph 291
Figure 12.11 Polyethylene pellets, V_b = 3 m/s, Position C, m_s = 37.8 tph 291
Figure 12.12 Iron ore, V_b = 2 m/s, m_s = 15.3 tph 293
Figure 12.13 Iron ore, V_b = 2 m/s, m_s = 63.8 tph 293

Figure B1 Hood and spoon elevation view 333
Figure B2 Hood and spoon isometric and sectional view 334
Figure B3 Transfer hood 335
Figure B4 Transfer hood Polystone Ultra liner 336
Figure B5 Transfer spoon 337
Figure B6 Transfer spoon Polystone Ultra liner 338

LIST OF TABLES

Table 2.1	Divergent coefficients (Golka et al., 2007)	18
Table 2.2	Summary of some of the applications DEM has been used to simulate, including corresponding authors	44
Table 2.3	Relative time to simulation various shapes (Potapov and Campbell, 1998)	47
Table 4.1	Belt speed calibration chart	77
Table 4.2	Belt speed calibration check	78
Table 5.1	Particle and bulk characteristics of polyethylene pellets	93
Table 5.2	Particle and bulk characteristics of iron ore	94
Table 5.3	Particle and bulk characteristics of corn	95
Table 5.4	Shear modulus values of materials and products	96
Table 5.5	Poisson's ratio values of materials and products	97
Table 6.1	Sensitivity analysis based on 100% particle restrain	110
Table 6.2	Sensitivity analysis based on 50% particle restrain	111
Table 6.3	Sensitivity analysis based on comparisons between 50% and 100% particle restrain	113
Table 6.4	Sensitivity analysis based on comparisons between 97.1% and 85.37% particle size distributions	114
Table 6.5	Variables used to investigate the sensitivity of the Rayleigh time step	120
Table 6.6	Sensitivity analysis settings for polyethylene pellets	122
Table 7.1	Critical belt speeds for the various methods	128
Table 7.2	Parameters used for comparisons	129
Table 7.3	Experimentally determined discharge angles for polyethylene pellets	138
Table 7.4	Discharge angles for polyethylene pellets determined from the trajectory models	138
Table 7.5	Discharge angles determined from the Chute Maven™ DEM simulations	139
Table 7.6	Discharge angles determined from the EDEM simulations for polyethylene pellets	140
Table 7.7	Experimentally determined discharge angles for corn	142
Table 7.8	Discharge angles for corn determined from the trajectory models	142
Table 7.9	Discharge angles determined from the EDEM simulations for corn	143
Table 8.1	Conveyor parameters used for comparisons	148
Table 8.2	Discharge velocities versus pulley diameter for a belt speed of V_b = 3.0 m/s	151

Table 8.3	Combinations of coefficient of static and kinetic friction used for the Korzen method (1989)	155
Table 8.4	Range of trajectory profiles for low-speed conditions by Booth (1934) and Korzen (1989)	155
Table 8.5	Selected equivalent spherical particle diameters for comparison	156
Table 8.6	Experimental trajectory test setups	161
Table 8.7	Mass flow rate calibration simulation of polyethylene pellets in EDEM	172
Table 8.8	Required number of polyethylene pellets to achieve the experimental mass flow rates	173
Table 8.9	Belt speed settings for all EDEM simulations	175
Table 8.10	Belt speeds used to generate the 'correct' particle discharge velocities for spherical particle simulations for the low mass flow rate	178
Table 8.11	Belt speeds used to generate the 'correct' particle discharge velocities for shaped particle simulations for the low mass flow rate	178
Table 8.12	Belt speeds used to generate the 'correct' particle discharge velocities for spherical particle simulations for the high mass flow rates	180
Table 8.13	Results of rolling friction sensitivity simulations for polyethylene pellets	181
Table 8.14	Additional rolling friction sensitivity simulations for polyethylene pellets	181
Table 8.15	Experimental trajectory test setups	189
Table 8.16	Mass flow rate calibration of corn in EDEM	199
Table 8.17	Required number of corn grains to achieve the experimental mass flow rates	199
Table 8.18	Results of rolling friction sensitivity simulations for corn	200
Table 8.19	Additional rolling friction sensitivity simulations for corn	201
Table 9.1	Product feed rates used in experimental tests	211
Table 9.2	Chute Maven™ DEM simulation parameters	221
Table 9.3	Average stream velocity from particle friction calibration	221
Table 9.4	Experimental geometries simulated with EDEM	224
Table 9.5	Results of rolling friction sensitivity simulations	236
Table 10.1	Experimental freefall tests	243
Table 10.2	Estimates of the experimental terminal velocity of polyethylene pellets	246
Table 10.3	Predicted terminal velocity of polyethylene pellets	247
Table 10.4	Range of Chute Maven™ DEM simulations performed	247
Table 10.5	Predicted terminal velocity of iron ore	252
Table 10.6	Predicted terminal velocity of corn	253
Table 11.1	Product feed rates used in experimental tests	257
Table 11.2	Product feed rates used in EDEM simulations	267

