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## Abstract

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## Keywords

coded, modulation, compensation, iterative, ofdm, soft, systems, clipping, superposition

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# Iterative Soft Compensation for OFDM Systems with Clipping and Superposition Coded Modulation

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**Abstract**—This paper deals with the clipping method used in orthogonal frequency-division multiplexing (OFDM) systems to reduce the peak-to-average power ratio (PAPR). An iterative soft compensation method is proposed to mitigate the clipping distortion, which can outperform conventional treatments. The impact of signaling schemes on the residual clipping noise power is studied via the symbol variance analysis. It is found that superposition coded modulation (SCM) can minimize the residual clipping noise power among all possible signaling schemes. This indicates that SCM-based OFDM systems are more robust to clipping effect than other alternatives when soft compensation is applied. It is also shown that a multi-code SCM scheme can further reduce the clipping effect and its overall performance can be quickly evaluated using a semi-analytical evolution method. Numerical examples are provided to verify the analysis.

**Index Terms**—Clipping, iterative decoding, orthogonal frequency-division multiplexing (OFDM), soft compensation, superposition coded modulation (SCM).

## I. INTRODUCTION

ORTHOGONAL frequency-division multiplexing (OFDM) is a multi-carrier transmission technique for broadband channels. It has attracted tremendous attention due to its high spectral efficiency and low receiver complexity. However, OFDM systems suffer from a high peak-to-average power ratio (PAPR) problem. This makes OFDM more prone to the non-linear effect of transmitter devices than single-carrier schemes.

Various PAPR reduction techniques have been investigated (see [1] and the references therein). They can be broadly classified into two categories. The first category involves redundancy (and so spectral efficiency loss), such as coding, partial transmit sequence, and selective mapping techniques. The second category does not involve redundancy, such as deliberate clipping [2]–[18].

Clipping may cause non-linear distortion which degrades performance. To treat such distortion, iterative techniques have

been studied. In [12]–[15], clipping distortion is estimated and then partially canceled at the receiver. A key step in the works in [12]–[15] is to compute the means for the original signals before clipping using decoder feedbacks. The technique developed in [28] involves both means and variances of the original signals, which offers noticeable performance gains. The focus of [28] is on superposition coded modulation (SCM) in single-carrier environments and the discussion on OFDM systems is very limited.

This paper is concerned with the treatment of clipping distortion for OFDM systems with coded modulation. The proposed technique combines the works in [12]–[15] and [28]: Price's result is used to remove the correlation between the original signals and clipping distortions; and both means and variances of the original signals computed from the decoder feedbacks are used to estimate the clipping distortions. This combined technique ensures the effectiveness of iterative detection and brings about considerable performance gains.

As discussed in [28], SCM generally exhibits high PAPR. However, since OFDM suffers from high PAPR anyway, the use of SCM in OFDM does not worsen the problem. On the contrary, based on a result from [41], we will show that SCM is advantageous (compared with other alternative coded modulation methods) for estimating the original signals. This advantage implies that SCM-OFDM systems may lead to better performance with a low clipping ratio and hence can help reduce the PAPR. Moreover, further performance improvement can be achieved using a multi-code SCM scheme. These properties are confirmed by simulation results.

We also develop a fast, semi-analytical technique to predict the performance of an iterative receiver. The presence of several different types of distortions, namely, additive white Gaussian noise (AWGN), frequency-selective fading, inter-layer interference (due to SCM) and clipping distortion, makes the problem complicated. We will develop a procedure to handle such difficulty. It involves (i) the approximation of the clipping distortions plus inter-layer interferences using additive Gaussian variables and (ii) a two-dimensional pre-calculated table to separate the different types of distortions. The resultant prediction method is reasonably accurate. The availability of a fast analysis technique provides a useful tool for searching-based system design.

Combining the above mentioned techniques, we obtain a promising solution to the PAPR problem in OFDM systems. For example, we show that performance within 1.35 dB away from the Shannon limit can be achieved with a severe clipping ratio (CR) as low as 0 dB.

The rest of this paper is organized as follows. Section II

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introduces the system model and iterative soft compensation method. The impact of signaling schemes on the residual clipping noise power and SCM schemes are discussed in Section III. Numerical results are presented in Section IV, followed by the conclusion in Section V.

## II. ITERATIVE SOFT COMPENSATION

### A. System Model

Consider a coded OFDM system illustrated in Fig. 1. At the transmitter, the information bits are first encoded by a binary encoder (ENC). The resultant coded bits are then randomly interleaved and packed into groups  $\{\mathbf{b}[n] = (b_1[n], b_2[n], \dots, b_K[n])\}$  of  $K$  bits. Each  $\mathbf{b}[n]$  is mapped to a symbol  $X[n]$  to be carried by the  $n$ th sub-carrier. (The related mapping rule will be detailed in Section III-A.)

Let  $N$  be the number of sub-carriers and define  $\mathbf{X} = [X[0], X[1], \dots, X[N-1]]^T$ . Then  $\{X[n]\}$  are modulated onto sub-carriers using the inverse discrete Fourier transform (IDFT). The resultant signal is over-sampled into  $\mathbf{x} = [x[0], x[1], \dots, x[LN-1]]^T$  with over-sampling factor  $L$ ,

$$\mathbf{x} = \mathbf{F}^\dagger \mathbf{X}, \quad (1)$$

where  $\mathbf{F}$  is an  $N \times LN$  matrix with  $(n, m)$ th entry given by  $e^{-j\frac{2\pi mn}{LN}}/\sqrt{NL}$ ,  $j = \sqrt{-1}$ , and “ $(\cdot)^\dagger$ ” denotes conjugate transpose of matrix. To reduce the PAPR, each entry  $x[m]$  of  $\mathbf{x}$  is deliberately clipped using the following clipping function,

$$g(x[m]) = \begin{cases} x[m], & \text{if } |x[m]| \leq A \\ Ax[m]/|x[m]|, & \text{if } |x[m]| > A \end{cases} \quad (2)$$

where  $A$  is a clipping threshold and  $|\cdot|$  represents absolute value. The clipping ratio in decibel is defined as  $\text{CR} = 10 \log_{10}(A^2/\mathbb{E}[|x[m]|^2])$  where  $\mathbb{E}[\cdot]$  represents mathematical expectation. With abuse of notation, we write the clipped signal vector as  $g(\mathbf{x})$ .

Clipping introduces in-band non-linear distortion and out-of-band radiation [3], [6]. This paper focuses only on the in-band distortion that degrades the bit-error-rate (BER) performance [2]–[18]. We assume that ideal bandpass filtering [3] is applied to  $g(\mathbf{x})$  and the out-of-band radiation is perfectly mitigated. Cyclic prefix is then added to the filtered signal to treat inter-block-interference (IBI) and the resultant signal is transmitted. For simplicity, we assume that

- clipping is the only source of non-linear distortion, which implies that linear power amplifiers are used;
- the cyclic prefix is sufficiently long so that the IBI can be perfectly removed; and
- the channel coefficients remain unchanged within each OFDM block and are perfectly known at the receiver.

Denote by  $\mathbf{y}$  the time-domain received signal vector. After removing the cyclic prefix and applying the discrete Fourier transform (DFT) to  $\mathbf{y}$ , we obtain a frequency-domain signal vector  $\mathbf{Y} = [Y[0], Y[1], \dots, Y[N-1]]^T$ . Considering (1) and (2), we model  $\mathbf{Y}$  as

$$\mathbf{Y} = \mathbf{H}\mathbf{F}g(\mathbf{x}) + \mathbf{W} \quad (3)$$

where  $\mathbf{H} = \text{diag}(H[0], H[1], \dots, H[N-1])$  consists of the fading coefficients on the  $N$  subcarriers and  $\mathbf{W}$  is a vector

of samples of a complex Gaussian noise with mean zero and variance  $\sigma_W^2/2$  per dimension.

