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# FLOW RATE OF SIMPLE BULK SOLIDS FROM MASS FLOW BINS

A thesis submitted in fulfilment of the requirements for the award of the degree of

DOCTOR OF PHILOSOPHY

from

The University of Wollongong

bу

A.G.McLean

B.E.(N.S.W), Grad, I.E. Aust.

Department of Mechanical Engineering
1979

This is to certify that this work has not been submitted for a degree to any other university or institution.

ARNOLD GRANT McLEAN

#### 1. Abstract

There is hardly an industry which does not employ gravity flow bins for the storage of bulk solids. Consequently there has been considerable activity and research to provide the engineer with information that will enable him to design bulk solid storage facilities for unobstructed and predictable flow. There also has been an increasing tendency in industry to handle finer materials which has led to considerable difficulties especially in cases where the actual flow rate was considerably less than plant capacities. Such misfortunes have prompted a number of investigations in this field in an endeavour to understand the flow of fine materials and to make suitable modifications to gravity flow bins to improve the flow situation.

This thesis is concerned with the prediction of the flow rates of fine bulk solids from mass flow bins and hoppers. In this initial treatment the two-phase nature of the flow will become apparent. The analysis requires the simultaneous consideration of the continuity of the bulk solid and the interstitial fluid, and the inclusion of the effects of the interstitial gas pressure gradients in the equation of motion of the flowing bulk solid.

This analysis will be applied initially to an incremental element of bulk solid, the flow through which is described by a number of partial differential equations whose solution requires considerable numerical effort. An approximate analysis is then be developed by considering the flow through an assumed element. Three non-linear total differential equations result which also require numerical techniques for

their solution.

By making suitable assumptions for the form of the flow stress field, analytical solutions for the flow rate, the interstitial gas pressure distribution and the flow stress field are obtained for channels with or without surcharge. Results for coarse bulk solids are then derived by neglecting the effects of interstitial fliud pressure gradients.

The predicted flow rates compare favourably with observed flow rates from an experimental plane flow bin.

It must be stressed that this is only an initial treatment, extensive work still is required to completely understand and predict the flow of general bulk solids from bins.

#### 2. Acknowledgements

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#### 1. Nomenclature

- exponent used to relate permeability coefficient to consolidation stress, eqn. (3.19)
- A area of flow channel
- A<sub>1</sub>, A<sub>2</sub>, A<sub>3</sub> variables defined by eqns.(6.27),(6.28),(6.29) respectively
- A,A,A variables defined by eqns.(6.44),(6.45),(6.46)
  respectively
- A,A,A variables defined by eqns.(7.23),(7.24),(7.25)
  respectively
- b exponent used to relate bulk density to consolidation stress, eqn. (3.18)
- $\overline{b_1}, \overline{b_2}$  body forces used in eqns.(4.30) and (4.31)
- B width or diameter of outlet of flow channel
- c permeability coefficient of bulk solid, eqn. (3.19)
- $c_0$  permeability coefficient of bulk solid when  $\sigma = \sigma_0$
- C constant relating mean stress at hopper outlet to dynamic head at outlet,eqn.(5.111)
- constant of integration used in eqn.(6.18), defined in eqn.(A5.18)
- constant of integration used in eqn.(7.9),
  defined in eqn.(7.17)
- $C_n$  constant of integration used in eqn. (7.10)
- constant of integration used in eqn.(8.16), defined in eqn.(8.20).
- C constant of integration used in eqn.(8.17)

```
f_1, f_2 fluid drag force, used in eqns. (4.30) and (4.31)
ff - flow factor for a converging channel
ff a - 'actual' flow factor for a flow situation

    unconfined yield strength of a bulk solid

FF - flow function of a bulk solid

    acceleration due to gravity

G_{1}, G_{2}, G_{3} - constants defined in eqns.(6.11),(7.7) & (8.15)
            respectively

    moisture content

I - integrand defined by eqn.(5.7) or by (5.14)
        respectively
    - coefficient
k
         k = +1 for the major principal stress
         k=-1 for the minor principal stress
         k=0 for the mean stress

    length of a plane flow hopper outlet

m - coefficient
        m=0 for plane flow (end effects neglected)
         m=1 for axisymmetric flow
     - suspended mass in experimental bin
    - bulk solid mass flow rate
    - vertical momentum component
n,n<sub>1</sub>,n<sub>2</sub> - exponents used in the analytical expressions for
```