NOMENCLATURE

ALL CHAPTERS (EXCEPT CHAPTERS 2.9 – 2.15)

a	acceleration or deceleration of material on a straight chute	m/s^2
a_1	height to material centroid	m
Ar	Archimedes number	-
A_1	initial cross-sectional area of material	m^2
A_2	exit cross-sectional area of material	m^2
A_a	cross-sectional area of material outgoing from impact plate	m^2
A_p	cross-sectional area of material inflow to impact plate	m^2
A_P	projected area of particle	m^2
b	belt thickness	m
b_d	width of the discharged material stream	m
b_w	width of material at discharge point	m
B	width of chute	m
B_0	initial width of chute	m
c	cohesion	kN/m^2
C	constant of integration	-
C_1	chute constant	-
C_D	drag coefficient	-
d_*	dimensionless particle diameter	-
d_c	diameter of a circle	m
d_k	equivalent spherical grain diameter	m
dm	elementary mass of material stream	kg
D_{sv}	equivalent volume diameter of a particle	m
E	elastic (Young's) modulus	Pa
F	frictional force	N
F_A	adhesive force	N
F_D	drag force	N
$F_{lateral}$	lateral force due to angled impact plate	N
F_N	normal force	N
F_S	shear force	N
F_x	horizontal component of force acting on impact plate	N
F_y	vertical component of force acting on impact plate	N
g	gravitational acceleration	m/s^2
G	shear modulus	Pa
h	material depth	m
h_1	initial height of material	m
h_2	exit height of material	m
h_d	material depth at discharge	m
h_p	material stream depth at the moment of impact with impact plate	m
H	height of material in chute	m
H_0	height of material in chute at a particular location	m
K	constant of integration	-
K_v	pressure ratio	-
L_t	length of conveyor transition	m
$\dot{m}$	mass flow rate	kg/s
N	normal force	N
P_1	initial material pressure	Pa

P_2	final material pressure	Pa
Q	mass flow rate	tph
r_p	particle radius	m
R	constant radius of curvature of chute	m
R_b	radius to outer belt surface	m
R_c	radius of material centroid/centre	m
R_d	radius of discharge	m
Re_∞	Reynolds Number	-
R_h	radius to outer depth of material surface	m
R_p	head pulley radius	m
R_t	radius of curvature of the trajectory	m
s	distance from head pulley axis to impact plate	m
s	distance around chute (equation 2.47)	m
s_0	distance from head pulley axis to centre of material element	m
t	increment time for trajectory path	s
T_k	kinetic frictional resistance on the belt surface	N
T_R	Rayleigh time step	s
T_s	static frictional resistance on the belt surface	N
U_*	dimensionless terminal velocity	-
v	velocity of mass element	m/s
$\dot{v}$	acceleration of mass element	m/s^2
v(x)	resultant velocity of inclined freefall	m/s
v_∞	terminal velocity	m/s
v_a	material outgoing velocity from impact plate	m/s
v_b	belt velocity	m/s
v_d	discharge velocity	m/s
v_{ip}	velocity of material inflow to impact plate	m/s
v_0	discharge velocity of upper boundary, = V_2	m/s
v_{0l}	discharge velocity of lower boundary, = V_1	m/s
v_{0y}	initial velocity in the y direction	m/s
$v_{p\eta}$	vertical component of v_{ip}	m/s
V_∞	terminal velocity	m/s
$V_{\infty\psi}$	terminal velocity adjusted for shape	m/s
V_1	discharge velocity of lower boundary	m/s
V_2	discharge velocity of upper boundary	m/s
V_b	belt velocity	m/s
V_{cr}	critical velocity	m/s
V_d	velocity of material at discharge point	m/s
V_{final}	vertical component of material velocity discharging from feeder	m/s
$V_{initial}$	velocity at drop height h at point of impact with chute	m/s
V_{p1}	initial material velocity	m/s
V_{p2}	exit material velocity	m/s
V_P	particle volume	m^3
V_s	tangential velocity of material at discharge point	m/s
w_b	belt width	m
X	distance travelled along tangent line of belt and pulley	mm
x	horizontal distance at which y(x), ξ(x) and v(x) are calculated	m
x_0	initial x co-ordinate for start of upper trajectory	m
x_1	x co-ordinates of trajectory for lower boundary	m
x_2	x co-ordinates of trajectory for upper boundary	m