The suboptimal receiver consists of an elementary signal estimator (ESE) and a decoder (DEC) connected by an interleaver ( $\Pi$ ) and a de-interleaver ( $\Pi^{-1}$ ). They operate iteratively following the principles of bit-interleaved coded modulation with iterative decoding (BICM-ID) [19], [20].

### B. Iterative Detection/Decoding

The clipping effect is handled by the soft compensation (SC) module in Fig. 1. Applying Price’s theorem for Gaussian-input memoryless non-linear systems [32], the clipped signal vector  $g(\mathbf{x})$  is approximated as

$$g(\mathbf{x}) = \alpha \mathbf{x} + \mathbf{d} \quad (4)$$

where  $\mathbf{d} = [d[0], \dots, d[LN-1]]^T$  and  $\alpha$  is a constant scalar computed as

$$\alpha = \frac{\mathbb{E}[\mathbf{x}^\dagger g(\mathbf{x})]}{\mathbb{E}[\|\mathbf{x}\|^2]} \quad (5)$$

and  $\|\cdot\|$  denotes the Frobenius norm of a vector. Then from (3) and  $\mathbf{F}\mathbf{F}^\dagger = \mathbf{I}$ , we have

$$\mathbf{Y} = \alpha \mathbf{H}\mathbf{X} + \mathbf{H}\mathbf{F}\mathbf{d} + \mathbf{W}. \quad (6)$$

Assume that the mean of  $\mathbf{d}$ , denoted by  $\bar{\mathbf{d}}$  below, is available. (At the beginning of decoding, a common choice is  $\bar{\mathbf{d}} = \mathbf{0}$ . The details in updating  $\bar{\mathbf{d}}$  will be given in (12)). To reduce the distortion related to  $\mathbf{d}$ , we subtract  $\mathbf{H}\mathbf{F}\bar{\mathbf{d}}$  from  $\mathbf{Y}$ , yielding

$$\mathbf{Z} = \mathbf{Y} - \mathbf{H}\mathbf{F}\bar{\mathbf{d}} = \alpha \mathbf{H}\mathbf{X} + \mathbf{\Xi} \quad (7)$$

where

$$\mathbf{\Xi} = \mathbf{H}\mathbf{F}(\mathbf{d} - \bar{\mathbf{d}}) + \mathbf{W} \quad (8)$$

is the residual clipping noise plus channel noise. Clearly,  $\mathbf{d} - \bar{\mathbf{d}}$  has zero mean, and so does  $\mathbf{F}(\mathbf{d} - \bar{\mathbf{d}})$ .

**Assumption 1:** The entries in  $\mathbf{F}(\mathbf{d} - \bar{\mathbf{d}})$  are independent, identically distributed (i.i.d.) Gaussian random variables that are uncorrelated with  $\mathbf{x}$ .

This assumption can be justified as follows.

- 1) Following general treatments, the entries of  $\mathbf{x}$  are approximately uncorrelated. Then the entries of  $\mathbf{d} - \bar{\mathbf{d}}$  are approximately uncorrelated, so are the entries of  $\mathbf{F}(\mathbf{d} - \bar{\mathbf{d}})$  since  $\mathbf{F}$  is orthogonal. According to the central limit theorem, the entries of  $\mathbf{F}(\mathbf{d} - \bar{\mathbf{d}})$  are approximately Gaussian since they are weighted sums of many uncorrelated random variables.
- 2) Due to the use of  $\alpha$  in (4),  $\mathbf{d} - \bar{\mathbf{d}}$  and  $\mathbf{x}$  are uncorrelated, i.e.,  $\mathbb{E}[(\mathbf{d} - \bar{\mathbf{d}})\mathbf{x}^\dagger] = \mathbf{0}$ , so are  $\mathbf{F}(\mathbf{d} - \bar{\mathbf{d}})$  and  $\mathbf{x}$ .

Let  $\{V[d[n]]\}$  be the variance of the entries in  $\mathbf{d}$ . Since the entries in  $\mathbf{F}$  have the same absolute value of  $1/\sqrt{LN}$ , the entries in  $\mathbf{F}(\mathbf{d} - \bar{\mathbf{d}})$  have the same variance denoted by  $v_d$  where

$$v_d = \frac{1}{LN} \sum_{m=0}^{LN-1} V[d[m]]. \quad (9)$$

We will consider the details in evaluating  $v_d$  in Section II-C. From Assumption 1 and (9), we can make the following assumption.

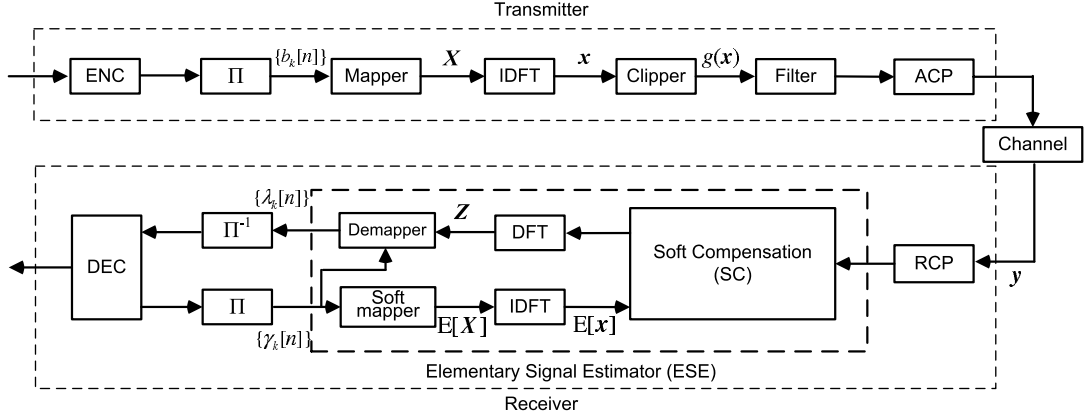


Fig. 1. Coded OFDM systems with clipping, where  $\Pi$  denotes interleaver and  $\Pi^{-1}$  de-interleaver. ACP denotes adding cyclic prefix and RCP removing cyclic prefix.

**Assumption 2:**  $\Xi = \{\Xi[n]\}$  are independent Gaussian random variables with mean  $E[\Xi[n]] = 0$  and variance  $V[\Xi[n]] = |H[n]|^2 v_d + \sigma_W^2$ .

Now return to the detection problem based on (7) and (8). From Assumption 1, the residual clipping noise  $\mathbf{F}(\mathbf{d} - \bar{\mathbf{d}})$  is uncorrelated with  $\mathbf{X}$  and symbol-by-symbol detection can be applied. The soft demapper outputs the so-called *extrinsic* log-likelihood ratios (LLRs):

$$\lambda_k[n] = \ln \left( \frac{\Pr(b_k[n] = 0 | Z[n], \{\gamma_k[n]\})}{\Pr(b_k[n] = 1 | Z[n], \{\gamma_k[n]\})} \right) - \gamma_k[n],$$

$$k = 1, 2, \dots, K, n = 0, 1, \dots, N-1, \quad (10)$$

where  $Z[n]$  denotes the  $n$ th entry of  $\mathbf{Z}$ , and  $\gamma_k[n]$  is the *a priori* LLR about  $b_k[n]$  which can be obtained as the *extrinsic* LLR from the DEC (see (11)). In general cases, the *a posteriori* probability (APP) demapping [19], [33] can be used to evaluate (10). For SCM, more details regarding evaluating (10) will be discussed in Section III.

