Inverse optimal control as a tool to understand human yoyo playing

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Keywords
optimal, playing, control, tool, understand, inverse, human, yoyo

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Inverse Optimal Control as a Tool to Understand Human Yoyo Playing

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Abstract. This paper presents an inverse optimal control approach to identify objective functions of human motion from motion capture measurements. We apply it to analyze human yoyo playing. Yoyo playing may seem easy to us to learn but it is a challenging problem from a mechanical point of view involving a hybrid dynamics model. We recorded vertical yoyo playing of humans measuring yoyo height and rotation angle as well as the corresponding hand motions. Results of inverse optimal control are presented showing a mixed criterion of cycle time and terms depending on yoyo and hand acceleration and velocity.

Keywords: Inverse optimal control, hybrid dynamics model, yoyo playing

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INTRODUCTION

It is a common assumption that motions of humans and animals are optimal due to evolution, learning and training. But in many situations, the specific optimization criterion is unknown. We describe a numerical inverse optimal control approach to identify optimality criteria from measurements. We apply it to gain understanding of the objectives of human yoyo playing. Yoyo playing may seem easy for a human to learn, but it is a challenging problem from a mechanical point of view. The dynamics of a yoyo are represented by a hybrid dynamics model with different phases of motion, state dependent phase switching events, and discontinuities of variables at impacts. We recorded vertical yoyo playing measuring yoyo height and rotation angle as well as the corresponding hand motions. This paper builds on our previous research [1] where we developed the yoyo model and used it to generate yoyo playing motions for the humanoid robot HRP-2.

FIGURE 1. (a) Recording human yoyo playing motion via motion capture technology (b) humanoid HRP-2 controlled to play yoyo (c) yoyo with reflective motion capture markers
CAPTURING HUMAN YOYO MOTION

We recorded human yoyo playing using a motion capture system (Motion Analysis Corporation, USA) with ten infra-red cameras, each collecting video data at a sampling rate of 100 Hz. In total, we measured yoyo playing motions of four participants, all of whom had previous experience with playing yoyos. To track the movement of the human, in particular its arm, we used eight tracking markers as shown in Fig. 1. The yoyo used was a Duncan Wheels free-wheeling type yoyo manufactured by Duncan, USA and weighing 66 gram. Three miniature reflective tracking markers were attached to the yoyo in order to track its trajectory during the oscillations (see Fig. 1-c). Two markers were attached symmetrically to one side of the yoyo, and the third marker was set in the middle of the other side. This configuration was chosen since it caused minimal perturbation during oscillations. Participants were instructed to play the yoyo at their preferred speed in a smooth fashion, moving the whole hand and nut just a finger, and not re-grasp the yoyo between trials. Before recording all participants were given about 5 minutes to practice yoyo playing with the given instructions. For each participant, we recorded three trials, each containing 90 seconds of up-down yoyo motion.

HYBRID DYNAMIC MODEL OF A YOYO

Figure 2 shows the different phases of the free-wheeling yoyo motion and the corresponding equations of motion (for a more detailed explanation, see [1]). In the first phase, the yoyo is in contact with the hand, and in the second phase the yoyo rolls down the string. After bottom impact, the rolling up phase starts, given the upward acceleration is large enough at this instant. The rolling-up phase lasts until the yoyo touches the hand. The acceleration of the hand $u_h$ is

\[
\dot{\bar{r}} = \bar{u}_h + \bar{\omega} \cdot \bar{r}
\]

\[
\Theta \omega - m r \dot{\bar{r}} = m g r - \gamma r \omega
\]

\[
\dot{\bar{v}} = \bar{v}_h + \bar{\omega} \cdot \bar{r}
\]

\[
\Theta \Omega - m r \dot{\bar{v}} = m g r - \gamma r \omega
\]

\[
\dot{\bar{\omega}} = \bar{\omega} - \dot{\bar{r}} \cdot \bar{r} - \delta \omega
\]

\[
\Theta \dot{\omega} - m r \dot{\bar{v}} = m g r - \gamma r \omega
\]