$y(x)$	y component of trajectory of particle freefall	m
y	y co-ordinate of conveyor trajectory	m
y_0	initial y co-ordinate for start of upper trajectory	m
y_1	y co-ordinates of trajectory for lower boundary	m
y_2	y co-ordinates of trajectory for upper boundary	m
Y	distance material falls below line of discharge	mm
z	error approximation	-
z	depth of conveyor transition	m

GREEK

α	initial material discharge angle measured from the vertical	°
α_b	conveyor belt inclination angle	°
α_c	angle of chute at tangent point	°
α_d	material discharge angle measured from the vertical	°
α_{d1}	material discharge angle measured from the vertical for lower trajectory	°
α_{d2}	material discharge angle measured from the vertical for upper trajectory	°
α_i	angle of exiting flow to the vertical	°
α_{ip}	angle of material inflow to impact plate	°
α_r	angle at which particle slip begins to occur	°
β	angle of impact chute	°
γ	specific gravity	-
ΔA	contact area	m^2
Δm	mass of element	kg
Δr	change in radius	m
ε_1	divergent coefficient	-
ε_2	divergent coefficient	-
ε_b	divergence or dispersion coefficient of bulk stream width	-
ε	transition angle, measured from the horizontal	°
η	coefficient of restitution	-
η_f	air viscosity	Ns/m^2
φ	angular coordinate of the mass element at flow-round zone of impact plate	°
θ	angle to vertical when normal force becomes zero, discharge angle	°
θ_0	angle at which material leaves belt	°
λ	angle of varying width chute	°
μ	coefficient of friction	-
μ_e	coefficient of equivalent friction	-
μ_f	absolute viscosity of air	Ns/m^2
μ_i	coefficient of internal friction	-
μ_k	coefficient of kinetic friction	-
μ_p	coefficient of external friction on impact plate	-
μ_s	coefficient of static friction	-
ν	Poisson's Ratio	-
$\xi(x)$	trajectory direction angle	°
ρ_b	loose-poured bulk density of material	kg/m^3

ρ_f	air density	kg/m^3
ρ_s	particle density	kg/m^3
σ_a	adhesive stress	kPa
ϕ	kinematic angle of sliding friction	°
ψ	wrap angle around discharge pulley	°
ψ	sphericity	-
ψ_A	particle shape coefficient	-

CHAPTER 2 – Section 2.9 to Section 2.15

C_N	normal damping coefficient	-
C_T	tangential damping coefficient	-
E^*	equivalent Young's modulus	GPa
F_C	cohesion force	N
F_N	normal contact force	N
F_T	tangential contact force	N
F_{T*}	magnitude of tangential force at start of current slip plane	N
G	shear modulus	GPa
G^*	shear modulus	GPa
K_0	initial tangential stiffness	N/m
K_1	spring constant for loading	N/m
K_2	spring constant for unloading	N/m
K_C	cohesion constant	N/m
K_N	stiffness of the spring in the normal direction	N/m
K_T	tangential stiffness coefficient	N/m
m_i	mass of particle i	kg
m_j	mass of particle j	kg
m_{ij}	mass of particles i and j	kg
R	radius of particle	m
$\tilde{\tilde{R}}$	radius for Hertz-Mindlin model	m

GREEK

α	relative approach (overlap) after initial contact	m
α_0	the value of α where the unloading curve goes to zero	m
α_r	empirical constant related to the coefficient of restitution	-
β	fixed parameter	-
γ	coefficient of critical damping	-
δ_C	cohesion displacement	m
δ_N	displacement of particles in the normal direction	m
δ_R	constant based on Poisson's ratio and coefficient of friction	-
δ_T	displacement of particle in the tangential direction	m
δ_{Tmax}	maximum displacement in the tangential direction	m
ε	coefficient of restitution	-
μ	coefficient of friction	-
μ_r	coefficient of rolling friction	-
ν	Poisson's Ratio	-
υ_N	normal component of relative velocity between particles	m/s
υ_{slip}	slip velocity	m/s
υ_T	tangential component of relative velocity between particles	m/s

SUBSCRIPTS

i	particle i
j	particle j