The DEC takes the de-interleaved version of  $\{\lambda_k[n]\}$  in (10) as inputs and performs standard APP decoding. The *extrinsic* LLRs are produced by the DEC based on coding constraint  $\mathcal{C}$ :

$$\gamma_k[n] = \ln \left( \frac{\Pr(b_k[n] = 0 | \mathcal{C}, \{\lambda_k[n]\})}{\Pr(b_k[n] = 1 | \mathcal{C}, \{\lambda_k[n]\})} \right) - \lambda_k[n],$$

$$k = 1, 2, \dots, K, n = 0, 1, \dots, N-1. \quad (11)$$

The DEC feedback  $\{\gamma_k[n]\}$  will be then used to update  $\bar{\mathbf{d}}$  in an iterative manner, as detailed in the next subsection.

### C. Proposed Clipping Noise Estimation Method

The following assumption is the key to the soft compensation method proposed in this paper.

**Assumption 3:** Each entry  $x$  in  $\mathbf{x}$  is a Gaussian random variable.

This assumption can be justified by the central limit theorem since  $x$  is a weighted sum of  $N$  independent random variables  $\{X[n]\}$ . It also leads to a low-cost method to compensate the distortion caused by  $\mathbf{d}$  at the receiver.

For simplicity, we assume that the real and imaginary parts of  $x$  have the same variance  $V[x]/2$ , where  $V[x] = E[|x - E[x]|^2]$  is referred to as the *symbol variance*. The mean  $E[x]$

and variance  $V[x]$  can be estimated from  $\{\gamma_k[n]\}$ , as will be detailed in Section III-A. With the Gaussian approximation, the distribution of  $x$  is fully characterized by  $E[x]$  and  $V[x]$ . According to [31], given *a priori* information about  $x$ , the optimal estimate of  $d = g(x) - \alpha x$  (in terms of minimization of the mean square error (MSE)) is the conditional mean of  $d$ . Therefore,  $\bar{d}$  (an entry in  $\bar{\mathbf{d}}$ ) can be computed as

$$\begin{aligned} \bar{d} &= E[g(x) - \alpha x] \\ &= \int \frac{g(x) - \alpha x}{\pi V[x]} \exp\left(-\frac{|x - E[x]|^2}{V[x]}\right) dx. \end{aligned} \quad (12)$$

The corresponding *residual* clipping noise power can be estimated by

$$\begin{aligned} V[d] &= E[|d - \bar{d}|^2] \\ &= \int \frac{|g(x) - \alpha x - \bar{d}|^2}{\pi V[x]} \exp\left(-\frac{|x - E[x]|^2}{V[x]}\right) dx, \end{aligned} \quad (13)$$

which can be used to generate the variance of the entries of  $\Xi$  in (8). In practice, (12) and (13) can be tabulated as functions of  $(E[x], V[x])$  for online evaluation, following [28], which involves two two-dimensional tables.

Note that in the proposed soft compensation method, we make no assumption regarding the distribution of  $\mathbf{d}$ . The reason is that, under Assumption 1,  $\mathbf{F}(\mathbf{d} - \bar{\mathbf{d}})$  contains uncorrelated, Gaussian entries, and so its distribution can be fully characterized by the means and variances in (12) and (13).

Based on the above discussions, we list the clipping noise estimation procedure as follows.

- (i) Estimate the mean  $E[X[n]]$  and variance  $V[X[n]]$  of  $X[n]$  (the  $n$ th entry of  $\mathbf{X}$ ) from  $\{\gamma_k[n]\}$ , as detailed in Section III-A.
- (ii) Generate the means and variances of the entries of  $\mathbf{x}$  based on the relationship  $\mathbf{x} = \mathbf{F}^\dagger \mathbf{X}$ . More specifically,  $E[\mathbf{x}] = \mathbf{F}^\dagger E[\mathbf{X}]$  where  $E[\mathbf{x}]$  and  $E[\mathbf{X}]$  are respectively the means of  $\mathbf{x}$  and  $\mathbf{X}$ . The variance of  $x[m]$ ,  $\forall m$ , is computed as  $V[x[m]] = 1/(LN) \sum_{n=0}^{N-1} V[X[n]]$ .
- (iii) Generate the means and variances of the entries of  $\mathbf{d}$  using (12) and (13). In this way, the variance of the entries of  $\mathbf{F}(\mathbf{d} - \bar{\mathbf{d}})$  in (8) is computed as  $v_d = 1/(LN) \sum_{m=0}^{LN-1} V[d[m]]$ .

Steps (i)-(iii) are performed respectively by the soft mapper, IDFT, and SC modules in the ESE. The ESE/DEC operations outlined above can be repeated iteratively.

#### D. Comparisons with Existing Methods

The following methods have been proposed in [3], [12]-[15] and [28] to treat the clipping effect.

- In [3], the clipping noise is approximated by a zero-mean Gaussian noise independent of the wanted signal. The variance of the clipping noise is estimated and the impact of clipping noise is treated together with channel noise. Decoder feedbacks are not required for this purpose.
- The clipping noise is estimated as  $\bar{\mathbf{d}} = g(\mathbb{E}[\mathbf{x}]) - \alpha \mathbb{E}[\mathbf{x}]$  in [12]-[15]. The variance of  $\mathbf{x}$  is not required in this approach.
- The method in [28] involves both mean and variance of  $\mathbf{x}$ , which makes better use of the decoder feedback. The focus of [28] is plain SCM and OFDM is only briefly mentioned in [28]. The clipping operation is modeled as  $g(\mathbf{x}) = \mathbf{x} + \mathbf{d}$  in [28] (without  $\alpha$  in (4)) where  $\mathbf{d}$  is correlated with  $\mathbf{x}$ . The correlation between  $\mathbf{d}$  and  $\mathbf{x}$  complicates the detection of  $\mathbf{x}$ . A hypothesis-dependant technique is devised in [28, Section IV-G] to handle this correlation problem. When applied to OFDM systems, however, this technique incurs excessively high complexity. A simplified method that ignores correlation is used for OFDM systems in [28] to save complexity, which, unfortunately, leads to noticeable performance degradation, as shown in Fig. 7 below.

The scheme outlined above combines the techniques in [12]-[15] and [28]. Removing the correlation between  $\mathbf{d}$  and  $\mathbf{x}$  means that  $\mathbf{d}$  can be more accurately modeled as additive noise samples, which has two beneficial consequences. First, the estimation of  $\bar{\mathbf{d}}$  and  $V[\mathbf{d}]$  based on (12) and (13) provides a much simpler solution than the hypothesis-dependent technique used in [28]. Second, we can apply a SNR evolution technique to analyze the performance of the iterative detection process and search for optimized system design, as will be discussed in Section III-C below.

Also, (approximate) maximum likelihood sequence detection is considered in [17] and [18] for uncoded OFDM systems involving clipping. This technique provides near-optimal performance in uncoded systems. However, it is difficult to apply this technique to coded cases due to excessively high complexity involved.

#### E. Effect of the Uncertainty on Clipping Distortion

We now analyze the effectiveness of the soft compensation technique in (7) and (8).

From Section II-C, the soft estimate  $\bar{\mathbf{d}}$  is computed from the DEC feedback  $\{\gamma_k[n]\}$ . Assumption 4 below follows the *extrinsic* information transfer (EXIT) chart principle [34], which is approximately true with sufficiently long random interleavers.