\[
\dot{\bar{\Omega}} = \bar{\omega} + \dot{\bar{r}} \cdot \bar{r} - \delta \omega
\]

\[
\Theta \dot{\Omega} - m r \dot{\bar{v}} = m g r - \gamma r \omega
\]

\[
\dot{\bar{\omega}} = \bar{\omega} - \dot{\bar{r}} \cdot \bar{r} - \delta \omega
\]
used as control variable for the yoyo. The state of the system is described by seven variables, namely hand height \( h \), yoyo height \( z \) and rotation angle \( \phi \), corresponding velocities \( v_h \), \( v_z \), and \( \omega \) as well as the effective radius of the string at the yoyo \( r \).

### The Inverse Optimal Control Approach

The goal of classical (forward) optimal control problems is to determine solutions that are optimal with respect to a given cost function and satisfy the constraints imposed by dynamic equations of motion. The inverse optimal control problem addresses exactly the opposite problem. It consists in determining, from a solution that is (partly) known from measurements, the optimization criterion that has produced this solution. For this, we make the assumption that a set of reasonable independent base functions \( \phi_i(t) \) for the objective function can be established. The relative contribution of these base functions - expressed by a weight factor \( \alpha_i \) - remains to be determined by the algorithm. The inverse optimal control problem can be formulated as bilevel problem:

\[
\min_{\alpha} \sum_{j=1}^{m} ||z^*(t_j; \alpha) - z_M(t_j)||^2
\]

where \( z^*(t; \alpha) \) is the solution of

\[
\min_{x, u, t} \int_0^T \left[ \sum_{i=1}^{n} \alpha_i \phi_i(x(t), u(t)) \right] dt
\]

s. t. \( \dot{x} = f(t, x(t), u(t)) \)
\( x(0) = x_0, \quad x(T) = x_e \)

While the lower level solves the optimal control problem for each given set of parameters \( \alpha \), the upper level serves to adjust \( \alpha \) by minimizing the distance of the lower level solution to measurement points. We propose a combination of efficient direct techniques for the solution of the lower level optimal control problems [2], and of an efficient derivative-free method [3] or the solution of the upper-level least-squares problem. This method is described in more detail in [4], where it has been used to identify human locomotion trajectories.

### Numerical Results for Inverse Optimal Control of Yoyo Playing

For the identification of objectives of human yoyo playing, we have used the following parameterized objective function involving six independent base functions:

\[
\min \alpha_0 T + \alpha_1 \int_0^T \dot{a}_h^2 \, dt + \alpha_2 \int_0^T v_h^2 \, dt + \alpha_3 \int_0^T |v_h| \, dt + \alpha_4 \int_0^T v_z^2 \, dt + \alpha_4 \int_0^T a_z^2 \, dt
\]

(1)
This objective function combines a minimization of cycle time, terms related to the motion of the hand (squared hand accelerations, squared hand velocity and absolute value of hand velocity), as well as terms related to the motion of the yoyo (squared velocity and acceleration). In this study, we have analyzed the data of one subject with the described approach. The combination of objective function weight parameters that best describes the measured data for this subject is $\alpha^1 = (0.14, -0.75, 7.27, -2.05, 0.33, 50)$, i.e the dominant factors are for cycle time minimization, minimization of the integral of hand velocity (which for this motion is proportional to the delta in hand height over the cycle) and maximization of yoyo velocity. Figure 4 shows the fit between experimental data and computed solution.

**DISCUSSION AND CONCLUSIONS**

The computational results of inverse optimal control show that human yoyo playing can be interpreted as a result of an optimization process based on a combined criterion of cycle time minimization and terms depending acceleration and velocity of the yoyo and the driving hand. We are currently exploring if additional terms in the objective function related to stability and energy loss in the yoyo motion might lead to an even better fit between theory and experiment. In addition, we are analyzing how sensitive these results are to individual preferences of subjects. It seems that every subject adds its personal style to yoyo playing and the observed solutions are slightly different. It remains to be evaluated how large the variations of objective function parameters between subjects are.

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**REFERENCES**