the flow stress field

N

- number of divisions over which approximate gas

pressure distribution is evaluated

```
p - interstitial gas pressure
```

- p(R) interstitial gas pressure at distance R from hopper vertex
- p(r) function defined by eqn. (A5.3)
- Q bulk solid mass flow rate, eqn. (6.34)
- Q dimensionless flow rate, used in eqn. (6.51)
- Q(r) function defined by eqn. (A5.4)
- r radial distance from the vertex of the flow channel
- r radial distance to hopper outlet, eqn. (6.30)
- r\* the distance from the vertex of the channel to where the stress is assumed to be zero, Fig. (1.5)
- R radial distance from the vertex to the transition of a hopper
- s stress function, used in eqns.(6.1),(7.1),(8.8)
  respectively
- t time
- T temperature
- T = tensile force in load cell in experimental
   installation,eqn.(9.1)
- T<sub>1</sub>,T<sub>2</sub>,T<sub>3</sub> variables defined by eqns.(6.13),(6.19),(6.20) respectively
- T<sub>4</sub>,T<sub>5</sub>,T<sub>6</sub>- variables defined by eqns.(6.21),(7.3),(8.10) respectively
- u interstitial fluid velocity

```
V - bulk solid velocity
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V<sub>e</sub> - representative velocity in experimental installation

V<sub>O</sub> - bulk solid velocity at the outlet of a channel

V1 - major consolidation load

W<sub>1</sub> - variable defined by eqn.(6.17)

x - variable used in Enstad theory, defined in eqn. (5.103)

Y - variable used in Enstad theory, defined in eqn. (5.104)

YY - variable defined in eqn. (5.82)

z - component direction

distance along a plane flow hopper, Fig. 3.2 meridional direction in axisymmetry, Fig. 3.3

Z<sub>1</sub> - variable defined by eqn.(6.14)

 $z_2, z_3, z_4$  variables defined in eqns.(8.26),(8.27),(8.28) respectively

 $\alpha$  - hopper half angle

- β angle between major principal stress and normal to hopper wall for flow conditions, defined in eqn. (3.16)
- $\Gamma$  porosity of a bulk solid, defined in eqn.(3.20)
- $\delta$  effective angle of internal friction of a bulk solid
- $\theta$  angular co-ordinate in plane radial co-ordinates and in the meridional plane in axial symmetry
- $\xi$  integration variable

- $\rho$  bulk density of a bulk solid, eqn. (3.18)
- $\rho_0$  bulk density when  $\sigma = \sigma_0$
- $\rho_{\text{f}}$  density of the interstitial fluid
- $\rho_g$  density of the interstitial gas
- $\rho_s$  density of solid particles
- σ mean stress
- σ mean stress at outlet of flow channel
- $\sigma(R)$  value of the mean stress at the transition
- $\sigma_1, \sigma_2$  major and minor consolidation stresses
- $\sigma_{\mathbf{r}}, \sigma_{\theta}$  consolidation stresses in the r and  $\theta$  direction respectively
  - $\varphi$  kinematic angle of friction between a bulk solid and a bin wall
  - $\tau$  shear stress
  - $\tau_{\rm w}$  shear stress along the wall
  - $\tau_{r\theta}$  shear stress component defined in Fig. 3.6
    - $\psi$  angle between the co-ordinate ray and principal stress direction, defined in Fig. 3.6
    - ψ\*- angle defined in Fig. A2.1
    - $\omega$  angle related to  $\theta$  and  $\psi$  ,defined in Fig. 3.6

#### Subscripts

- r r component
- $\theta$  component
- e experimental
- f interstitial fluid
- g interstitial gas
- s solid particles
- o pertaining to the outlet of the flow channel
- R pertaining to distance R from vertex of flow channel
- T pertaining to the transition of the flow channel
- w denotes conditions at the wall
- c conical
- p plane flow

#### Superscripts

denotes Enstad co-ordinate system, defined
in Fig.5.1

#### Prefix

 $\Delta$  - finite change