**Assumption 4:**  $\{\gamma_k[n]\}$  are i.i.d. random variables.

Define  $\mathbb{E}_\gamma[\cdot]$  the expectation with respect to the distribution of  $\{\gamma_k[n]\}$ , and  $\sigma_d^2$  the average residual clipping noise power

$$\sigma_d^2 \equiv \mathbb{E}_\gamma[\|\mathbf{F}(\mathbf{d} - \bar{\mathbf{d}})\|^2]/N = \mathbb{E}_\gamma[\|\mathbf{d} - \bar{\mathbf{d}}\|^2]/(LN). \quad (14)$$

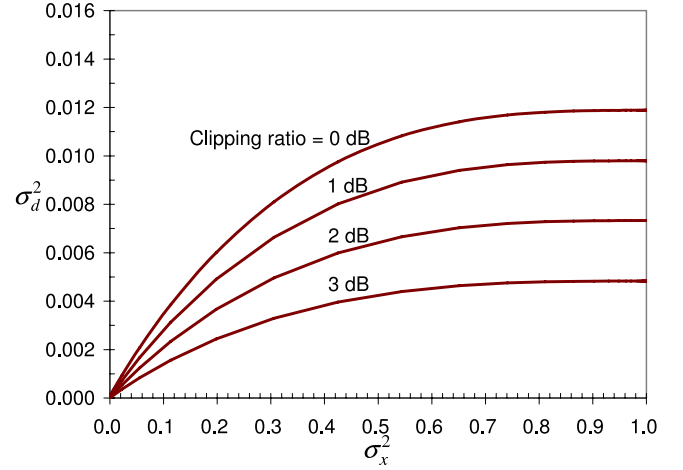


Fig. 2.  $\sigma_d^2$  versus  $\sigma_x^2$ . The oversampling factor  $L = 4$ . The number of sub-carriers  $N = 256$ . The average signal power  $\mathbb{E}[|X[n]|^2]$  is normalized to 1.

Here,  $\sigma_d^2$  represents the uncertainty on clipping distortion. If  $\sigma_d^2 = 0$ , i.e.,  $\mathbf{d} = \bar{\mathbf{d}}$ , then the clipping distortion is completely removed from the received signal using (7). Thus we can use  $\sigma_d^2$  as a measure for the effectiveness of the compensation operation in (7).

With interleaving, we assume that  $\{X[n]\}$  are i.i.d.. Define symbol variance as

$$\begin{aligned} \sigma_x^2 &\equiv \mathbb{E}_\gamma[V[X[n]]] = \mathbb{E}_\gamma[\|\mathbf{X} - \mathbb{E}[\mathbf{X}]\|^2]/N \\ &= \mathbb{E}_\gamma[\|\mathbf{x} - \mathbb{E}[\mathbf{x}]\|^2]/N. \end{aligned} \quad (15)$$

The last equation in (15) holds since  $\mathbf{x} = \mathbf{F}^\dagger \mathbf{X}$ ,  $\mathbb{E}[\mathbf{x}] = \mathbf{F}^\dagger \mathbb{E}[\mathbf{X}]$  and  $\mathbf{F}\mathbf{F}^\dagger = \mathbf{I}$ . Clearly,  $\sigma_x^2$  represents the uncertainty about  $\mathbf{x}$ . From (12) and (13), the estimation of  $\mathbf{d}$  is obtained based on that for  $\mathbf{x}$ . Therefore,  $\sigma_d^2$  is an implicit function of  $\sigma_x^2$ :

$$\sigma_d^2 = \phi(\sigma_x^2). \quad (16)$$

Fig. 2 shows the  $\sigma_d^2$  versus  $\sigma_x^2$  curves (obtained by the Monte Carlo method) for different clipping ratios. We can observe that  $\sigma_d^2$  is a monotonically increasing function of  $\sigma_x^2$  but we are not able to rigorously prove this monotonicity yet. This observation is well within expectation: a more accurate estimate of  $\mathbf{x}$  would lead to a more accurate estimate of  $\mathbf{d}$ . We have also observed in our simulation that  $\phi(\cdot)$  defined in (16) is nearly independent of the signaling schemes employed for generating  $\mathbf{X}$  from the coded bits  $\{b_k[n]\}$ . This is because when  $N$  is sufficiently large, from the central limit theorem, the IDFT output  $\mathbf{x}$  is always approximately Gaussian-distributed, regardless of the signaling schemes. As a consequence of these two observations, minimizing  $\sigma_d^2$  is equivalent to minimizing  $\sigma_x^2$ .

In the next section, we will show that  $\sigma_x^2$  and so  $\sigma_d^2$  can be minimized using properly designed signaling schemes.

### III. SUPERPOSITION CODED MODULATION (SCM)

In this section, we discuss the impact of signaling scheme on the performance of the iterative soft compensation method and show the optimality of the SCM signaling. We also show the advantage of multi-code SCM and analyze its performance.

### A. Optimal Signaling Scheme for Soft Compensation

Let  $\mathcal{S}$  be a constellation of  $2^K$  points. Let  $b = (b_1, b_2, \dots, b_K)$  be a binary  $K$ -tuple with  $b_k \in \{0, 1\}$  and  $\mathcal{B}$  the set of  $2^K$  such  $K$ -tuples. A signaling scheme is defined by  $(\mathcal{S}, \mathcal{R})$ , where  $\mathcal{R}$  is the mapping from  $\mathcal{B}$  to  $\mathcal{S}$ . Some examples of  $(\mathcal{S}, \mathcal{R})$  can be found in [19]–[23]. Another example is the SCM [24]–[30] that generates  $X$  as a superposition of  $K$  bipolar modulated signals,

$$X = \sum_{k=1}^K \beta_k (-1)^{b_k}, \quad (17)$$

where the weighting factors  $\{\beta_k\}$  are complex constants. We will refer to the operation in (17) as “SCM signaling”. We can apply the SCM signaling to the transmitter in Fig. 1, which will result in a special case of BICM-ID [27].

Following the turbo principle,  $\{b_k\}$  can be treated as binary random variables and so the mapped symbol  $X$  is also random. Suppose a set of *a priori* LLRs  $\{\gamma_k\}$  about  $\{b_k\}$  are available.

$$\gamma_k \equiv \ln \left( \frac{\Pr(b_k = 0)}{\Pr(b_k = 1)} \right), \quad k = 1, 2, \dots, K. \quad (18)$$

(In the soft mapper,  $\{\gamma_k\}$  are the feedback LLRs from the DEC.) Then the *a priori* probability for each  $b_k$  can be computed as  $\Pr(b_k = 0) = 1 - \Pr(b_k = 1) = \frac{e^{\gamma_k}}{1 + e^{\gamma_k}}$ ,  $k = 1, 2, \dots, K$ . Let  $\{s_0, s_1, \dots, s_{2^K-1}\}$  be the set of the signaling points in  $\mathcal{S}$ . Now the *a priori* probability that  $s_m \in \mathcal{S}$  is the transmitted symbol can be computed as  $\Pr(X = s_m) = \prod_{k=1}^K \Pr(b_k)$ , where  $\Pr(b_k)$  is either  $\Pr(b_k = 0)$  or  $\Pr(b_k = 1)$ , depending on  $\mathcal{R}$ . (We have assumed that  $\{b_1, b_2, \dots, b_K\}$  are independent.) Finally, the mean and variance of  $X$  are, respectively,

$$\mathbb{E}[X] = \sum_{m=0}^{2^K-1} s_m \Pr(X = s_m) \quad (19a)$$

$$\mathbb{V}[X] = \sum_{m=0}^{2^K-1} |s_m - \mathbb{E}[X]|^2 \Pr(X = s_m). \quad (19b)$$

Note that  $\mathbb{E}[X]$  and  $\mathbb{V}[X]$  in (19) are computed for fixed  $\{\gamma_k\}$ . Similarly to the treatment in Section II-E, assume that  $\{\gamma_k\}$  are random and denote by  $\mathbb{E}_\gamma[\cdot]$  the mathematical expectation over the joint distribution of  $\{\gamma_k\}$ . We are interested in the impact of signaling scheme  $(\mathcal{S}, \mathcal{R})$  on  $\mathbb{E}_\gamma[\mathbb{V}[X]]$ . Now let  $X$  be any entry  $X[n]$  in  $\mathbf{X}$ . Comparing the definitions in (15) and (19), we can see that

$$\sigma_x^2 = \mathbb{E}_\gamma[\mathbb{V}[X]]. \quad (20)$$

To make a fair comparison of different  $(\mathcal{S}, \mathcal{R})$  and following [34], we have two assumptions.

**Assumption 5:**  $(\mathcal{S}, \mathcal{R})$  is unbiased with unit average power:  $2^{-K} \sum_{m=0}^{2^K-1} s_m = 0$ ,  $2^{-K} \sum_{m=0}^{2^K-1} |s_m|^2 = 1$ .

**Assumption 6:**  $\{\gamma_k\}$  are i.i.d. and their probability density function satisfies the symmetric condition [34]:

$$p_\gamma(\gamma) = p_\gamma(-\gamma), \quad \forall \gamma \in \{\gamma_k\}. \quad (21)$$

We define the variance of a bit after bipolar modulation as

$$\rho = \mathbb{E}_\gamma[\mathbb{V}[(-1)^{b_k}]] = \mathbb{E}_\gamma[1 - \tanh^2(\gamma_k/2)], \quad \forall k. \quad (22)$$

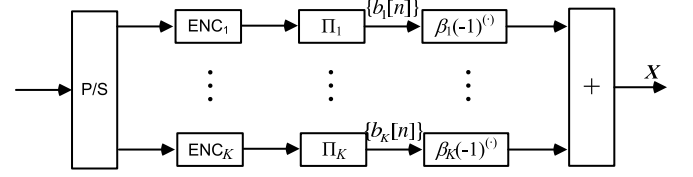


Fig. 3. Transmitter of multi-code SCM.

Here  $\rho$  is not a function of  $k$  since  $\{\gamma_k\}$  are i.i.d.

The following property states that SCM is optimal among all possible signaling schemes  $(\mathcal{S}, \mathcal{R})$  in the sense of minimizing  $\mathbb{E}_\gamma[\mathbb{V}[X]]$ . (See [41] for the proof.)

**Property 1:** Under Assumptions 5 and 6, the minimum symbol variance is given by

$$\min_{\mathcal{S}, \mathcal{R}} \sigma_x^2 = \min_{\mathcal{S}, \mathcal{R}} \mathbb{E}_\gamma[\mathbb{V}[X]] = \rho. \quad (23)$$

Furthermore, SCM achieves this minimum.

Property 1 indicates that the SCM signaling is an optimal solution to minimize  $\sigma_x^2$  among all possible signaling schemes. Assuming that the function  $\sigma_d^2 = \phi(\sigma_x^2)$  (see the discussion below (16)) is monotonically increasing, the SCM signaling is also optimal to minimize  $\sigma_d^2$ . Note that Property 1 holds for arbitrary constellation sizes, indicating that SCM provides a unified solution to minimize  $\sigma_d^2$  for systems with arbitrary rates.

It is important to note that the optimality of the SCM signaling is with respect to the clipping noise compensation only. SCM is not necessarily optimal in combating the channel noise, which, together with the clipping noise, determines bit-error rate (BER). We observed that SCM is particularly attractive when transmission rate is high and/or clipping is deep where the clipping effect becomes dominant, as demonstrated by the numerical results in Section IV.

### B. Multi-Code SCM

A multi-code SCM scheme [25], [28] is illustrated in Fig. 3. For a fixed  $k$ , we call the set  $\{(-1)^{b_k[n]}\}$  as layer- $k$ . All the bits in a layer are encoded by a single code and separate encoders are used for different layers. The transmit signal is generated using (17). For the multi-code SCM scheme, the following assumption is approximately true.

**Assumption 7:**

- (i) the DEC feedback  $\{\gamma_k[n]\}$  for the bits in each layer are i.i.d., and
- (ii)  $\{\gamma_k[n]\}$  may have different distributions for different layers.

We now show that the multi-code SCM scheme can lead to a smaller symbol variance  $\mathbb{E}_\gamma[\mathbb{V}[X]]$  and thus potentially further improved performance. Denote the symbol variance for the signals in layer- $k$  as (see (22))

$$\rho_k = \mathbb{E}_\gamma[\mathbb{V}[(-1)^{b_k[n]}]] = \mathbb{E}_\gamma[1 - \tanh^2(\gamma_k[n]/2)]. \quad (24)$$

A smaller  $\rho_k$  implies that the DEC feedback from layer- $k$  is more reliable. We also define the average of  $\{\rho_k\}$  as

$$\rho \equiv \frac{1}{K} \sum_{k=1}^K \rho_k. \quad (25)$$

From (17) and (24), the symbol variance for a multi-code SCM scheme is given by

$$\mathbb{E}_\gamma[V[X]] = \sum_{k=1}^K |\beta_k|^2 \rho_k. \quad (26)$$

When  $\{|\beta_k|\}$  are unequal, without loss of generality, we order  $\{|\beta_k|\}$  as

$$1 \geq |\beta_1| \geq |\beta_2| \geq \dots \geq |\beta_K| \geq 0. \quad (27)$$

The outputs of a DEC for a layer with a larger  $|\beta_k|$  should naturally have a smaller variance. Therefore,  $\{\rho_k\}$  will follow the order below.

$$0 \leq \rho_1 \leq \rho_2 \leq \dots \leq \rho_K \leq 1. \quad (28)$$

Recall the average power constraint  $\sum_{k=1}^K |\beta_k|^2 = 1$ . From (25), (27), (28) and Chebyshev's inequality [40],

$$\mathbb{E}_\gamma[V[X]] \leq \rho. \quad (29)$$

This can be compared with  $\mathbb{E}_\gamma[V[X]] = \rho$  in Property 1. (Note that Property 1 is based on the assumption that  $\{\gamma_k[n]\}$  are all i.i.d., and so  $\{\rho_k\}$  are equal, i.e.,  $\rho_k = \rho, \forall k$ , but  $\{\rho_k\}$  with (29) can be unequal.) From (29), when the overall quality (characterized by  $\rho$ ) of the DEC feedback is the same, the multi-code SCM scheme may lead to a smaller symbol variance compared with a BICM-ID scheme.

At the start of iterative process, without decoder feedbacks, we can initialize  $\rho = 1$  for the single-code scheme in Fig. 1 and  $\rho_k = 1, \forall k$  for the multi-code one in Fig. 3. From (29), as the iterative process proceeds, the symbol variance of the multi-code scheme will always be equal to or less than the single-code one, indicating the former may potentially outperform the latter.

### C. Evolution Analysis

We now outline a semi-analytical SNR evolution technique to predict the BER of multi-code SCM-based OFDM systems with clipping and iterative soft compensation. It is an extension of the EXIT chart method [34] to multi-code systems. For the OFDM system under consideration, the problem becomes very complicated due to the presence of different types of distortions, including AWGN, frequency-selective fading, clipping distortion, and inter-layer interference related to SCM. In the following, we will show how to characterize these distortions.

From (17), a SCM signal can be expressed as  $X[n] = \sum_{k=1}^K \beta_k X_k[n]$ , where each  $X_k[n] = (-1)^{b_k[n]} \in \{+1, -1\}$  is a binary phase-shift keying (BPSK) signal. We focus on a particular  $X_k[n]$  and rewrite the  $n$ th entry of  $\mathbf{Z}$  in (7) as

$$Z[n] = \alpha H[n] \beta_k X_k[n] + \zeta_k[n] \quad (30)$$

where

$$\zeta_k[n] = \alpha H[n] \sum_{m \neq k} \beta_m X_m[n] + H[n] D[n] + W[n] \quad (31)$$

is the distortion component. For notational simplicity, we assume that  $\alpha$ ,  $\{\beta_k\}$  and  $H[n]$  in (30) are real numbers. (The results below can be easily generalized when they take

complex values.) We treat approximately the distortion term  $\zeta_k[n]$  as an independent, Gaussian variable. The statistics of  $\{\zeta_k[n]\}$  in (30) can be found using the DEC feedback  $\{\gamma_m[n]\}$  [35]. Then the *extrinsic* LLR defined in (10) can be approximated as

$$\lambda_k[n] = \frac{2\alpha\beta_k H[n]}{V[\text{Re}(\zeta_k[n])]} (\text{Re}(Z[n]) - \mathbb{E}[\text{Re}(\zeta_k[n])]). \quad (32)$$

Substituting (30) into (32), we can rewrite the ESE output into a signal-plus-distortion form as

$$\lambda_k[n] = \frac{2\alpha\beta_k H[n]}{V[\text{Re}(\zeta_k[n])]} \times (\alpha H[n] \beta_k X_k[n] + \text{Re}(\zeta_k[n]) - \mathbb{E}[\text{Re}(\zeta_k[n])]). \quad (33)$$

In (33),  $\text{Re}(\zeta_k[n]) - \mathbb{E}[\text{Re}(\zeta_k[n])]$  represents the distortion component with respect to the useful signal  $X_k[n]$ . Its average power can be measured using the symbol variance

$$\begin{aligned} \mathbb{E}_\gamma[V[\text{Re}(\zeta_k[n])]] &= \mathbb{E}_\gamma[|\text{Re}(\zeta_k[n]) - \mathbb{E}[\text{Re}(\zeta_k[n])]|^2] \\ &= |\alpha\beta_k|^2 (|H[n]|^2 P_{I,k} + P_{W,k}) \end{aligned} \quad (34)$$

where  $P_{I,k}$  and  $P_{W,k}$  are the relative power (normalized by  $|\alpha\beta_k|^2$ ) of the interference and noise components, respectively, as given below:

$$P_{I,k} = \sum_{m \neq k} \frac{|\beta_m|^2}{|\beta_k|^2} \mathbb{E}_\gamma[V[X_m[n]]] + \frac{\sigma_d^2}{2|\alpha\beta_k|^2}, \quad (35)$$

$$P_{W,k} = \frac{\sigma_W^2}{2|\alpha\beta_k|^2}, \quad (36)$$

where  $\sigma_W^2$  is the average power of the complex channel noise. Then, the SNR for (33) with respect to  $X_k[n]$  is

$$\text{snr}_k[n] = \frac{|\alpha\beta_k H[n]|^2}{\mathbb{E}_\gamma[V[\text{Re}(\zeta_k[n])]]} = \frac{|H[n]|^2}{|H[n]|^2 P_{I,k} + P_{W,k}}. \quad (37)$$

The following assumption can greatly simplify the analysis problem.

#### Assumption 8:

- (i) The inputs  $\{\lambda_k[n]\}$  to the DEC at different time are independent. Similarly, the feedbacks  $\{\gamma_k[n]\}$  from the DEC at different time are also independent.
- (ii) The input sequence of the DEC is characterized by  $\{\text{snr}_k[n]\}$  in (37).
- (iii) The distribution of  $\{H[n]\}$  is given.

Note that (i) is true when infinite-length random interleavers are assumed, and (ii) holds when the Gaussian assumption is applied to  $\{\zeta_k[n]\}$ . In (iii), a typical case is that  $\{H[n]\}$  follows Rayleigh distribution. Based on Assumption 8 and from (37), the pair  $(P_{I,k}, P_{W,k})$  fully determines the DEC performance. In the iterative decoding process,  $P_{W,k}$  is a constant but  $P_{I,k}$  may decrease as the iteration proceeds. We discuss below how to track  $P_{I,k}$ .

From our earlier definitions,  $\rho_k = \mathbb{E}_\gamma[V[X_k[n]]]$ ,  $k = 1, 2, \dots, K$ . Now,  $\sigma_x^2 = \sum_{k=1}^K |\beta_k|^2 \rho_k$ . Therefore, from (16),  $\sigma_d^2$  can be found from  $\{\rho_k\}$  as

$$\sigma_d^2 = \phi \left( \sum_{k=1}^K |\beta_k|^2 \rho_k \right), \quad (38)$$



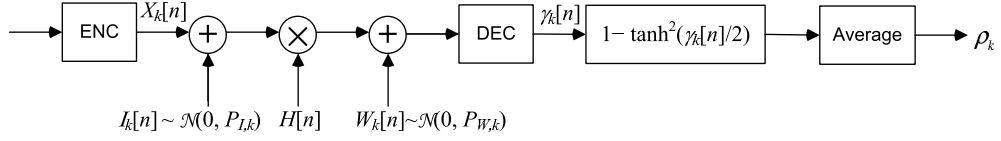


Fig. 4. Block diagram of the Monte Carlo simulation for the global variance  $\rho_k$ .

where  $\phi(\cdot)$  is defined in (16). From (35) and (38),  $P_{I,k}$  is fully determined by  $\{\rho_k\}$ . On the other hand, since  $\rho_k$  is the variance of the DEC feedback of the  $k$ th layer, it is a function of  $P_{I,k}$  and  $P_{W,k}$  that characterize the inputs to the DEC of the  $k$ th layer. We write this function as

$$\rho_k = f(P_{I,k}, P_{W,k}). \quad (39)$$

Note that here the decoder is characterized by a bivariate function  $f(\cdot)$ , differing from the treatments with univariate  $f(\cdot)$  in [35], [36]. This is because, for OFDM over frequency-selective channels, the clipping noise plus inter-layer interference and AWGN (characterized by  $P_{I,k}$  and  $P_{W,k}$ , respectively) have different impact on  $\{\text{snr}_k[n]\}$  in (37).

In general,  $f(\cdot)$  cannot be expressed in a closed form, but it can be characterized by a look-up table created by the Monte Carlo simulation. The block diagram of the simulation is depicted in Fig. 4, where we have used an equivalent channel model

$$Z_k[n] = H[n](X_k[n] + I_k[n]) + W_k[n], \quad (40)$$

where  $X_k[n] \in \{+1, -1\}$  is the coded BPSK signal,  $I_k[n] \sim \mathcal{N}(0, P_{I,k})$  and  $W_k[n] \sim \mathcal{N}(0, P_{W,k})$ , respectively, represent the (normalized) interference and channel noise. As shown in Fig. 4,  $\rho_k$  is estimated using the average of the DEC outputs. Similarly, the BER performance of the DEC can be characterized by a function as

$$\text{BER}_k = g(P_{I,k}, P_{W,k}). \quad (41)$$

To summarize, we can characterize the iterative decoding process using the following procedure. (We assume  $Q$  iterations and denote by  $(\cdot)^{(q)}$  the  $q$ th iteration.)

#### SNR Evolution:

- (i) Initialization: Set  $q = 1$  and  $\rho_k^{(q)} = 1$ ,  $P_{W,k} = \frac{\sigma_W^2}{2|\alpha\beta_k|^2}$ ,  $k = 1, 2, \dots, K$ .
- (ii) For the  $q$ th iteration:

Find the normalized interference power for the ESE:

$$P_{I,k}^{(q)} = \sum_{m \neq k} \frac{|\beta_m|^2}{|\beta_k|^2} \rho_m^{(q)} + \frac{\phi\left(\sum_{m=1}^K |\beta_m|^2 \rho_m^{(q)}\right)}{2|\alpha\beta_k|^2},$$

$$k = 1, 2, \dots, K.$$

Find the output variance of the DEC:

$$\rho_k^{(q+1)} = f(P_{I,k}^{(q)}, P_{W,k}), \quad k = 1, 2, \dots, K.$$

- (iii) Recursion: If  $q < Q$ , set  $q \leftarrow q + 1$  and go to (ii); otherwise, go to (iv).
- (iv) Find the BER for each layer:

$$\text{BER}_k = g(P_{I,k}^{(q)}, P_{W,k}), \quad k = 1, 2, \dots, K.$$

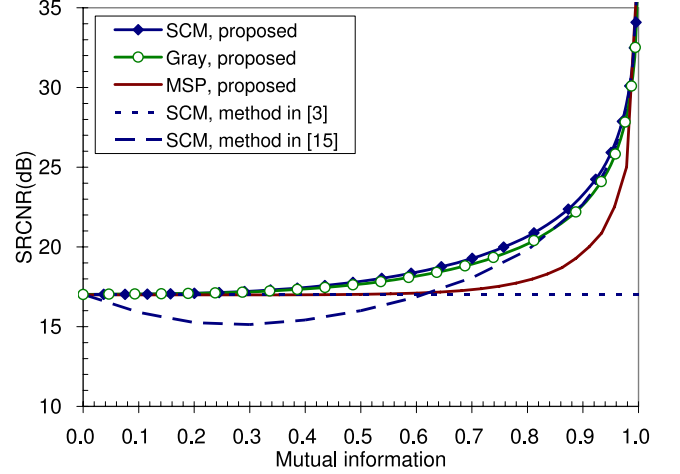


Fig. 5. Impact of detection methods and signaling schemes on SRCNR.  $L = 4$ ,  $N = 256$ , and  $\text{CR} = 0$  dB.

With the above technique, only the binary-input system in Fig. 4 needs to be simulated. Then using the stored look-up tables to characterize (16), (39) and (41), one can predict the performance of multi-code SCM with arbitrary weighting factors  $\{\beta_k\}$  and channel SNR. This is more convenient than the EXIT chart method in BICM-ID where Monte Carlo simulations of a multi-ary system have to be performed whenever the signaling scheme or channel SNR is changed. The SNR evolution technique can also be used for optimizing the weighting factors  $\{\beta_k\}$ . The related optimization technique is beyond the scope of this paper. Interested readers may refer to [25], [35] for related discussions.

#### IV. NUMERICAL RESULTS

This section presents numerical results. We take OFDM systems based on the BICM-ID [19], [20] and multi-code SCM schemes [24]–[28] as examples. For SCM, we assume even  $K$  and  $\beta_{k-1} = i\beta_k$  with  $i = \sqrt{-1}$  and  $\beta_{k-1}$  being a real number for  $k = 2, 4, \dots, K$ . The APP and Gaussian approximation (GA) [28] demapping methods are applied to the BICM-ID and multi-code SCM schemes, respectively. The number of subcarriers  $N = 256$  and the oversampling factor  $L = 4$ .

1) *SRCNR*: We first show the effectiveness of the proposed soft compensation method by comparing it with the background work in [3] and [15], where the clipping noise  $d$  modeled in (4) are treated as an additive Gaussian noise and estimated as  $g(E[x]) - \alpha E[x]$ , respectively. (See also Section II-D.) Define the signal to residual clipping noise ratio (SRCNR) from (7) and (8) as

$$\text{SRCNR} = \frac{|\alpha|^2 E[|X[n]|^2]}{\sigma_d^2}.$$

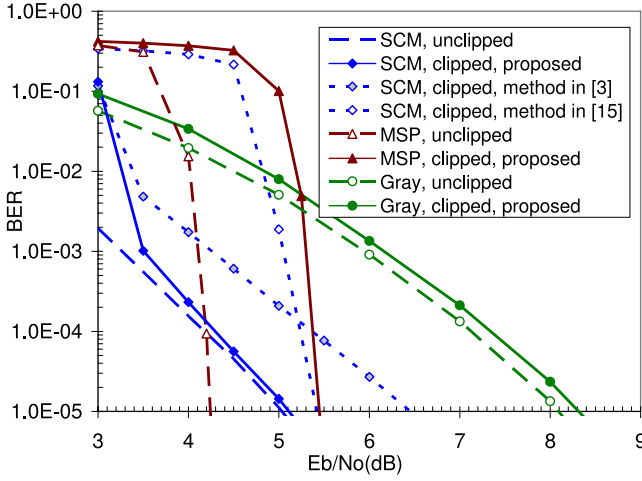


Fig. 6. Comparison of different detection methods and signaling schemes for clipped BICM-ID-OFDM schemes over AWGN channels. For SCM,  $K = 4$ ,  $\{\beta_k\} = \{1 \times 4\}$ .  $L = 4$ ,  $N = 256$ ,  $CR = 0$  dB. The rate-1/2 convolutional code  $(23, 35)_8$  is used. The system rate is  $R = 2$  bits/symbol. The frame length is 4096. (Each frame consists of 16 OFDM symbols.) The number of iterations is 12.

Here, the residual clipping noise power  $\sigma_d^2$  defined in (14) is measured using Monte-Carlo simulation and applying the different methods to the SC module in Fig. 1. We assume i.i.d., consistent Gaussian DEC feedbacks  $\{\gamma_k[n]\}$ , which is reasonable in BICM-ID. Then, the mutual information ( $I_\gamma$ ) between  $\{\gamma_k[n]\}$  and the coded bits  $\{b_k[n]\}$  can characterize the distribution of  $\{\gamma_k[n]\}$ .

The SRCNR achieved by different methods for an SCM signaling with  $K = 4$  are compared in Fig. 5. The proposed method consistently outperforms the alternatives in [3] and [15]. This is because it estimates  $\mathbf{d}$  as the conditional mean  $\bar{\mathbf{d}} = E[g(\mathbf{x}) - \alpha\mathbf{x}]$ , which is optimal when the *a priori* distribution of  $\mathbf{x}$  is available. The SRCNR with the method in [15] drops as  $I_\gamma$  increases from 0 to 0.25 and is even worse than the method in [3] where  $\mathbf{d}$  is treated as a Gaussian noise without mitigation. This indicates that the method in [15] does not necessarily yield better estimate of  $\mathbf{d}$  if the estimates of  $\mathbf{x}$  are improved. By contrast, Fig. 2 shows that the proposed method does lead to better estimates of  $\mathbf{d}$  with better estimates of  $\mathbf{x}$ .

We also compare the SCM signaling and the 16-QAM signaling schemes with Gray and MSP mappings [19] for the proposed method. From Fig. 5, the SCM signaling is clearly the best choice. This confirms the discussion in Section III that the SCM signaling is optimal for minimizing the residual clipping noise power  $\sigma_d^2$ . Note that the Gray signaling can also achieve good SRCNR according to Fig. 5.

2) *BER in AWGN Channels*: Fig. 6 compares BERs of a BICM-ID-OFDM system with the different detection methods and signaling schemes in Fig. 5, where we have defined  $E_b/N_0 = 10 \log_{10} (E[|Fg(\mathbf{x})|^2]/(NR\sigma_W^2))$  in decibel with  $R$  being the system rate. The proposed soft compensation method based on (12) and (13) significantly outperforms the other two approaches. It has roughly the same computational complexity as the alternatives (except the extra memory required to store the look-up tables). Clearly, SCM is more

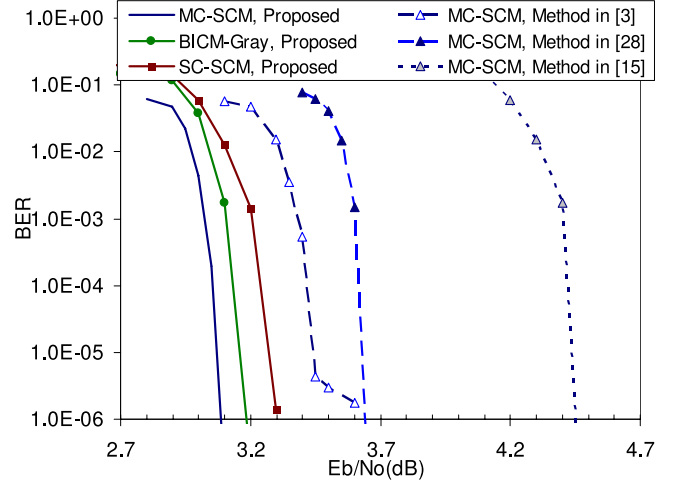


Fig. 7. Comparisons of LDPC-coded BICM-ID-OFDM, SC-SCM-OFDM and MC-SCM-OFDM with clipping over AWGN channels.  $L = 4$ ,  $N = 256$ ,  $CR = 0$  dB. The frame length is 32768.  $R = 2$ . For BICM-Gray and SC-SCM, the number of outer iterations (between the ESE and DEC) is 1000 and the number of inner iterations (of LDPC decoding) is 1. For MC-SCM, the number of outer iterations is 10 and the number of inner iterations is 20. For all the SCM schemes,  $\{\beta_k\} = \{1 \times 2, 1.5 \times 2\}$ . The constellation of BICM-Gray is 16-QAM.

robust against the clipping effect than its alternatives, which is in line with the SRCNR results in Fig. 5.

Fig. 7 compares BICM-ID with the single- and multi-code SCM schemes with a rate-1/2 LDPC code and  $K = 4$ . Following [43], the performance with LDPC codes can be improved by optimizing the node degree distributions. The best results (for  $CR = 0$  dB with the proposed soft compensation method) known to us are applied in Fig. 7:

- *Single-code SCM (SC-SCM)*: the degree profile of variable nodes  $\lambda(x) = 0.2394x^2 + 0.1824x^3 + 0.0682x^5 + 0.0175x^7 + 0.1907x^8 + 0.3018x^{30}$ , and the degree profile of check nodes  $\rho(x) = 0.3x^8 + 0.7x^9$ .
- *BICM-Gray*:  $\lambda(x) = 0.3965x^2 + 0.2272x^3 + 0.0583x^{10} + 0.0842x^{11} + 0.2338x^{30}$  and  $\rho(x) = 0.2x^6 + 0.8x^7$ .
- *Multi-code SCM (MC-SCM)*:  $\lambda(x) = 0.25x^2 + 0.1854x^3 + 0.0031x^4 + 0.1499x^5 + 0.1616x^8 + 0.0055x^{10} + 0.0423x^{20} + 0.2022x^{21}$  and  $\rho(x) = x^8$ .

From Fig. 7, the MC-SCM scheme yields the best performance (within 1.35 dB from the Shannon limit) with the proposed soft compensation method. This scheme also has lower complexity than the alternatives in Fig. 7 as a smaller number of iterations are applied. BICM-Gray outperforms the SC-SCM slightly by 0.12 dB, which indicates that the BER performance is not determined by Property 1 alone. (Recall that, from Fig. 5, the difference in SRCNR between the SCM and Gray signaling schemes is very small.) From Fig. 7, the proposed soft compensation technique provides noticeable performance improvement compared with those proposed in [3], [15] and [28]. This confirms our discussions in Section II-D.

3) *BER in Fading Channels*: We also consider Rayleigh fading channels. The multi-code SCM and BICM-ID schemes at rate  $R = 4$  bits/symbol are compared. For SCM, the component code is a concatenation of the rate-1/2 convolutional code  $(23, 35)_8$  with a length-4 repetition code [35];  $K = 32$ . For BICM-ID, the 64-QAM with set-partitioning (SP) mapping

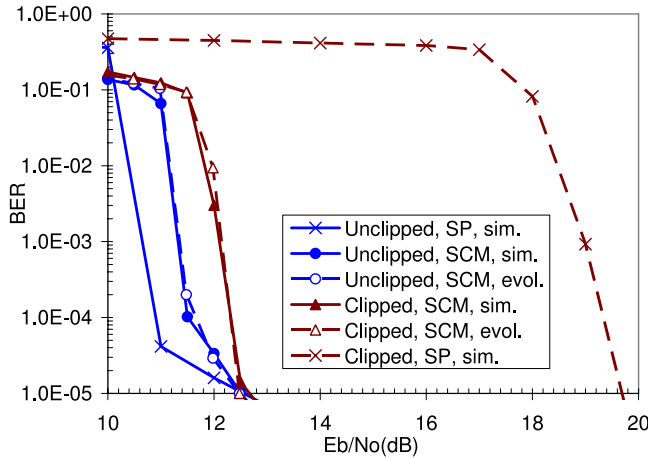


Fig. 8. Comparison of convolutional-coded multi-code SCM-OFDM and BICM-ID-OFDM with clipping and soft compensation over fully interleaved Rayleigh fading channels.  $L = 4$ ,  $N = 256$ , and  $CR = 2$  dB. The frame length is 16384.  $R = 4$ . The number of iterations is 12. The weighting factors of SCM are given by  $\{\beta_k\} = \{1 \times 12, 1.58 \times 4, 2.10 \times 6, 2.49 \times 2, 2.73 \times 2, 3.58 \times 4, 3.93 \times 2\}$ .

and  $K = 6$  is used [38] and the above convolutional code is punctured to rate  $2/3$  to achieve  $R = 4$ . The APP and GA [28] demapping methods at complexities of  $\mathcal{O}(2^K)$  and  $\mathcal{O}(K)$  are applied to the BICM-ID and SCM schemes, respectively. As  $K = 6$  for the BICM-ID and  $K = 32$  for the SCM, the schemes compared here have comparable demapping complexities. The decoding complexities of the component codes are also comparable [35].

From Fig. 8, without clipping, the BICM-ID scheme with SP signaling and the SCM scheme have similar performance. However, when clipping is used, SCM significantly outperforms BICM-ID. In this case, the iterative compensation is not very effective for the BICM-ID scheme with the SP signaling; hence, its performance is dramatically degraded by the clipping effect. Soft compensation still works well with the SCM signaling in this case.

Fig. 8 also compares the simulation results and the BER predicted by the SNR evolution technique outlined in Section III-C. From Fig. 8, they agree well in both the clipped and unclipped cases. This clearly demonstrates the effectiveness of the SNR evolution technique.

## V. CONCLUSION

Clipping can alleviate the high PAPR problem in OFDM systems but causes serious performance degradation when not treated properly. In this paper, we propose an iterative soft compensation method to mitigate the clipping effect, which can outperform conventional approaches. We analyze the impact of signaling schemes on performance and show the optimality of SCM signaling for clipping noise compensation. We also show that the performance can be improved by using multi-code SCM schemes whose performance can be predicted and optimized based on a fast evolution technique. Both the analytical and numerical results show that the SCM scheme with the proposed soft compensation method can outperform other alternatives, especially when clipping is severe and/or transmission rate is high.

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